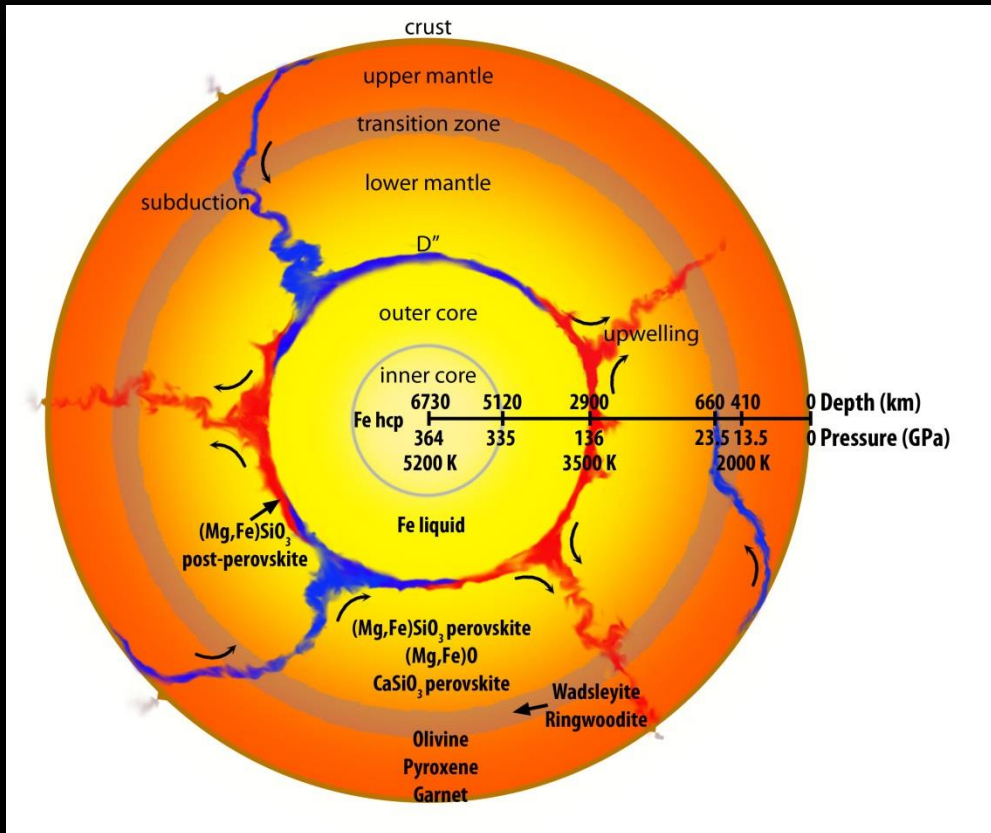# MINERAL PHYSICS 4: MANTLE RHEOLOGY

RHEOLOGY

Marble deformed in torsion (L. Burlini)

- Study of the flow of matter (plastic deformation)

- In geoscience flow of rocks

  - what is the stress strain-rate response?

