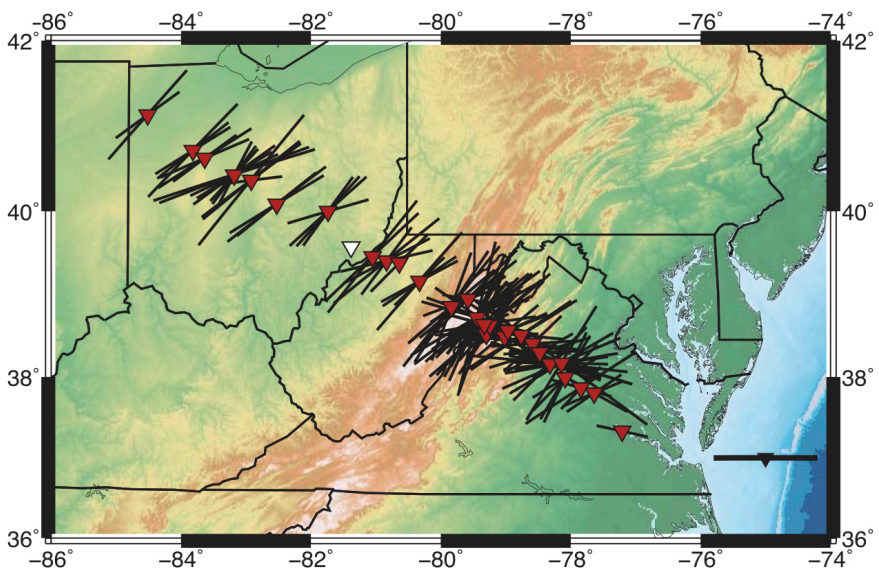
Seismology 2: Anisotropy and Attenuation

what they are, how we study them, and what they can tell us about how the Earth's interior works

Maureen D. Long, Yale University



Anisotropy and attenuation variations across the MAGIC array, Central Appalachians; Aragon et al., 2017; Byrnes et al., in prep.

CIDER 7/11/18

The Wave Equation

- Using Hooke's law for an isotropic medium $\sigma_{ij} = \lambda\theta\delta_{ij} + 2\mu\varepsilon_{ij}$
- And the definition of the strain tensor
- And ignoring body force (assuming far-field from an earthquake)
- And using $\nabla^2\boldsymbol{x} = \nabla(\nabla\cdot\boldsymbol{x}) - \nabla\times(\nabla\times\boldsymbol{x})$

We get the ***Seismic Wave Equation*** for an isotropic medium:

$$\rho\frac{\partial^2\boldsymbol{u}}{\partial t^2} = \nabla\lambda(\nabla\cdot\boldsymbol{u}) + \nabla\mu\cdot[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] + (\lambda + 2\mu)\nabla\nabla\cdot\mathbf{u} - \mu\nabla\times\nabla\times\mathbf{u}$$

Slide from Sanne's talk on Monday

Several assumptions are made in the derivation of this equation; ignore lateral heterogeneity, assume isotropy, and assume a perfectly elastic medium.

Road map for today's lecture

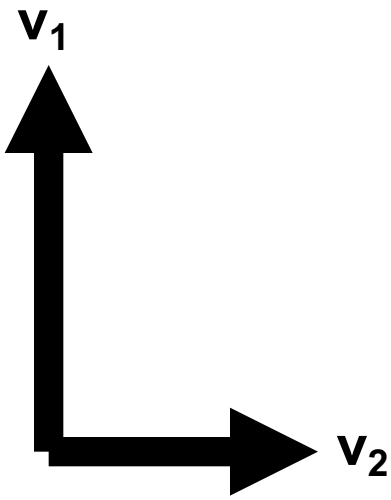
- Part I: Seismic anisotropy, in gory detail
  - What it is
  - Why we care
  - How to measure it: body waves, surface waves, normal modes
  - Caveats and challenges
  - Some key results in different parts of the Earth's mantle

- Part II: Attenuation, in somewhat less detail
  - What it is
  - Why we care
  - How to measure it: normal modes, surface waves, body waves
  - Caveats and challenges
  - A couple of examples

Part I: Seismic anisotropy

Olivine: highly anisotropic; dominant upper mantle constituent

The **speed** at which a seismic wave propagates depends on its **direction** (of propagation, or of polarization)**.**



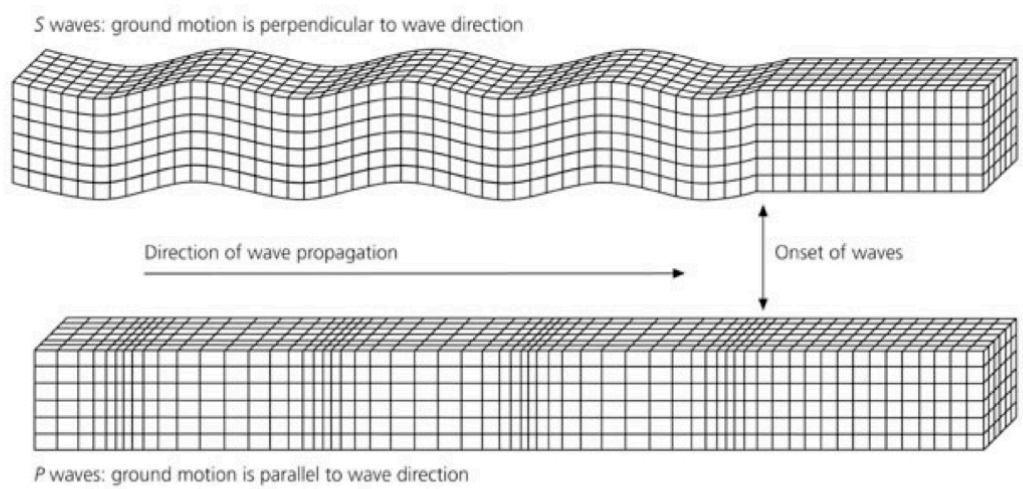
Think back to Monday's seismology lecture and what we learned about body waves…

In an isotropic solid only two elastic moduli are independent, and there are two types of waves, P and S

$$\beta = v_S = \sqrt{\frac{\mu}{\rho}}$$

$$\alpha = v_P = \sqrt{\frac{\kappa + \frac{4}{3}\mu}{\rho}}$$

Figure 2.4-3: Displacements for *P* and *S* waves.



modified from Stein and Wysession, 2009

Slide from Goran Ekstrom, CIDER 2013

If the material is anisotropic…

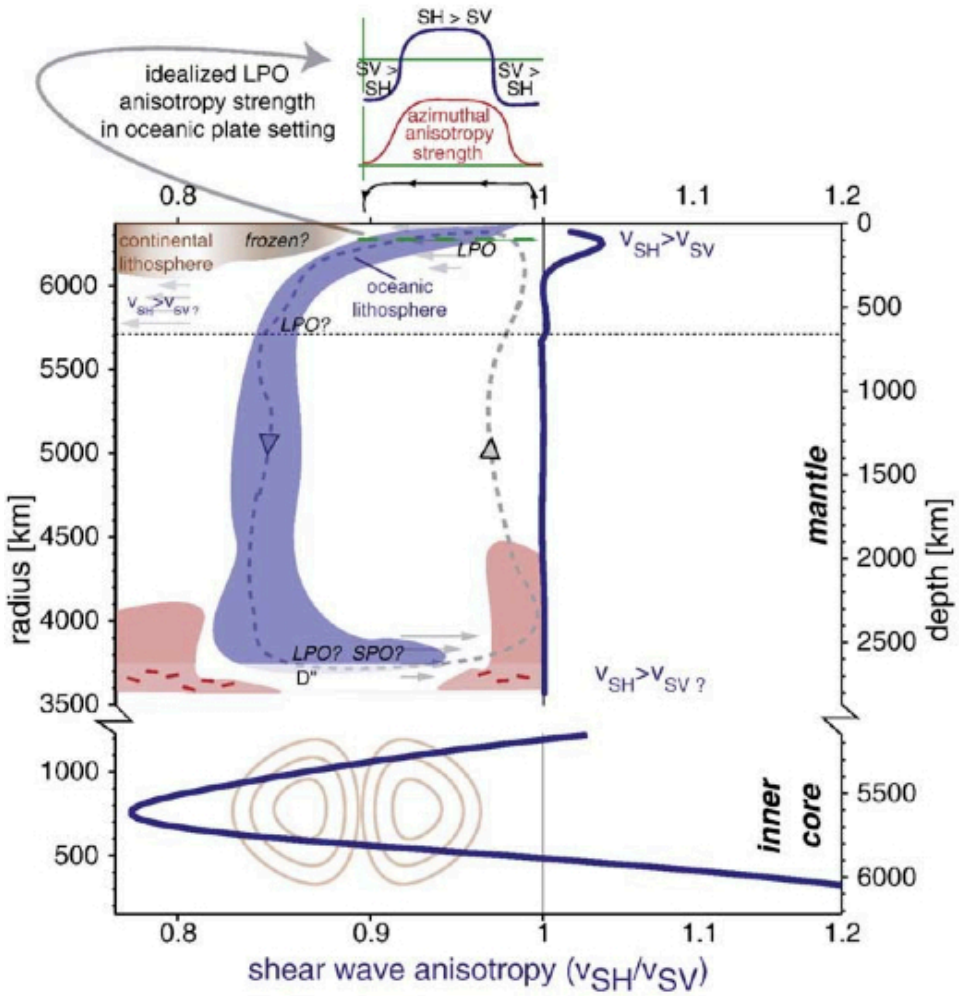
Hooke's law: $\sigma_{ij} = c_{ijkl} e_{kl}$

21 independent terms:

$$C_{mn} = \begin{pmatrix} c_{1111} & c_{1122} & c_{1133} & c_{1123} & c_{1113} & c_{1112} \\ c_{2211} & c_{2222} & c_{2233} & c_{2223} & c_{2213} & c_{2212} \\ c_{3311} & c_{3322} & c_{3333} & c_{3323} & c_{3313} & c_{3312} \\ c_{2311} & c_{2322} & c_{2333} & c_{2323} & c_{2313} & c_{2312} \\ c_{1311} & c_{1322} & c_{1333} & c_{1323} & c_{1313} & c_{1312} \\ c_{1211} & c_{1222} & c_{1233} & c_{1223} & c_{1213} & c_{1212} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{pmatrix}$$

The equations get quite tricky, but here's the upshot: wavespeed varies with direction; there are now 3 possible polarizations (quasi-P and two quasi-S waves)

Where in the Earth do we have anisotropy?



Long, M. D., Becker, T. W., 2010. Mantle dynamics and seismic anisotropy, *Earth Planet. Sci. Lett., 279,* 341-354.

Some useful terms

- **Radial anisotropy** – difference in propagation speed between horizontally and vertically polarized waves (SH vs. SV, or Love vs. Rayleigh).

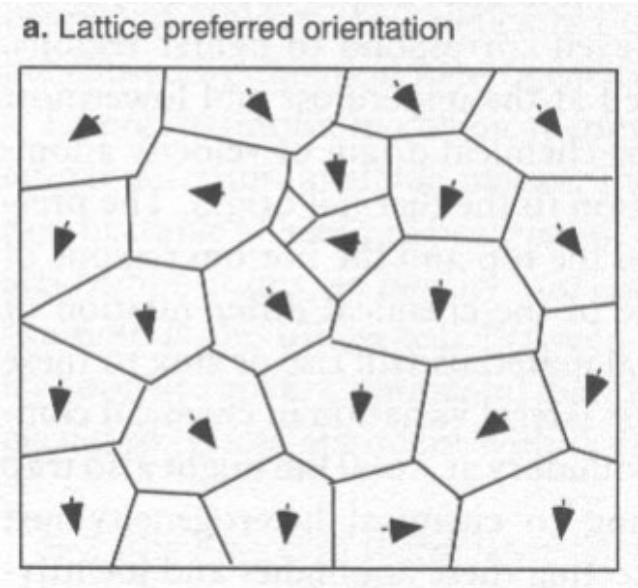
- **Azimuthal anisotropy** – directional dependence of wavespeed with azimuth (in horizontal plane)

- **Transverse isotropy** – equivalent to hexagonal anisotropy; simplified description of mantle anisotropy with 5 independent parameters. Can have vertical (VTI), horizontal (HTI), or tilted (TTI).

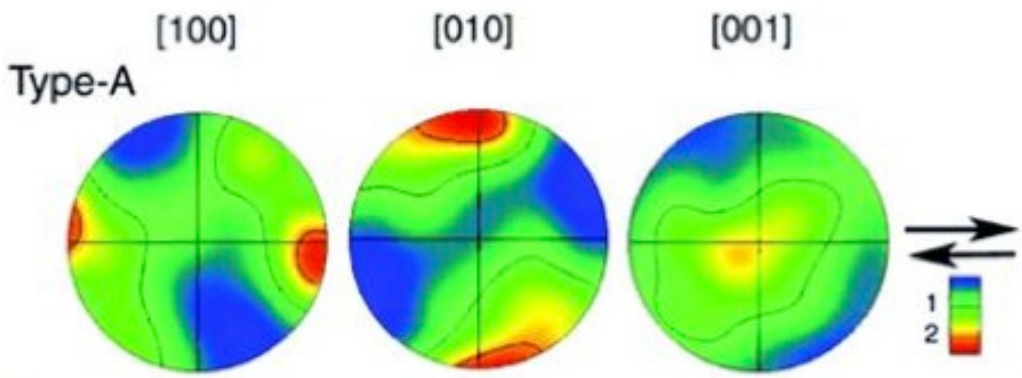
- **Polarization anisotropy** – anisotropy that manifests in the polarization behavior of waves

Why is the (upper) mantle anisotropic?

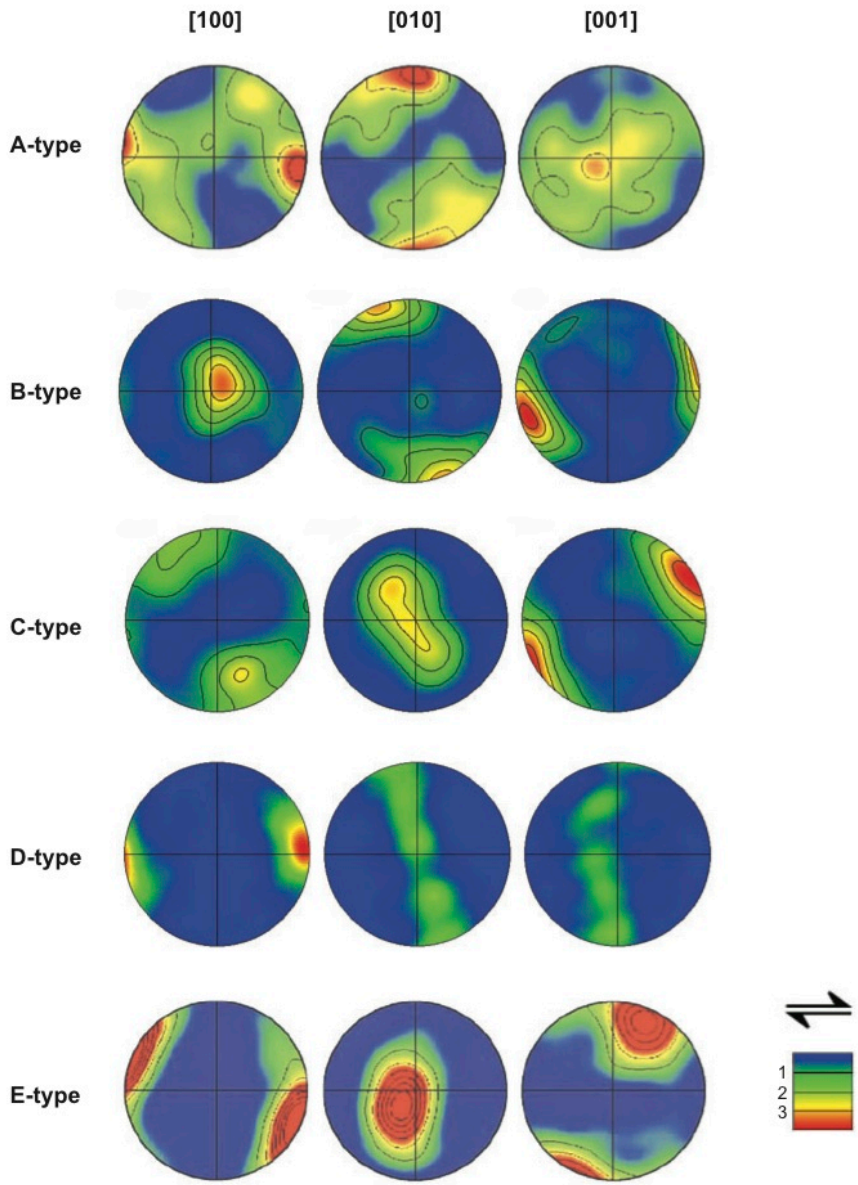
When a collection of mineral crystals (a rock) undergoes **deformation** (under certain conditions, specifically dislocation creep), individual crystals tend to align in certain directions. This is known as **lattice preferred orientation** (LPO or CPO) and results in seismic anisotropy that is "felt" by seismic waves.

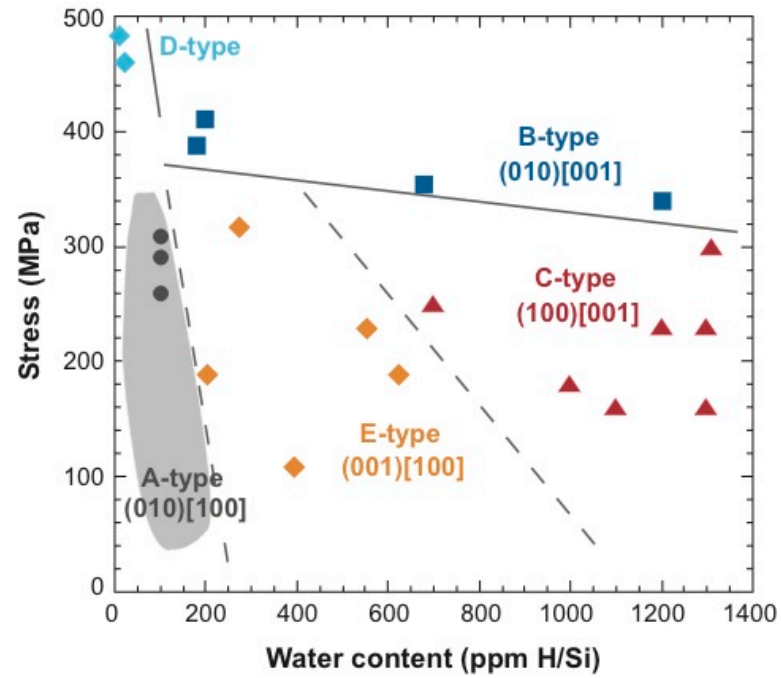


Karato, 2003



Jung and Karato, 2001





Constraints on olivine LPO from laboratory experiments

Review papers by Karato et al. (Annu. Rev. Earth Planet. Sci., 2008) and Skemer & Hansen (Tectonophysics, 2016)

Karato et al., 2008

Another mechanism for anisotropy: shape preferred orientation (SPO)

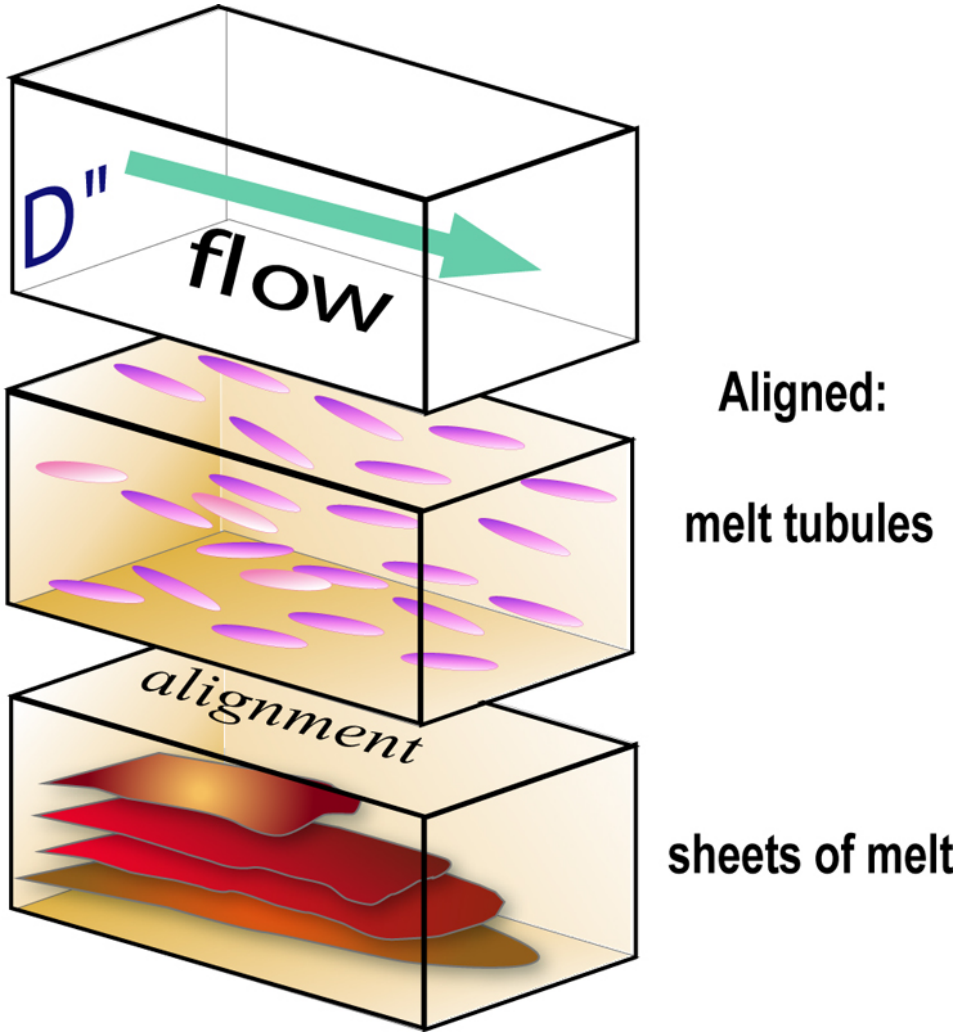
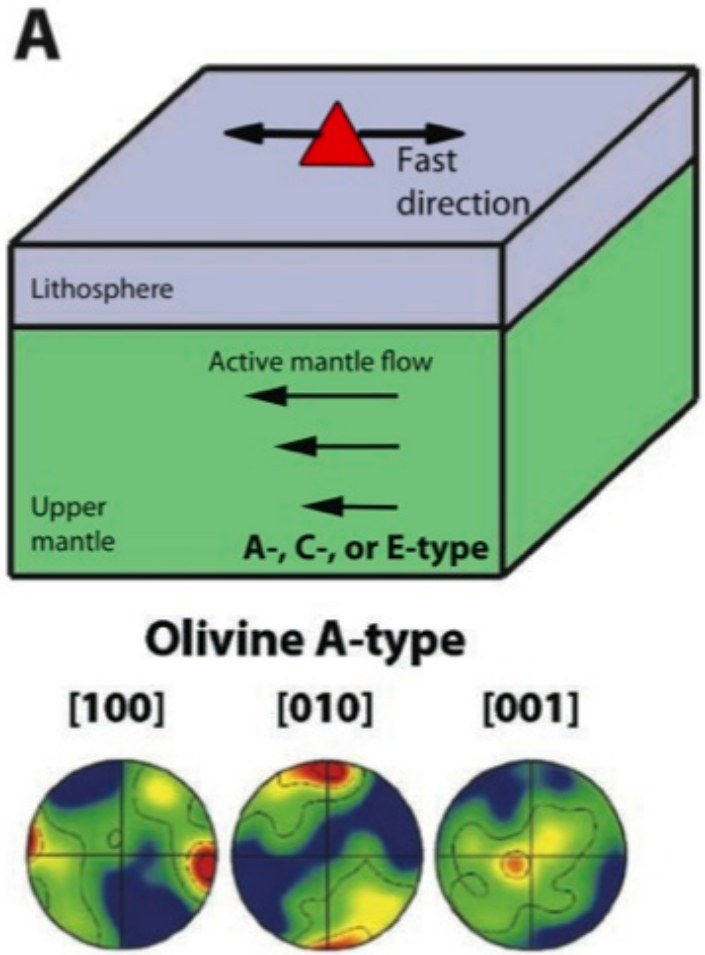
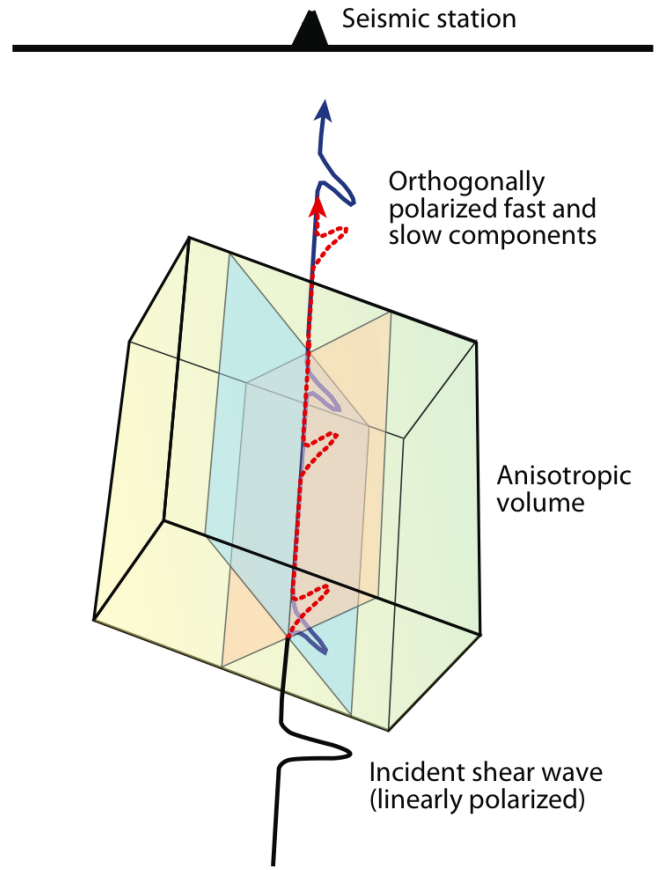


