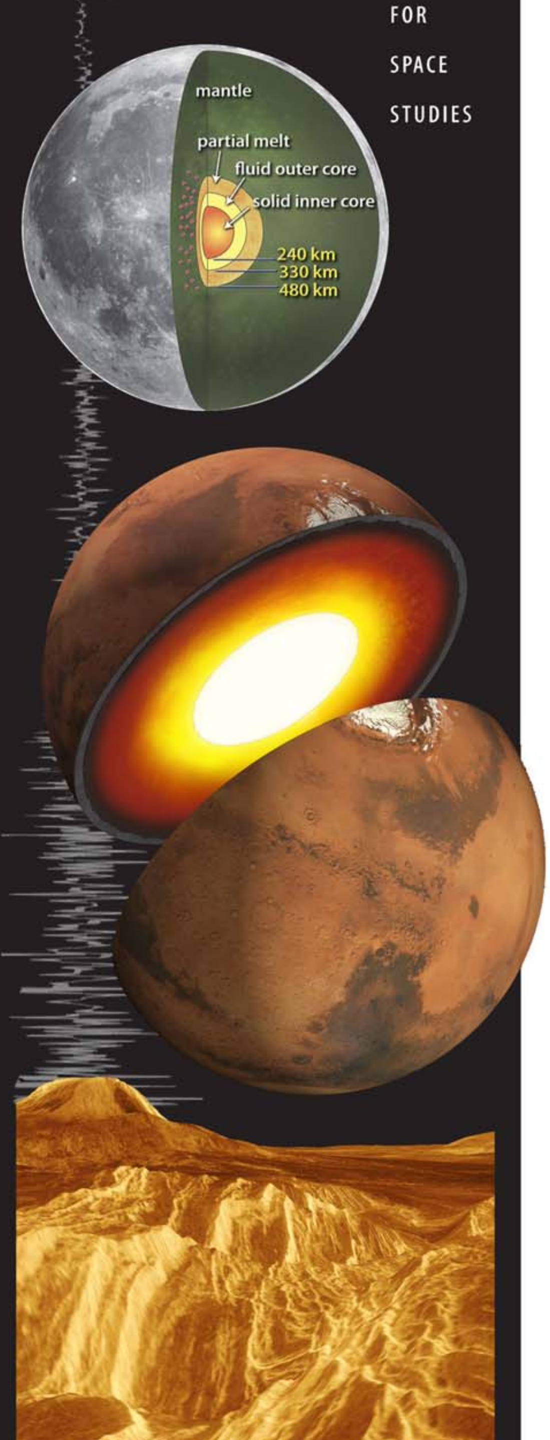


# Seismology Tutorial 1: Normal modes of telluric planets

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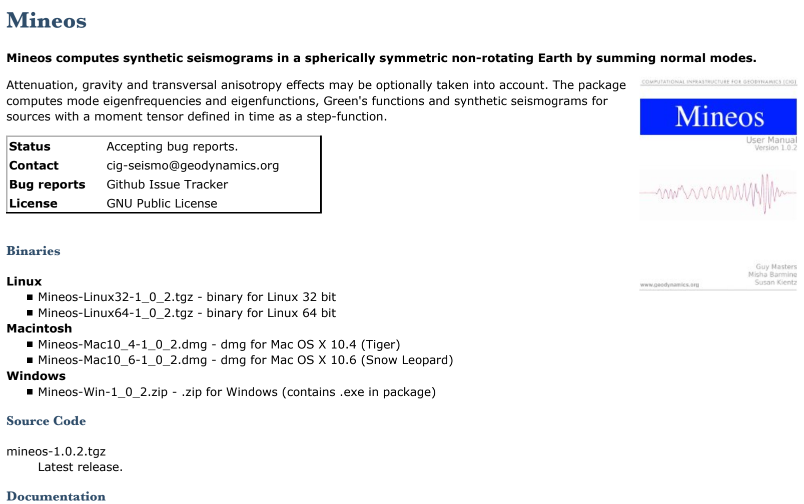
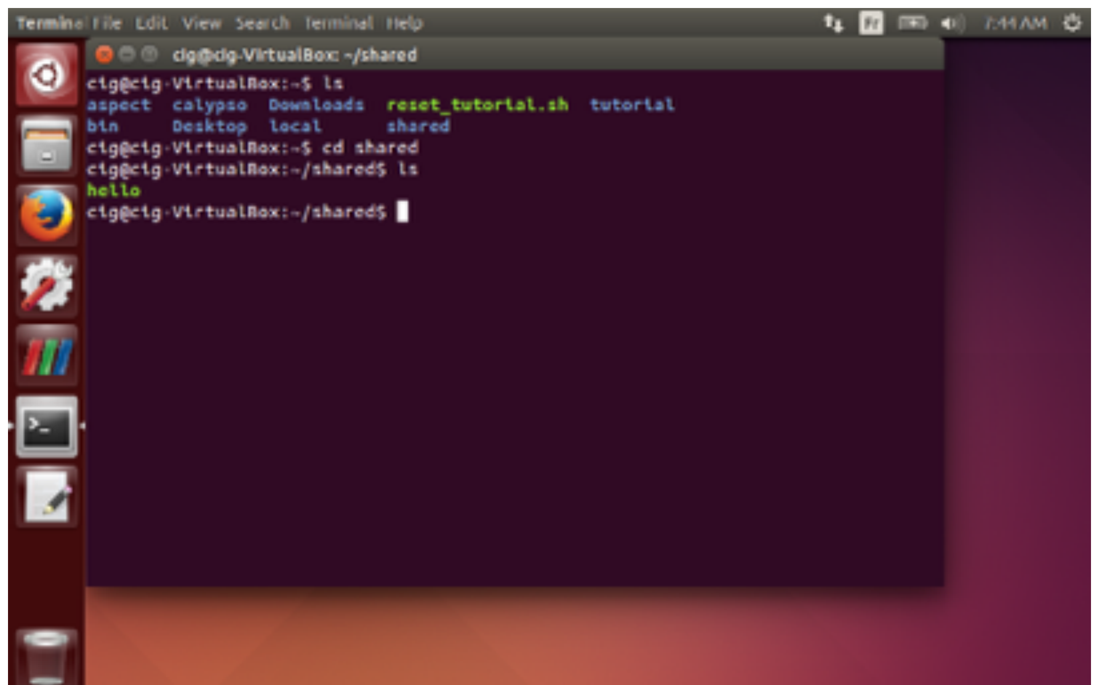
# Tutorial software

- The following tutorial is based on minos normal modes program
- A version of minos is available at CIG
  - http://geodynamics.org/archive/software/mineos
- The version used in this tutorial is NOT the CIG one but have been modified in order to compute modes and seismograms for different planets, and also to compute modes for higher order perturbation theories software (HOPT package)
  - http://www.ipgp.fr/fr/pss/3d-synthetic-seismogram-package

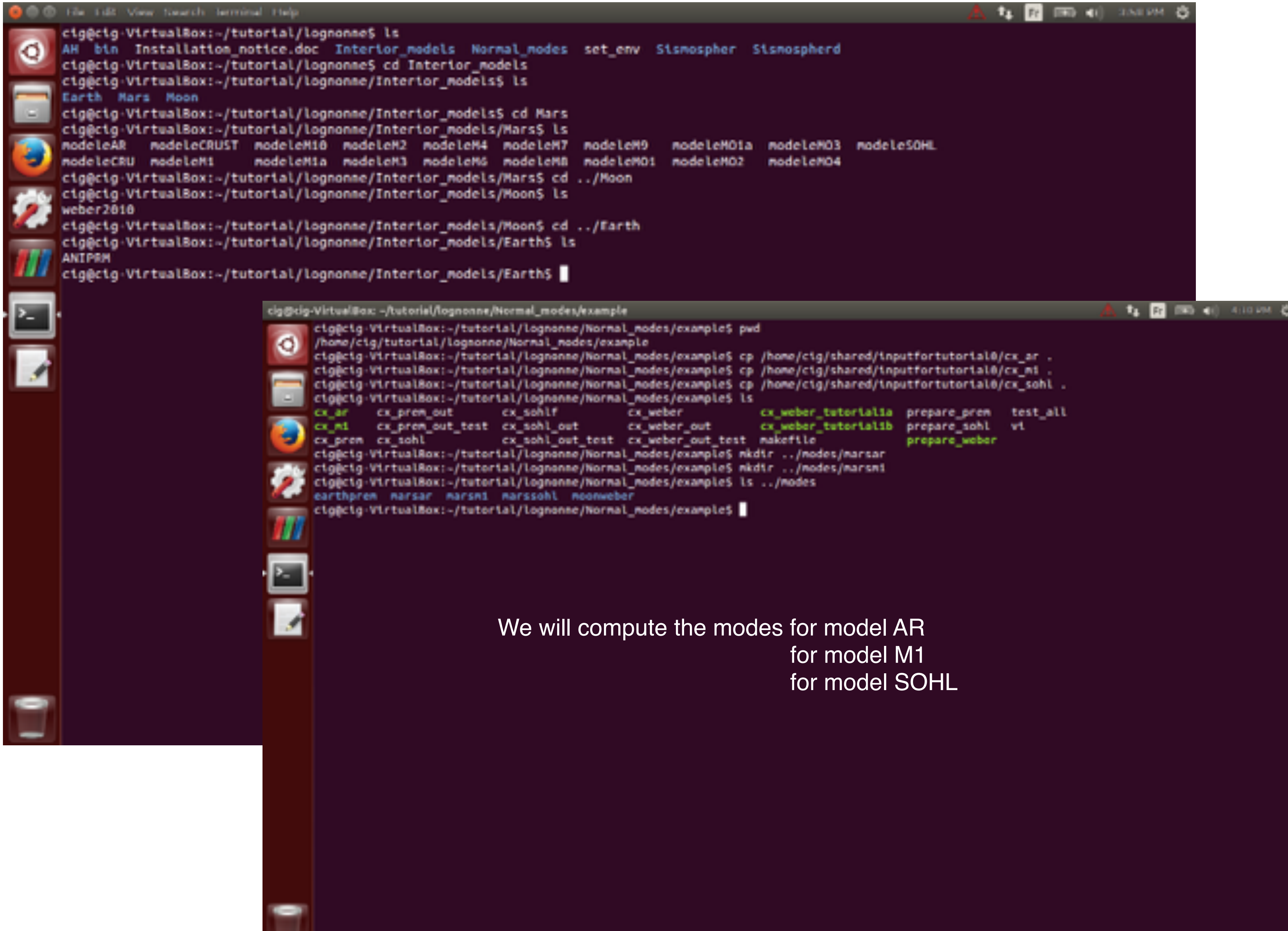



# setting up

- Setting up your VM
  - Start the VM
  - create a shared folder CIG_SHARED ( to use results for matlab and other application)
  - put a file “hello” inside
  - launch the CIG CIDER_1
  - if green, then put folders inputfortutorial0 and inputfortutorial1
  - make outputtutorial0 and outputtutorial1