  - How strong are different rocks

  - Microstructure/texture development during deformation

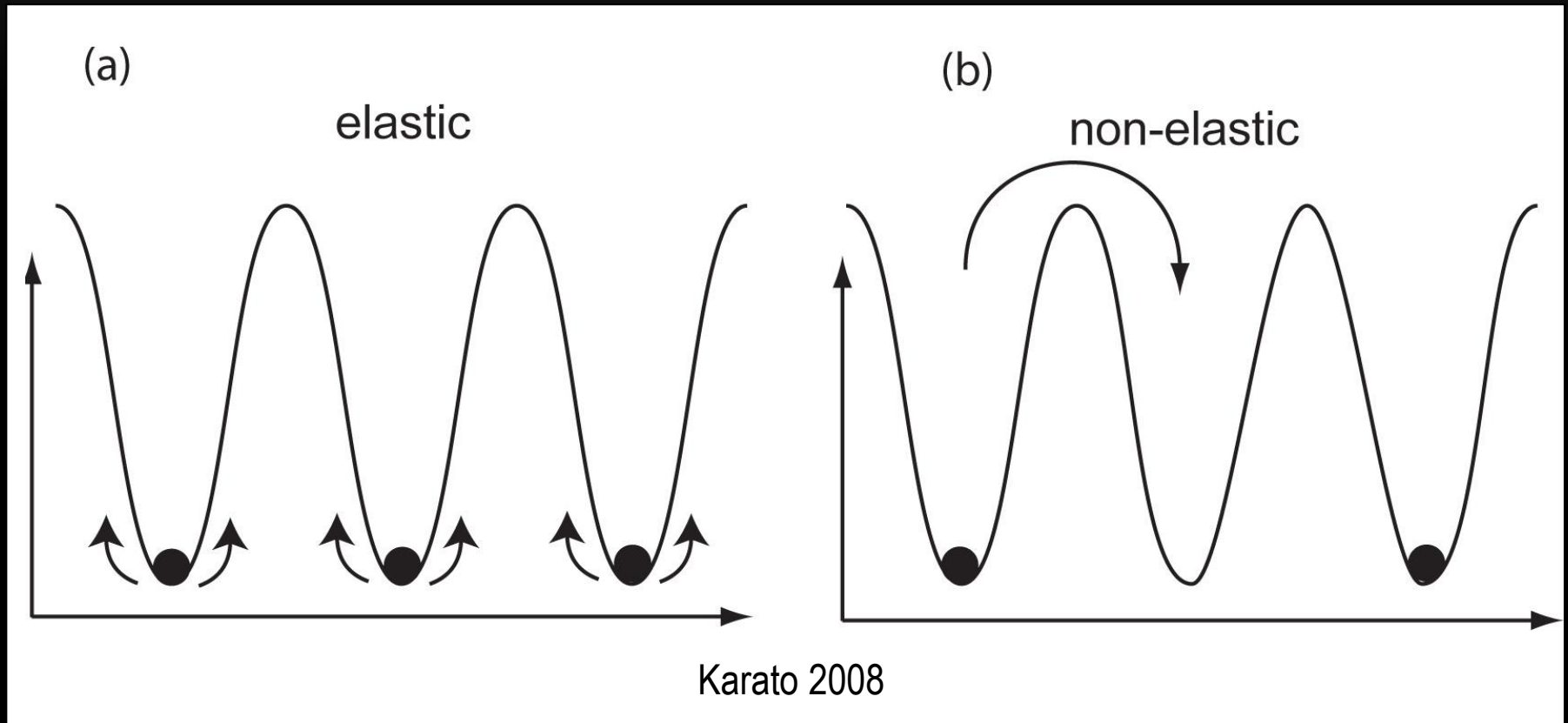# WHY CARE ABOUT DEFORMATION BEHAVIOR IN THE DEEP EARTH?