Image from Ed Garnero

Why do we care? - Seismic anisotropy is a key tool for understanding mantle flow



Long & Becker, EPSL, 2010



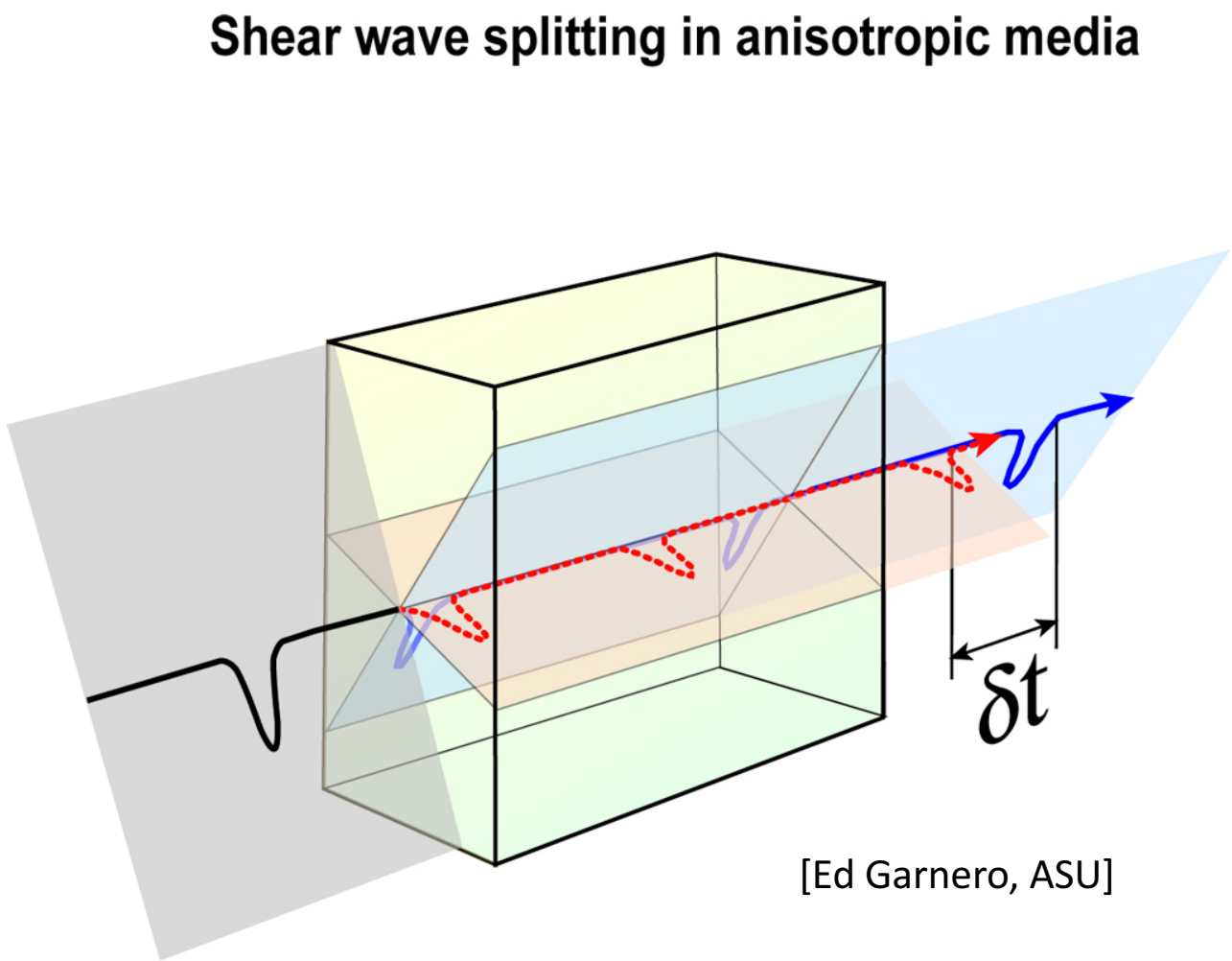
Mantle flow ➔ LPO of anisotropic minerals ➔ seismic anisotropy

(OVER)SIMPLIFIED RULE OF THUMB: fast direction = direction of horizontal mantle flow beneath station

Methods for observing anisotropy

- Shear wave splitting
- Receiver function analysis
- Surface wave tomography
- Normal modes

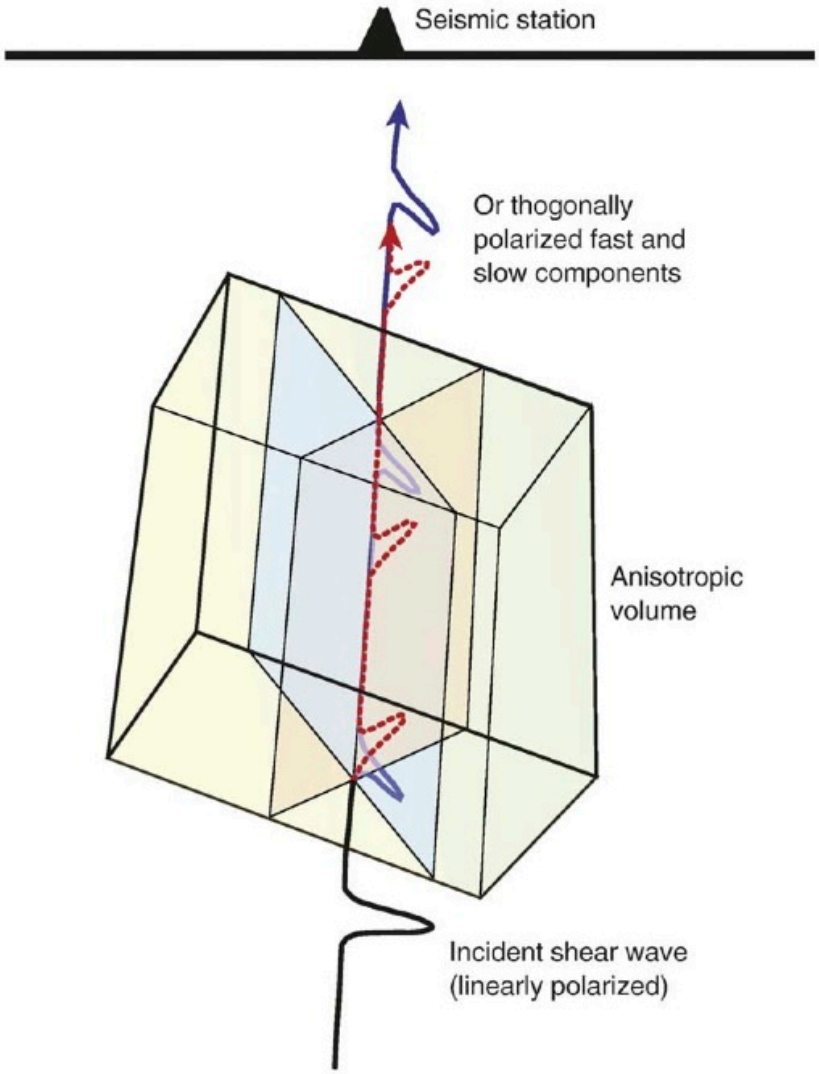
What is shear wave splitting?



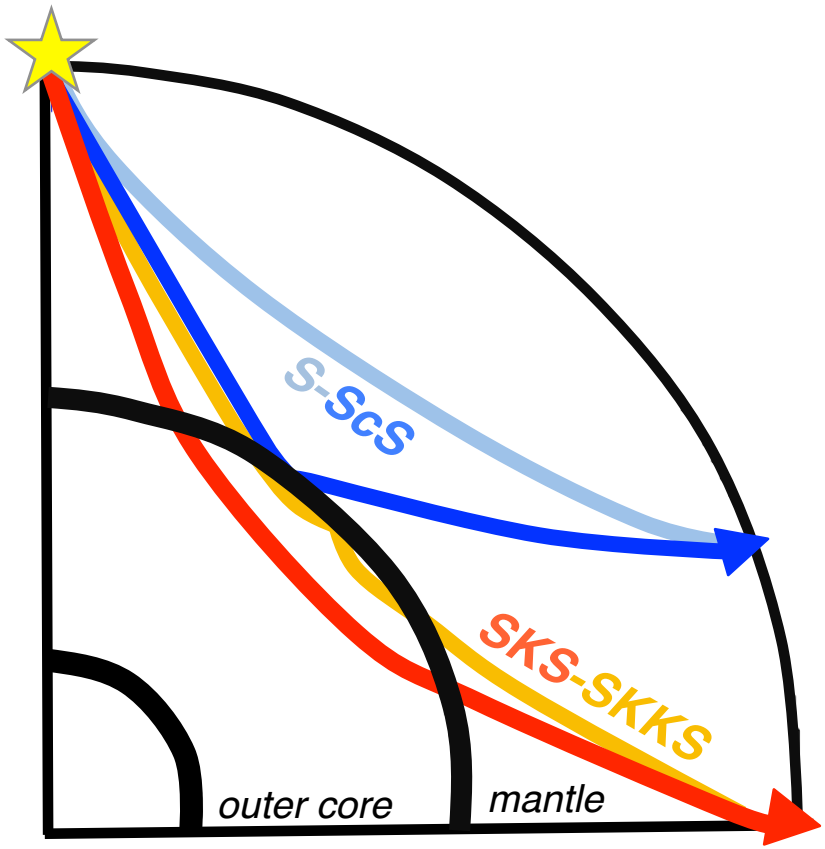
[Ed Garnero, ASU]

A shear wave is split into two orthogonal components that travel with different wavespeeds. The fast polarization direction (ϕ) and time separation (δt) depend on the characteristics of the anisotropic medium.

What is shear wave splitting?

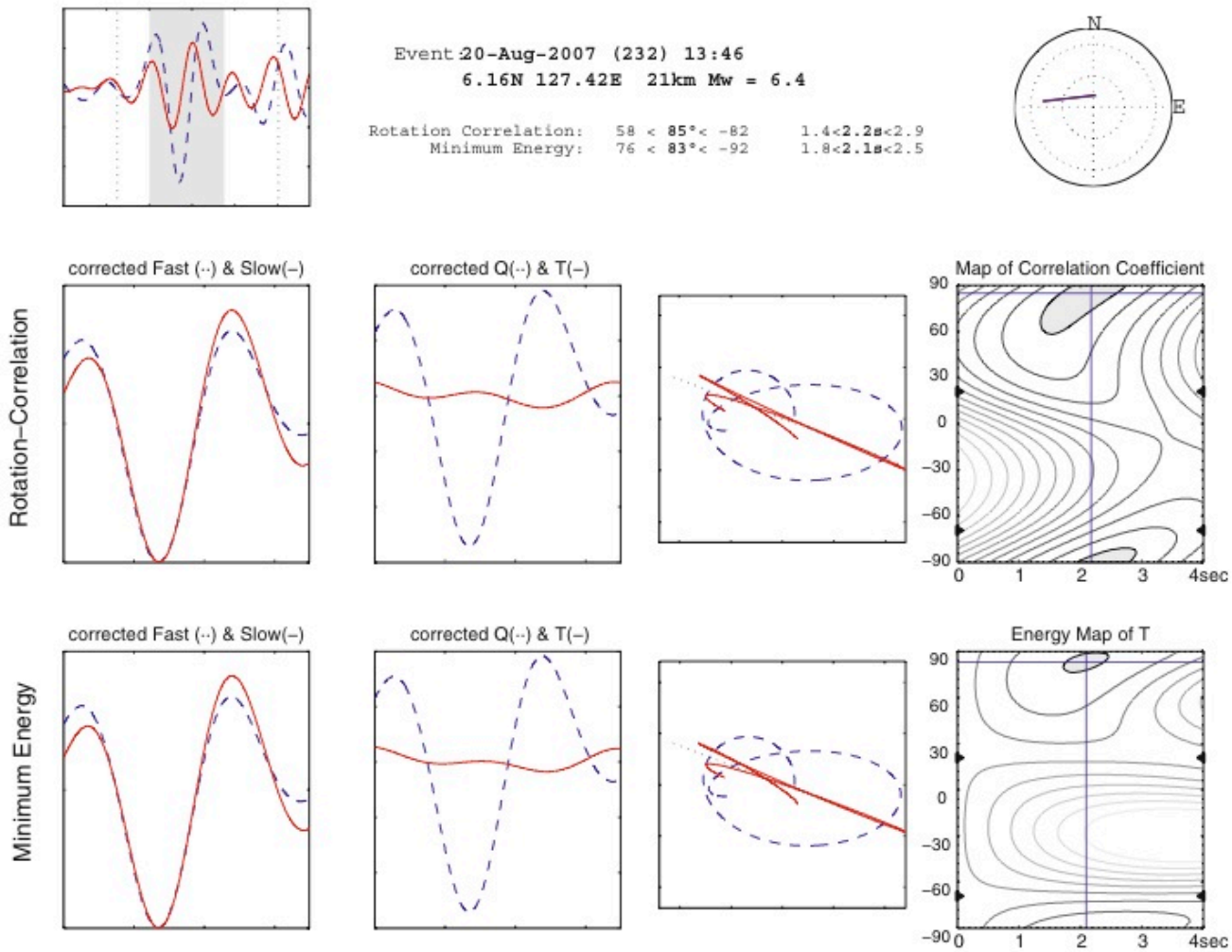


Long & Becker, 2010



Creasy et al., 2017

An example of a shear wave splitting measurement

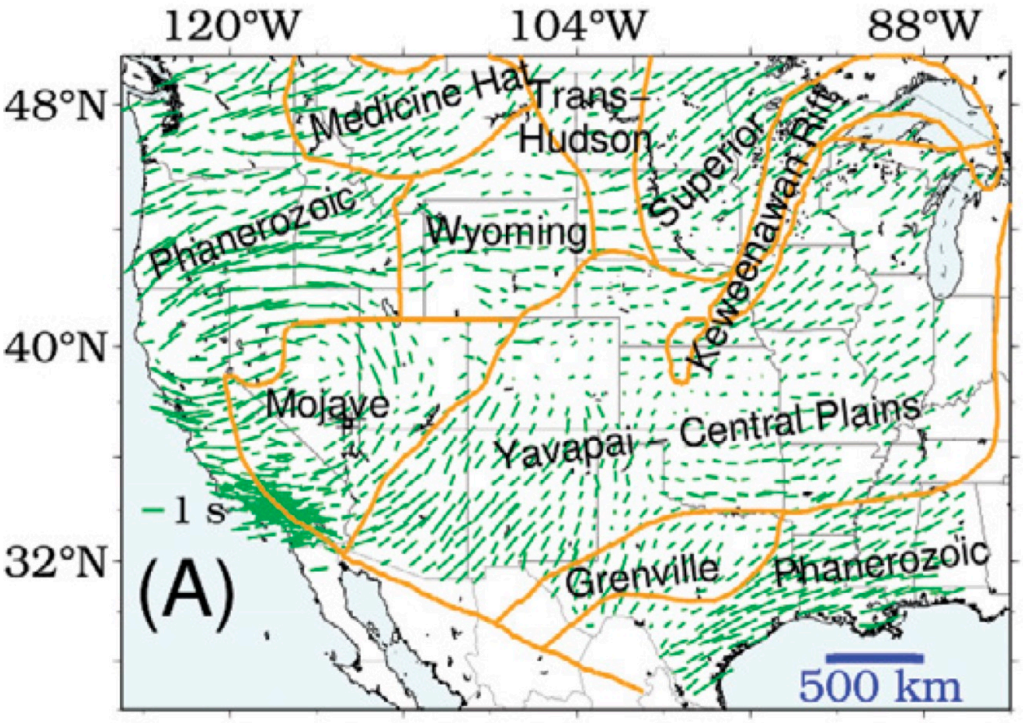


Your challenge: look for split SKS waves in the seismology tutorial next week.

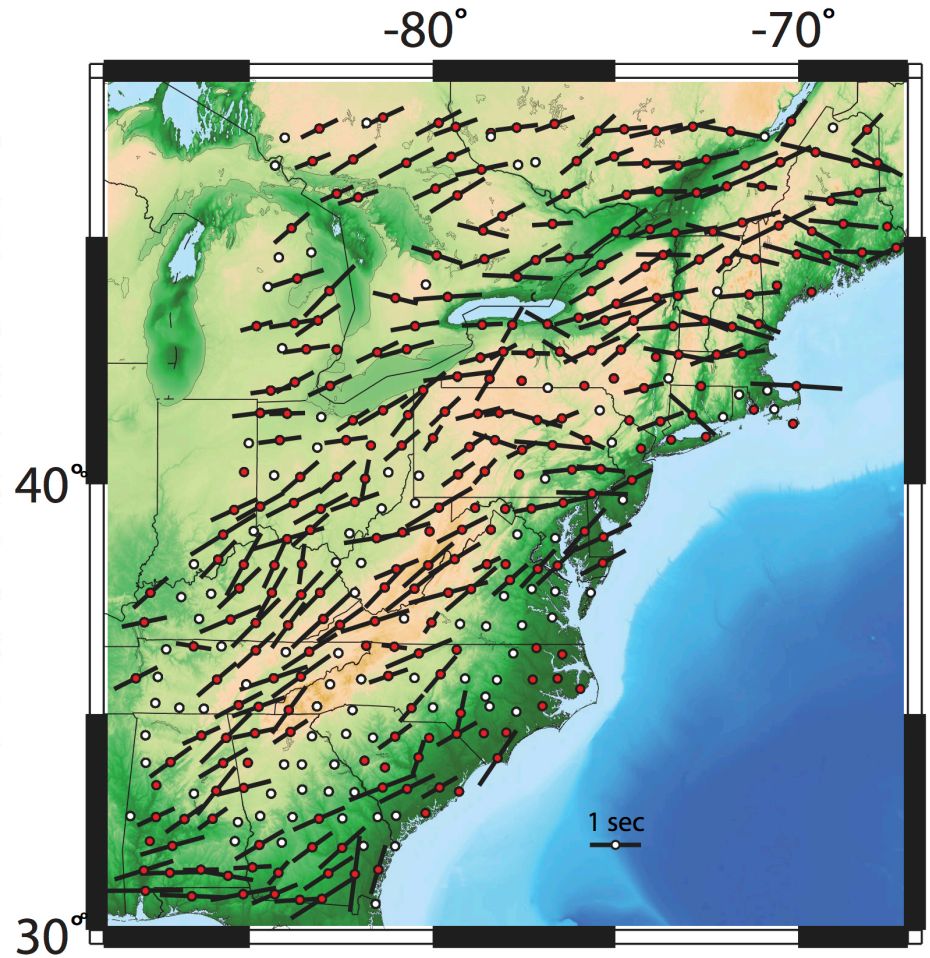
Key indicators: transverse component energy (**looks like radial component derivative**); elliptical particle motion.

Long & Silver, 2009

Examples of SKS splitting data sets: USArray



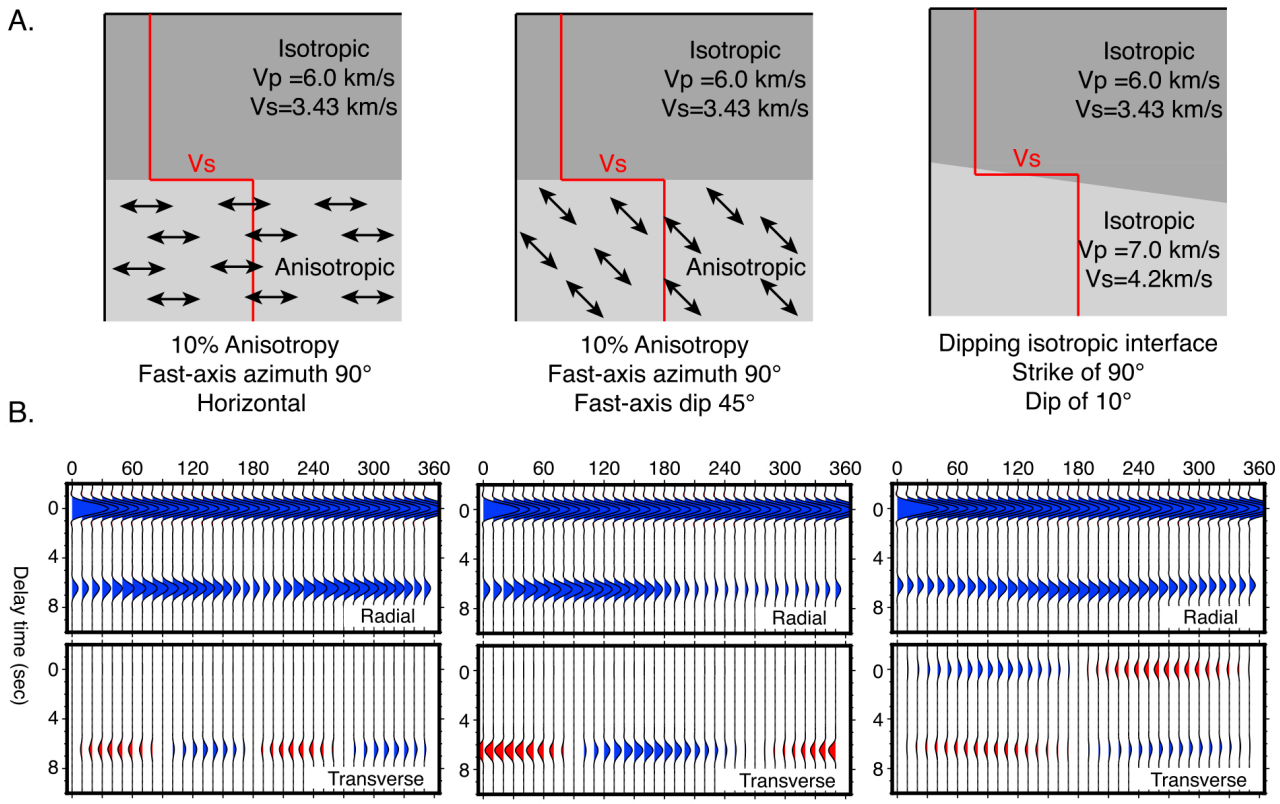
Hongsresawat et al., 2015



Long et al., 2016

New results from USArray: excellent geographic coverage, can be combined with surface waves/receiver functions to produce joint models (ongoing challenge). Key question: contributions from lithosphere vs. asthenosphere? Multiple layers? Present-day mantle flow vs. past lithospheric deformation?

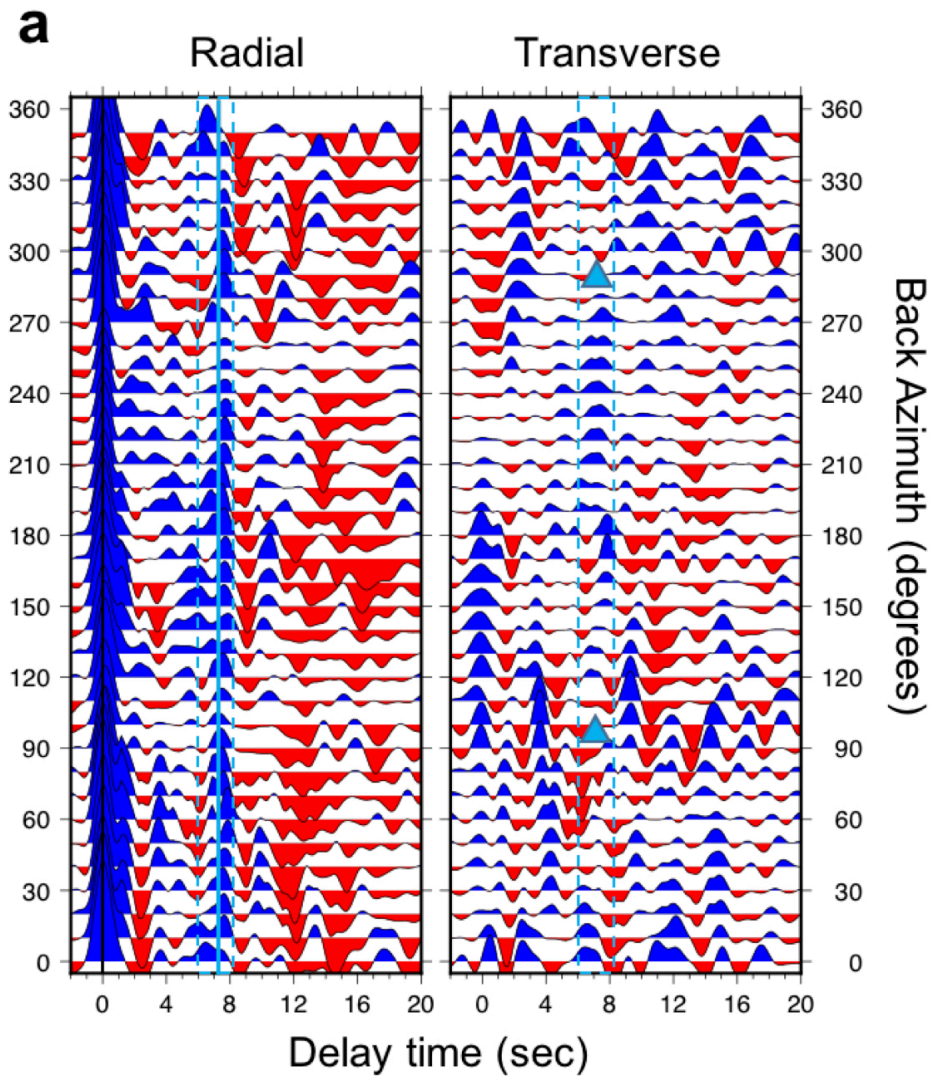
Anisotropy-aware receiver function analysis

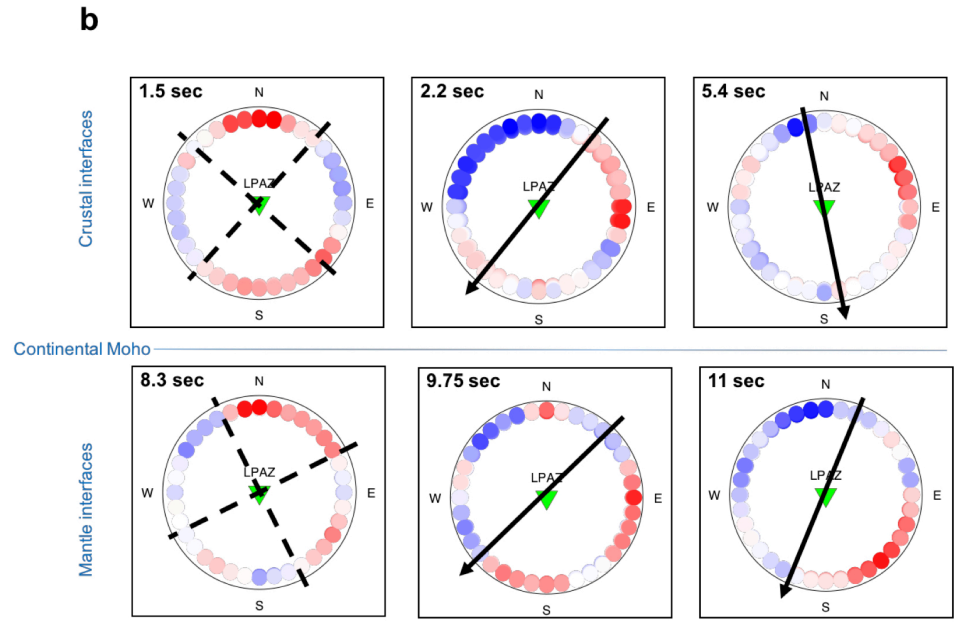


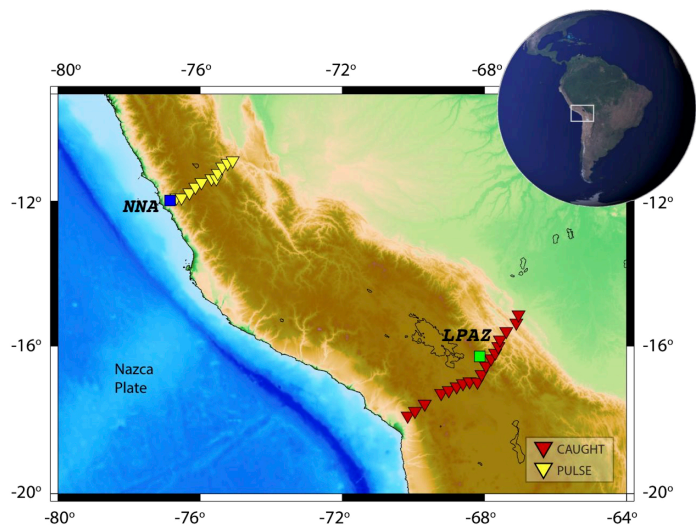
Ford et al., 2016

For the case of flat-lying layers and no anisotropy, theory predicts no energy on transverse component receiver functions. In the presence of anisotropy, can use azimuthal variability to constrain sharp contrasts in anisotropy at depth.