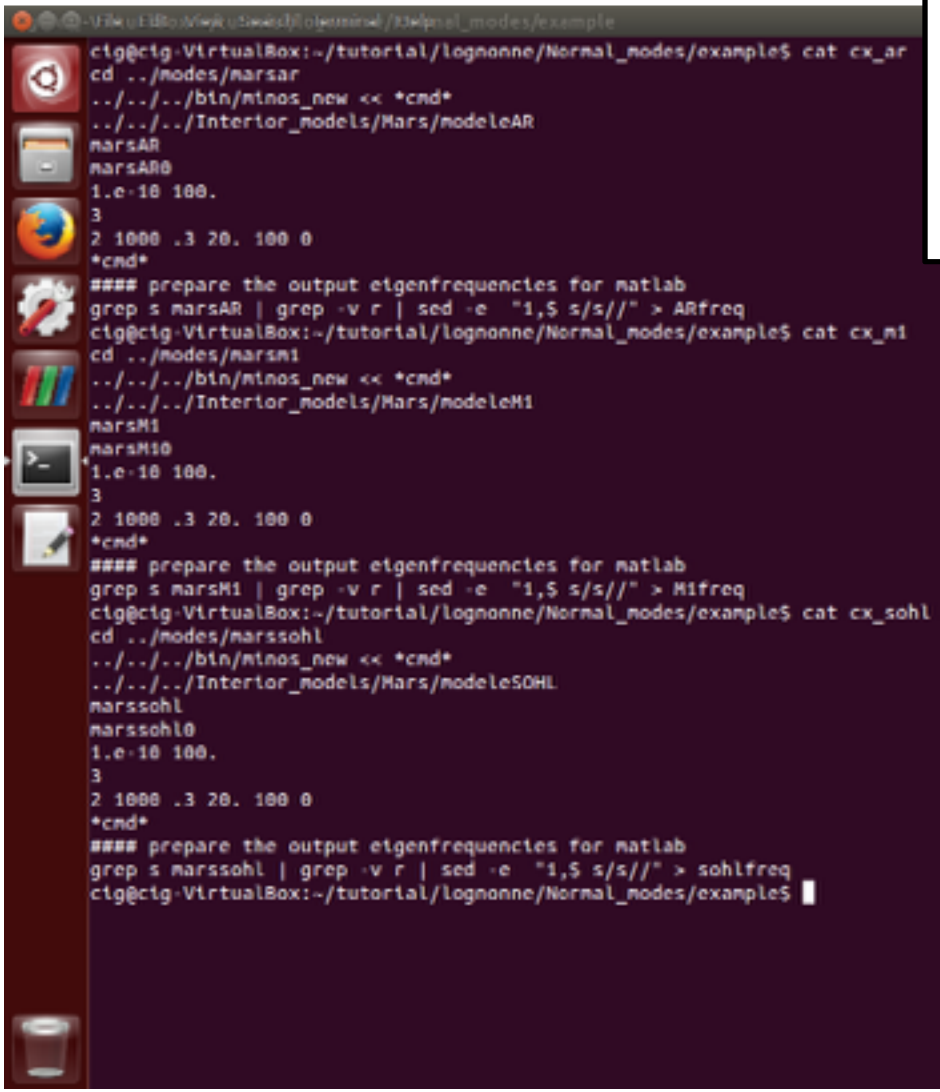


# Tutorial exercice #0:  Normal modes for different Mars models




We will compute the modes for model AR
for model M1
for model SOHL

# Tutorial exercice #0: Normal modes for different Mars models

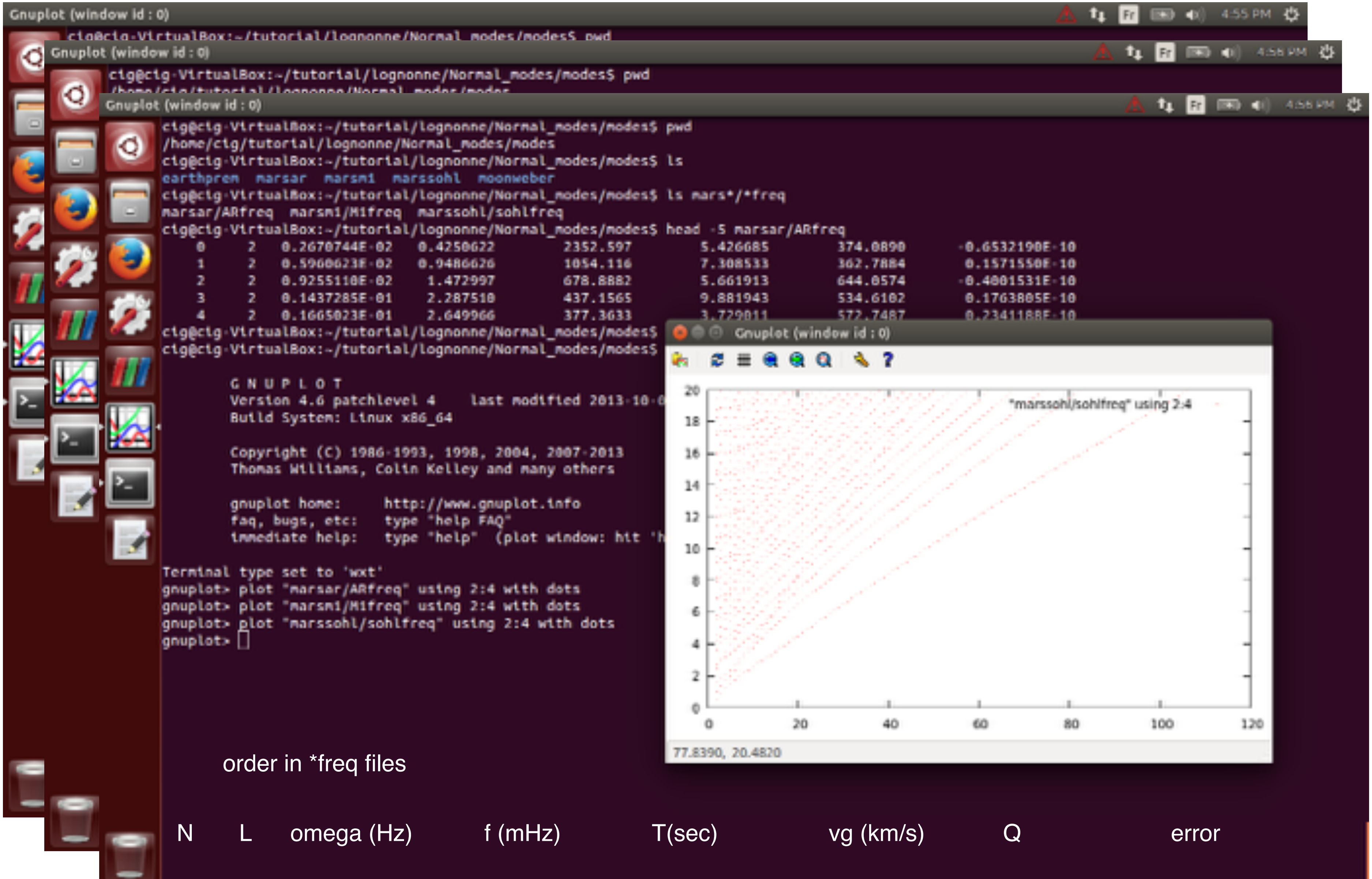



Principle:
Modes catalogue is put in a folder
Modes are computed in a frequency bandwidth
in a angular order bandwidth
and up to n overtones are computed
Modes are either spheroidal, radial or toroidal

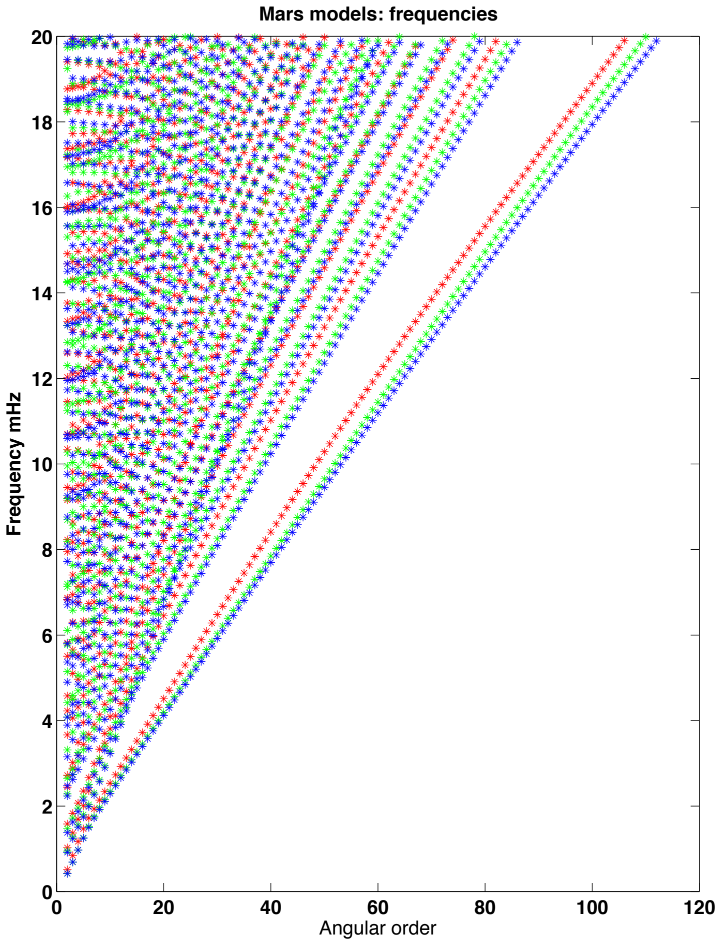
Command file
go in directory and execute minos_new
input parameters:
catalogue directory name
output file name
catalogue file generic name
error gravity-cutoff (sec-1)
type of mode ( 3 for S, 1 for R, 2 for T)
Lmin Lmax fmin fmax 0

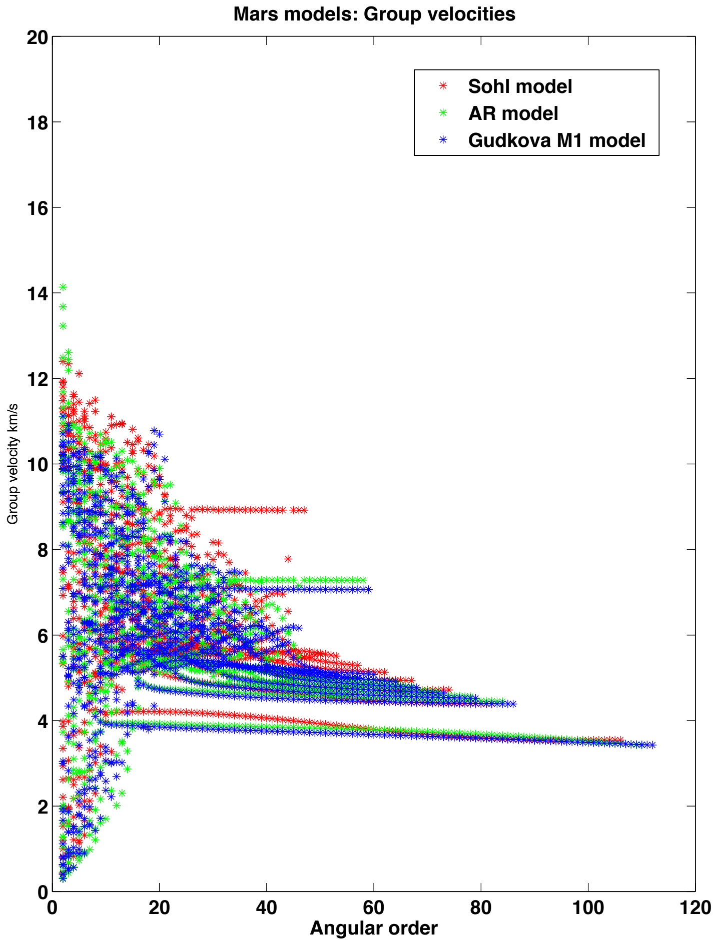
last command is to prepare the output for figures

# Tutorial exercice #0: Frequencies with gnu plot



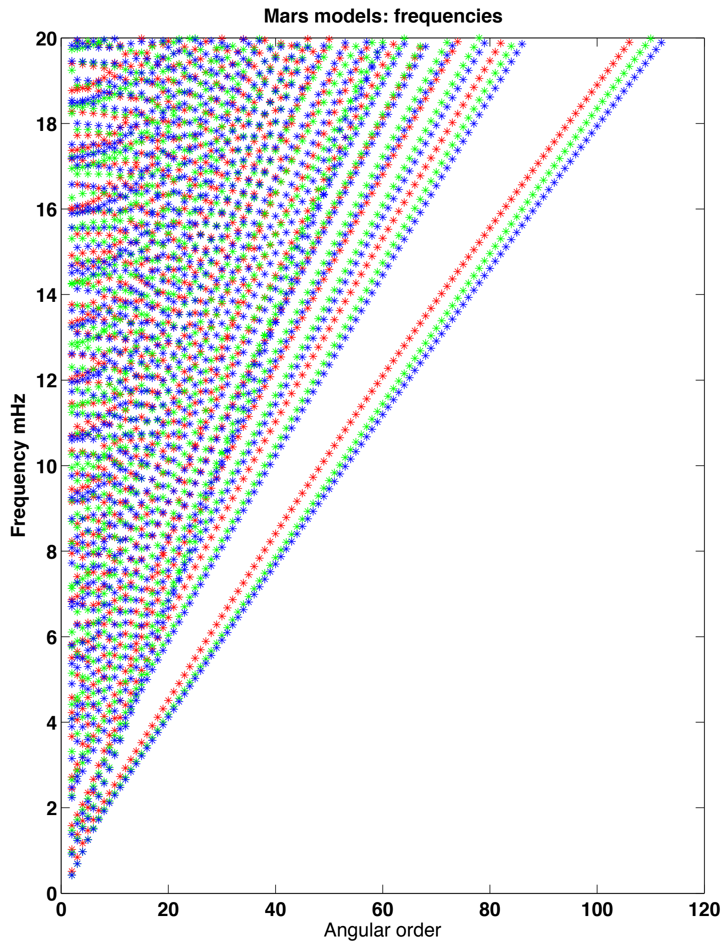
# Tutorial exercice #0: Frequencies with matlab (1/2)