- What are the controls on slab subduction

- Geochemical reservoirs

- Do things from the very deep earth mix with the upper mantle
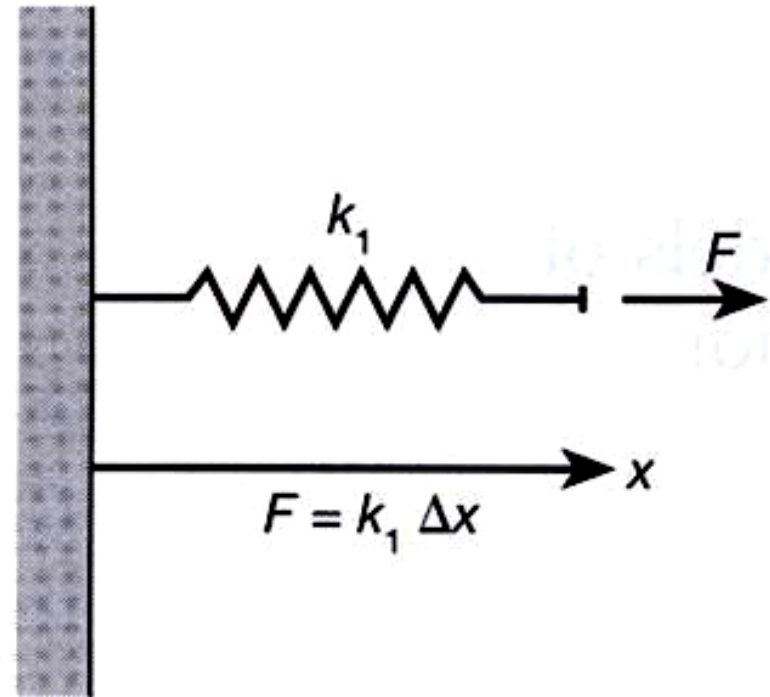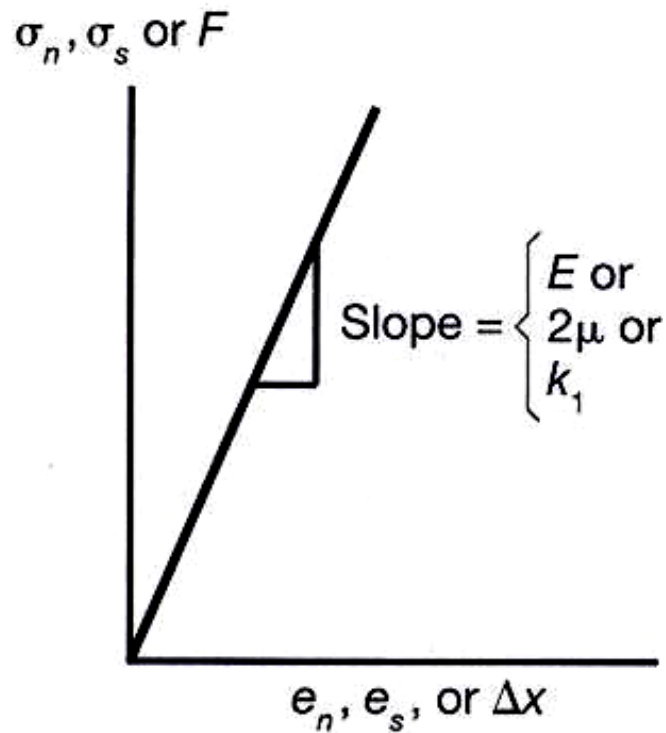
CONSTITUTIVE MODELS

ELASTIC VS PLASTIC DEFORMATION

Karato 2008

Elastic – perturbation of atoms from potential well

Non-Elastic – permanent of atoms i.e. moving from one well to another
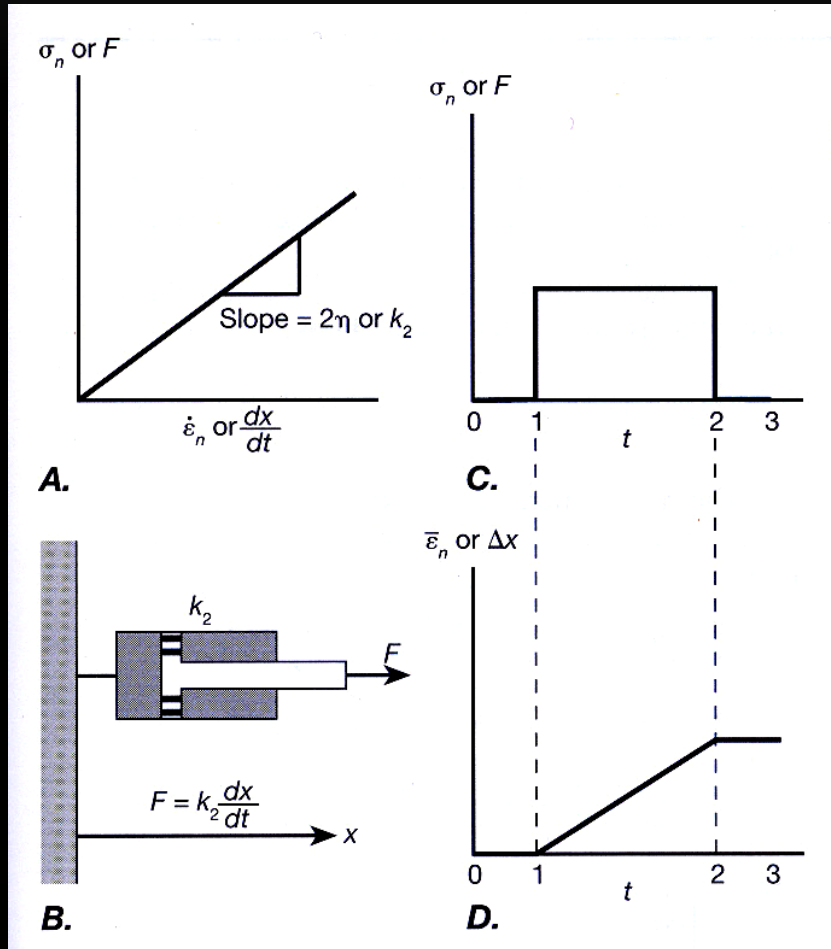
LINEAR ELASTICITY