Anisotropic RF analysis: an example

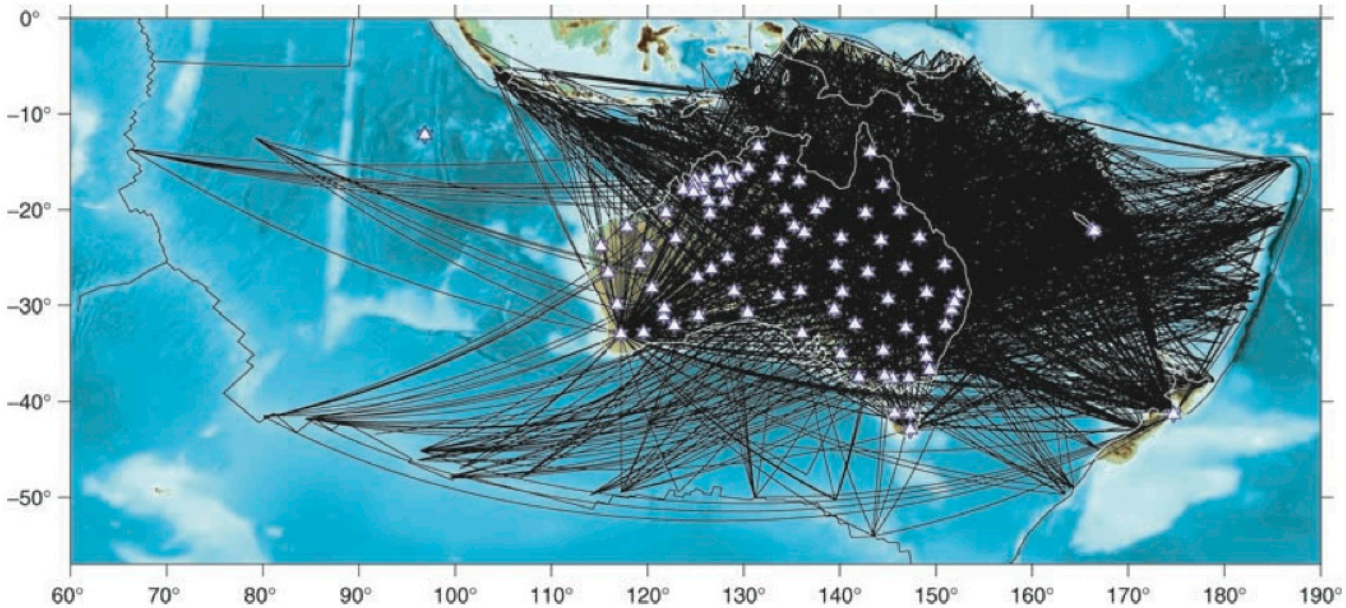


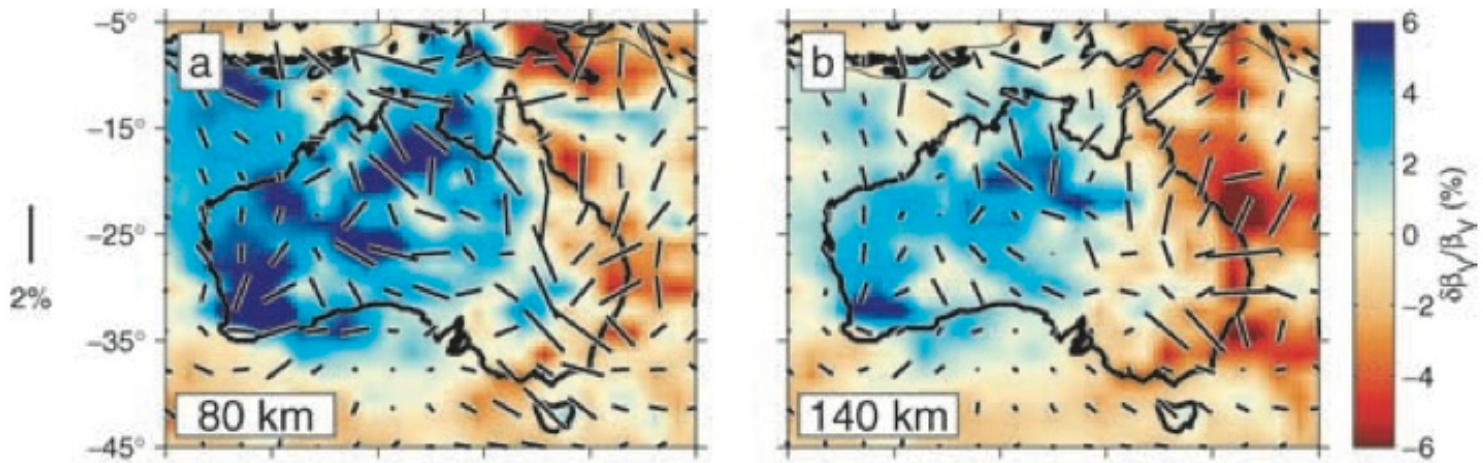




Bar et al., in review

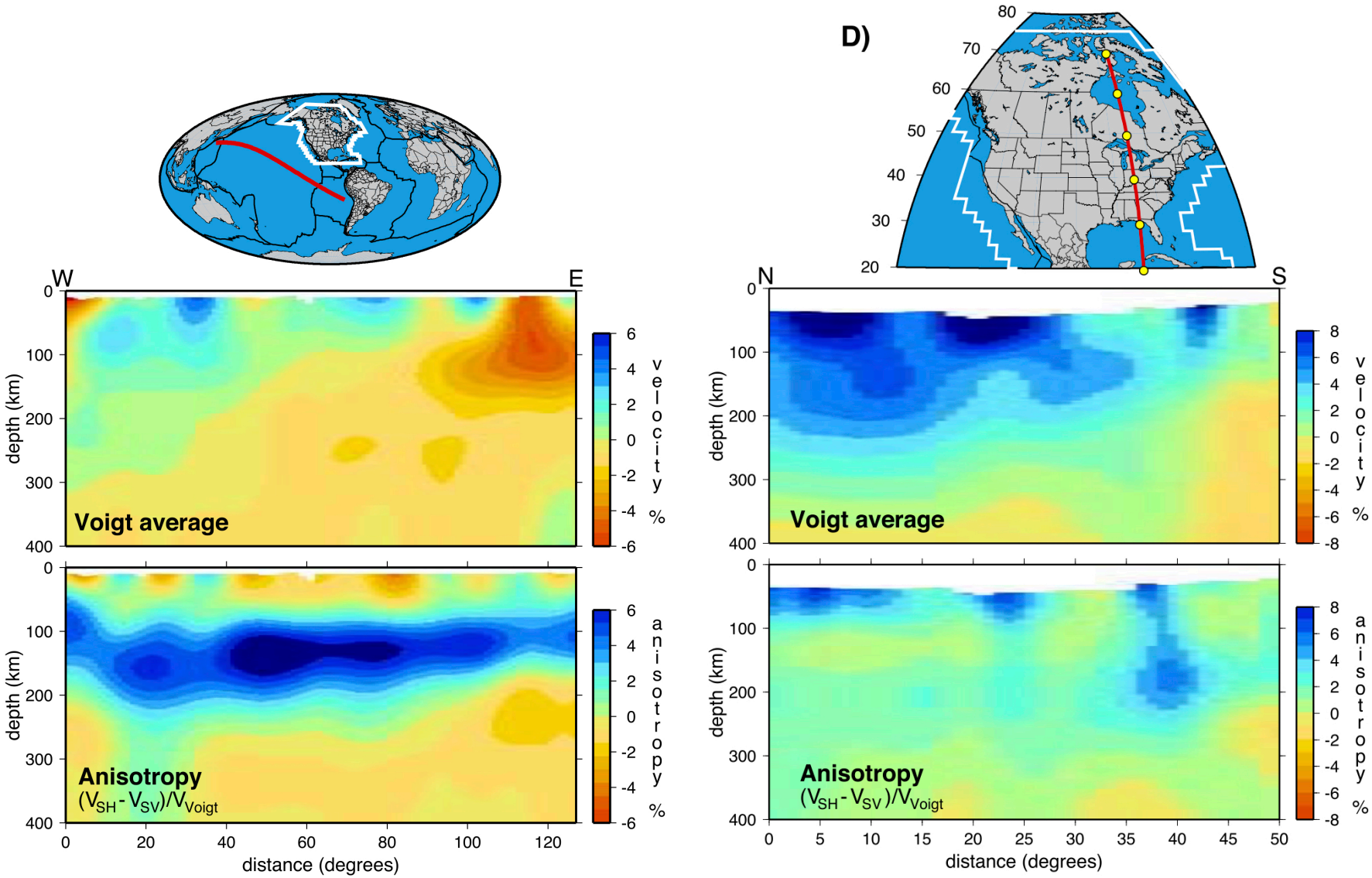
Azimuthal anisotropy from surface waves





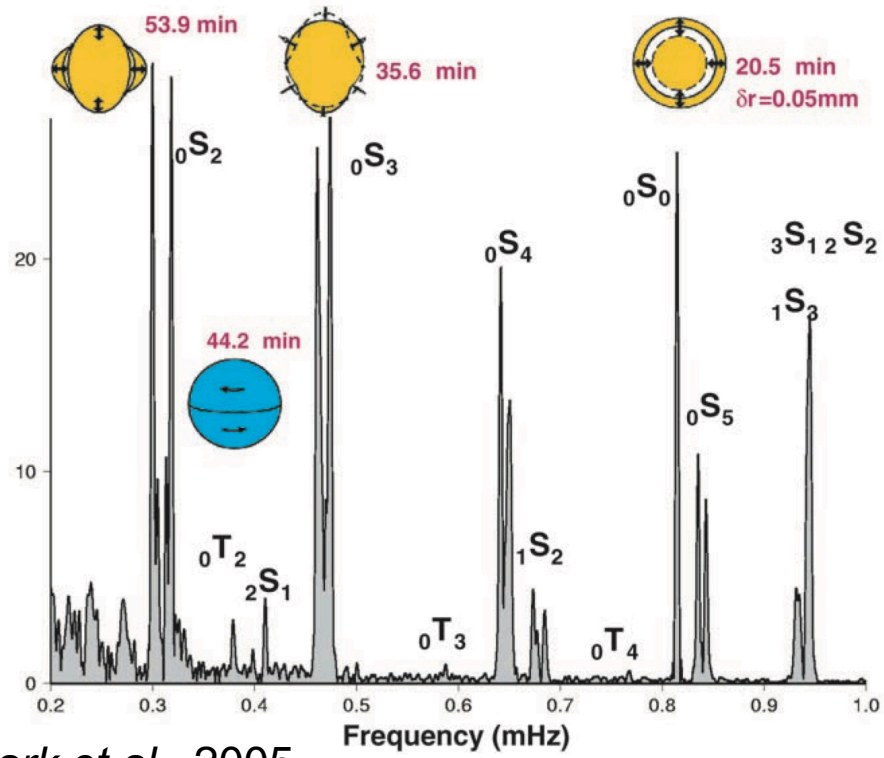
Simons et al., 2002

Radial anisotropy from surface waves

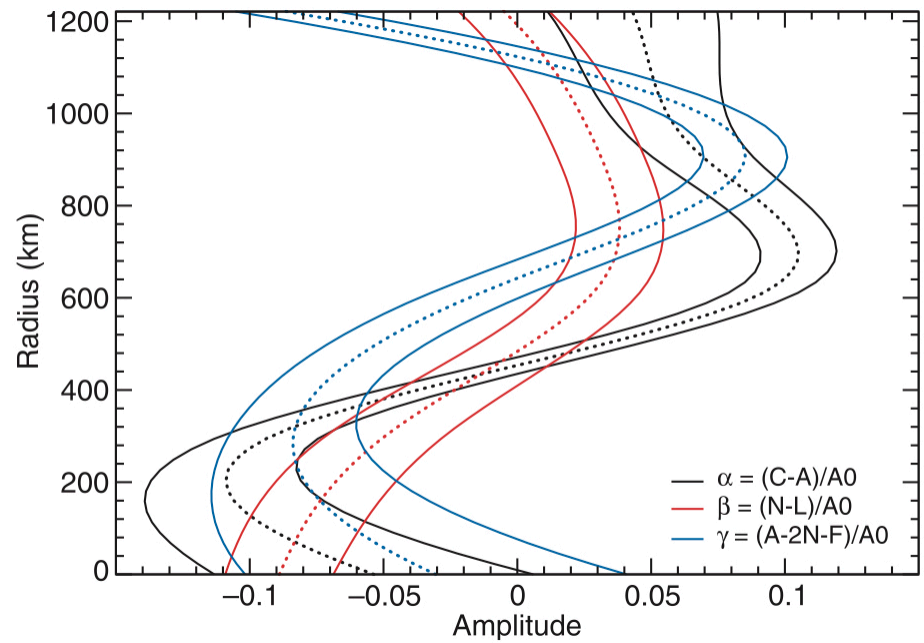


Nettles and Dziewonski, 2008

Anisotropy from normal mode splitting



Park et al., 2005

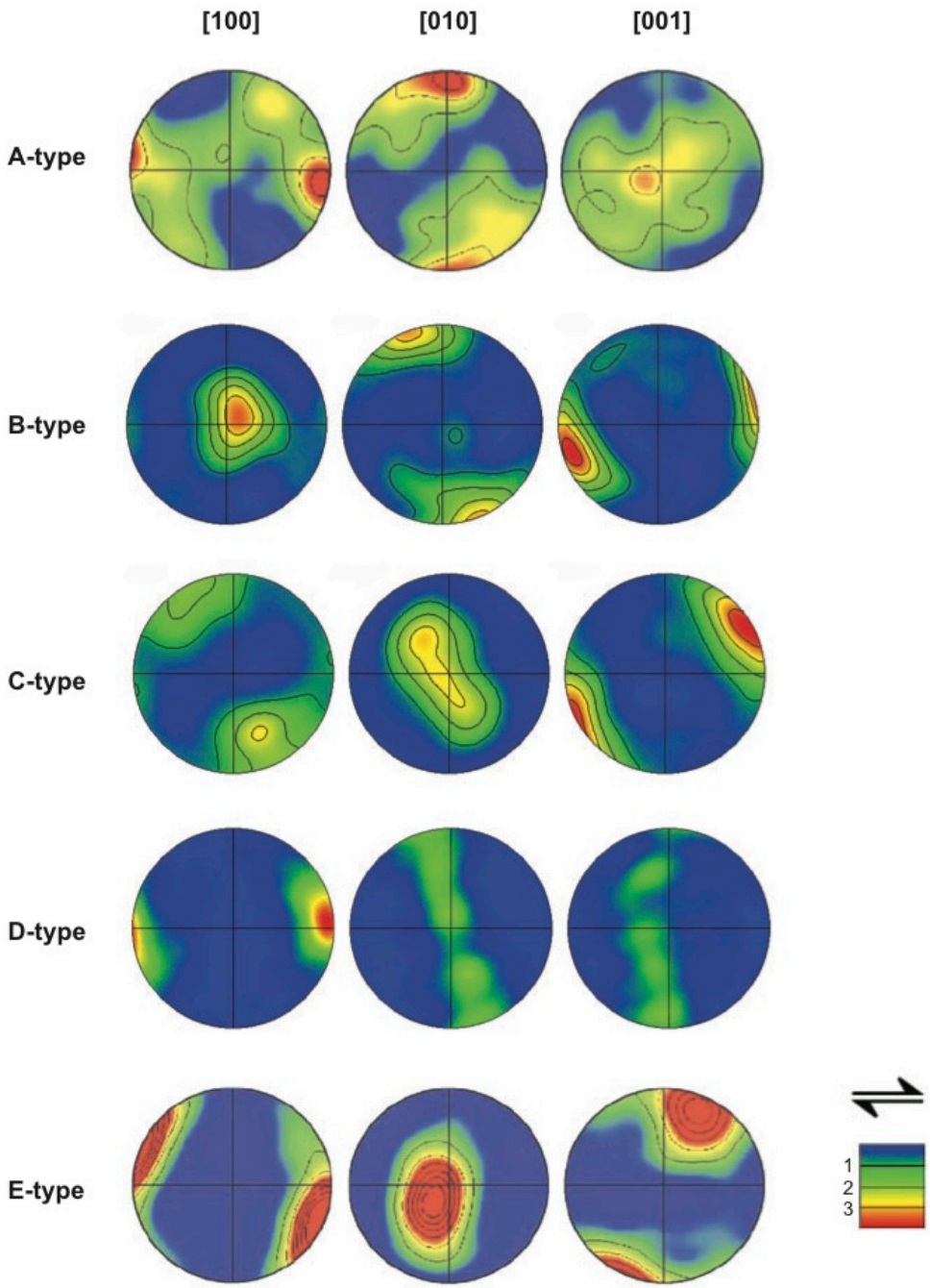


Beghein and Trampert, 2003

Model for depth-dependent seismic anisotropy of the inner core (*Beghein and Trampert,* 2003; also work by, e.g., Tromp, Ishii, Deuss, Irving, others). We will undoubtedly hear more about this during Jessica's lecture next week...

So: observations of seismic anisotropy have the potential to tell us about dynamic processes in the Earth's mantle

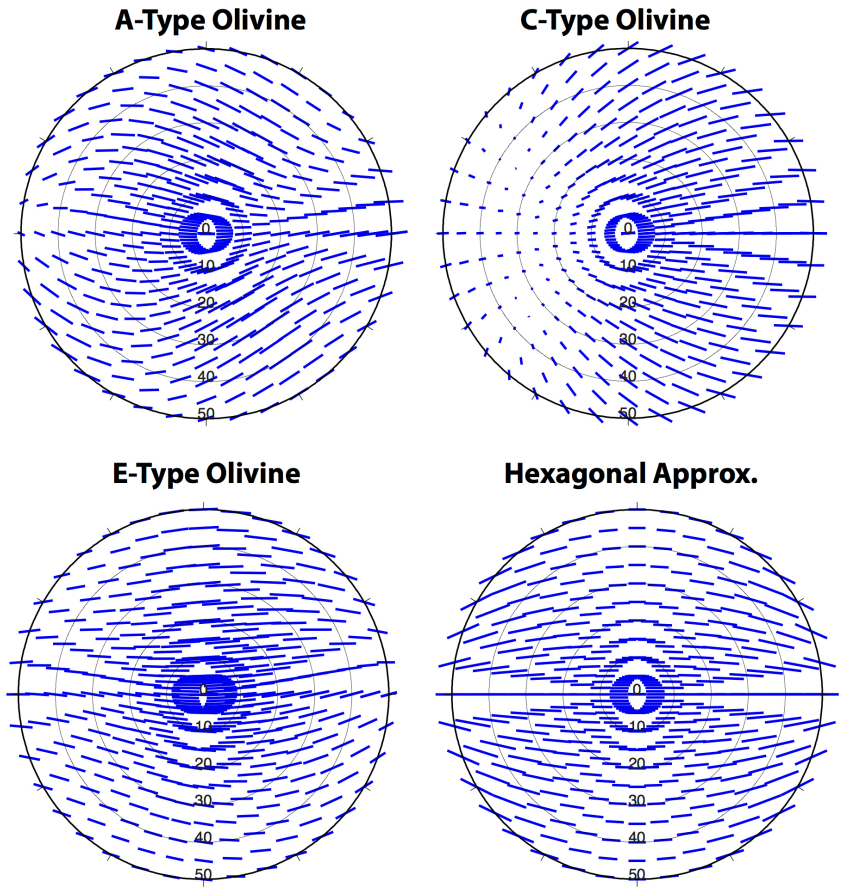
What are the caveats?



Many different olivine fabric types, each of which will have a different effect on the overall anisotropic signature.

Some differences are subtle, others major.

Karato et al., 2008

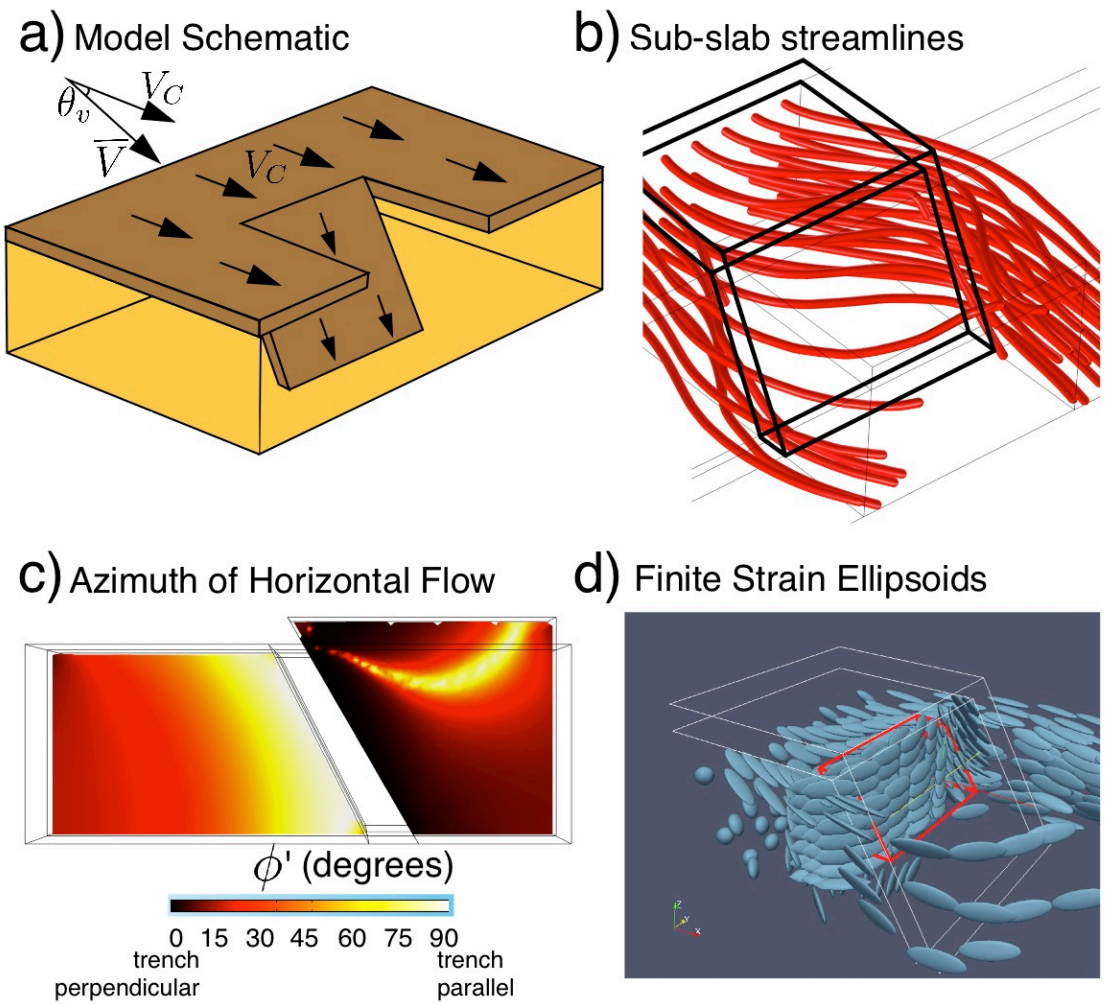


Shear Direction

Many different olivine fabric types, each of which will have a different effect on the overall anisotropic signature.

Some differences are subtle, others major.