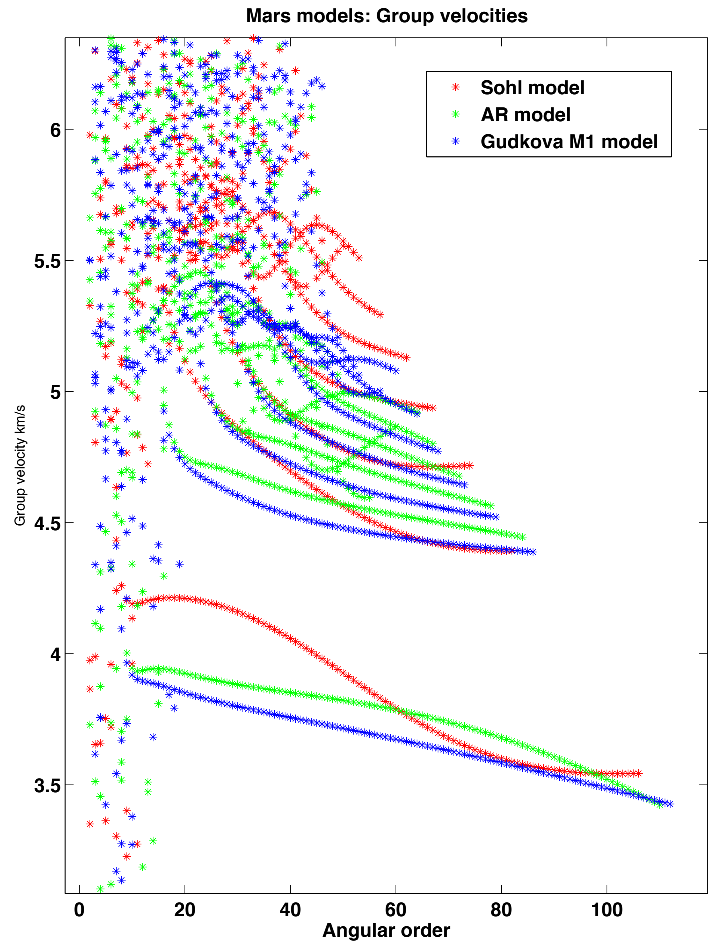


# Tutorial exercice #0: Frequencies with matlab (1/2)

- Both frequencies and group velocities shows large sensitivity to the model used…



# Tutorial exercice #1

- **WHAT ARE THE AMPLITUDE OF 15 sec SEISMIC WAVES GENERATED BY A SHALLOW MOONQUAKE ON THE MOON ?**
  - Choose a Lunar Model
  - Compute Normal modes: **Why ?**
  - Define a seismic source, i.e. moonquake with
    - position (including depth)
    - Magnitude
  - Compute seismograms at different stations…
  - Estimate the amplitude

# Tutorial exercice #1: background

- Why computing normal modes….?
- Reason is mathematical…
- Normal modes are a complete basis of ALL solutions of the seismic equation with given boundary limits, which practically means that ANY solution of the seismic equation (such as our solution for a deep moonquake) can be expressed in this basis

**3D Basis ( our 3D space)**

Constant Position and 3D coordinates

$$\vec{u} = x\vec{i} + y\vec{j} + z\vec{k}$$

Time dependent Position and 3D coordinates

$$\vec{u}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

Coordinates depends on time

unit vector does not depend on time

# Tutorial exercice #1: background

- Why computing normal modes….?
- Reason is mathematical…
- Normal modes are a complete basis of ALL solutions of the seismic equation with given boundary limits, which practically means that ANY solution of the seismic equation (such as our solution for a deep moonquake) can be expressed in this basis

**Normal mode Basis ( the space of seismic waves)**

3D field of position, including surface displacement

$$\vec{u}(t,r,\theta,\varphi) = \sum_k c(t)\vec{U}_k(r,\theta,\varphi)$$

Time dependent Position and 3D coordinates

$$k = \begin{pmatrix} \ell & m & n \end{pmatrix}$$

Coordinates depends on time

unit vector does not depend on time

# Tutorial exercice #1: background

The response of the Earth to an impulsive equivalent body force is obtained from the summation over all positive and negative indices, i.e., positive frequencies as well as negative frequencies. This finally allows recovery of a real amplitude for the Green tensor, equal to

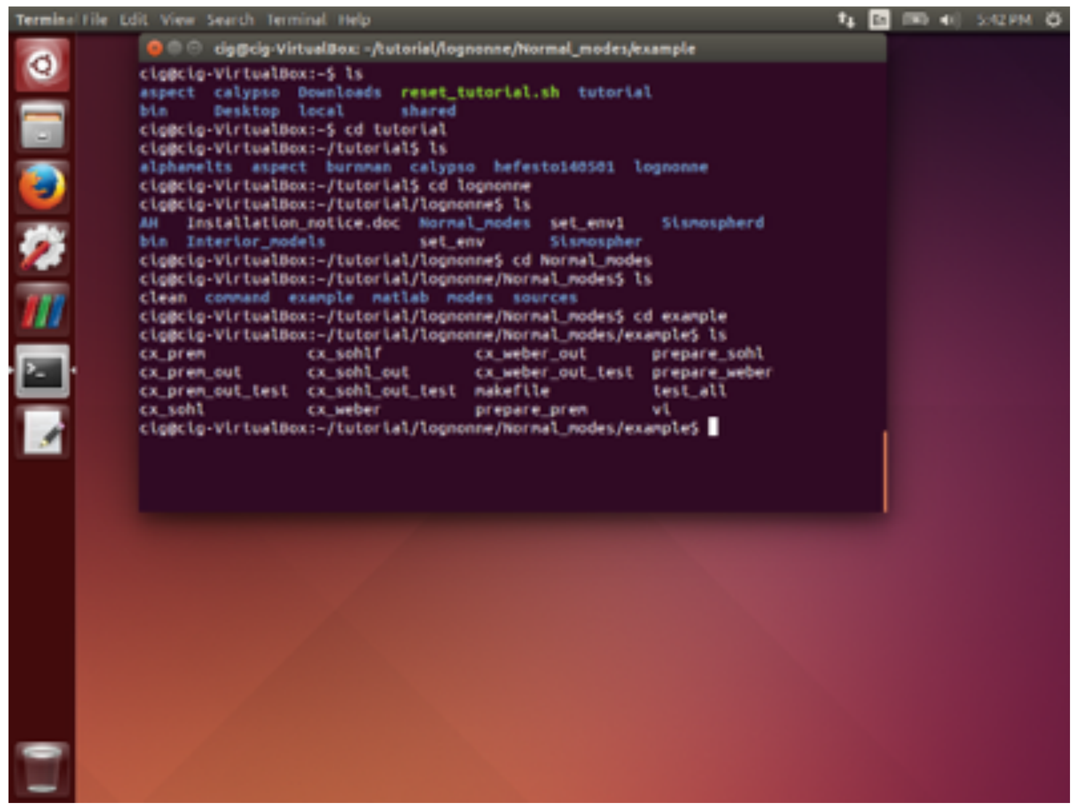
$$|\mathbf{u}(t)\rangle = H(t)\sum_{k>0}\Re e\left(\frac{1}{i\sigma_k}\langle \mathbf{v}_k|\mathbf{f}\rangle e^{i\sigma_k t}|\mathbf{u}_k\rangle\right) \tag{43}$$

**TABLE 4** Amplitude of the ground displacement as described by a summation of normal modes for a local double couple. The index $s$ denotes the position of the source

| Source Function | Displacement Expression |
|---|---|
| Seismic tensor with general source function $\|\mathbf{f}(t)\rangle = -\mathbf{M}(t)\nabla\delta(\mathbf{r}-\mathbf{r}_s)$ | $\|\mathbf{u}(t)\rangle = \sum_{k>0}\Re e\left(\frac{1}{i\sigma_k}\int_0^t dt'\,\mathbf{M}(t') : \nabla\,\mathbf{v}_k(\mathbf{r}_s)e^{i\sigma_k(t-t')}\|\mathbf{u}_k\rangle\right)$ |
| Seismic tensor with Heaviside source function $\|\mathbf{f}(t)\rangle = -\mathbf{M}_0 H(t)\nabla\delta(\mathbf{r}-\mathbf{r}_s)$ | $\|\mathbf{u}(t)\rangle = H(t)\sum_{k>0}\Re e\left(\frac{1}{\sigma_k^2}\mathbf{M}_0 : \nabla\,\mathbf{v}_k(\mathbf{r}_s)(1-e^{i\sigma_k t})\|\mathbf{u}_k\rangle\right)$ |

# Tutorial exercice #1

- go and update the command file for normal modes computations
- cx_prem (Prem model for Earth)
- cx_sohl ( Sohl model for Mars)
- cx_weber ( Weber model for Moon)



# Tutorial exercice #1  : update in VM

- update the cx_weber into two tutorial for  S modes between 0.1 and 67 mHz ( cx_weber_tutorial1a) and radial modes (cx_weber_tutorial1b)   *both are in inputfortutorial1*

folder where the modes will be stored and file describing the model

output file

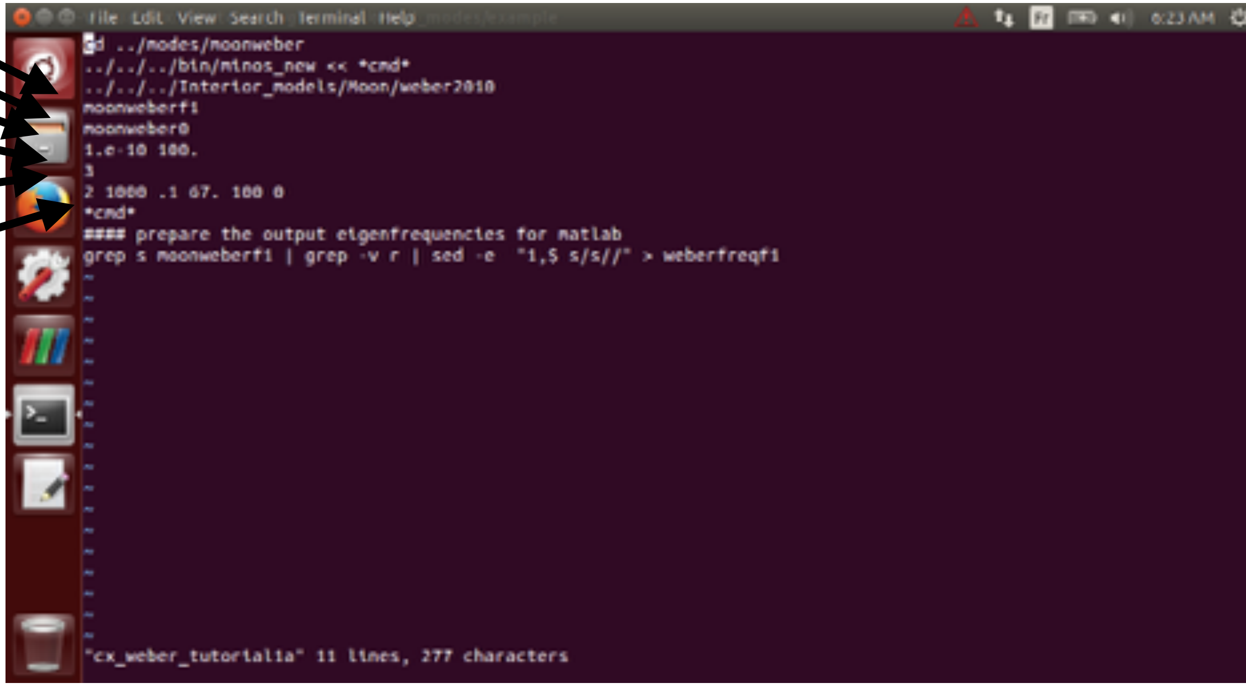
Generic name for model

error and angular
frequency above which
gravity is neglected

Spheroidal (3), Toroidal (2)
Radial(1)