Twiss and Moores 1992

$$\sigma_n = E\varepsilon_n \qquad \sigma_s = 2G\varepsilon_s$$

CONSTITUTIVE MODELS AND FLOW

Linear (Newtonian) viscous model

Twiss and Moores 1992

- Non-recoverable strain – material deforms in response to σ, but if we remove σ we don't return to initial state

- **Cannot** relate σ to ε
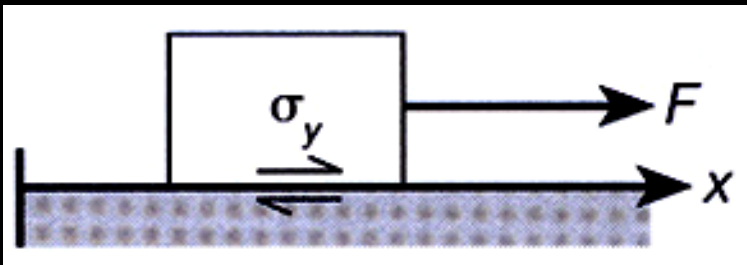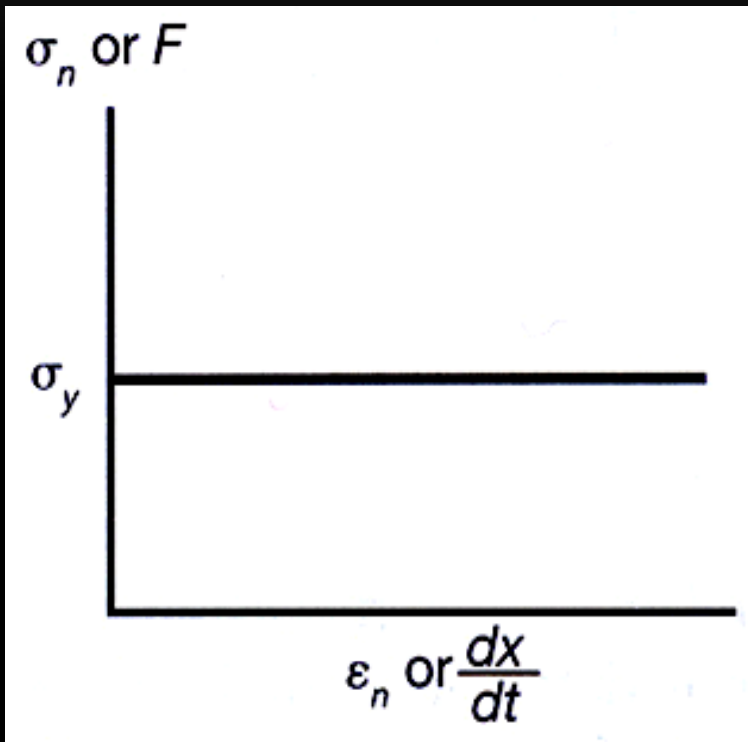
- Instead