Lynner et al., 2017

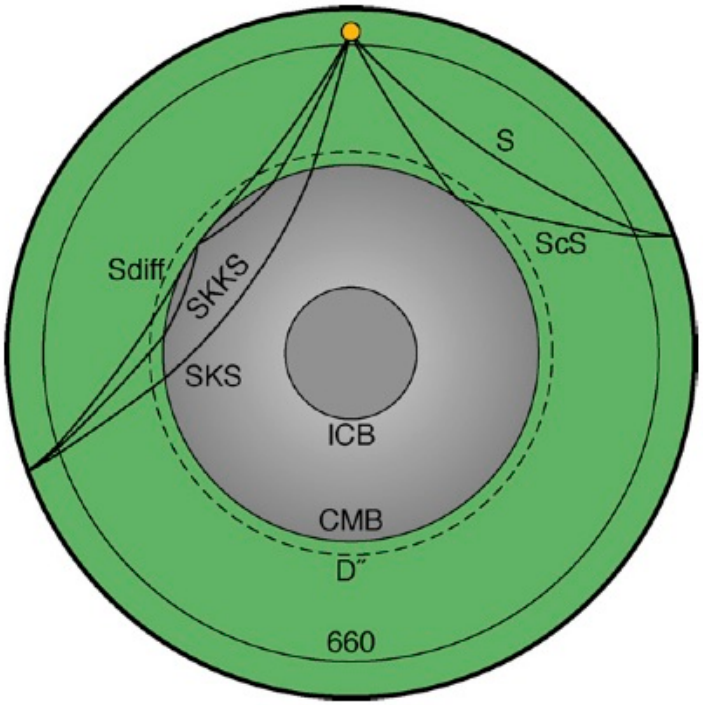


Paczkowski et al., 2014

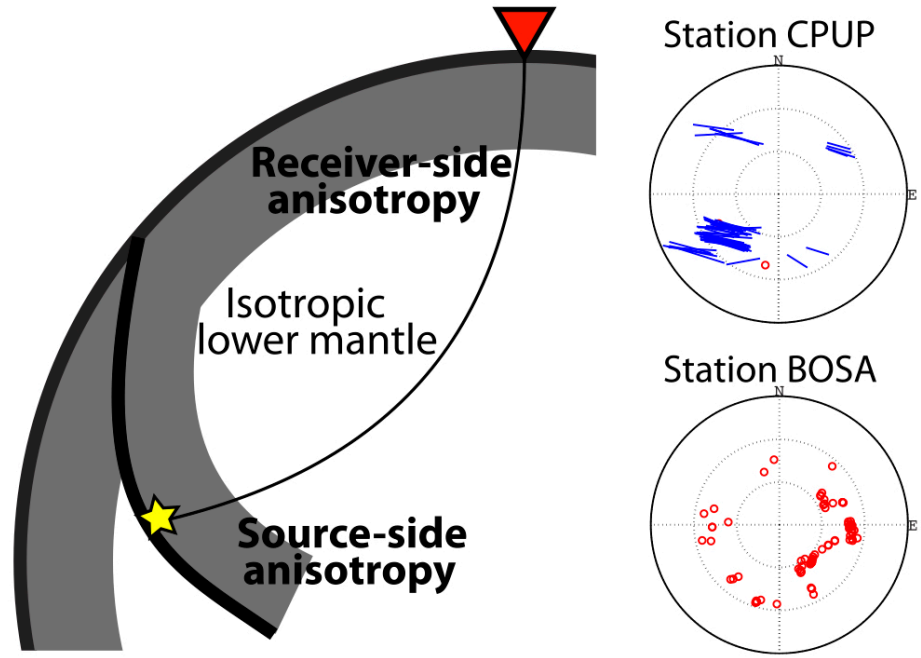
Anisotropy in the mantle is a complicated function of the (time-integrated) mantle strain, fabric type, etc.

"Rules of thumb" relationships between mantle flow direction and fast anisotropy directions are useful to a point, but they are simplifications!

Another major challenge for body wave studies: raypath coverage and isolating anisotropy along the path



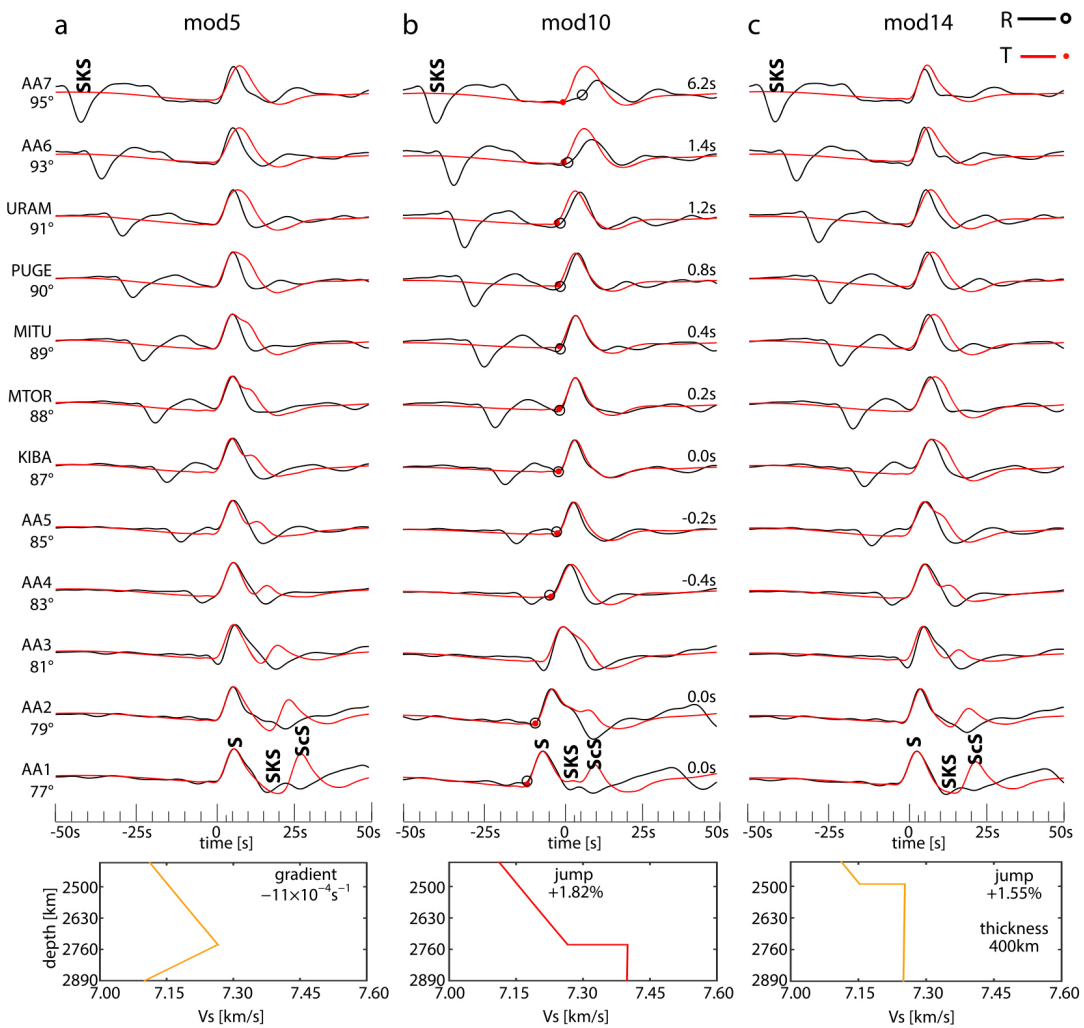
Nowacki et al., 2011



Lynner and Long, 2014

All body wave phases suffer from the same limitation: shear wave splitting is a path-integrated measurement - isolating anisotropy is difficult, and for studies of the deep mantle, correcting for anisotropy on the receiver side is important.

Another challenge: for some shear wave phases, isotropic structure can yield “apparent” splitting

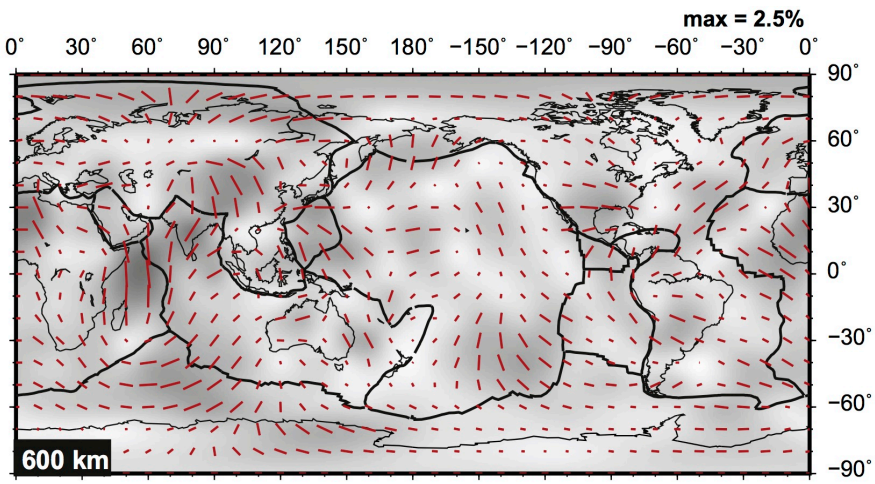


Parsi et al., 2017

For some phases (S, Sdiff, Scs) isotropic structure can yield “contamination” of the seismograms (via, e.g., phase interference) in a way that can mistaken for shear wave splitting, if care is not taken with the analysis.

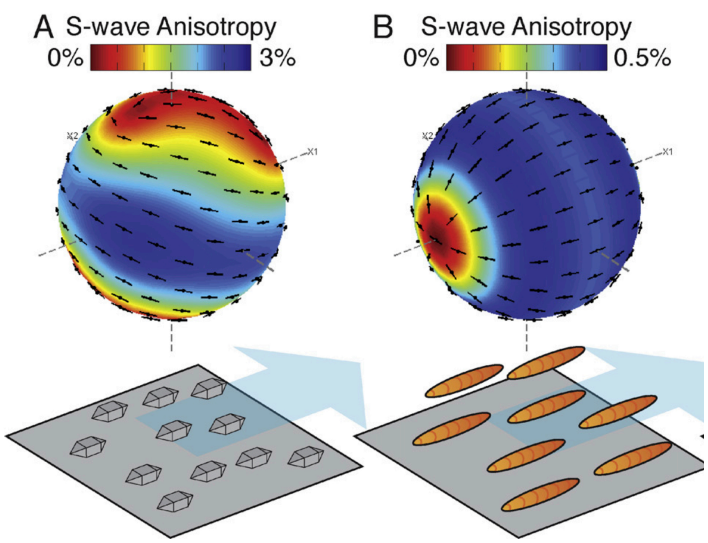
Solution: move towards a full-waveform synthetic framework for interpreting observations; implement careful analysis procedures to ensure that waveforms are interpreted correctly (e.g., does transverse component waveform match radial component derivative?).

And yet another limitation: mechanisms for deep mantle anisotropy (TZ, uppermost lower mantle, D”) poorly known



Yuan and Beghein, 2013

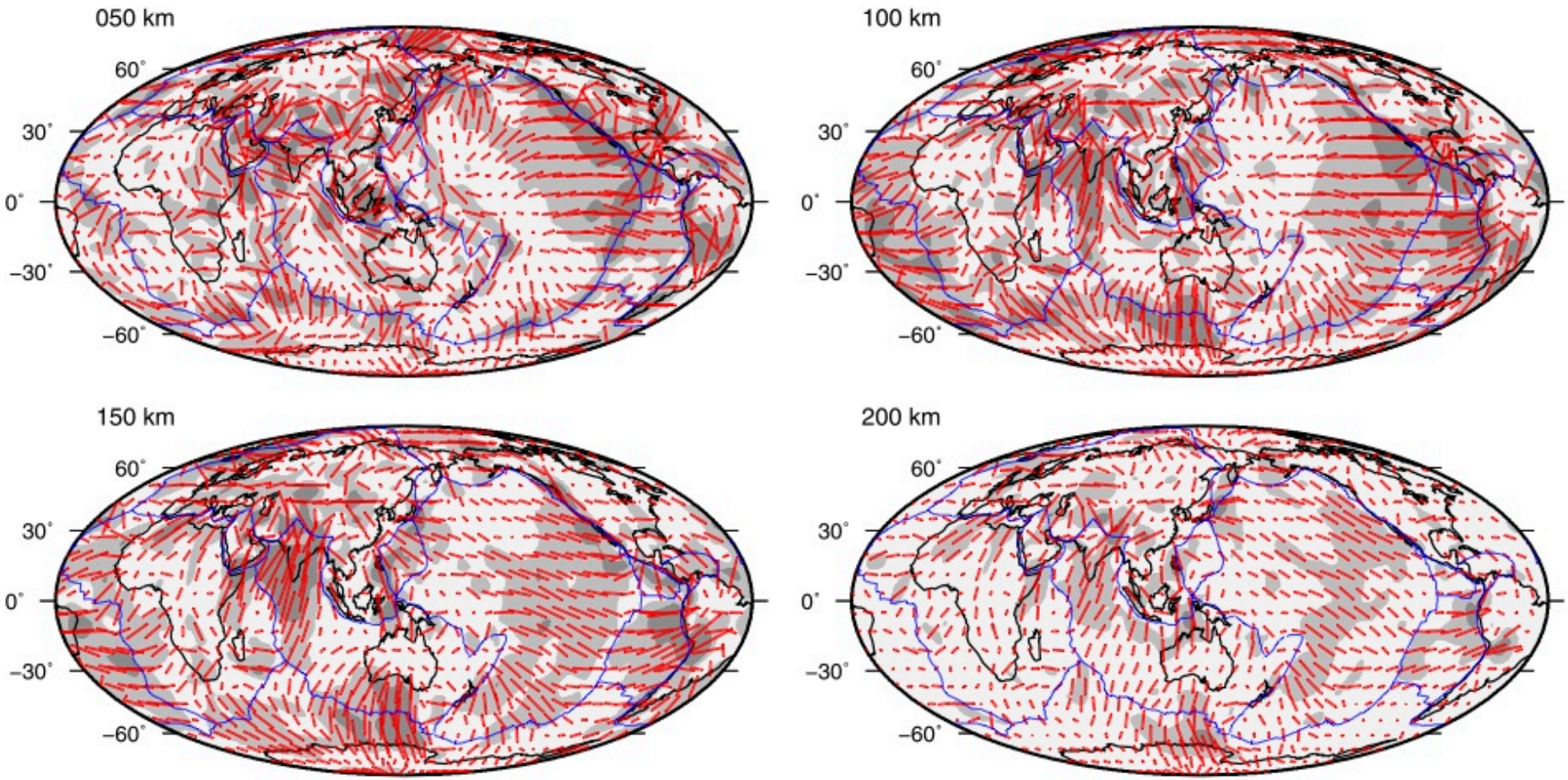
For the mid-mantle: which minerals contribute? What are relationships between strain and resulting anisotropy?



Nowacki et al., 2011

For the lowermost mantle: Is it LPO, SPO, or a combination? Which phases/materials contribute? What are single crystal elastic constants? Dominant slip systems? LPO patterns?

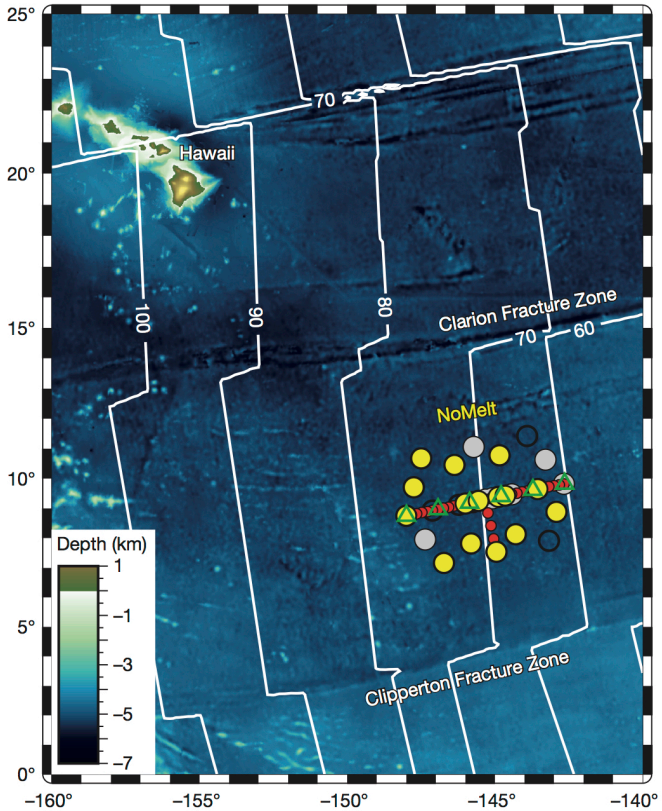
So, now that we've covered the limitations… what are some of the things we can ***do*** with anisotropy observations?



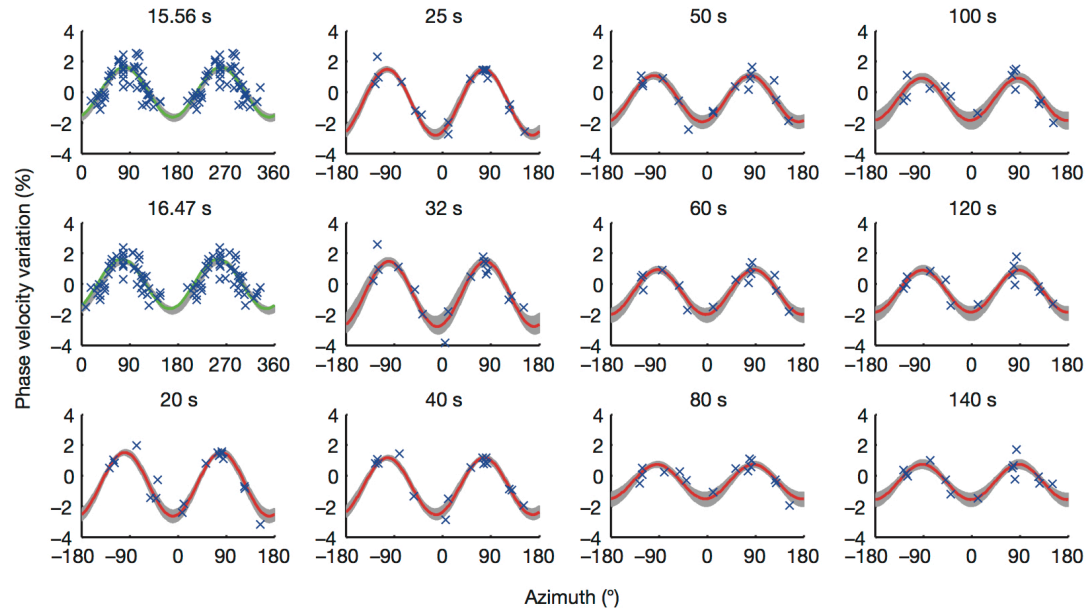
Debayle & Ricard, 2013

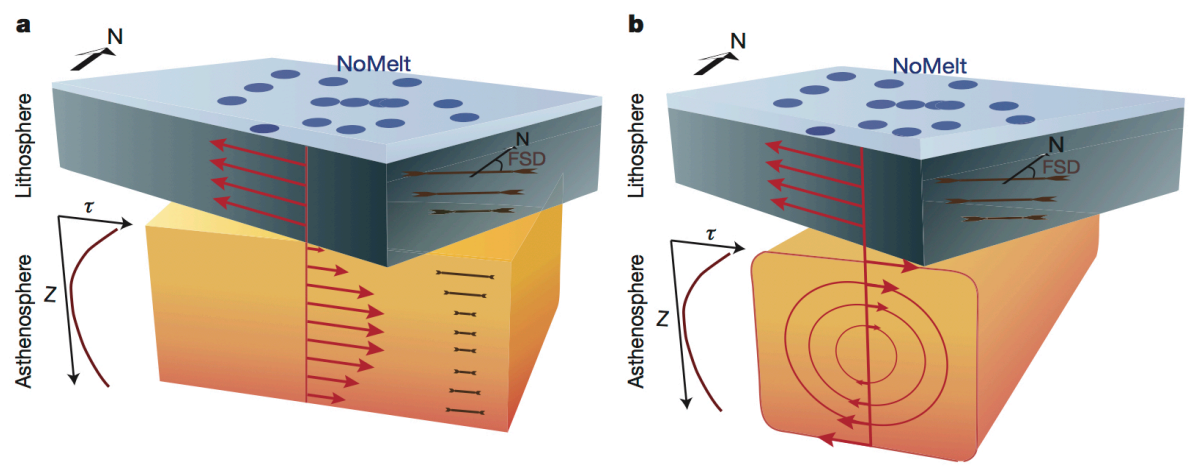
Anisotropy beneath ocean basins

Anisotropy beneath ocean basins



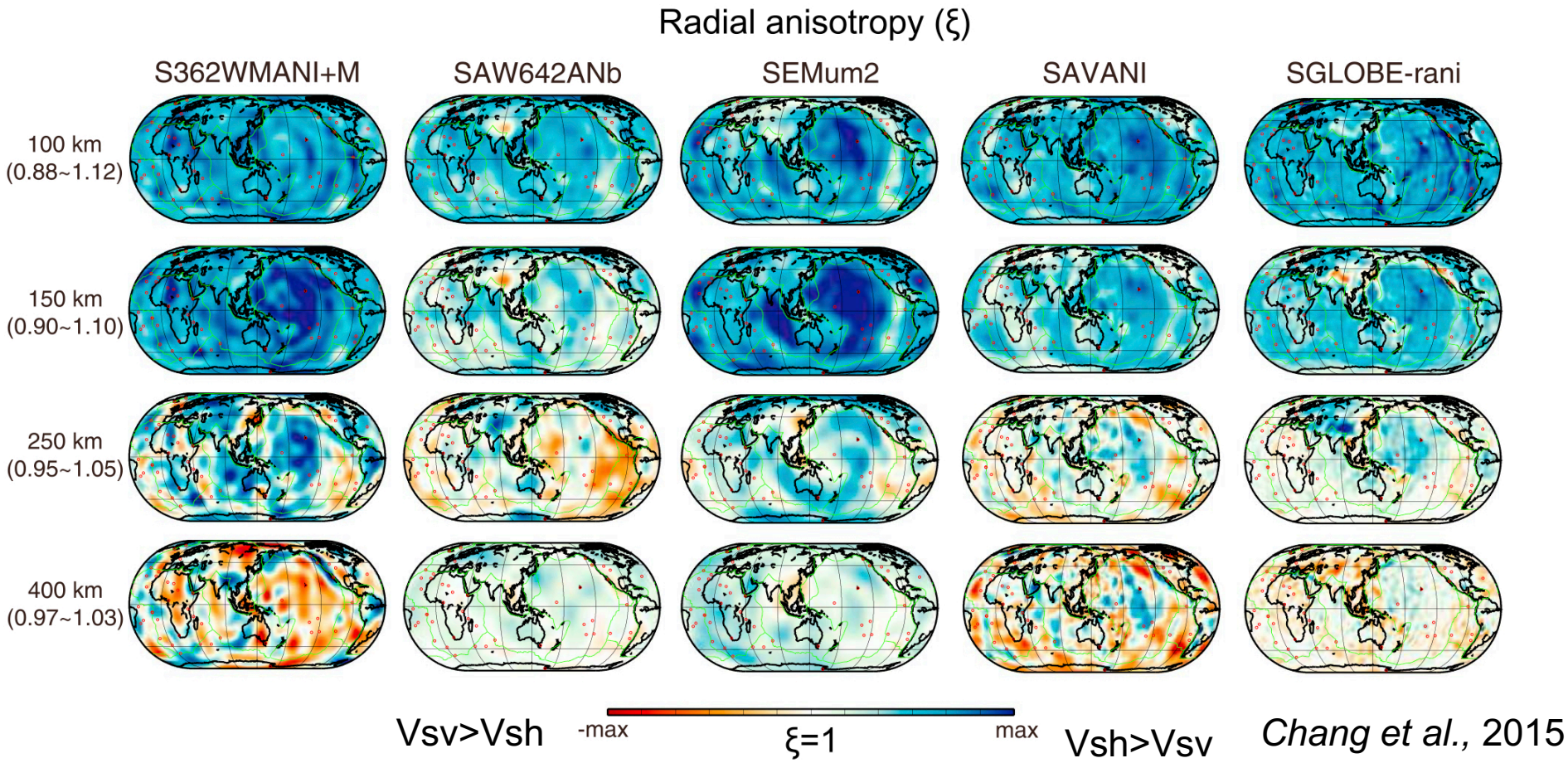
Lin et al., 2016





Anisotropy of the oceanic lithosphere-asthenosphere system beneath the NoMelt experiment – some surprises!

Global models for mantle anisotropy

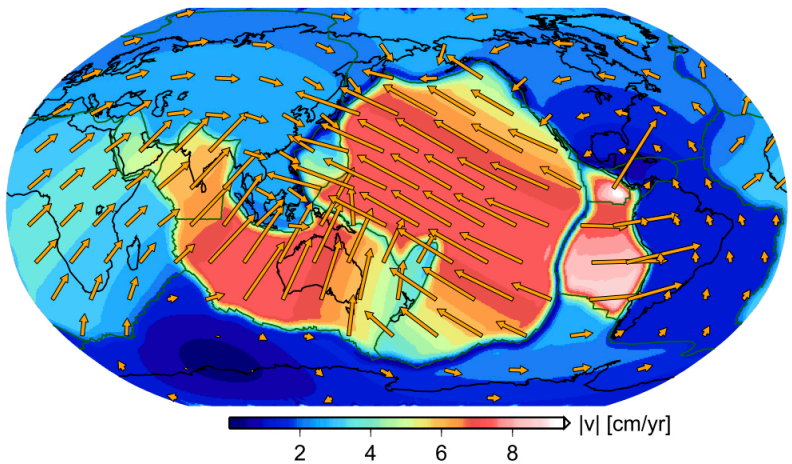


Chang et al., 2015

Considerably less consensus on radial anisotropy structure of the mantle than isotropic structure. Crustal corrections extremely important; detailed and accurate crustal corrections are necessary.