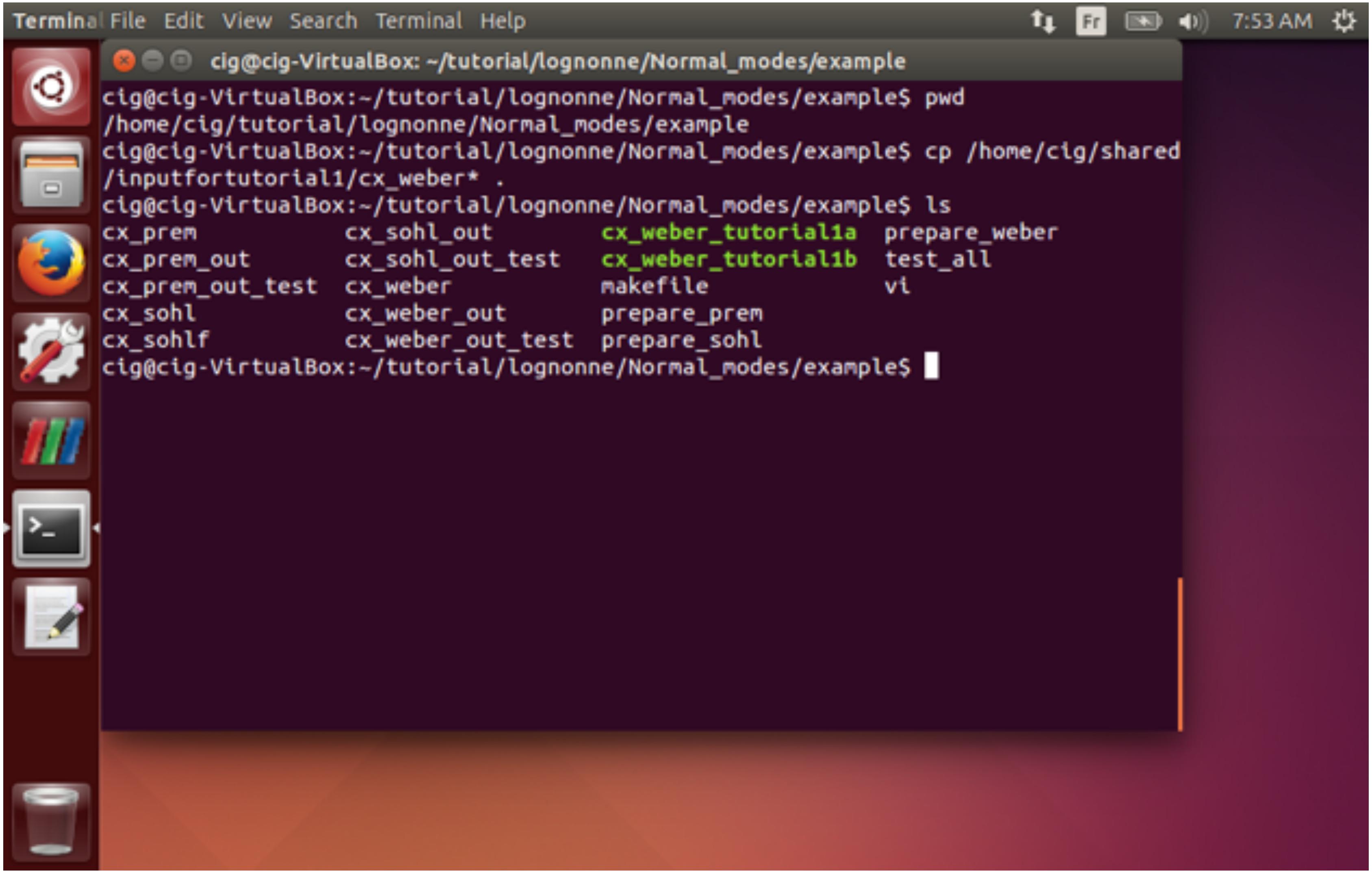
Lmin, Lmax, fmin, fmax, N 0

Lmin:  minimum angular number
Lmax: maximum angular number
fmin : minimum frequency
fmax: maximum frequency
Number of overtones computed




for 50 mHz, Lmax = 500
for 100 mHz, Lmax =1000

# Tutorial exercice #1 : update through shared folder



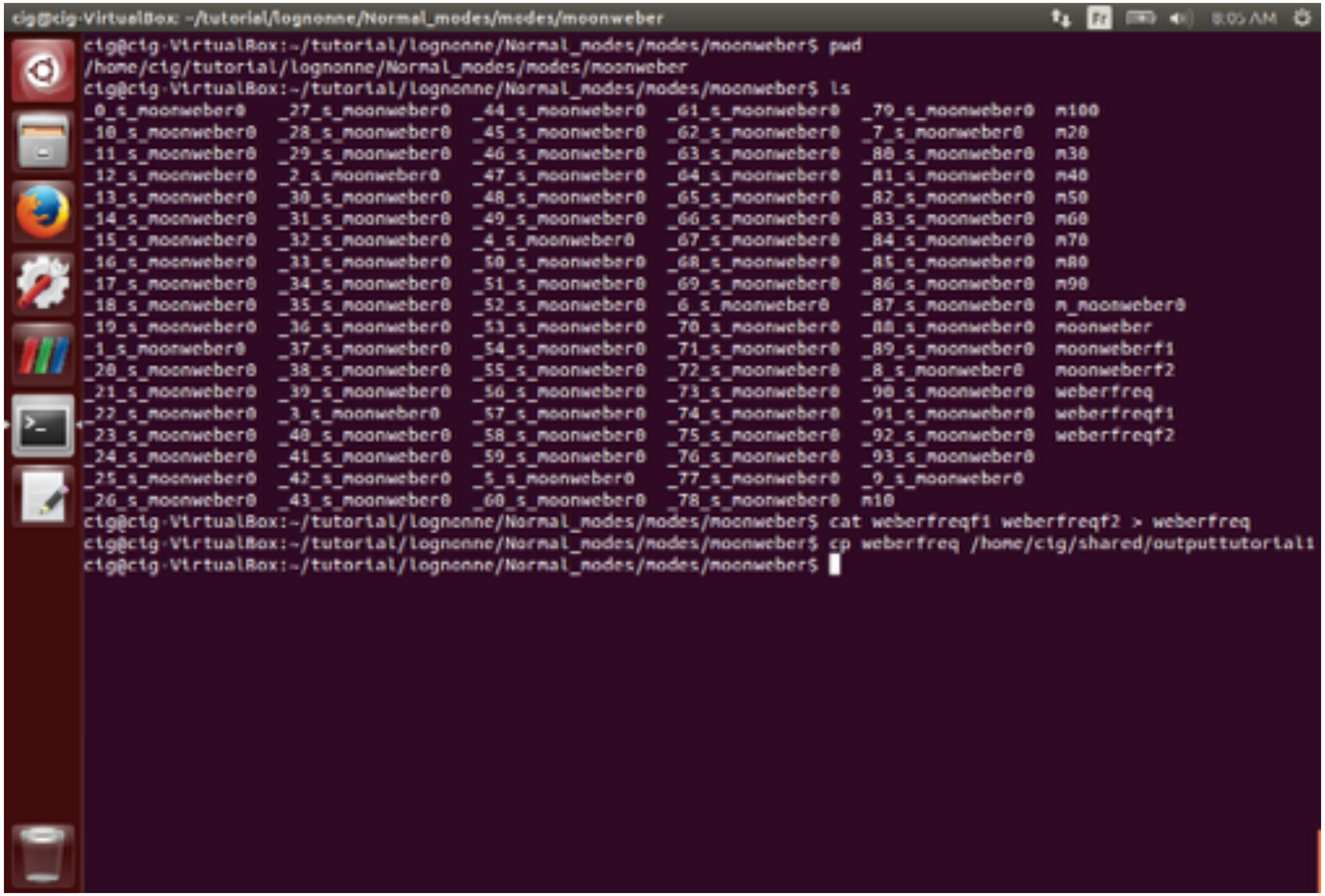
# Tutorial exercice #1 : compute normal modes

- execute the script by typing
  - ./cx_weber_tutorial1a
  - ./cx_weber_tutorial1b

- Normal modes can be computed to higher frequencies
- This will however need a vertical model with a thinner mesh
- The key in the computation is to check the error, which must never be larger than 1e-3 ( which is already large, precise computation must have errors of at least 1e-6 )

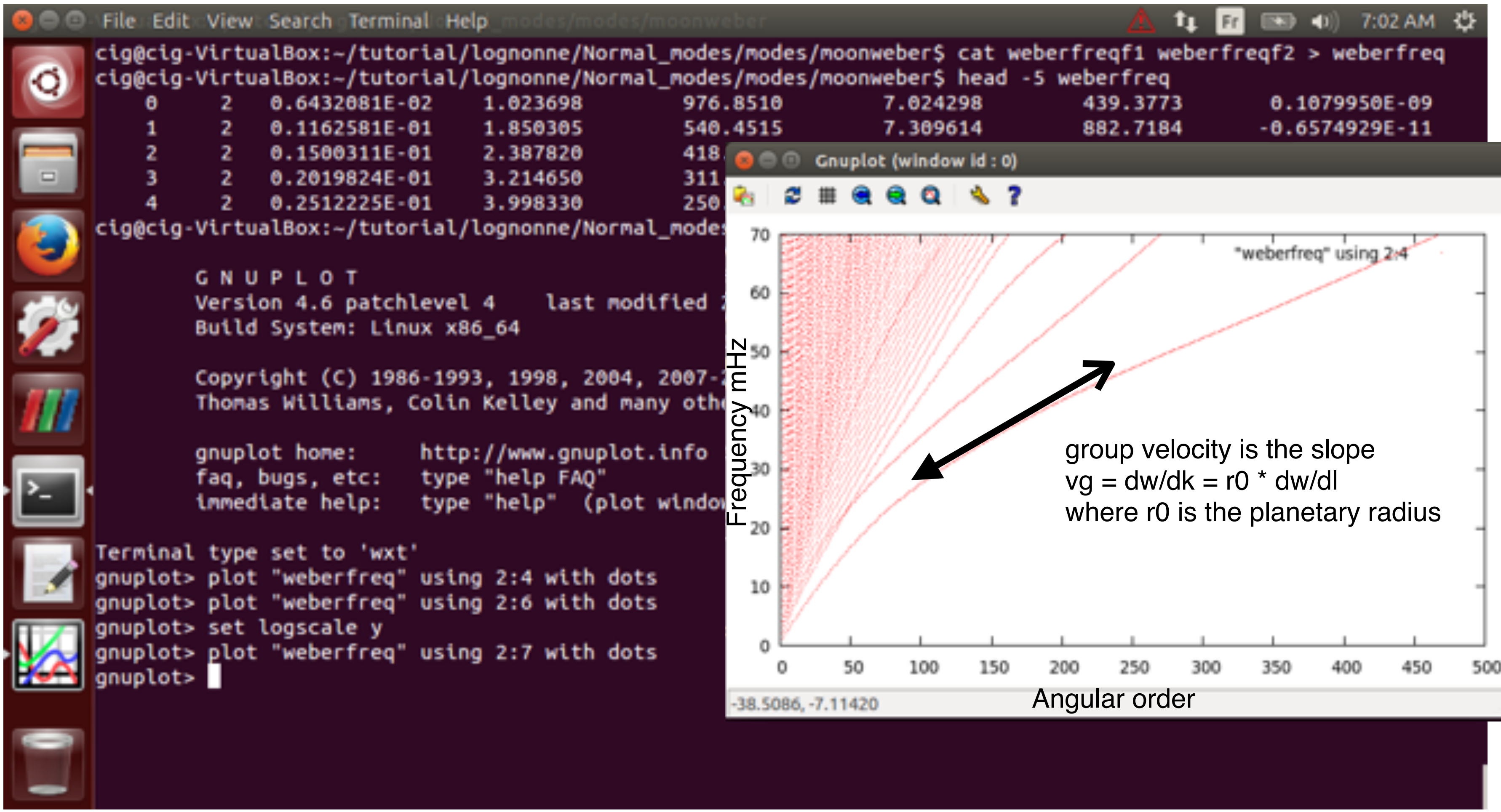
# Tutorial exercice #1

- Results  are in  Normal_modes/modes/moonweber
- These are files for all n orders, in which the normal modes functions and frequencies are stored
- create a frequency file with the two runs
- The results can be exported in the shared outputtutorial1