$$\Delta\sigma_n = 2\mu\dot{\varepsilon}_n$$

$$\sigma_s = 2\mu\dot{\varepsilon}_s$$

CONSTITUTIVE MODELS AND FLOW
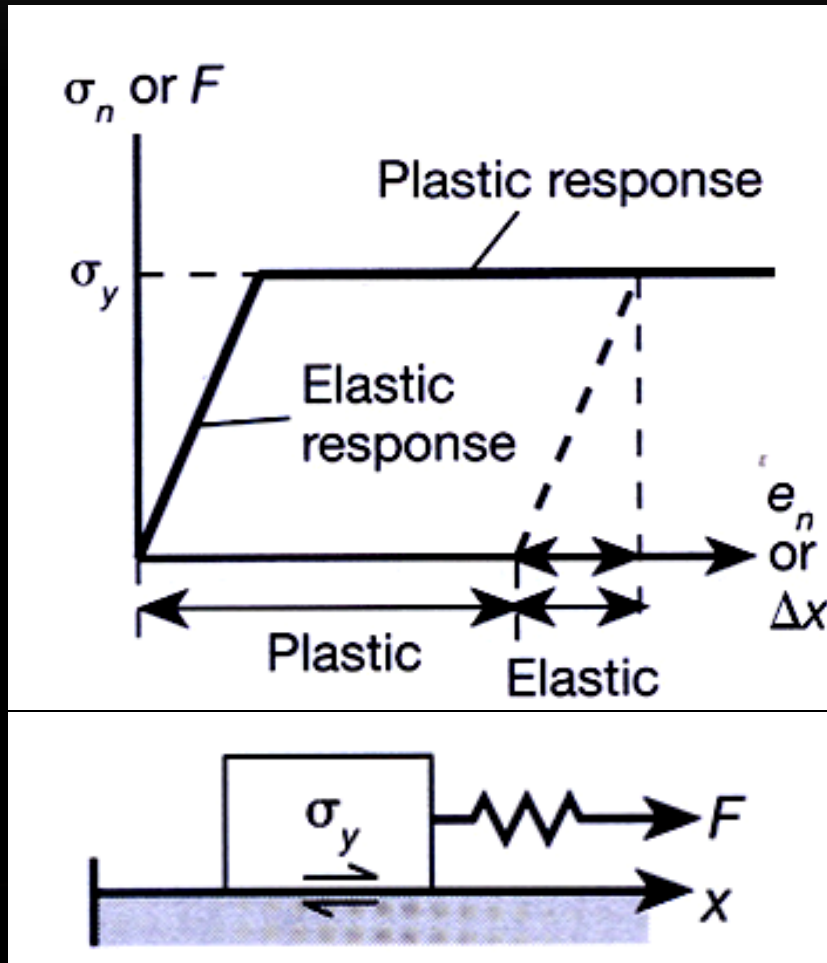
Rigid/Perfectly Plastic model