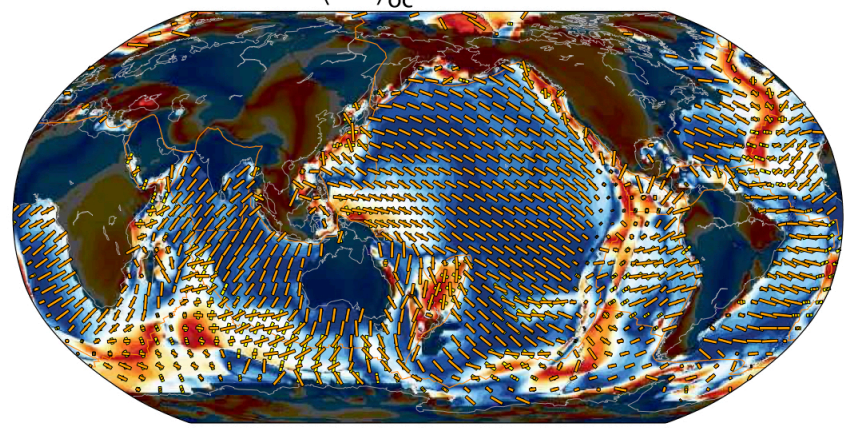
Global azimuthal anisotropy patterns: testing mantle convection models, ideas about driving forces, rheology…

a) surface velocities, prescribed plate motions



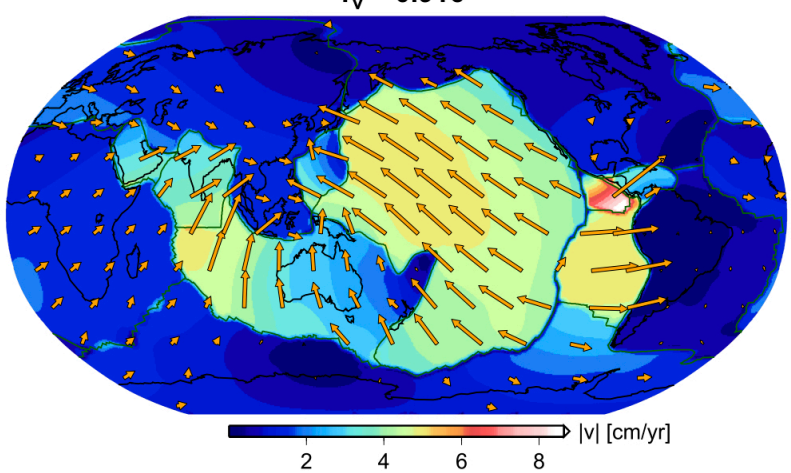
b) anisotropy misfit for model a)

$\langle \Delta\alpha \rangle_{oc}$ = 26.0°



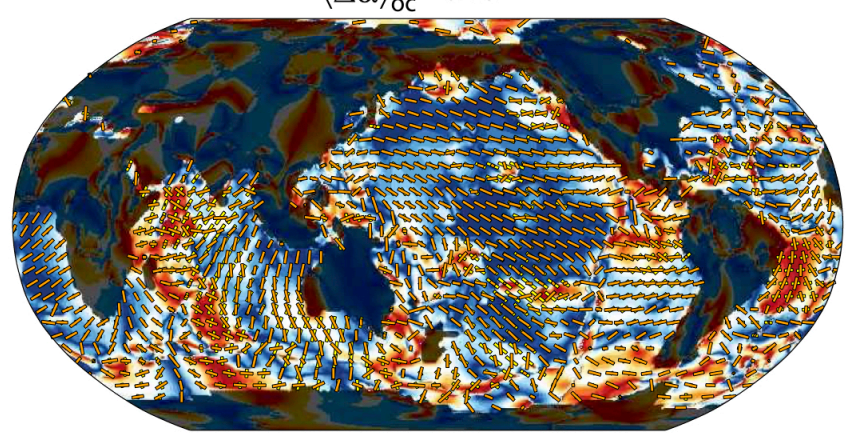
c) surface velocities, slabs and upper mantle anomalies

r_v = 0.916



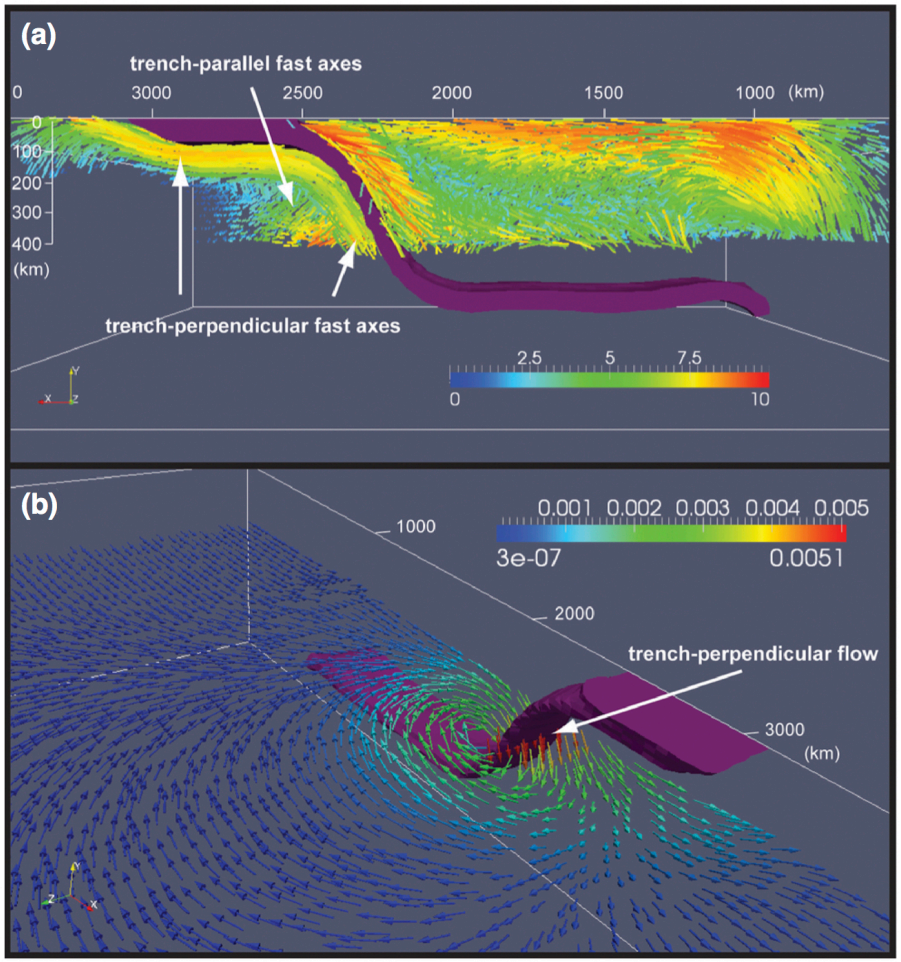
d) anisotropy misfit for model c)

$\langle \Delta\alpha \rangle_{oc}$ = 32.9°

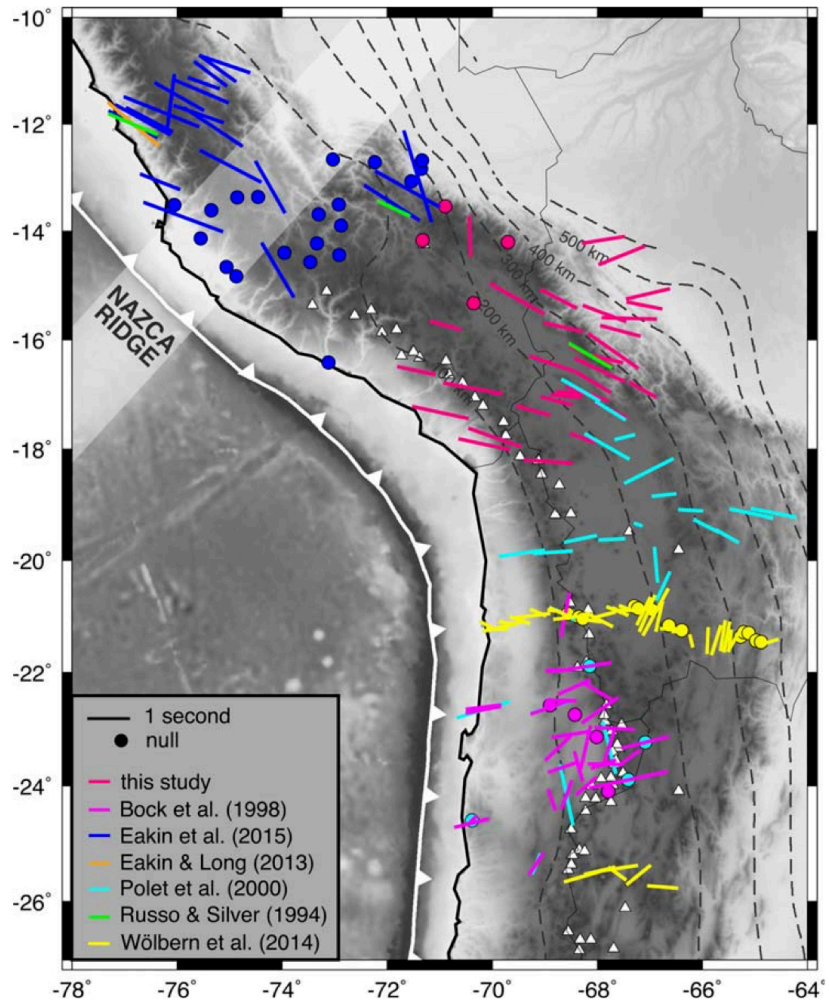


Becker, 2017

Subduction systems and dynamics



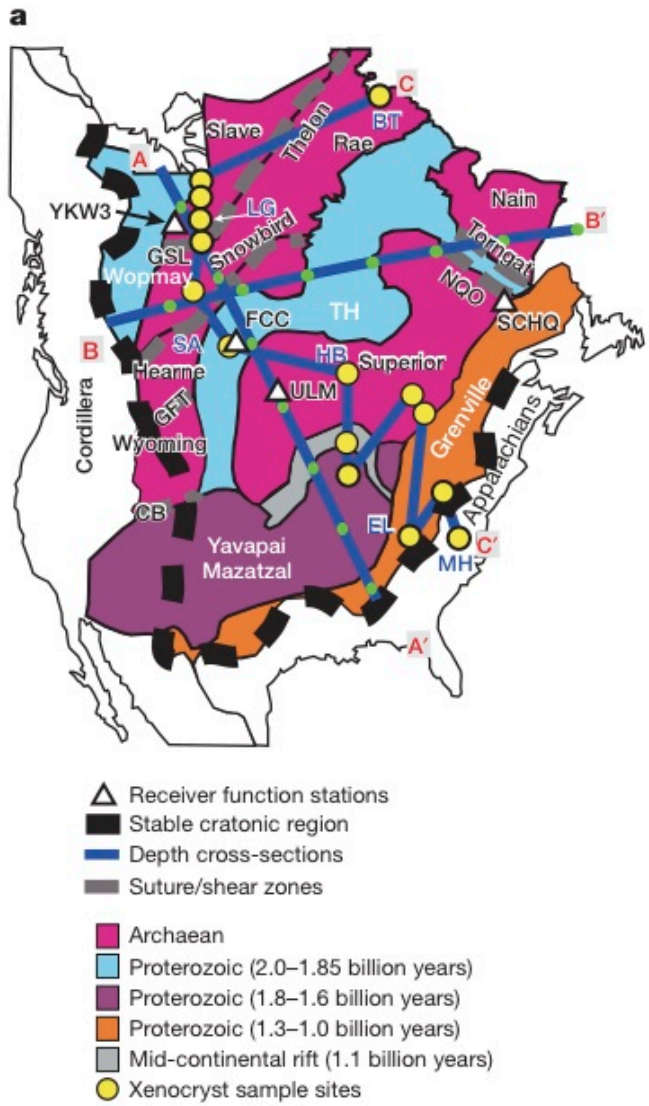
Faccenda and Capitanio, 2012

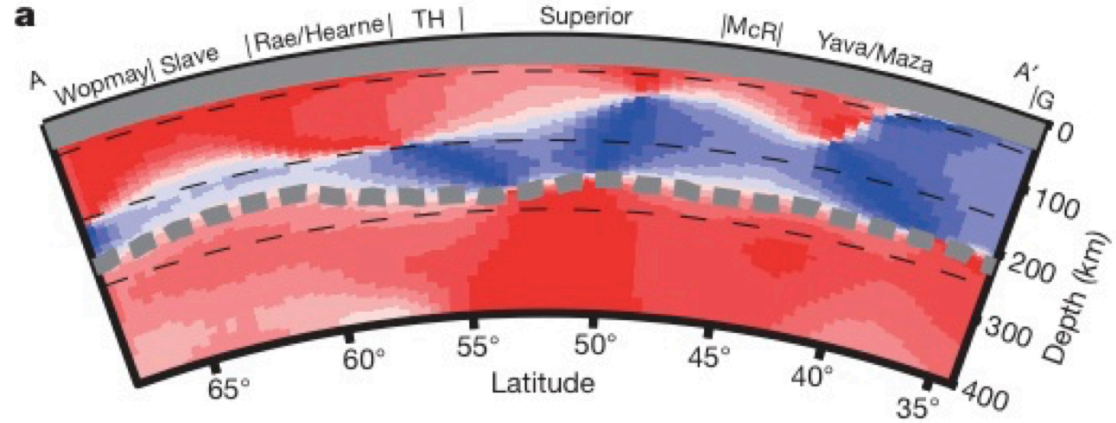


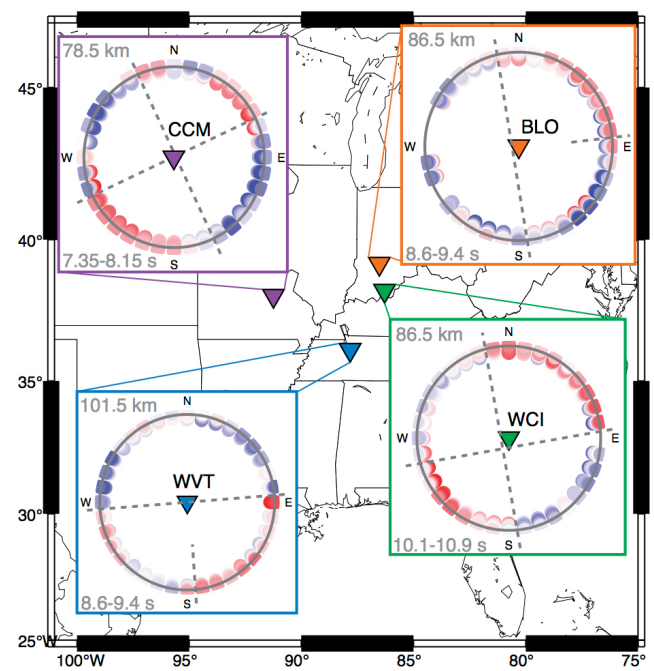
Long et al., 2016

Our ideas about subduction zone flow patterns are evolving beyond a 2D paradigm…but still many unsolved problems!

Continental deformation and evolution



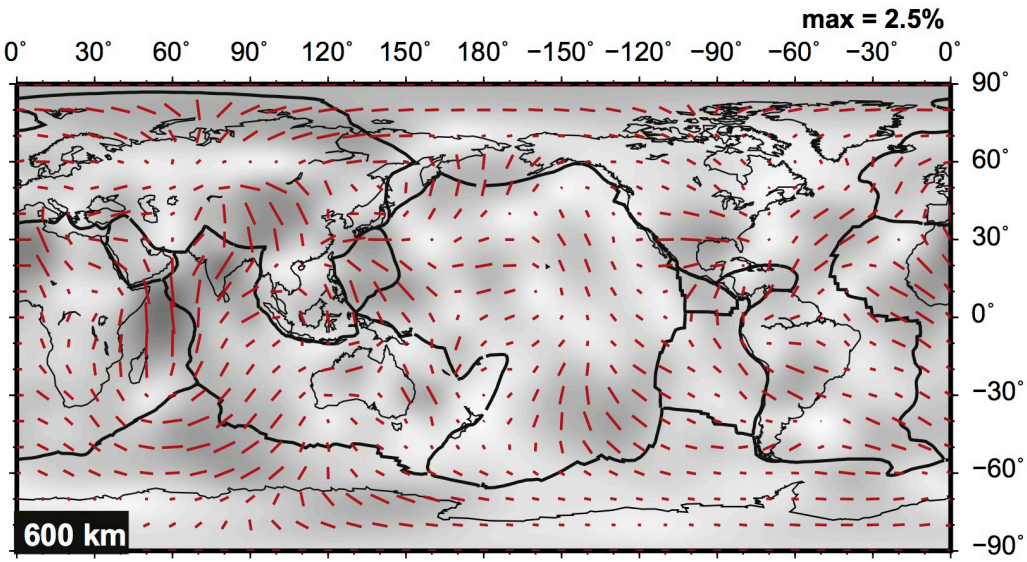




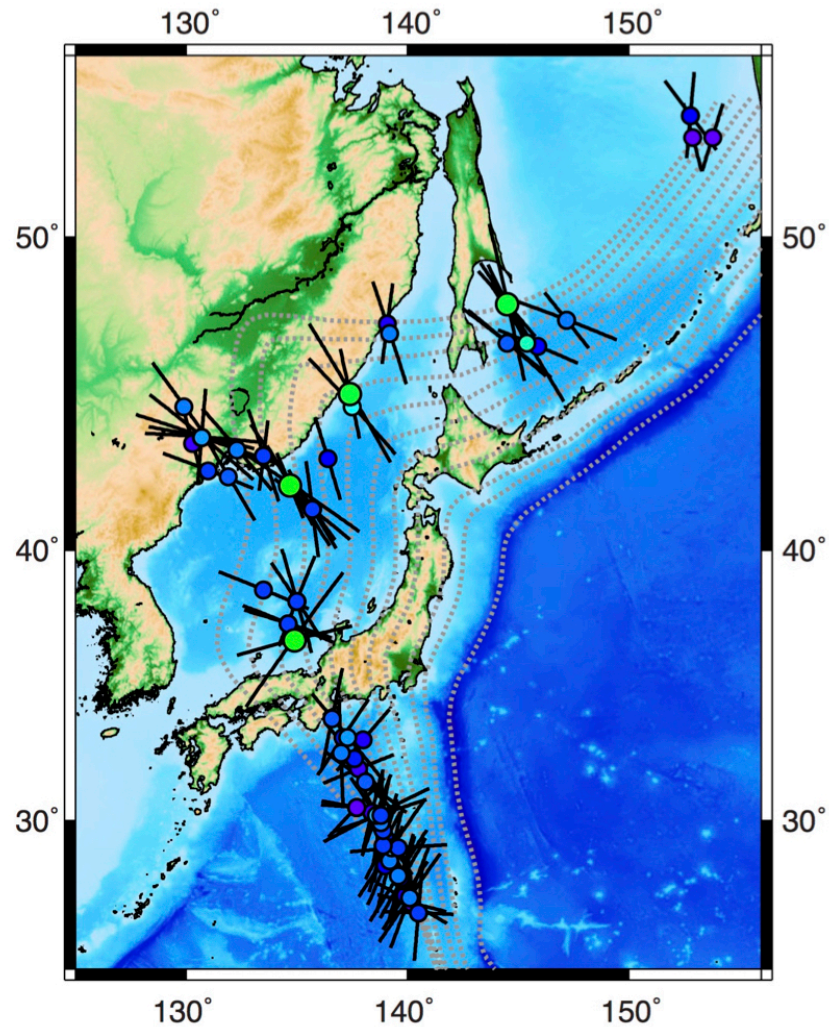
Yuan and Romanowicz, 2010

Wirth and Long, 2014

Deep mantle anisotropy: understanding transition zone dynamics



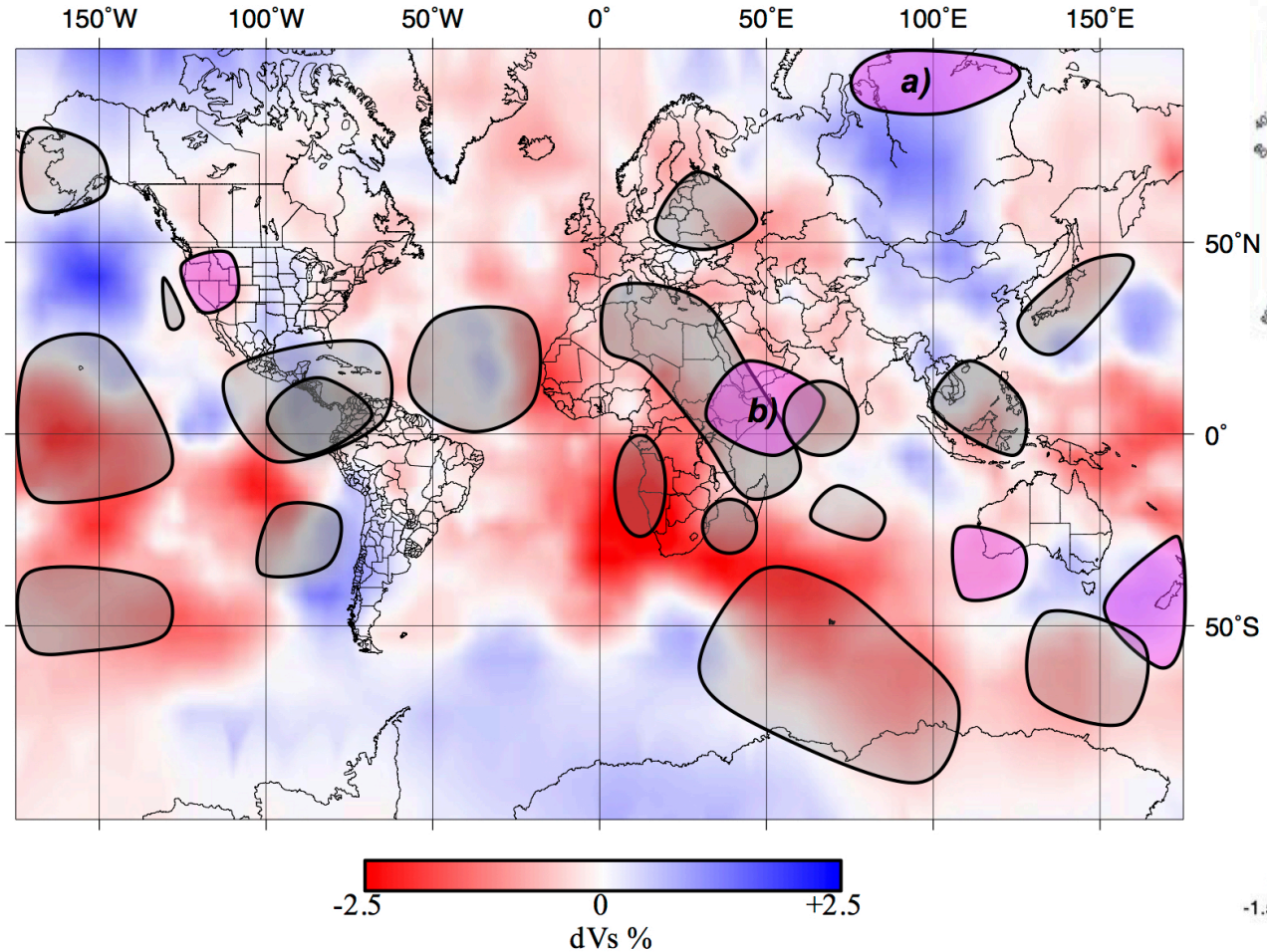
Yuan and Beghein, 2013

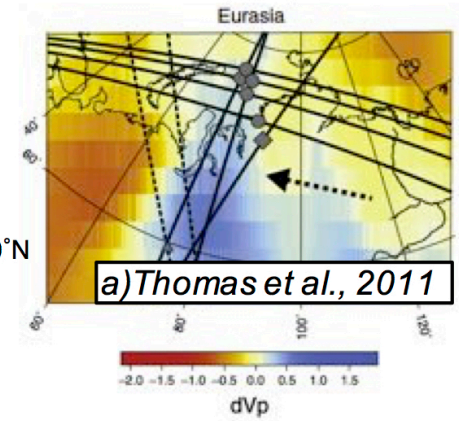


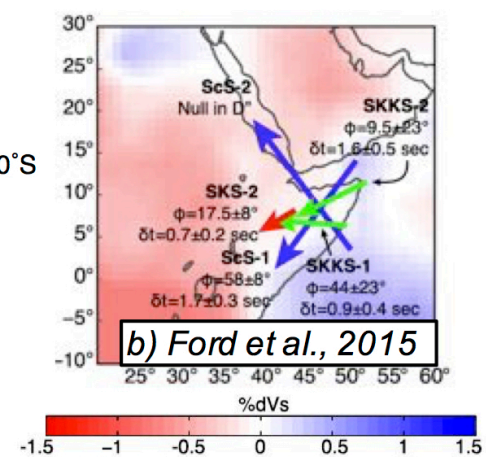
Lynner and Long, 2015

Several lines of evidence for anisotropy (both radial and azimuthal) in the mantle transition zone, and likely the uppermost lower mantle as well. Clues to the dynamics of subducting slabs in the mid-mantle?