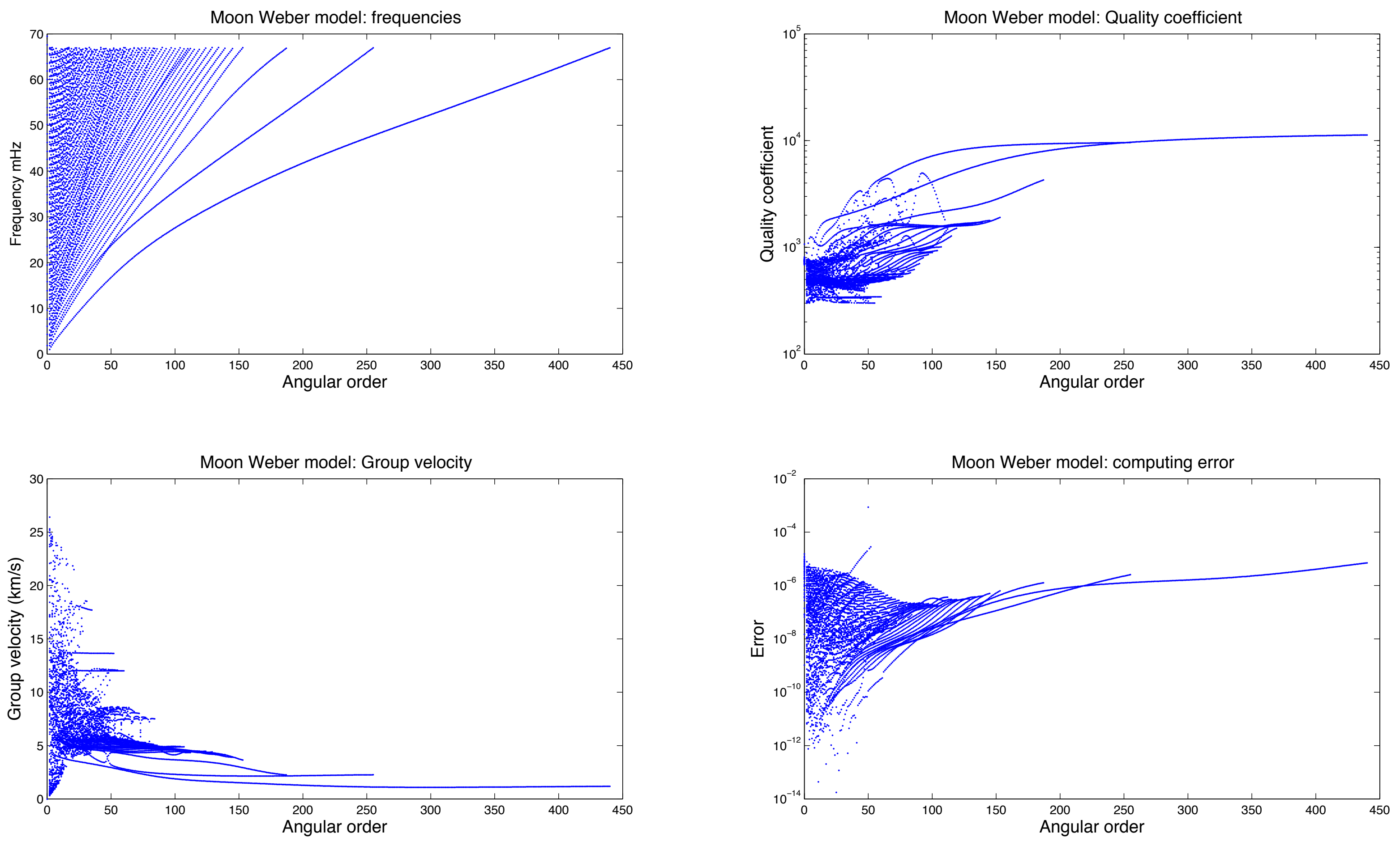
# Tutorial exercice #1 : plotting results with gnu plot

N L omega (Hz) f (mHz) T(sec) vg (km/s) Q error



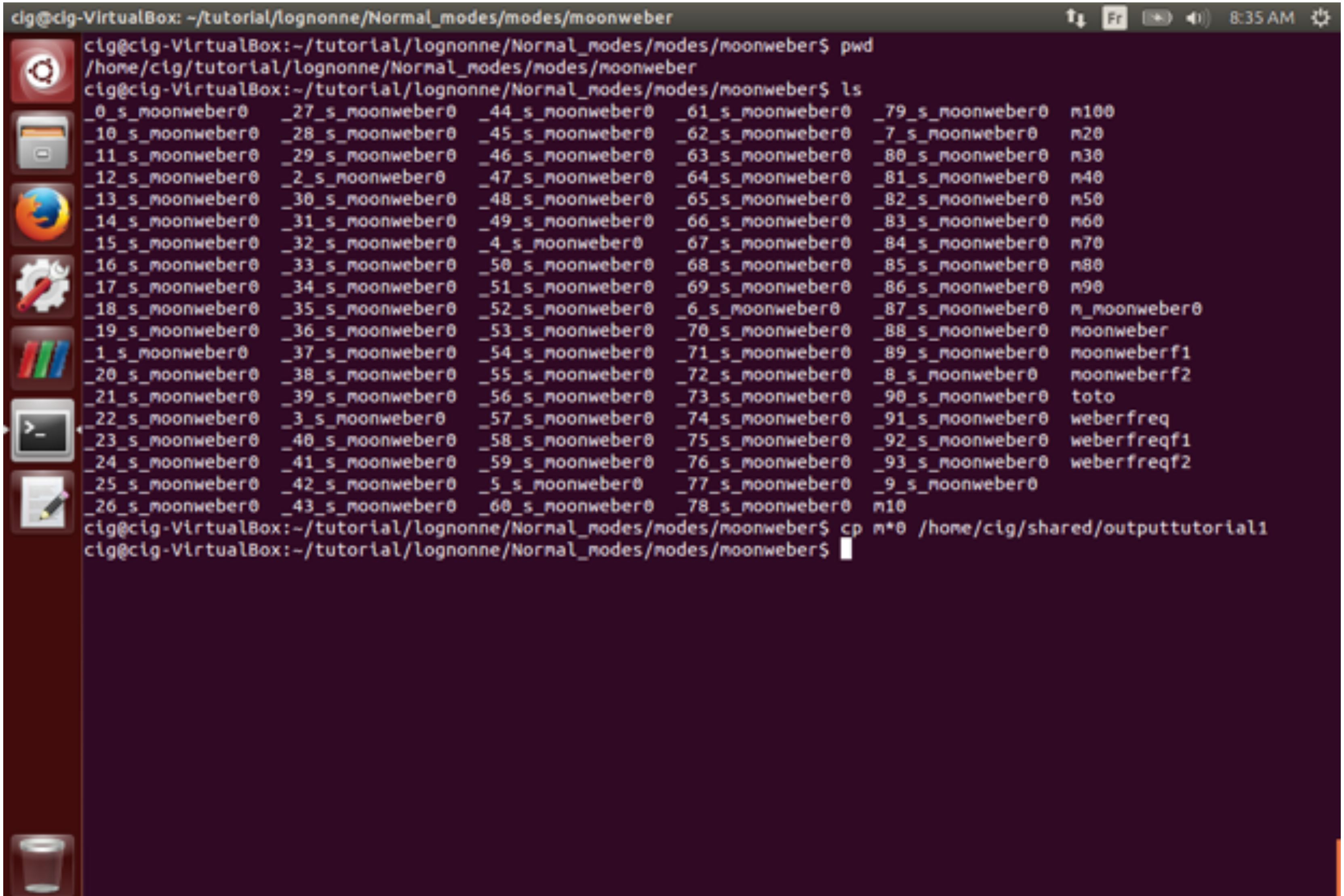
# Tutorial exercice #1 : plotting results with matlab

- matlab alternative ( plot_weber.m in  inputfortutorial1 folder)