Twiss and Moores 1992

- Below yield stress – no strain

- Above yield stress – plastic unrecoverable strain

- Stress does not exceed yield strength except during acceleration of deformation
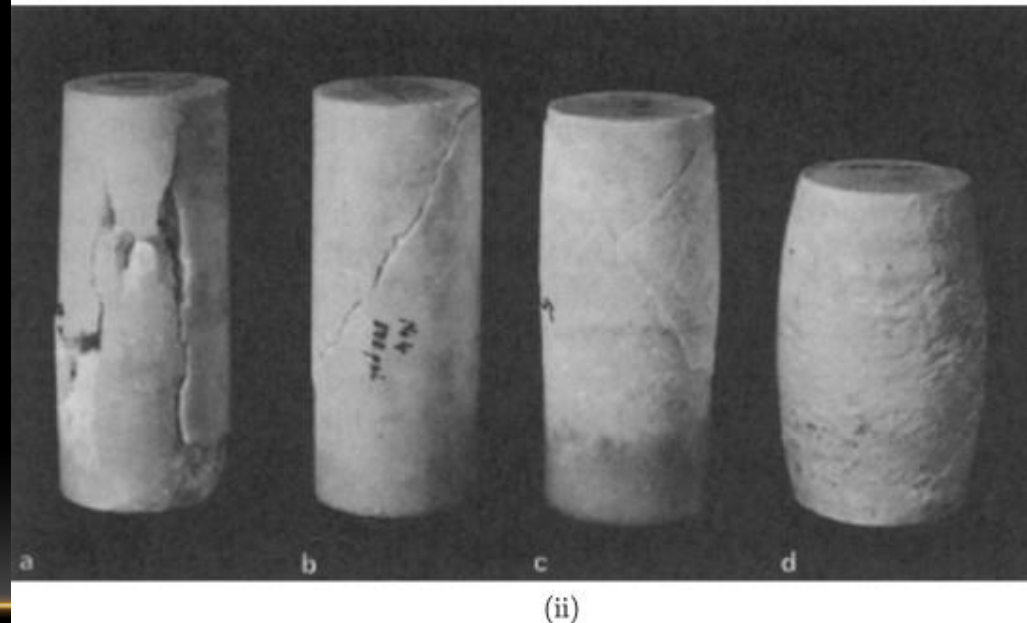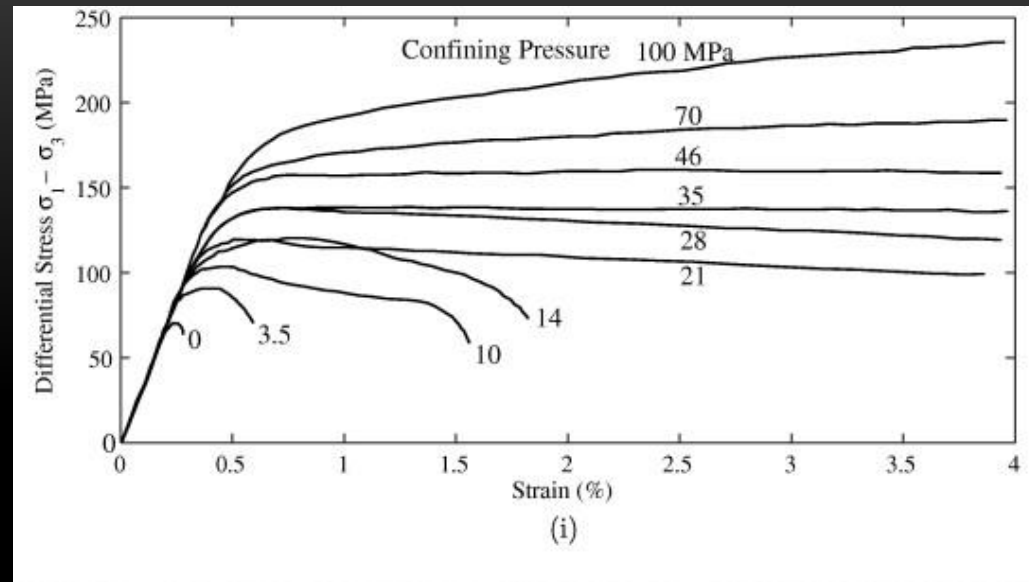
CONSTITUTIVE MODELS AND FLOW

Elasto-plastic (Prandtl) model

Twiss and Moores 1992

- Below yield stress – elastic recoverable strain

- Above yield stress – plastic unrecoverable strain

- Stress does not exceed yield strength except during acceleration of deformation
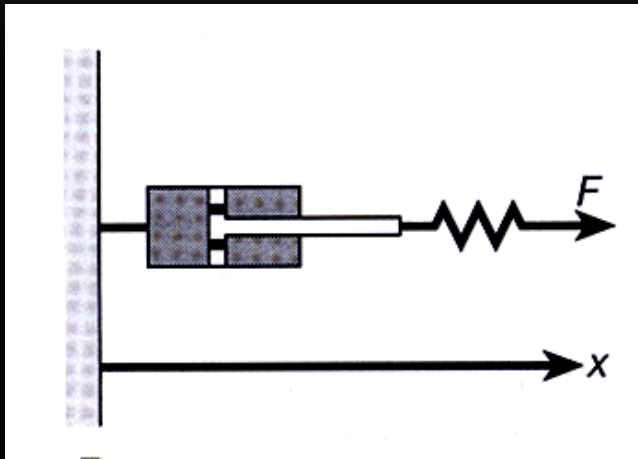
- Many crystal behave this way