Deep mantle anisotropy: understanding D” dynamics

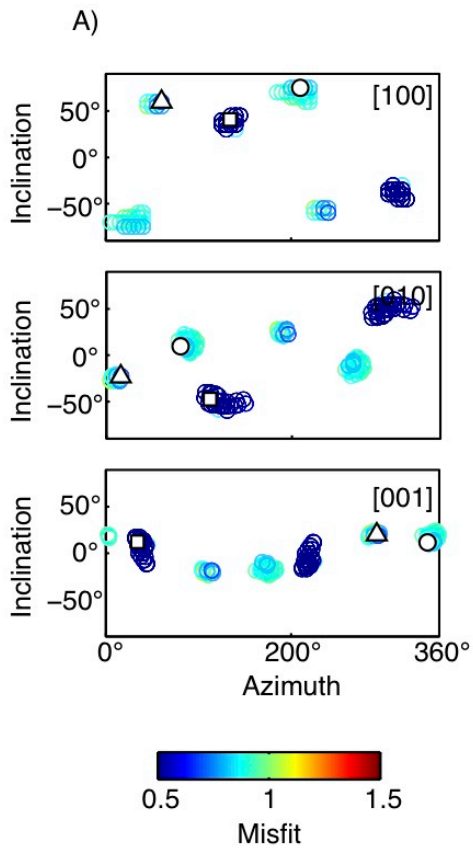


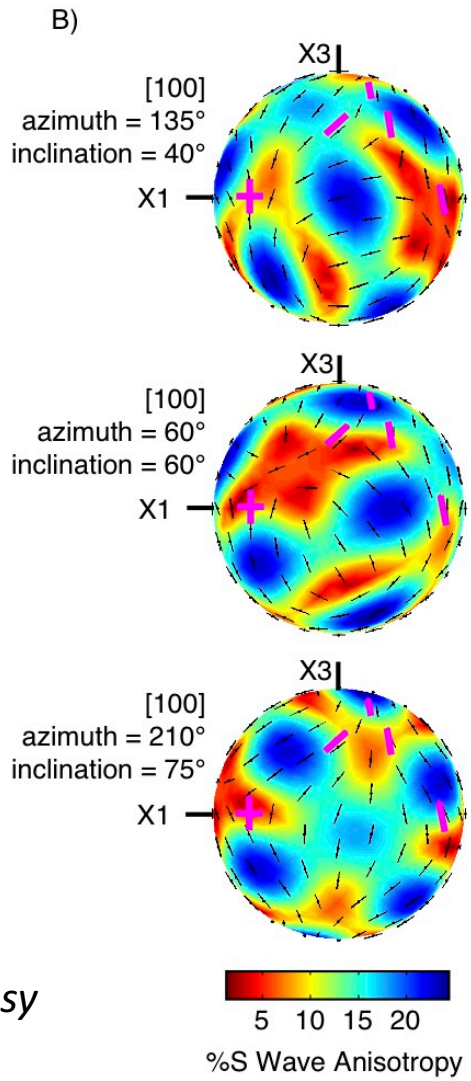


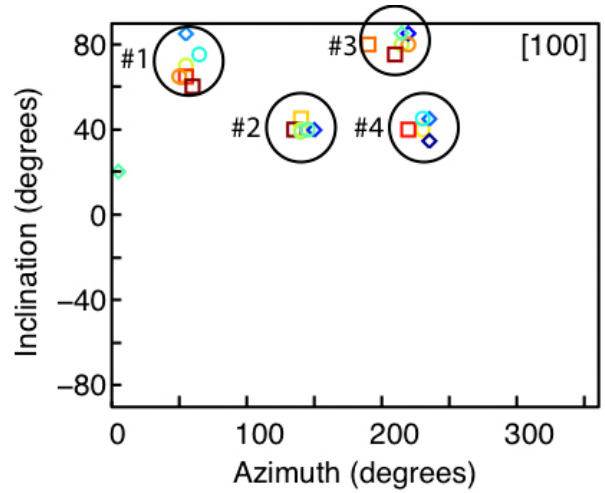


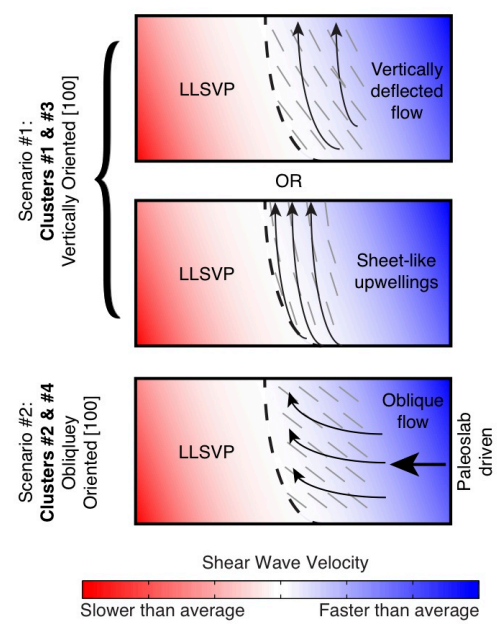
Creasy et al., in review; map modified from *Nowacki et al.* (2011)

Using D” anisotropy to understand flow patterns at the base of the mantle





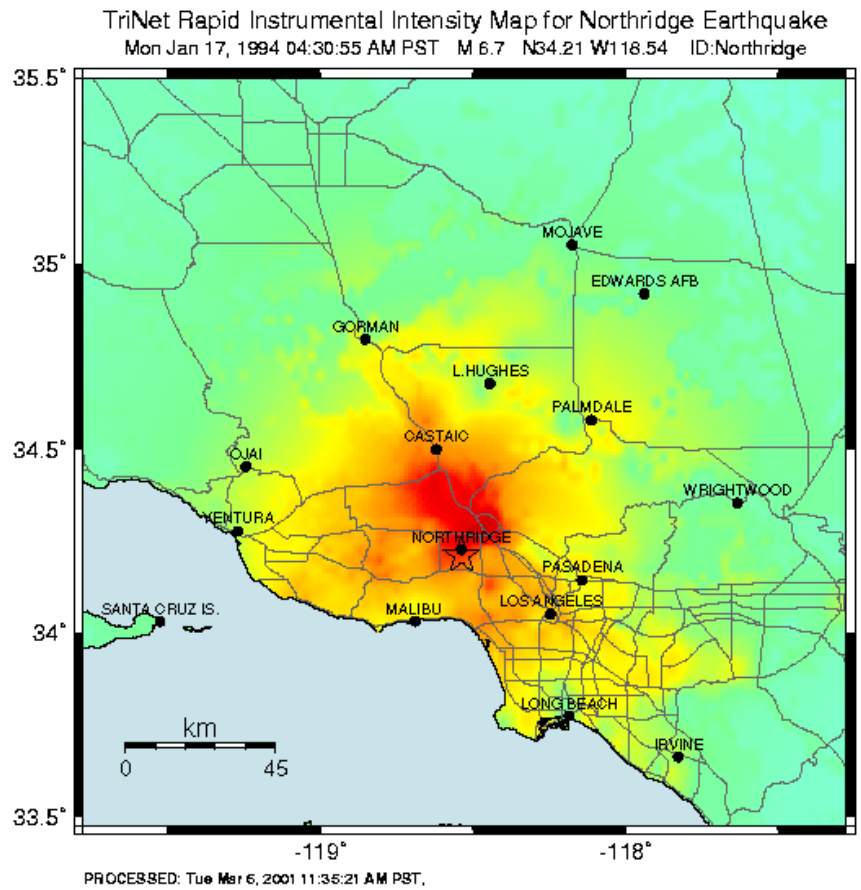




Ford et al., 2015; see also *Creasy et al.,* 2017; *Wolf et al.,* in revision; *Creasy et al.,* in review

Part II: Seismic attenuation

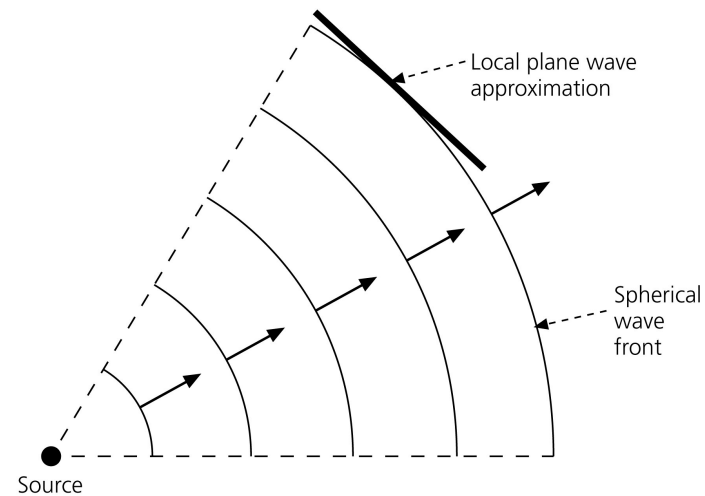
Seismic waves attenuate (lose amplitude) as they propagate. There are many reasons for this (more in a minute), but one of them is anelasticity (also known as intrinsic attenuation). NB: when seismologists say "attenuation," they often mean anelasticity.



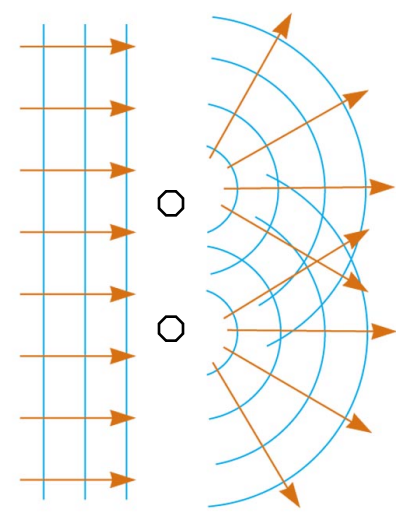
PERCEIVED SHAKING	Not felt	Weak	Light	Moderate	Strong	Very strong	Severe	Violent	Extreme
POTENTIAL DAMAGE	none	none	none	Very light	Light	Moderate	Moderate/Heavy	Heavy	Very Heavy
PEAK ACC.(%g)	<.17	.17-1.4	1.4-3.9	3.9-9.2	9.2-18	18-34	34-65	65-124	>124
PEAK VEL.(cm/s)	<0.1	0.1-1.1	1.1-3.4	3.4-8.1	8.1-16	16-31	31-60	60-116	>116
INSTRUMENTAL INTENSITY	I	II-III	IV	V	VI	VII	VIII	IX	X+

What affects the amplitude of seismic waves?

- Reflection and transmission at a sharp boundary
- Geometrical spreading
- Scattering
- Multipathing (also known as focusing/defocusing)
- Anelasticity



The first four of these are **elastic** processes (energy in seismic wavefield is conserved). In contrast, anelasticity involves the conversion of seismic energy to heat.



Stein & Wysession, 2003

ALL OF THESE PROCESSES ARE IMPORTANT FOR SEISMIC WAVES

The first four are described by elastic wave theory, and can increase or decrease an arrival's amplitude by shifting energy within the wave field.

In contrast, anelasticity only reduces wave amplitudes, because energy is lost from the elastic waves.

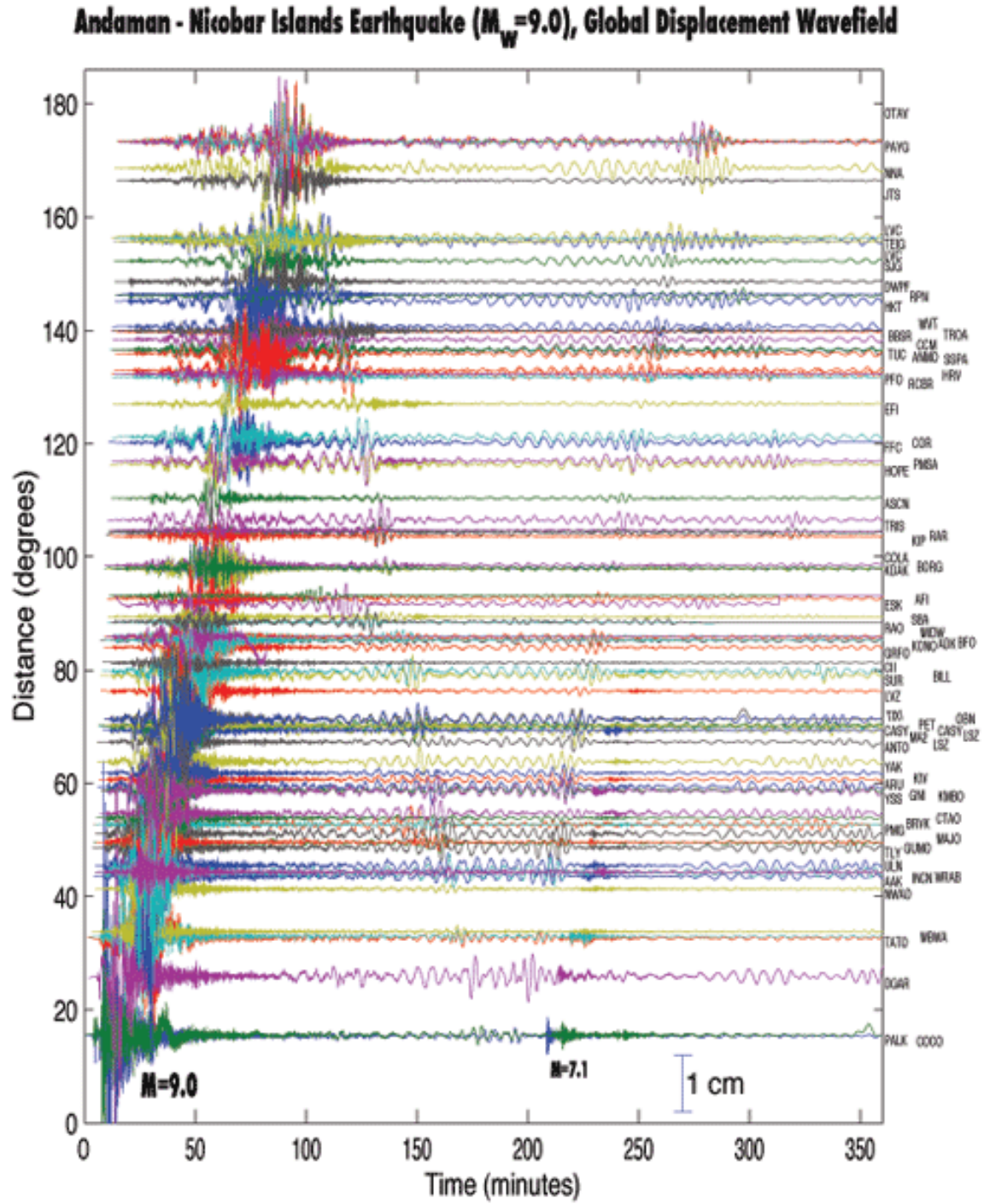
So much of seismology is built upon the approximation that the earth responds elastically during seismic propagation that it is easy to forget that the earth is not perfectly elastic.

However, without anelasticity seismic waves from every earthquake that ever occurred would still be reverberating -> this is NOT happening, obviously.

Elasticity is a good approximation for the earth's response to seismic waves, but there are many important implications and applications of anelasticity.

From geometric spreading alone, expect minimum at a distance of 90º, and maxima at 0º and 180º

Also have effects of anelasticity



Model of intrinsic attenuation:
damped harmonic oscillator composed of a spring and dashpot

Newton's Law: $\mathbf{F} = m\mathbf{a}$

Case for no damping:

$$m \frac{d^2 u(t)}{dt^2} + k\, u(t) = 0$$ where k is the spring constant.

Solution is perpetual harmonic oscillation:

$$u(t) = Ae^{i\omega_0 t} + Be^{i\omega_0 t} \quad \text{or} \quad u(t) = A_0 \cos(\omega_0 t)$$

(A and B are constants)

The mass moves back and forth with a natural frequency $\omega_0 = (k/m)^{1/2}$

Once the motion is started, the oscillation continues forever.

Case of damping:

The damping force is proportional to the velocity of the mass and opposes its motion.

$$m\,\frac{d^2u(t)}{dt^2} + \boxed{\gamma\, m\,\frac{du(t)}{dt}} + k\,u(t) = 0 \qquad (\gamma \text{ is the } \textit{damping factor.})$$

To simplify, define the *quality factor* $Q = \omega_0/\gamma$ to get:

$$\frac{d^2u(t)}{dt^2} + \frac{\omega_0}{Q}\,\frac{du(t)}{dt} + \omega_0^2\,u(t) = 0$$

Alternatively, look at successive peaks one full period $T = 2\pi/\omega_0$ apart:

$$A_1(t_1) = A_0 \exp\left(-\omega_0 t_1/2Q\right)$$

$$A_2(t_1 + T) = A_0 \exp\left(-\omega_0(t_1 + T)/2Q\right)$$

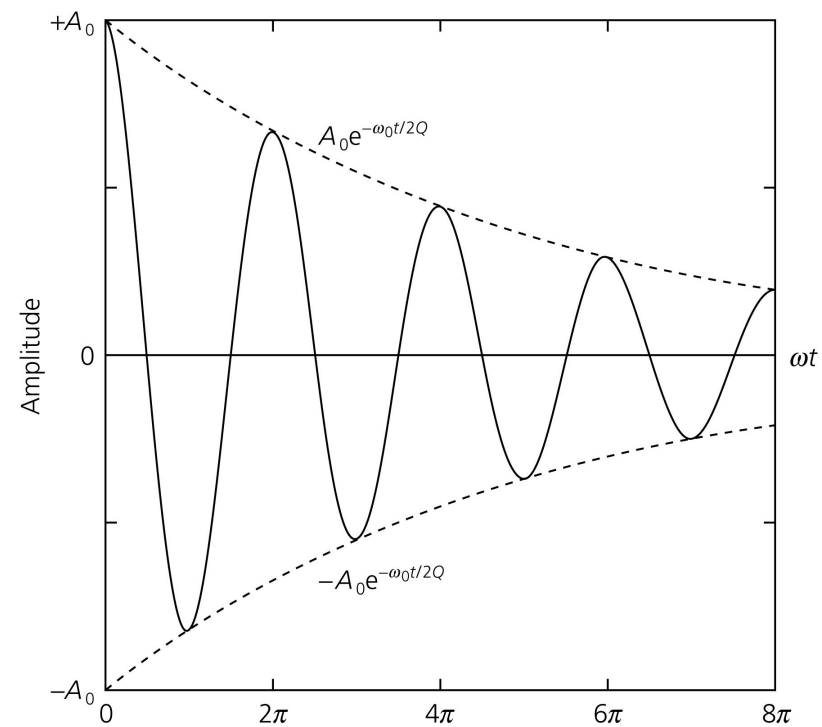
Their ratio is:

$$A_1/A_2 = \exp\left[-\omega_0 t_1/2Q - \omega_0(t_1 + T)/2Q\right] = e^{\pi/Q}$$

This gives $\quad Q = \pi \,/\, \ln\left(A_1/A_2\right)$

In Figure 11 the second peak, at $\omega t = 2\pi$, is about 2/3 of the first peak, at $\omega t = 0$. Therefore, $Q \approx \pi/\ln(3/2) \approx 8$.

Figure 3.7-11: Wave amplitude for a damped harmonic oscillator.



Some terminology: Q and Q^{-1}

The solution for the damped harmonic oscillator incorporated the damping through the quality factor Q.

Attenuation for seismic waves (and a variety of other physical phenomena) are often discussed in terms of Q or Q^{-1}.

Although Q has more convenient values, Q^{-1} has the advantage that is directly rather than inversely proportional to the damping.

No attenuation -> $Q = \infty$ $1/Q = 0$
High attenuation -> Q low 1/Q high

Where does the anelastic behavior come from?

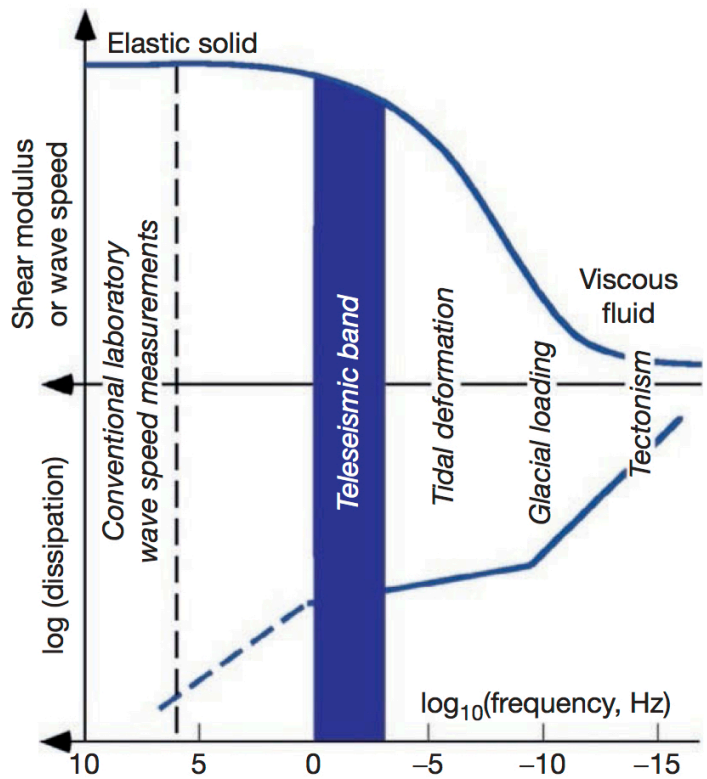


Figure 1 A schematic mechanical relaxation spectrum for the Earth. The mechanical behavior of the Earth's hot interior and its constituent materials changes progressively from that of an elastic solid to that of a viscous fluid with decreasing frequency or increasing timescale. This transition involves the thermally activated mobility of crystal defects including vacancies, dislocations, twin-domain and grain boundaries, and phase boundaries and intergranular melt. The resulting viscoelastic behavior is manifest in the dissipation of strain energy and associated frequency dependence of the shear modulus and wave speed. Adapted

Jackson, 2015

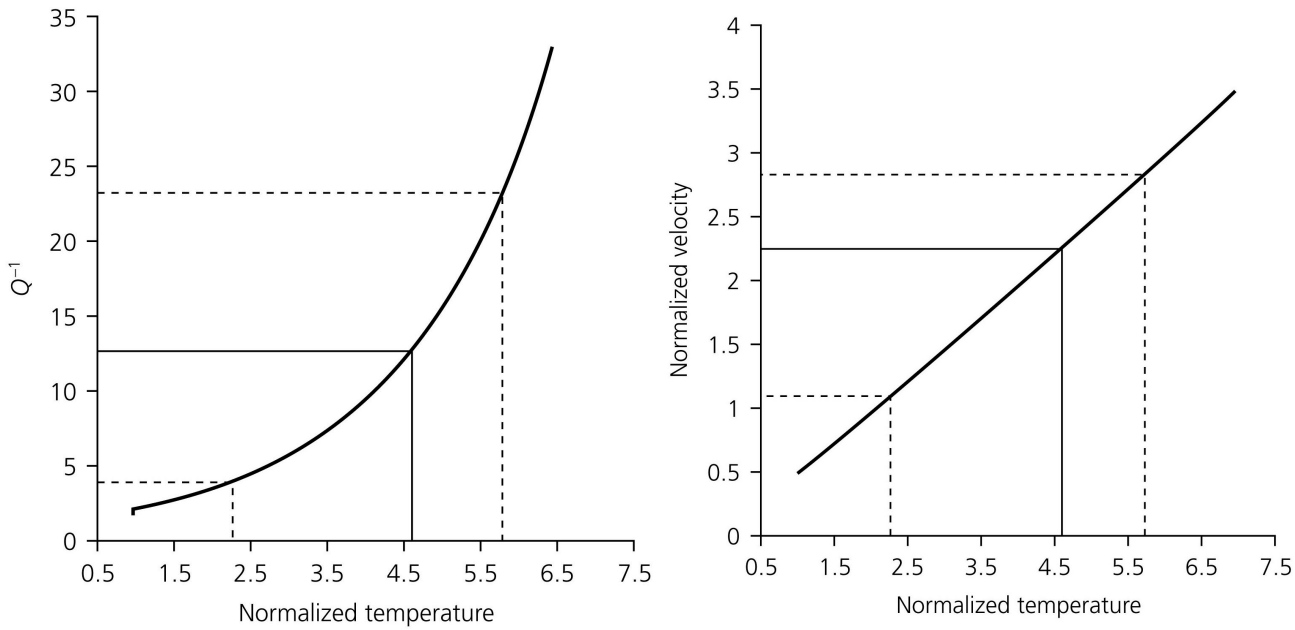
The Earth deforms elastically at short timescales and as a viscous fluid at long timescales, and there is a transition between these two endmember behaviors.

The relaxation of the shear modulus, and associated dissipation of seismic energy, is governed by some extremely complicated microphysics (intragranular processes such as defect redistribution, intergranular processes such as grain boundary sliding, melt squirt or melt enhancement of GBS, etc.)

Why do we care about attenuation?

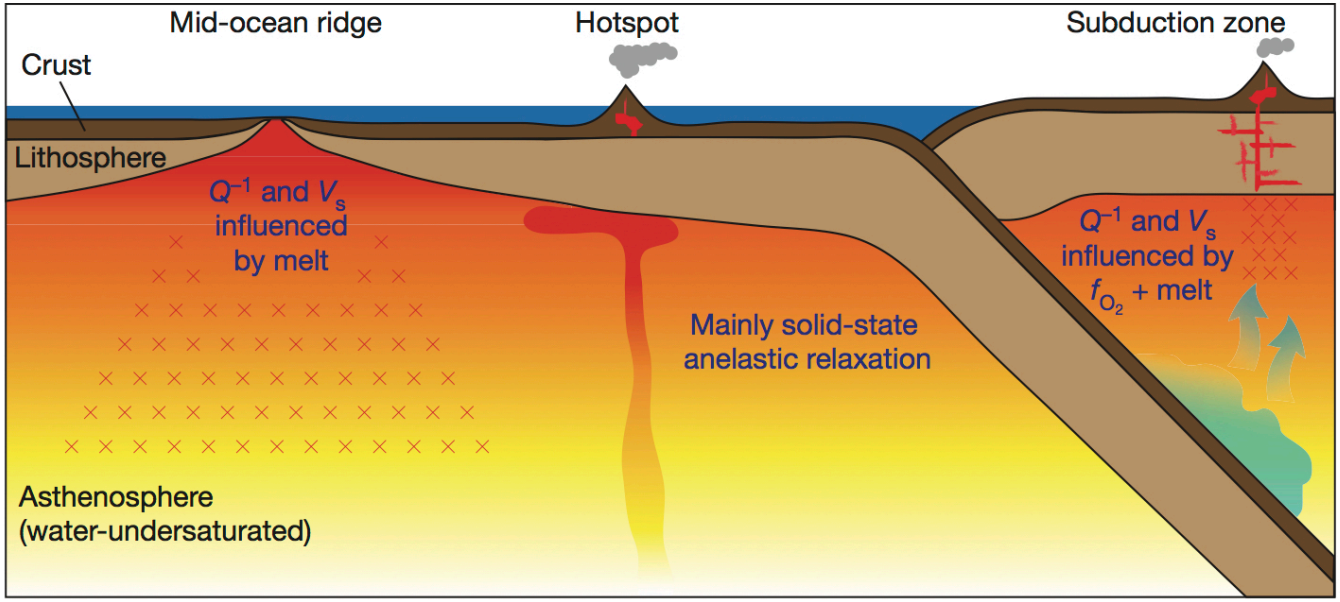
Several reasons: (intrinsic) attenuation is affected by quantities we want to study, such as temperature, grain size, partial melt, water (perhaps), oxygen fugacity, etc. Observations of attenuation are thus complementary to other observables such as seismic velocity or electrical conductivity. Furthermore: if you want to correctly scale seismic velocities to T, you have to take into account the softening of the shear modulus due to anelasticity -> departs significantly from a purely elastic calculation.

Figure 3.7-2: Cartoon of the effects of temperature on velocity and attenuation.



Stein and Wysession, 2003

Mechanisms: what factors affect attenuation?



Cline et al., 2018

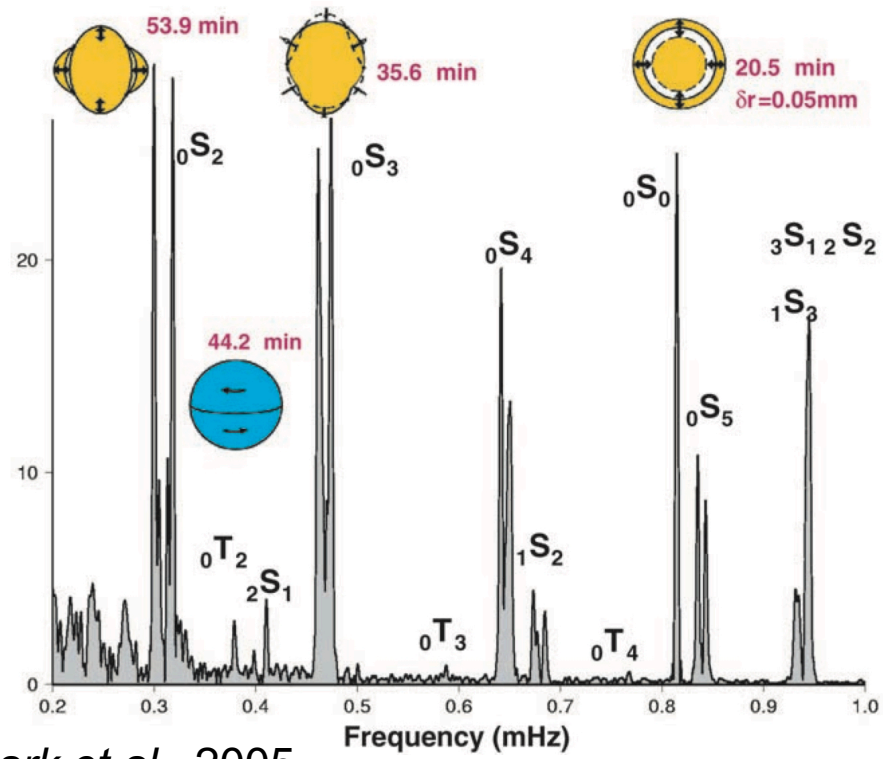
A large number of factors may contribute, including temperature, grain size, presence of partial melt and its configuration/behavior (melt squirt vs. melt enhancement of grain boundary sliding), presence of water (maybe!?!?!?), oxygen fugacity… The situation is further complicated by the fact that many of these factors are linked, and thus not independent – leading to a substantial modeling challenge.