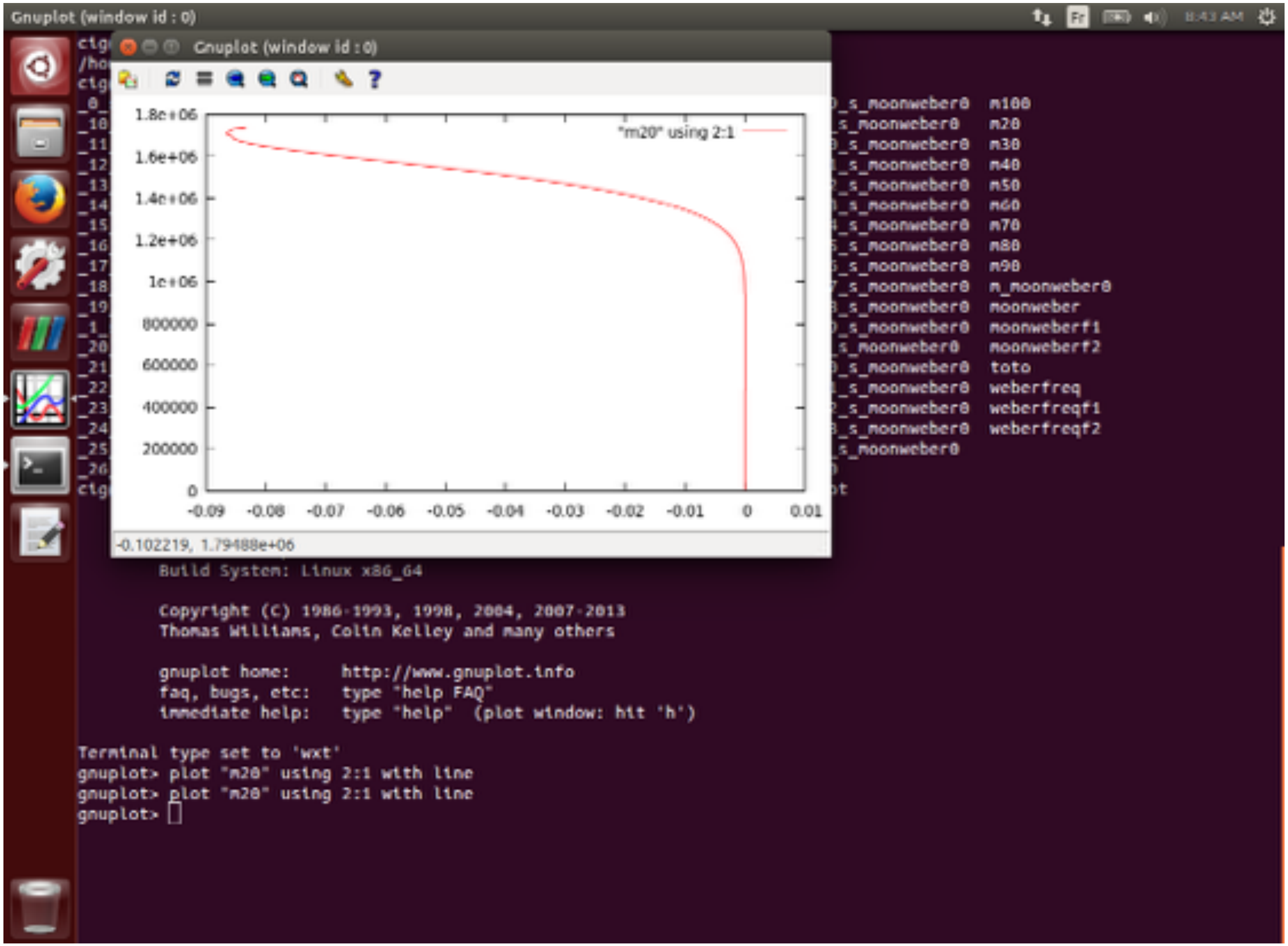


# Tutorial exercice #1 : plotting mode amplitudes with matlab

- Transfert modes amplitudes (m10,m20… files) in shared folder

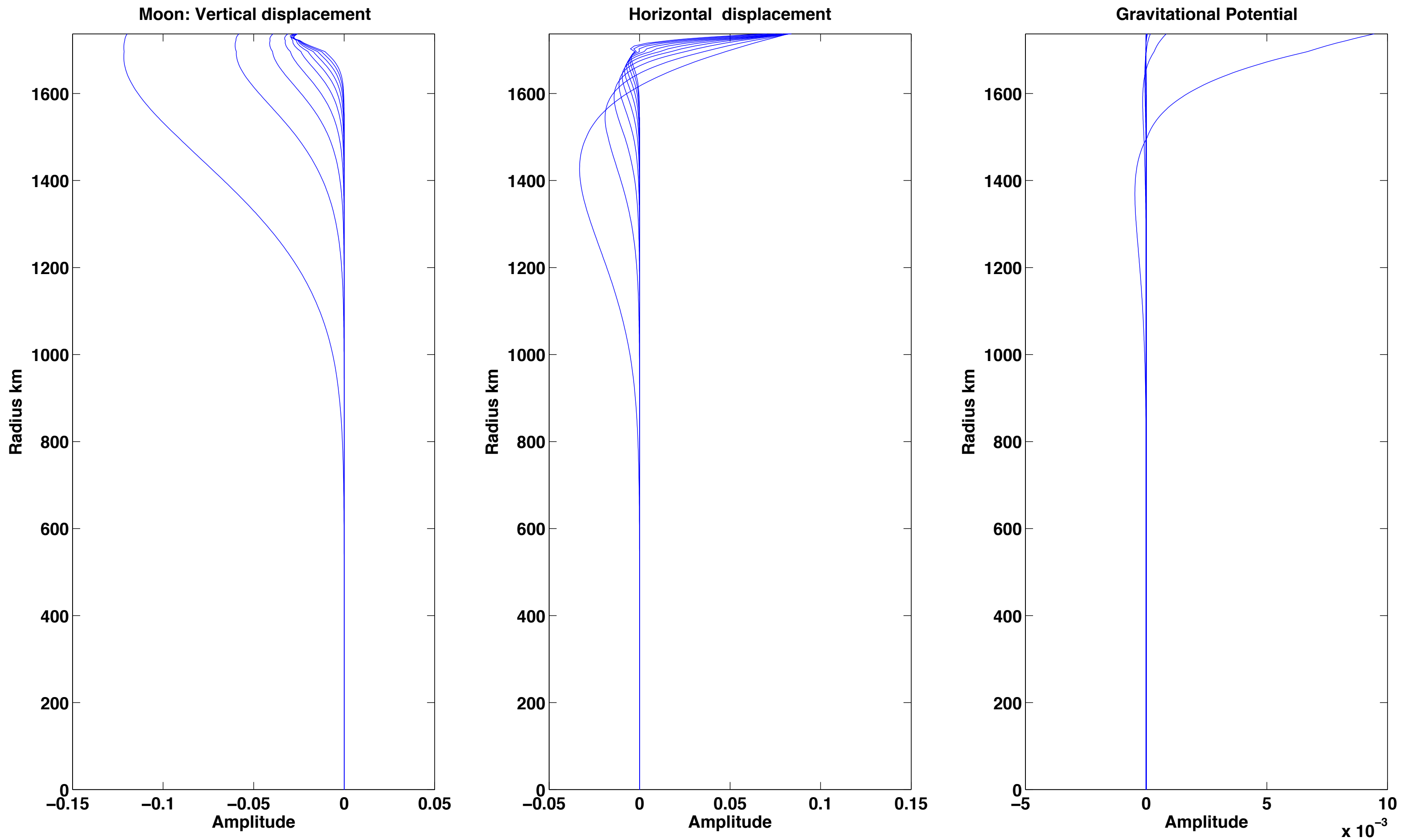


# Tutorial exercice #1 : plotting mode amplitudes with gnu plot



# Tutorial exercice #1 : plotting mode amplitudes with matlab

- plotmode_weber.m

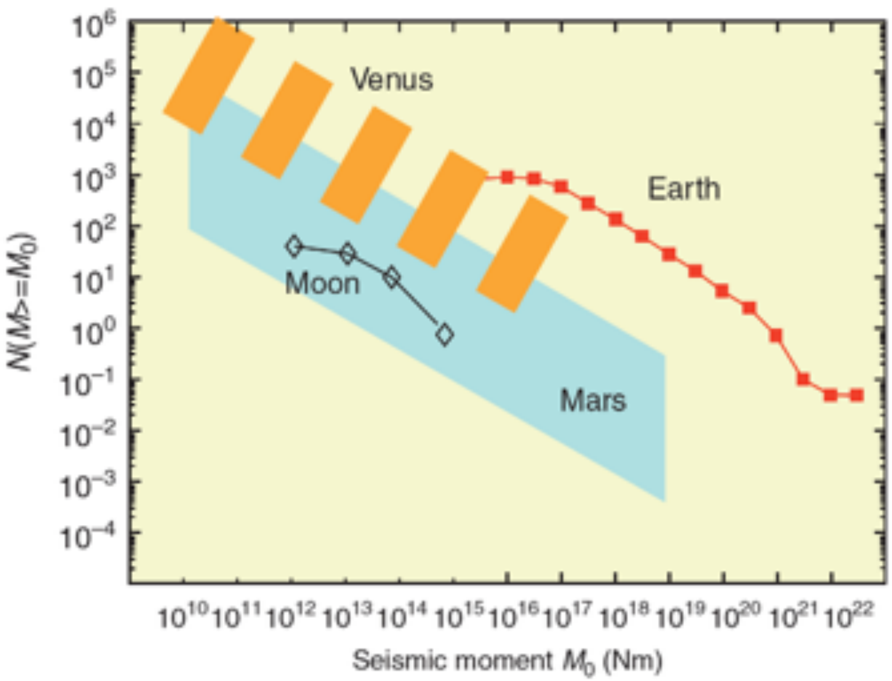


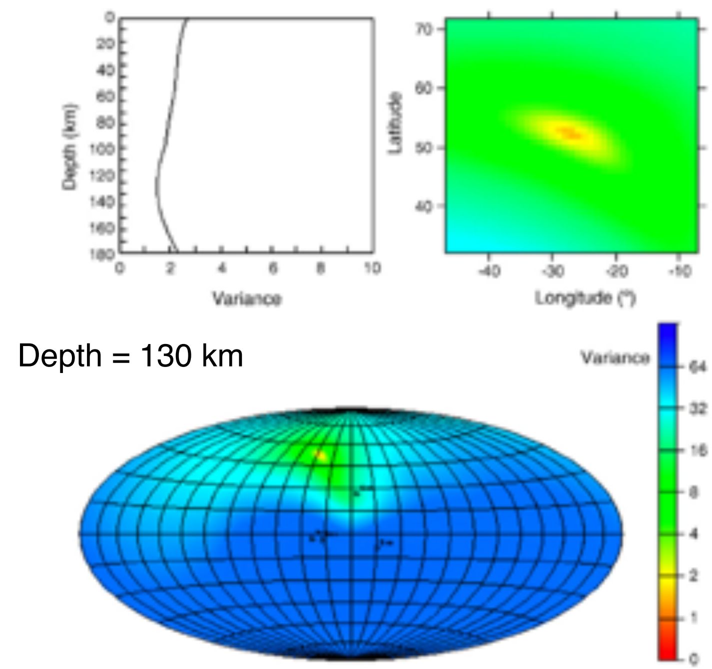
# Tutorial exercice #1

- **WHAT ARE THE AMPLITUDE OF 10 sec CORE PHASE, 20 sec SURFACE WAVES AND NORMAL MODES GENERATED BY A SHALLOW MOONQUAKE ON THE MOON ?**
  - Choose a Lunar Model
  - Compute Normal modes: Why ?
  - **Define a seismic source, i.e. moonquake with**
    - **position (including depth)**
    - **Magnitude**
  - Compute seismograms at different stations…
  - Estimate the amplitude

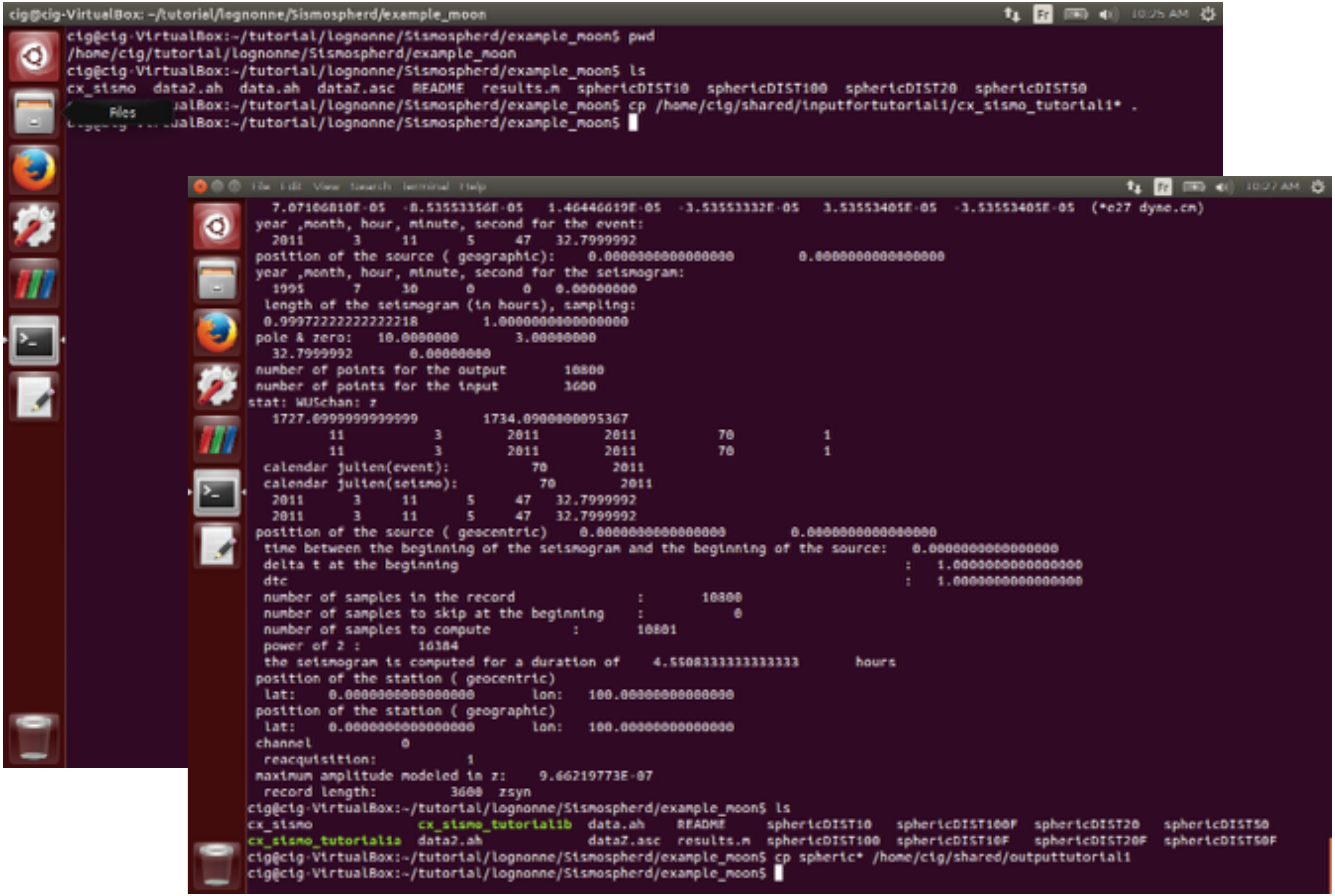
# Tutorial exercice #1 : Seismic moment for Moonquake

- Largest deep Moonquake ~ 5  $10^{13}$ Nm
- Largest Shallow Moonquake   3 $10^{14}$ Nm- 1.6 $10^{14}$ Nm
- Tohoku Earthquake  ~5.3 $10^{22}$ Nm
  - Moon DMQ moments are $10^{9}$ smaller than the largest Earth quakes…
  - We take a $10^{14}$ Nm quake ( $10^{21}$ dyne cm)



# Tutorial exercice #1 : preparing the input files (1/2)



# Tutorial exercice #1 : preparing the input files (2/2)

```
sed -e "1,$ s/DIST/10./g" < dataZ.asc | ../../bin/asc2ah >data.ah
../../bin/ahmodhead -e 0. 0. 10. 2011 3 11 05 47 32.8 -m 45. 45. 45. 1e-6 1. 1. -i data.ah > data2.ah #  reference
../../bin/sismospherd0 << *cmd*
moonweber0
../../Normal_modes/modes/moonweber
0 0 0 -1 1. 66.
1
2 0 0 1
sphericDIST10F
2 0
data2.ah
-1 -1
3
*cmd*
sed -e "1,$ s/DIST/20./g" < dataZ.asc | ../../bin/asc2ah >d
../../bin/ahmodhead -e 0. 0. 10. 2011 3 11 05 47 32.8 -m 45
../../bin/sismospherd0 << *cmd*
moonweber0
../../Normal_modes/modes/moonweber
0 0 0 -1 1. 66.
1
2 0 0 1
sphericDIST20F
2 0
data2.ah
-1 -1
3
*cmd*
sed -e "1,$ s/DIST/50./g" < dataZ.asc | ../../bin/asc2ah >d
../../bin/ahmodhead -e 0. 0. 10. 2011 3 11 05 47 32.8 -m 45
../../bin/sismospherd0 << *cmd*
moonweber0
../../Normal_modes/modes/moonweber
0 0 0 -1 1. 66.
1
2 0 0 1
sphericDIST50F
2 0
data2.ah
-1 -1
3
"cx_sismo_tutorial1a" 56 lines, 1314 characters
```