BRITTLE VS DUCTILE

- High confining pressure tends to suppress brittle failure

  - Pressure of brittle ductile transition depends on rock type
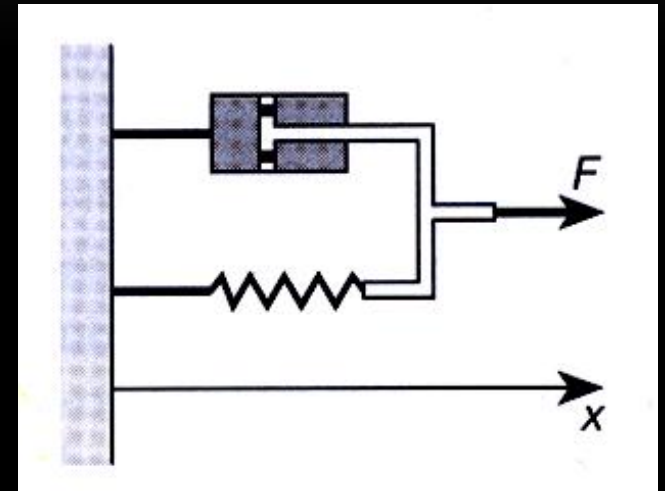
- High temperature suppresses brittle failure

Paterson (1958)

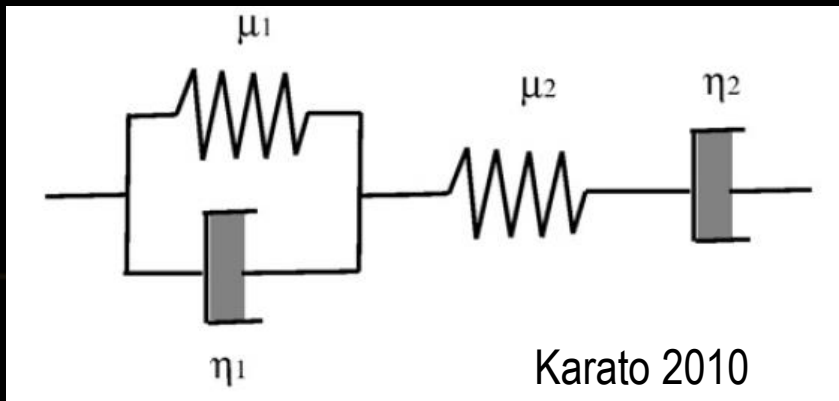# CONSTITUTIVE MODELS AND FLOW: MANY OTHERS

Maxwell model

Twiss and Moores 1992

Kelvin-Voigt Model

Twiss and Moores 1992

Burgers Model

Karato 2010

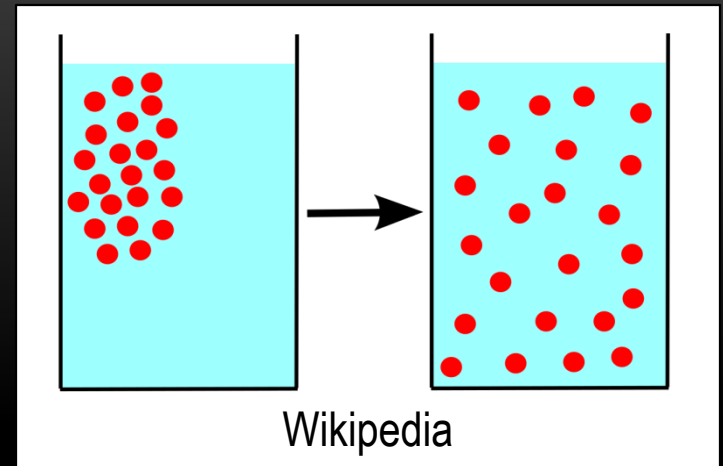# MICROSCOPIC MECHANISMS OF DEFORMATION 1: DIFFUSION

DIFFUSION

Wikipedia

Fick's 1st law

$$J = -D \frac{\partial c}{\partial x}$$

$J \equiv$ flux of atoms $\left(\frac{moles}{m^2 \cdot s}\right)$

$D \equiv$ Diffusion coefficient (m^2/s)

$c \equiv$ concentration atoms per unit volume $\left(\frac{moles}{m^3}\right)$

In 3 D

$$J = -D\nabla c$$

For non-steady state

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

Again in 3 D

$$\frac{\partial c}{\partial t} = D\nabla^2 c$$

NOTE: in the above equations D is assumed to be isotropic. In general it is not! It is a 2nd rank tensor D_{ij}