How do we measure attenuation?

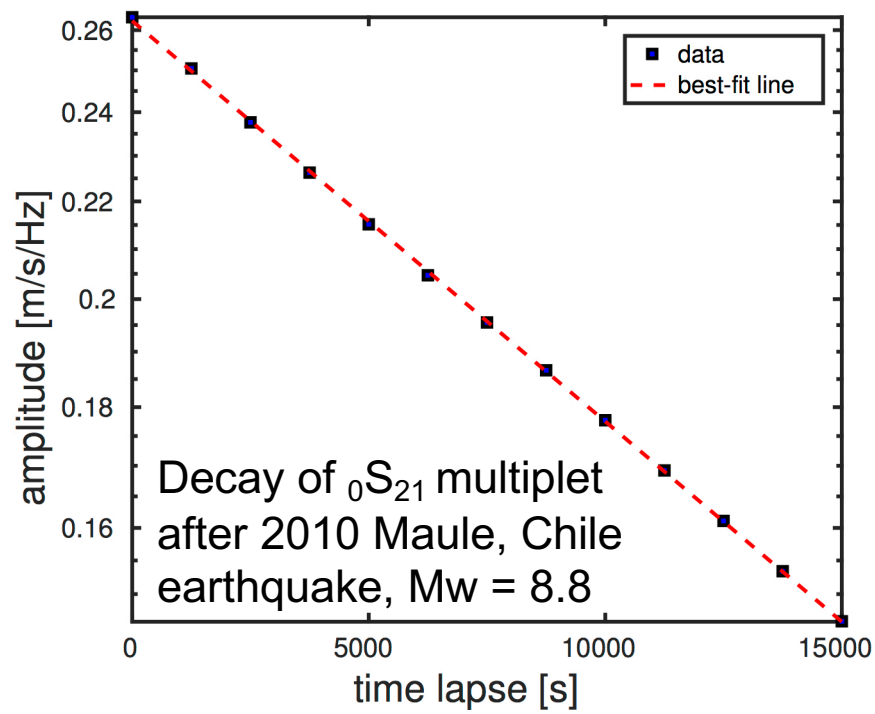
Use many of the same data types as for seismic velocity and anisotropy studies – the observational seismology toolkit!

- Normal modes
- Surface waves
  - Global & regional
- Body waves (P & S)
  - Global & regional

Normal Modes



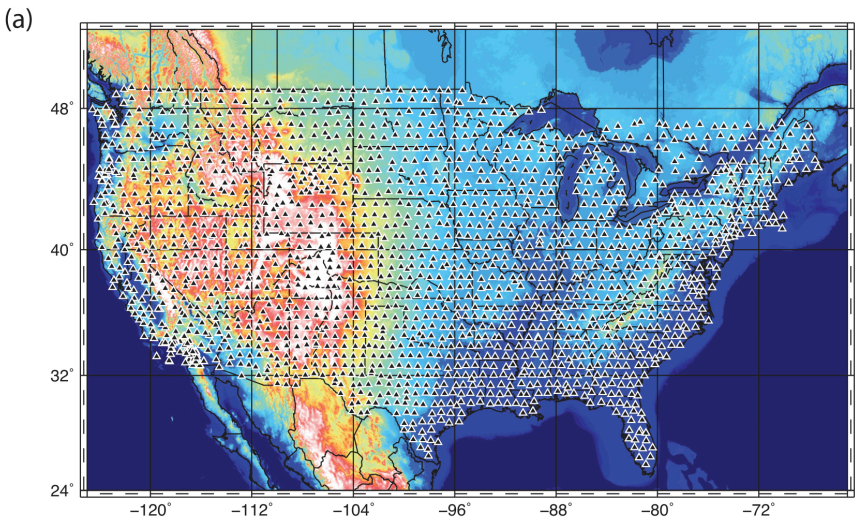
Park et al., 2005

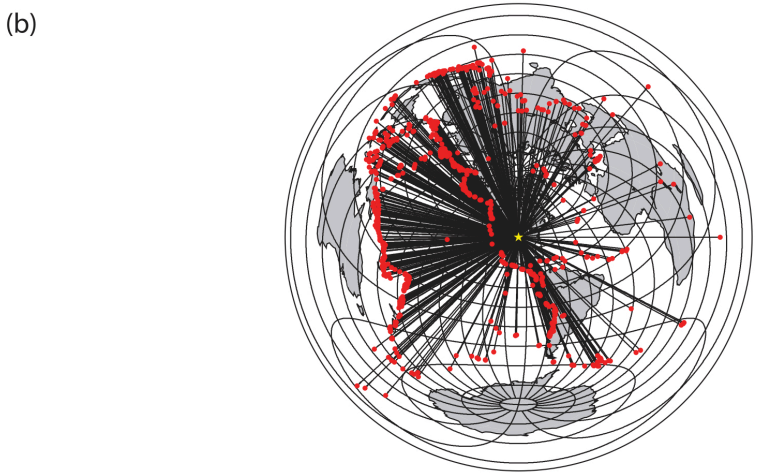


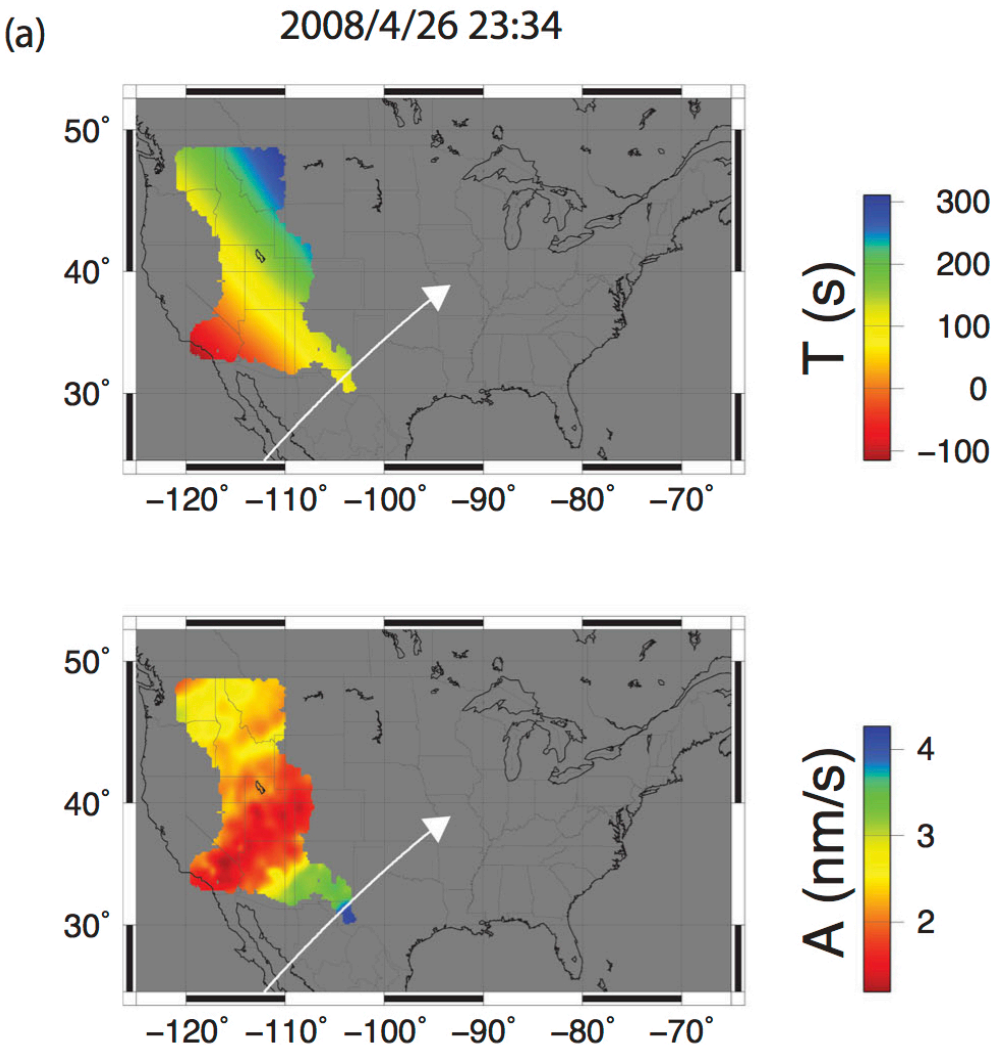
Chen (& Park), Yale Ph.D. thesis

Examine how the amplitudes of different modes decay with time after large earthquakes; because different modes are sensitive to different depth ranges, you can build up a global model of Q.

Surface Waves



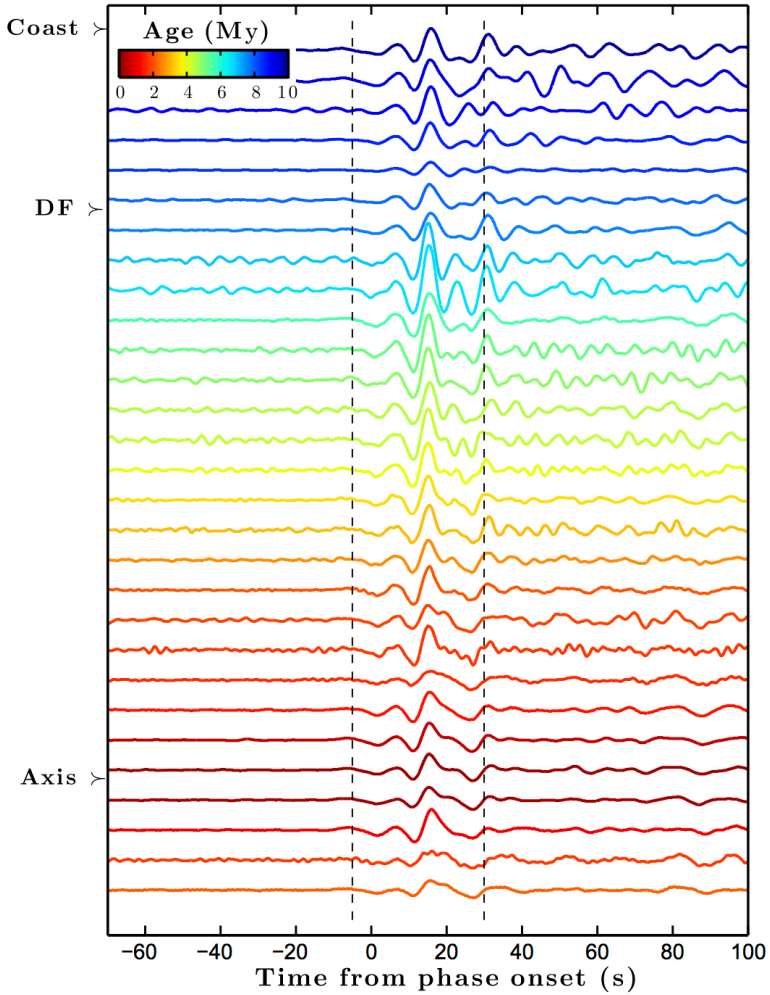


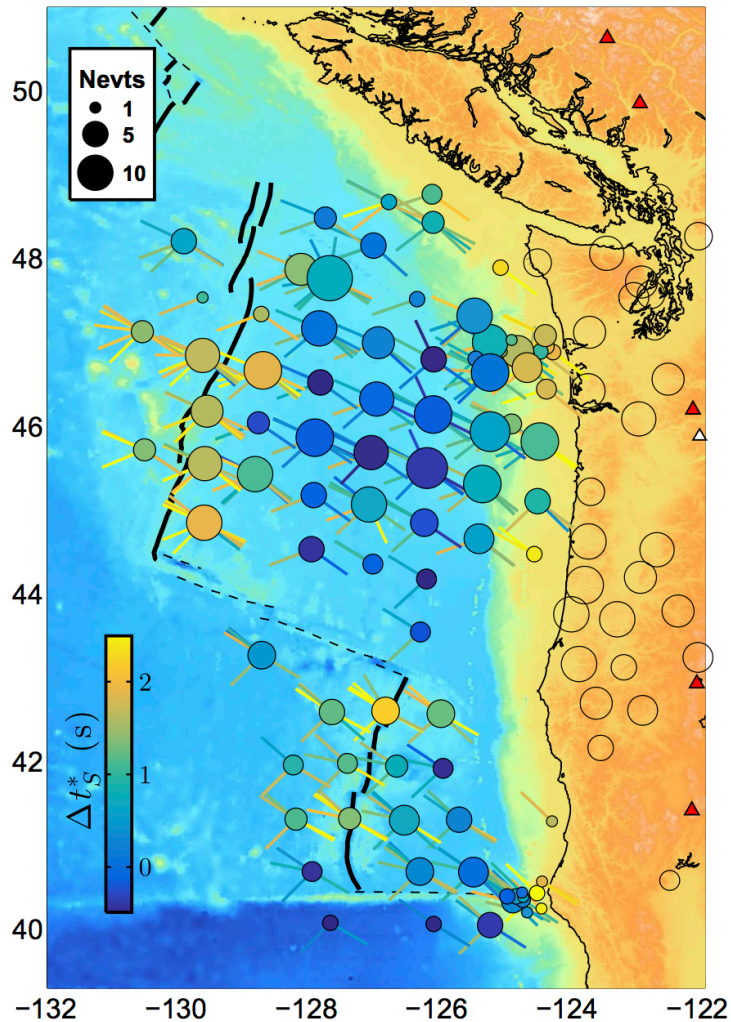


Measurements of surface wave phase velocities and amplitudes using USArray data

Bao et al., 2016

Body Waves

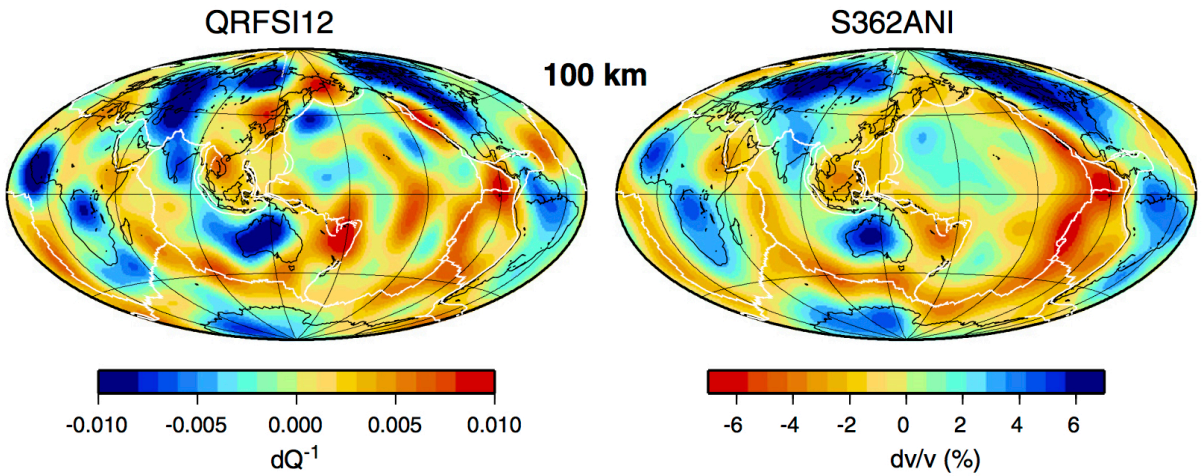


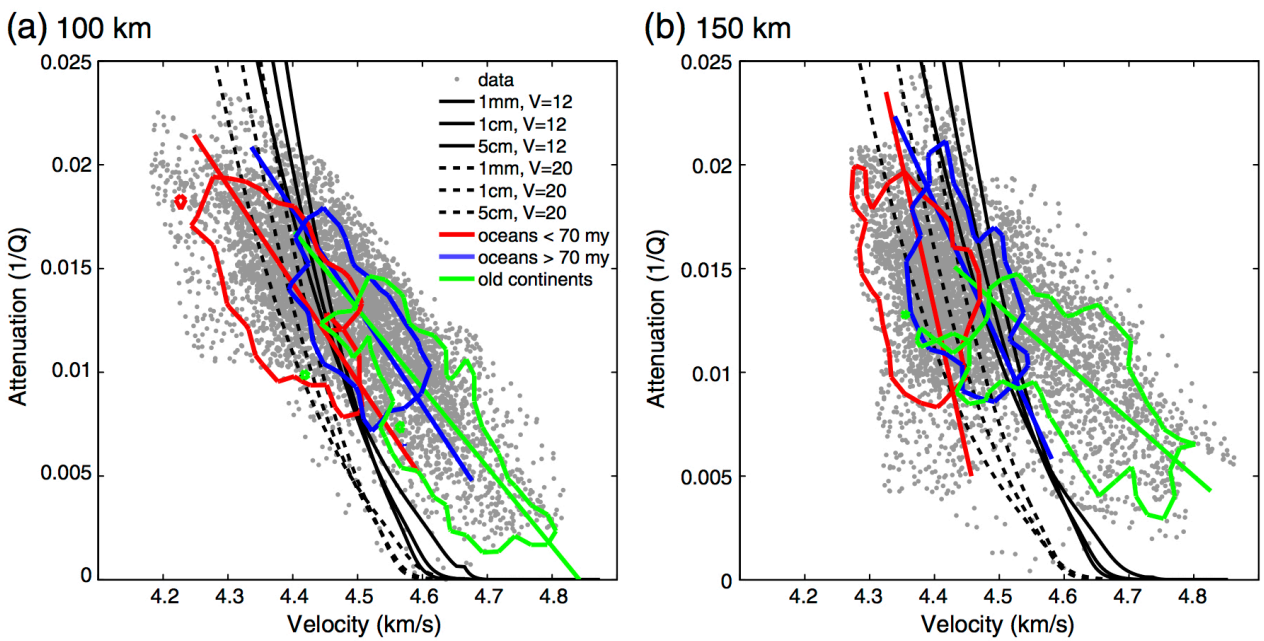


Eilon and Abers, 2017

Common to estimate the attenuation parameter t* (attenuation divided by wavespeed, integrated over the ray).

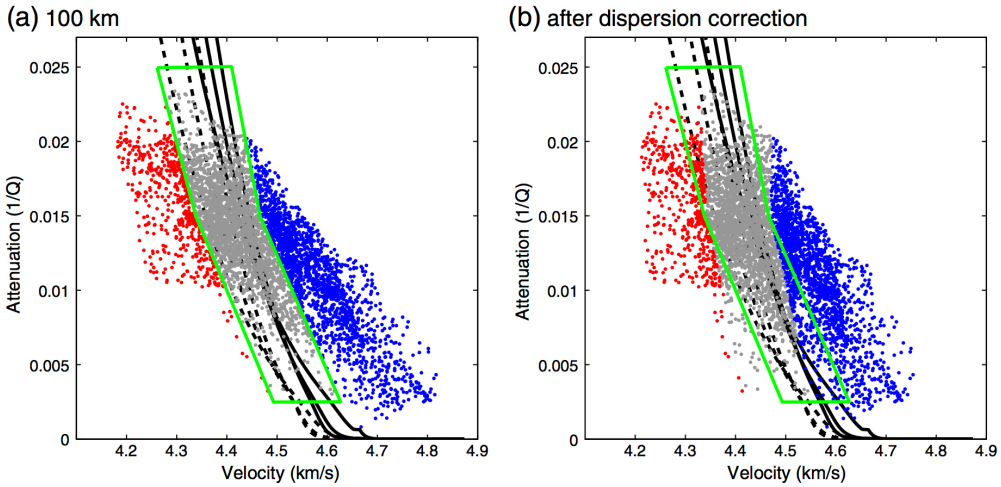
How do we interpret/use attenuation data?

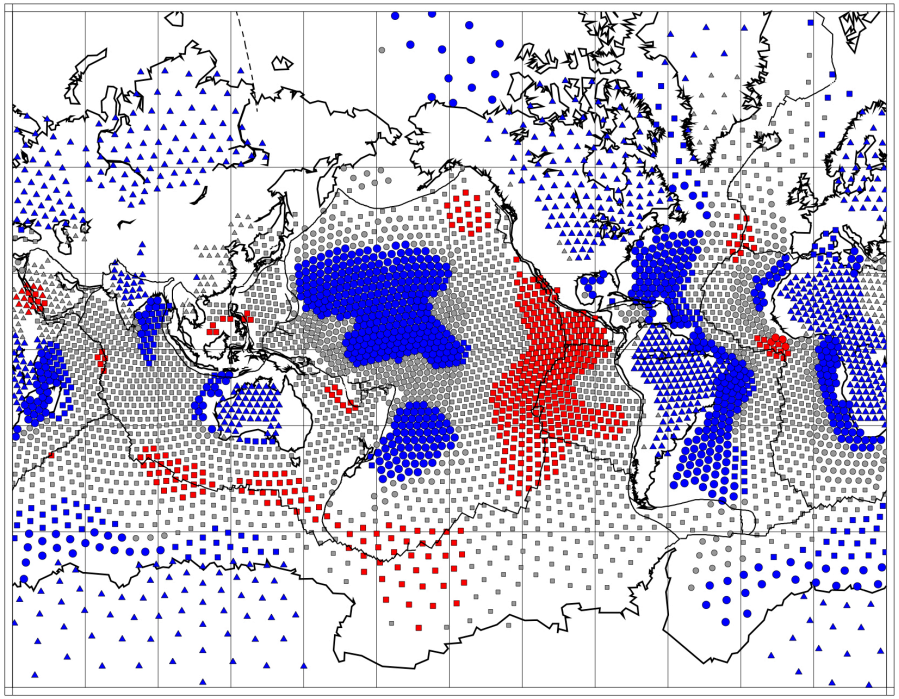




Dalton and Faul, 2010; also *Dalton et al.,* 2009

How do we interpret/use attenuation data?

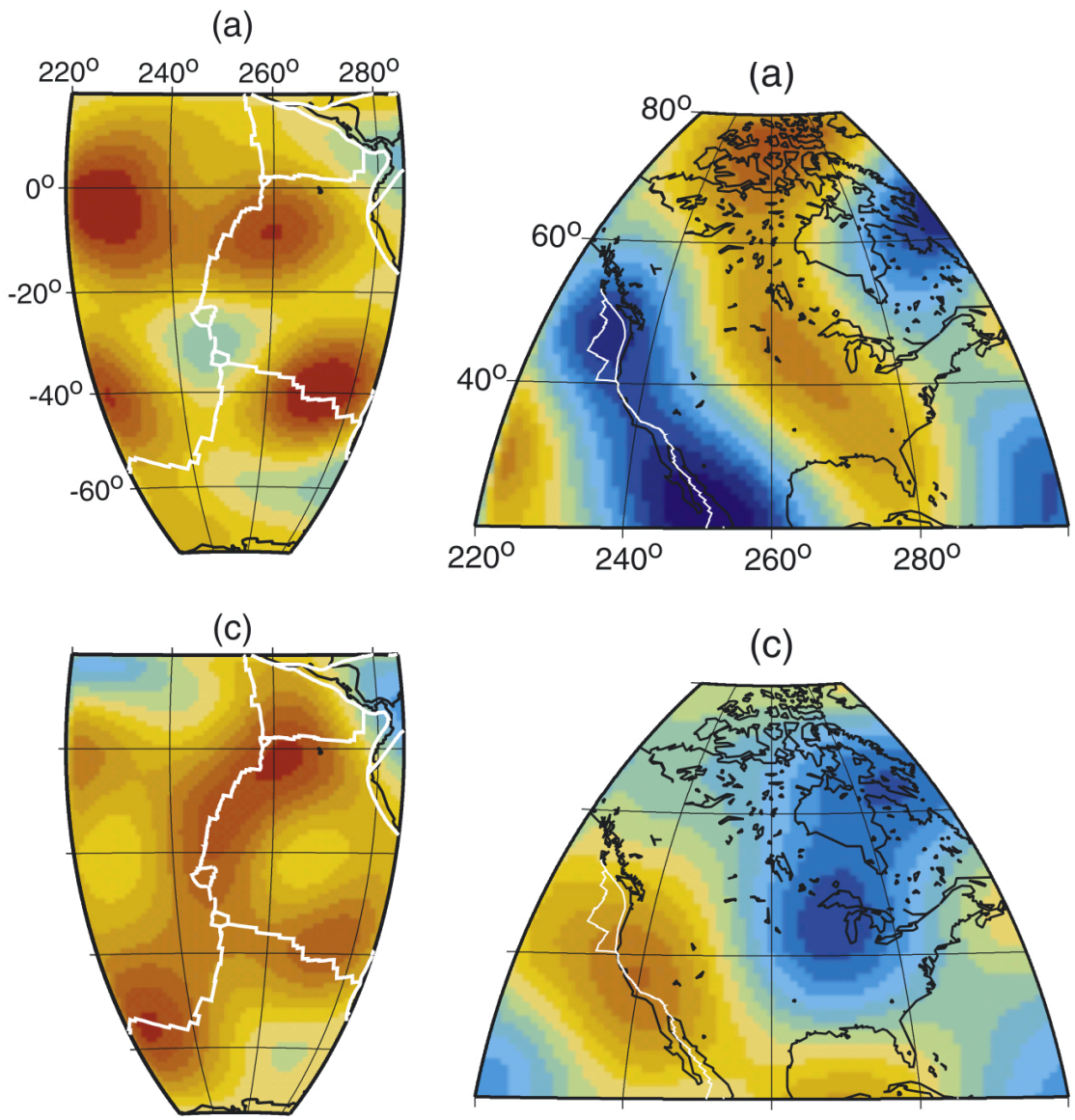




The approach: identify regions/depth ranges where Vs and Q are consistent with the mineral physics based predictions from Faul and Jackson (2010), which take into account thermal effects on melt-free olivine polycrystals. Regions with deviations point to other effects (e.g., partial melt beneath MOR, compositional variations beneath continents).