Folder with the mode

generic name

NSmin NSmax NTmin NTmax fmin fmax

Number of seismograms *(here 1)*

ivit igrav isec ioutput *(ivit 0,1,2 for displacement, velocity, acceleration; igrav=0,1 for gravity or not, ised =1,0 for static term or not, output =1,0 for output or not)*

input output *(input 0 (hand input), 1 through ah, 2 change duration, output 0 (only synthetics), 1 data and synthetics)*

output file

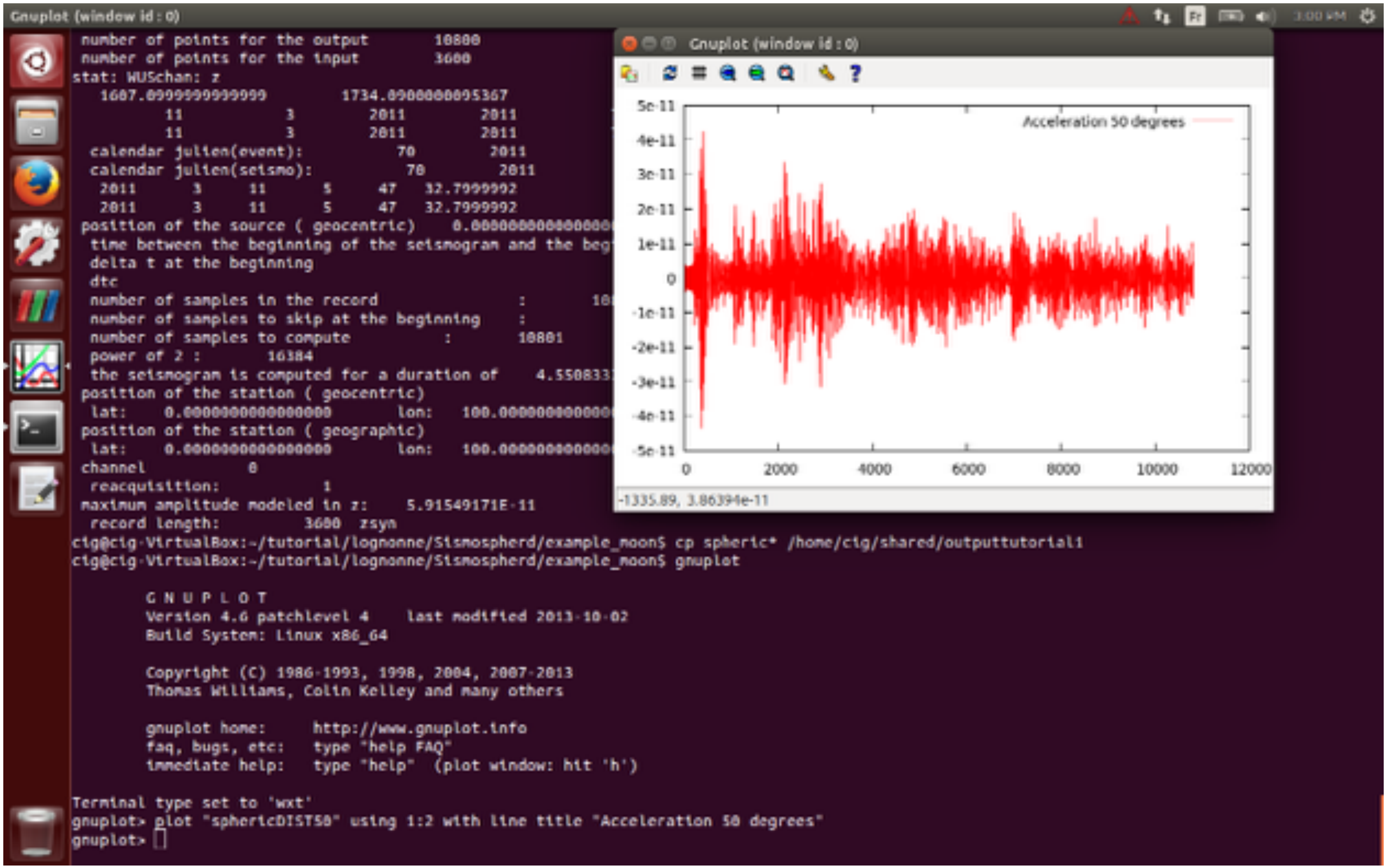
depth altitude *(if -1 -1, those of the input file)*

time of seismogram in hr

# Tutorial exercice #1 : compute seismograms

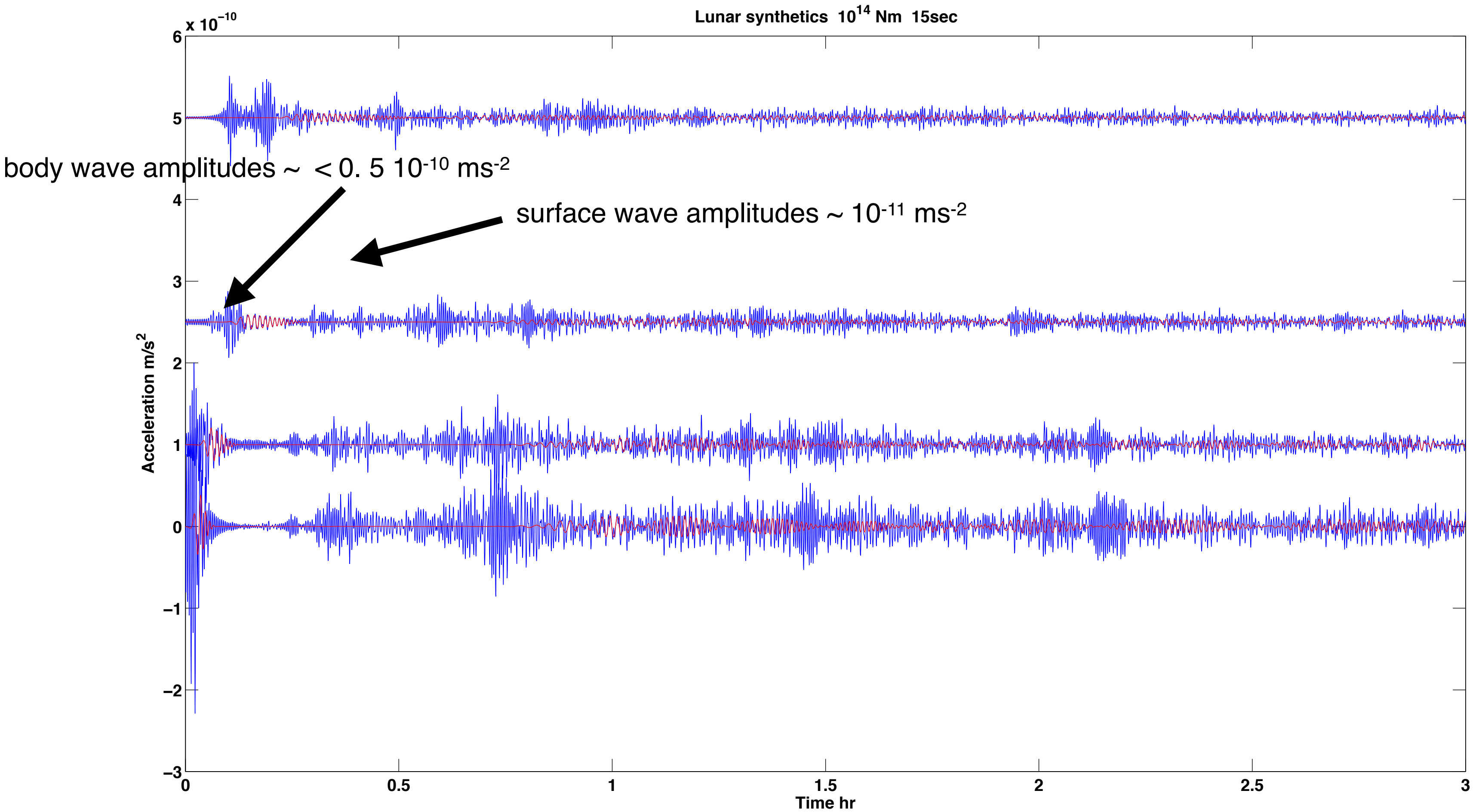
- type
  - ./cx_sismo_tutorial1a ( for fundamental only)
  - ./cx_sismo_tutorial1b ( for all modes )

# Tutorial exercice #1 : plotting the seismograms on gnuplot



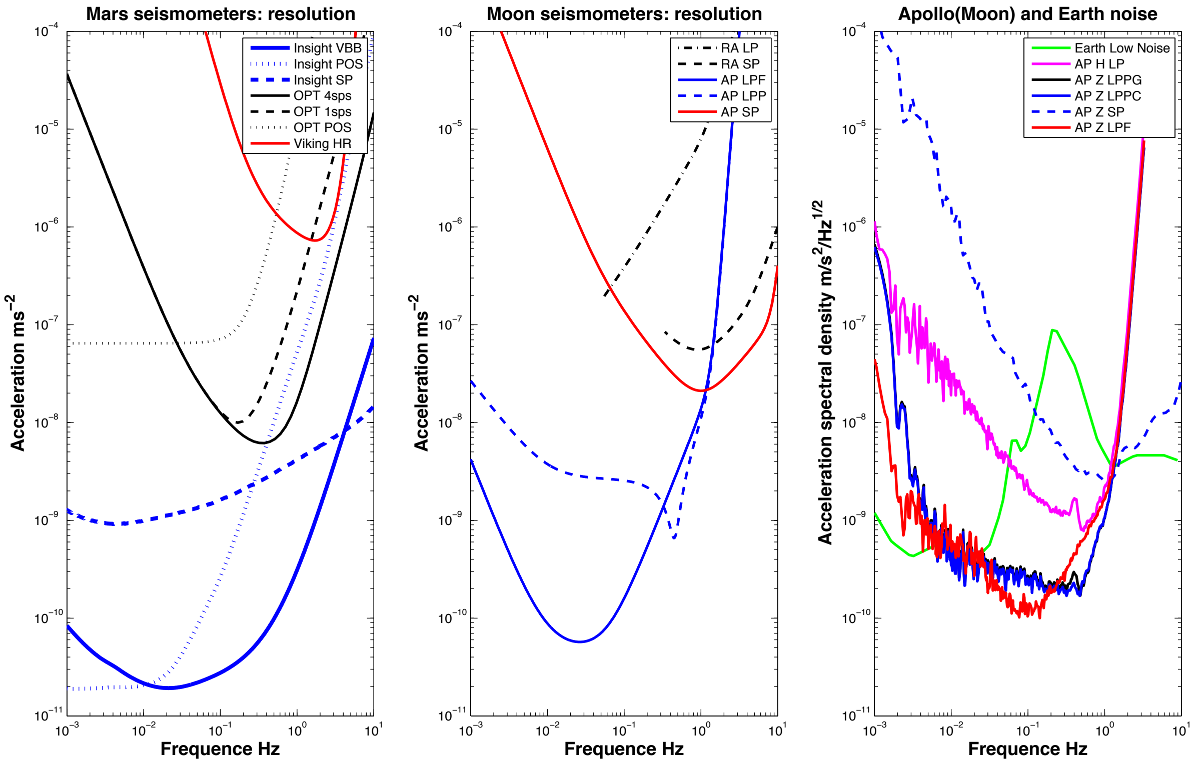
# Tutorial exercice #1 : plotting the seismograms on matlab