Dalton and Faul, 2010

Caveats and challenges: getting at Q

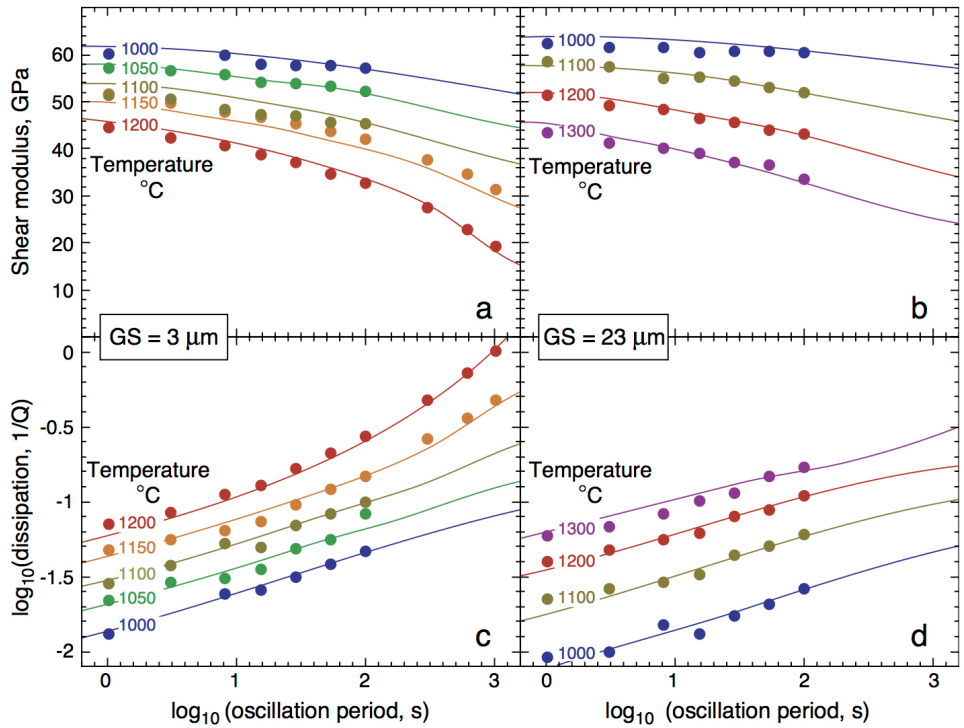


Dalton and Ekstrom, 2006

The measurement of amplitudes on a seismogram is (at least sort of) straightforward – but amplitudes are affected by many things, including scattering, source effects, local site effects, and focusing/defocusing. Accounting for these effects, and isolating the effect of anelasticity, is quite challenging.

Caveats and challenges: using laboratory measurements to interpret

Torsion apparatus at ANU, Ian Jackson's lab



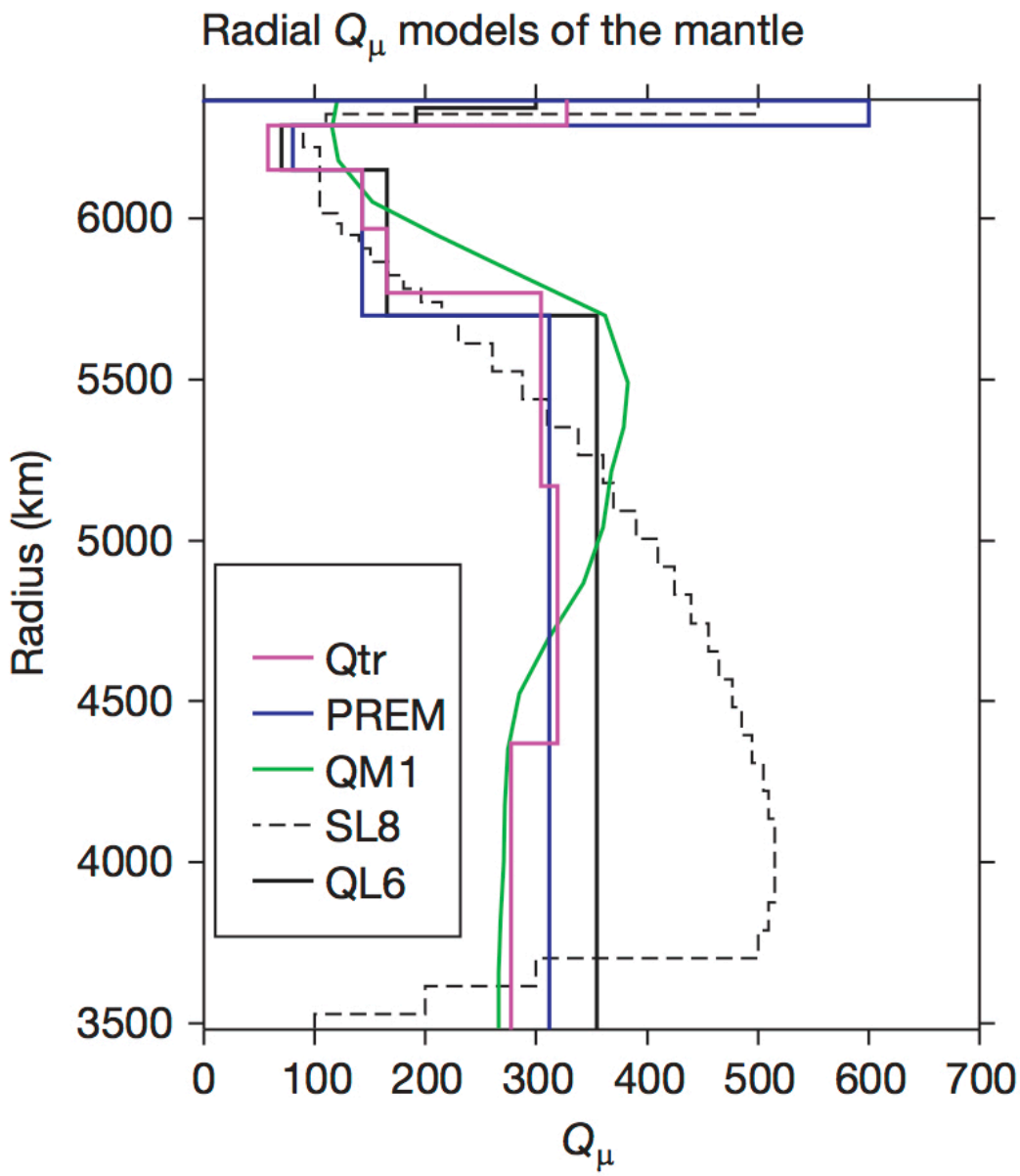
Faul and Jackson, 2010

Data for melt-free polycrystalline olivine; shear modulus and dissipation depend on T and grain size, as well as oscillation period.

Experiments must be done at seismic frequencies – difficult!

In order to extrapolate (pressure, grain size) from lab to Earth, must have detailed knowledge of the mechanisms - difficult!

Some results: Models for Q in the mantle

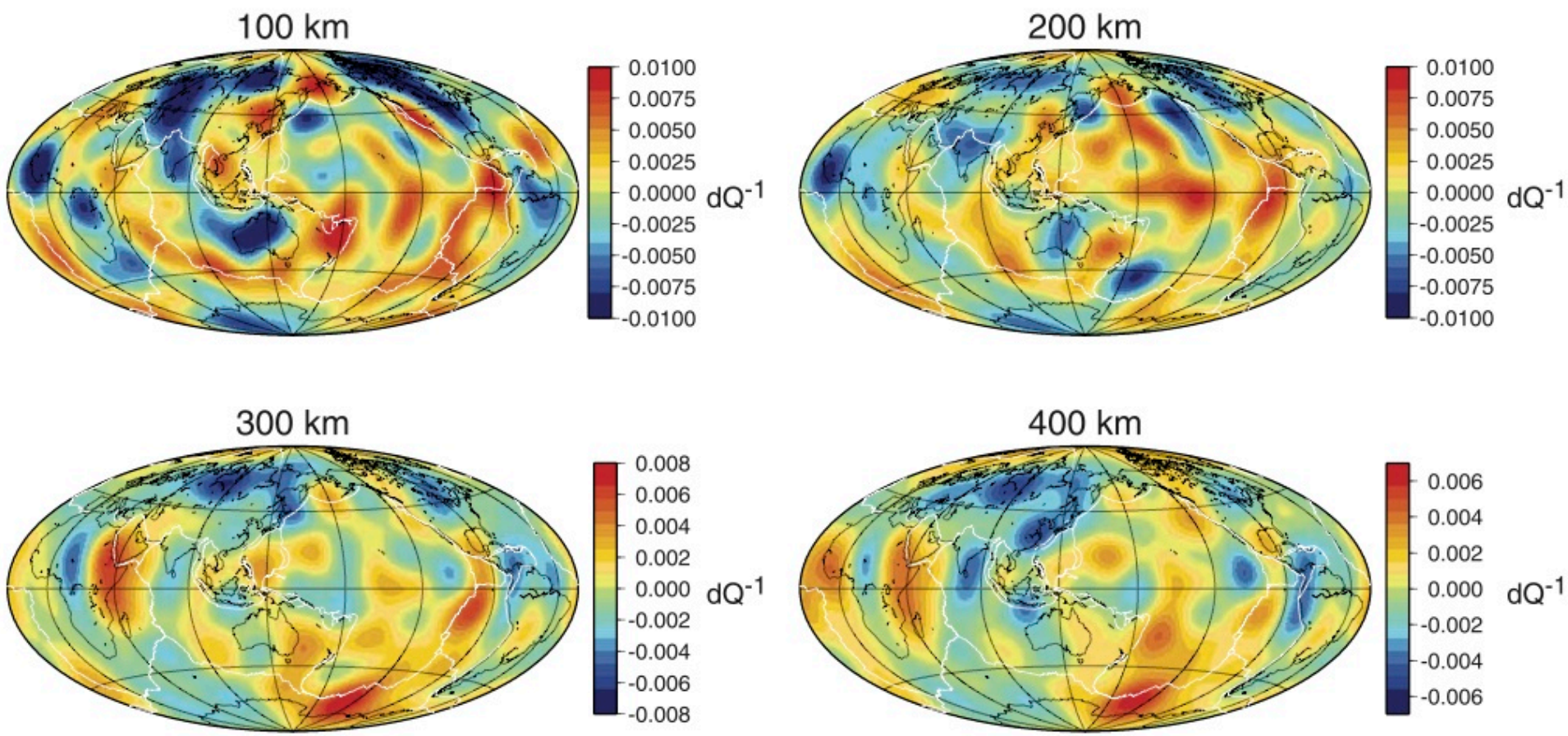


Major features of 1D models for Q:

- Attenuation low in the lithosphere
- Attenuation high in the upper mantle low-velocity zone
- Increase in Q with depth below ~200 km
- Generally higher Q in the lower mantle than in the upper mantle

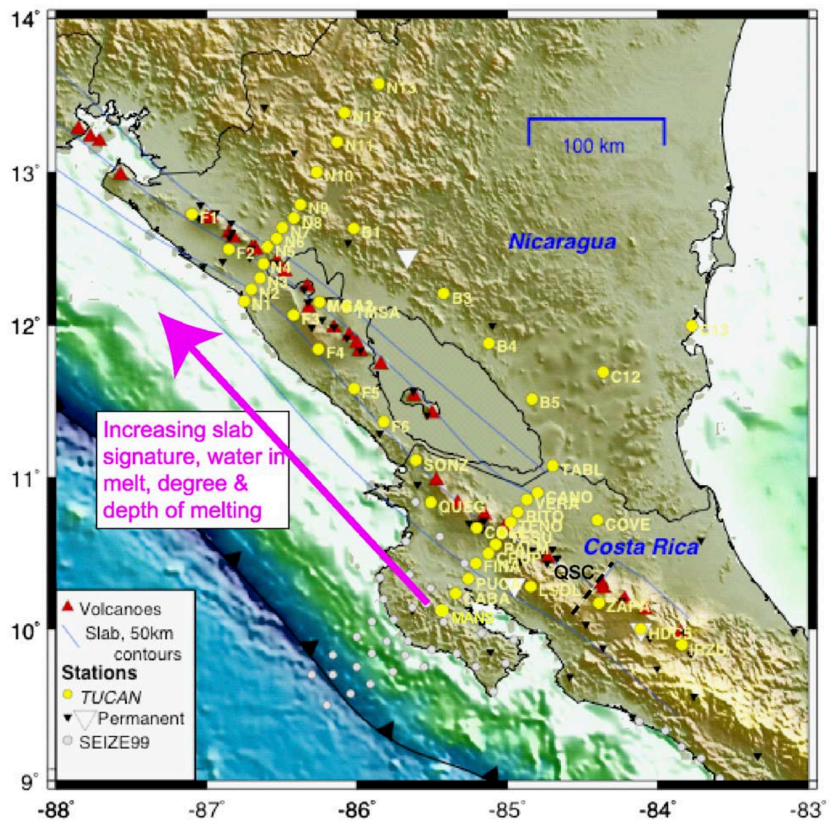
Romanowicz and Mitchell, 2015

Some results: Models for Q in the mantle

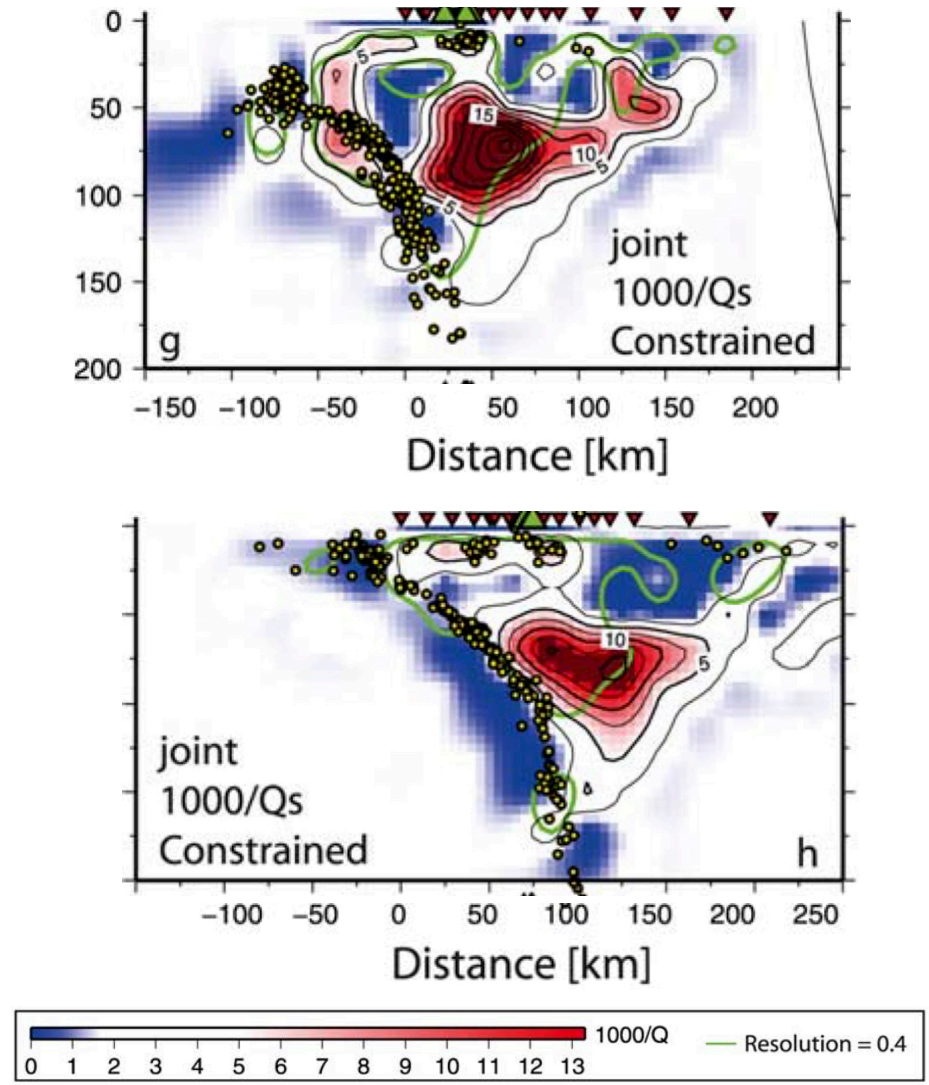


Dalton et al., 2008

Some results: Models for Q in subduction systems

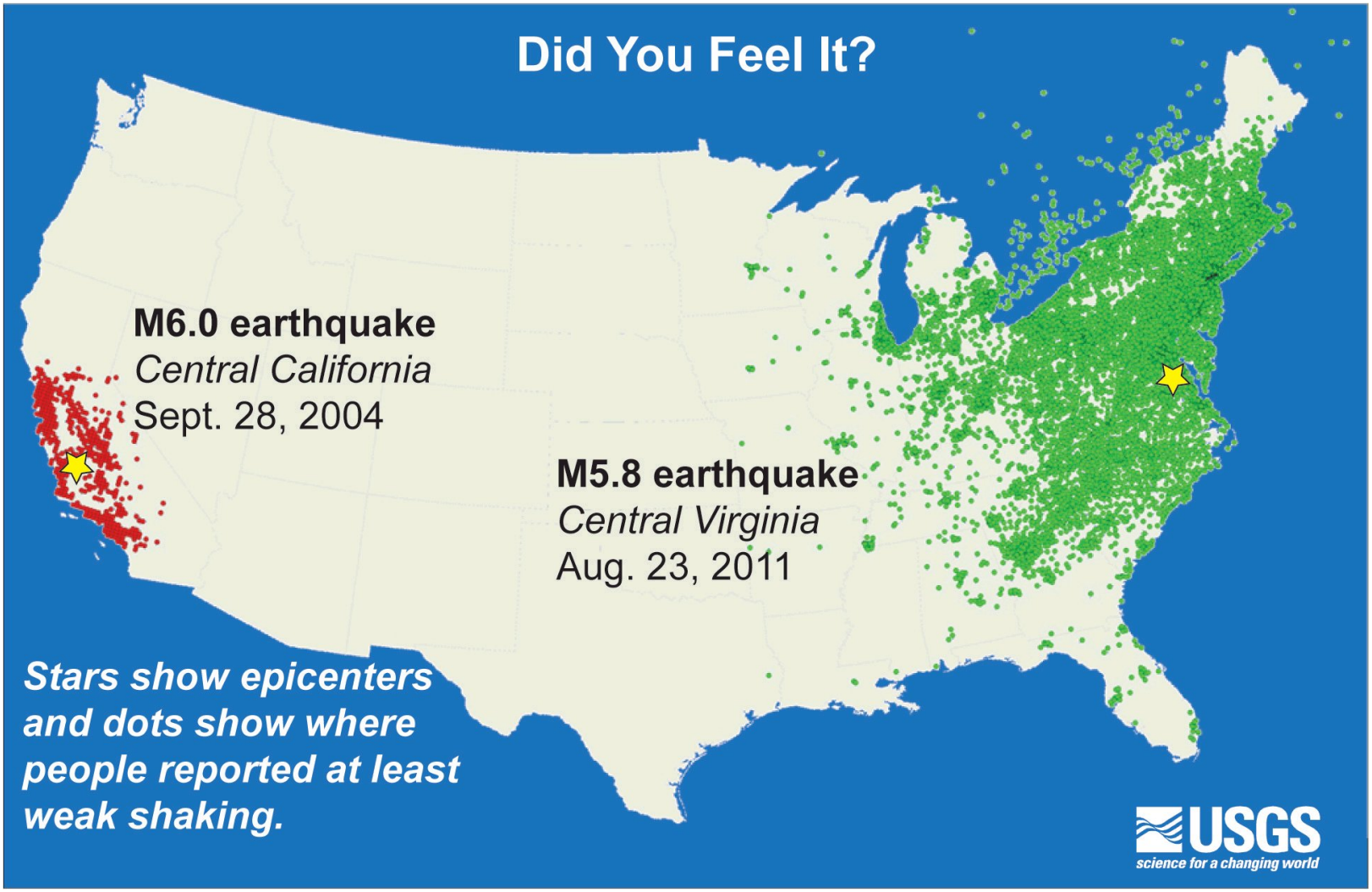


High attenuation zone within the mantle wedge, with a larger low-Q zone beneath Nicaragua (more hydrated melting, perhaps to greater depths).

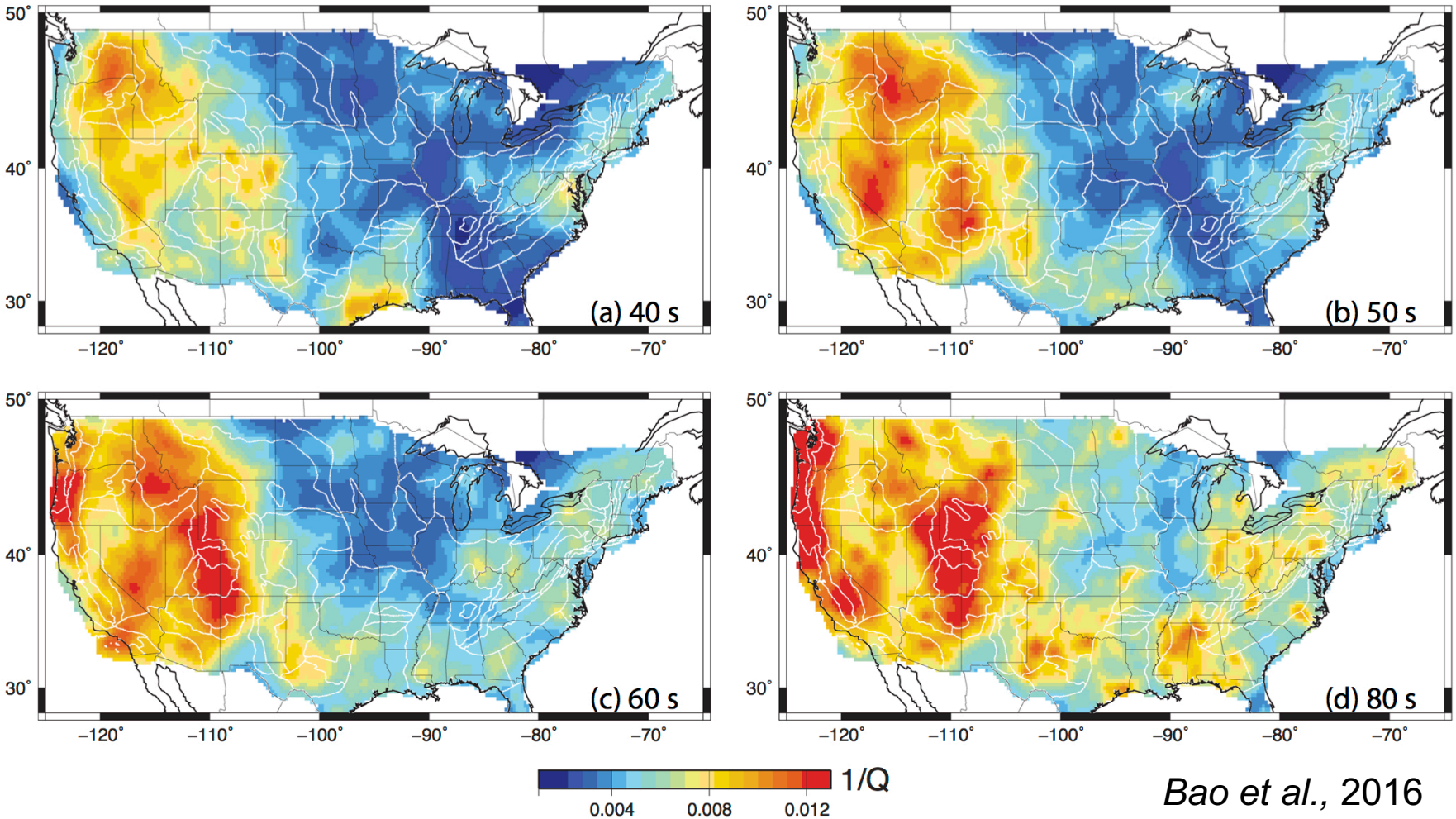


Rychert et al., 2008

Some results: Models for Q beneath continents



Some results: Models for Q beneath continents

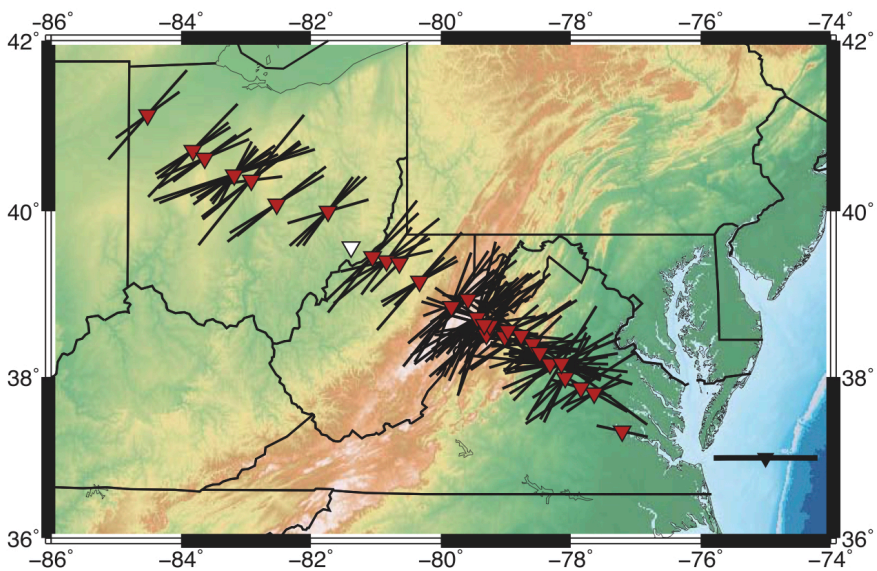


Bao et al., 2016

Rayleigh wave phase velocity maps beneath N. America from USArray data

A parting thought: anisotropy and attenuation

A lot of cutting-edge seismology research gets done by assuming an isotropic, perfectly elastic Earth. Anisotropy and attenuation are challenging to observe/constrain, and challenging to interpret in terms of physical processes in the Earth. However, these observations can yield constraints on the deep Earth that are not available via other methods. Challenging problems, but big potential payoffs!



Anisotropy and attenuation variations across the MAGIC array, Central Appalachians; Aragon et al., 2017; Byrnes et al., in prep.