- plot_sismo.m

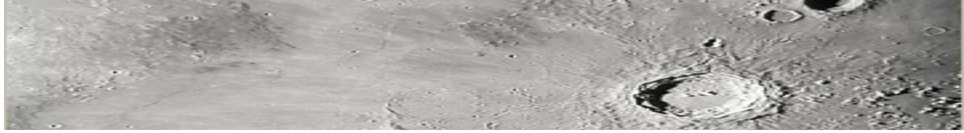


# Tutorial exercice #1 : did Apollo got this signal ?

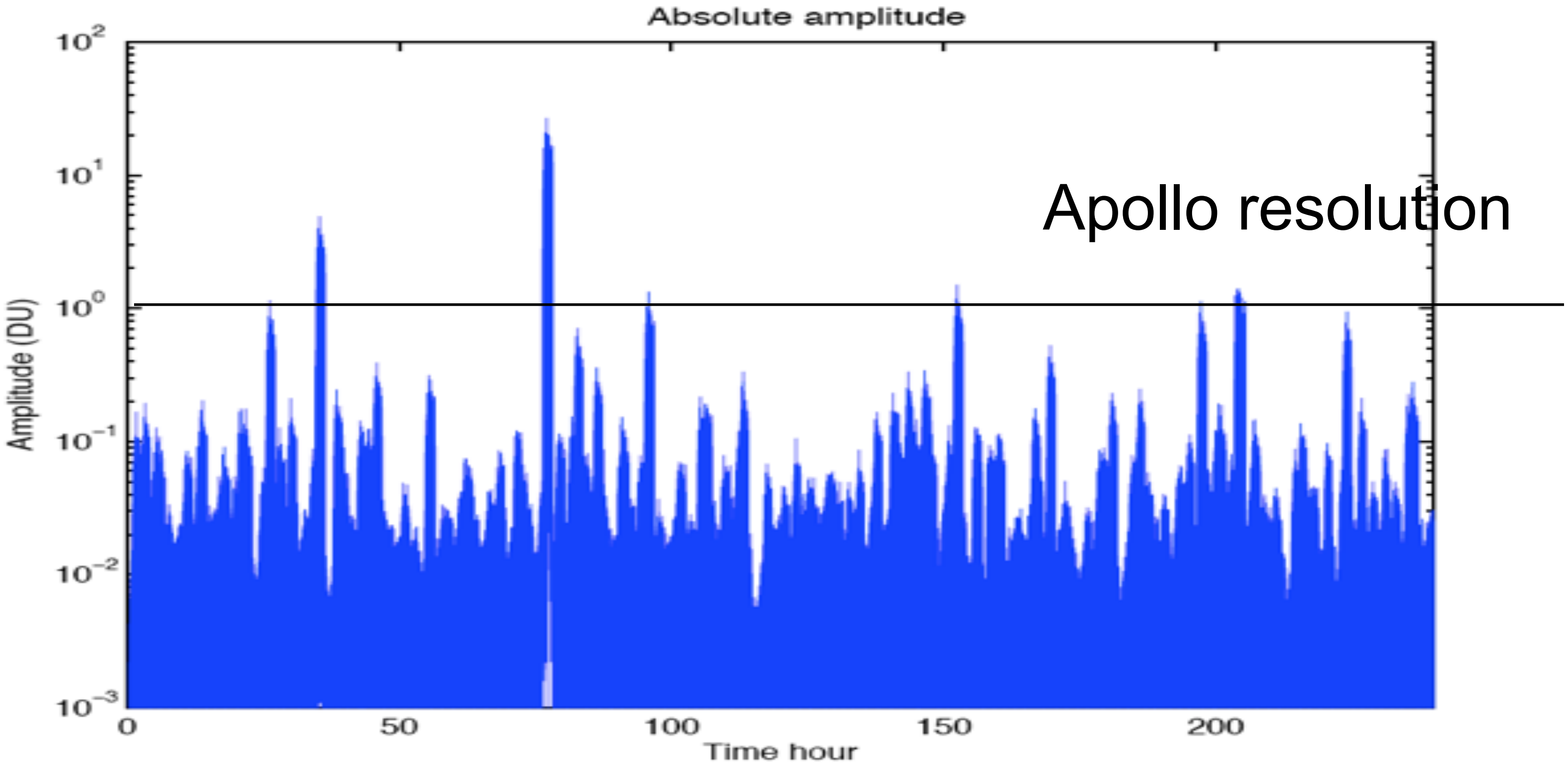
- Apollo noise at 20 sec ( 0.05 Hz) : ASD= 4 x $10^{-10}$ $ms^{-2}/Hz^{1/2}$
- Equivalent noise in the 0.01 Hz-0.06 Hz (15sec-100sec):
- $ASD*\sqrt{Df} = 0.9\ 10^{-10}\ ms^{-2}$ rms
- Apollo was not able to detect these long period waves… and practically, no waves below ~5 sec are detected…



# Seismic noise estimations



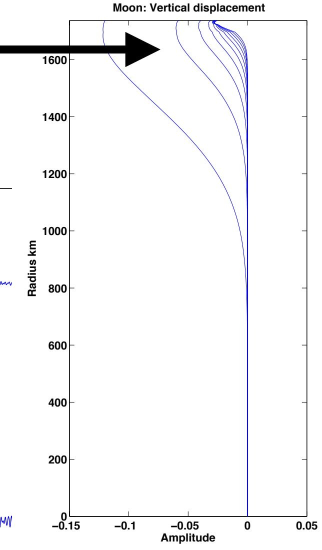
- A continuous source of noise on the Moon can be related to the continuous impacts of micro-meteorites (Lognonné et al., 2009)
- We find a seismic background continuous noise in the range of $10^{-2}$-$10^{-1}$ Apollo DU at 0.5 Hz, corresponding to amplitudes of $5 \times 10^{-11}$ to $5 \times 10^{-12}$ $ms^{-2}$

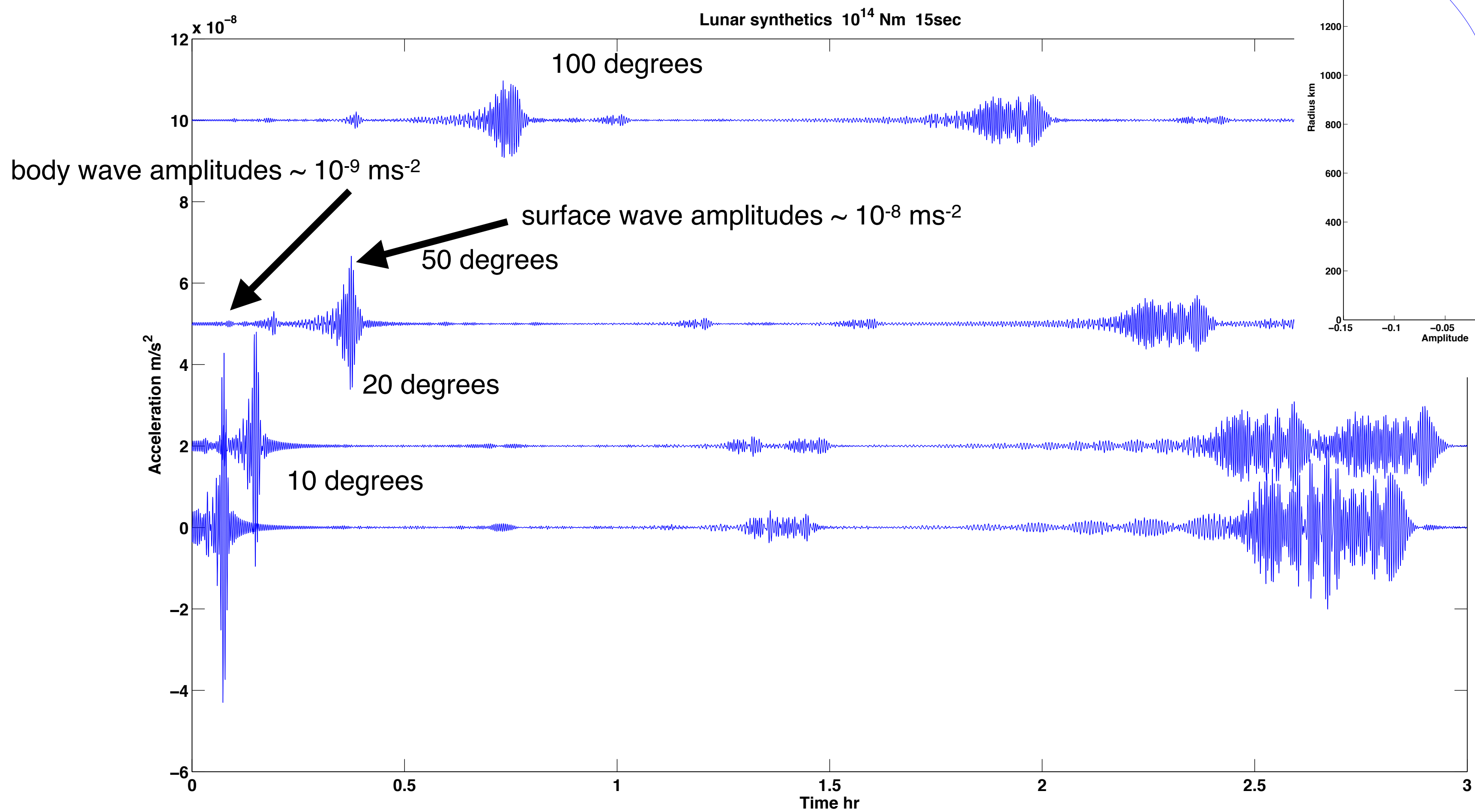


# Tutorial exercice #1 : source depth

- Home work… re compute a seismograms for 10 km depth and get much larger amplitudes

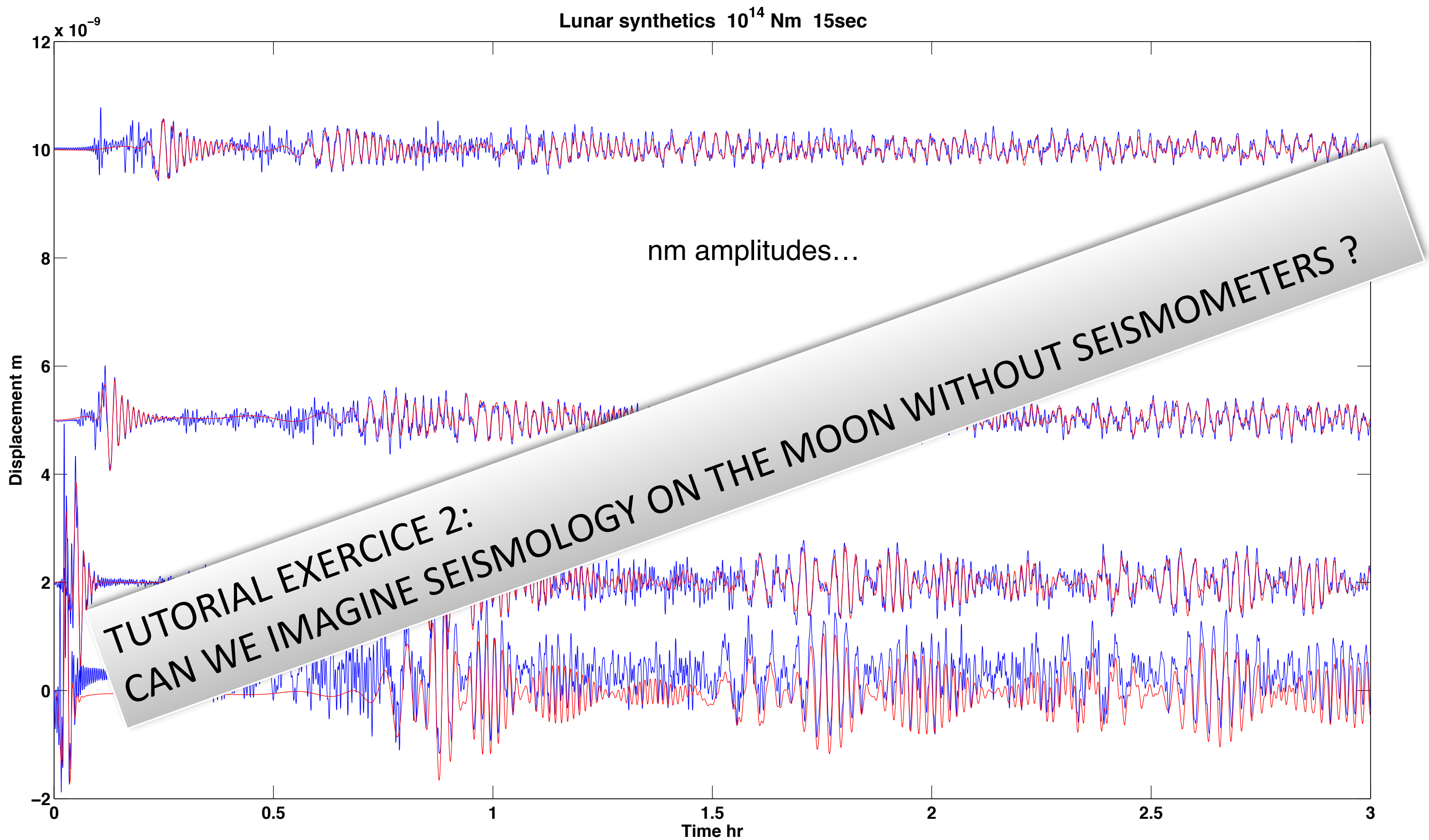
Surface waves ARE concentrated at the surface



# Tutorial exercice #1 : Displacement seismograms

- type
  - ./cx_sismo_tutorial1c ( for fundamental only)
  - ./cx_sismo_tutorial1d ( for all modes )




TUTORIAL EXERCICE 2:
CAN WE IMAGINE SEISMOLOGY ON THE MOON WITHOUT SEISMOMETERS ?