DIFFUSION LENGTH SCALE

$$D \approx \frac{l^2}{t}$$

In minerals D can range from something like 10^{-10} m^2/s for H diffusion to something like 10^{-21} m^2/s for U-Th
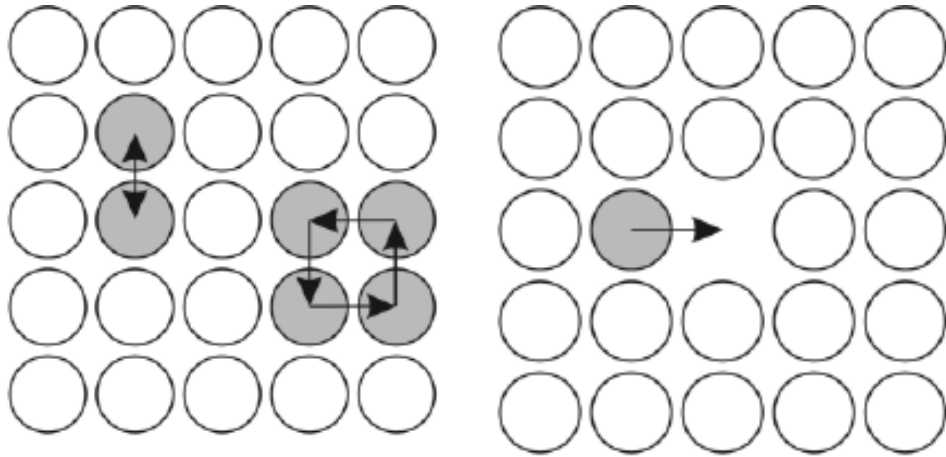
- So H can move 1m in ~300 years – very fast
- U-Th can move ~1mm over the age of the Earth – very slow

In general diffusion length scales are small, i.e. inefficient way to homogenize.

DIFFUSION IN CRYSTALS: THE ROLE OF DEFECTS

Perfect crystal   Vacancy Point Defect

Mühlbacher 2015

We are dealing with the probability to jump from one atomic site to another

- Very difficult in a perfect crystal

- Diffusion is related to defect concentration

$$D = \frac{1}{6}a^2\Gamma$$

$a \equiv length\ scale\ of\ a\ jump$
$\Gamma \equiv frequency\ of\ a\ jump\ (probablity)$

$$\Gamma = \Gamma_f \Gamma_m$$

$\Gamma_f \equiv defect\ fraction$
$(probablity\ of\ finding\ a\ defect)$

$\Gamma_m \equiv probability\ to\ jump$
$when\ a\ defect\ is\ present$

PRESSURE AND TEMPERATURE DEPENDENCE

- Both defect fraction and jump probability have an Arrhenius type temperature dependence
- Pressure tends to decrease vacancies and

Munis 2007
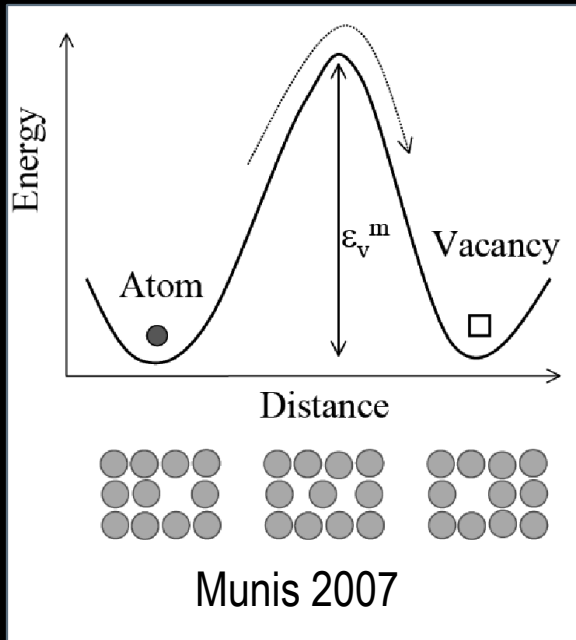
$$D = D_0 exp\left(-\frac{E_a + PV_a}{RT}\right)$$

$D_0 \equiv frequency\ factor$

$R \equiv gas\ constant = 8.314510\ \dfrac{J}{mol \cdot K}$

$E_a \equiv activation\ energy$

$V_a \equiv activation\ volume$
-takes into account the effects of pressure

HOMOLOGOUS TEMPERATURE SCALING
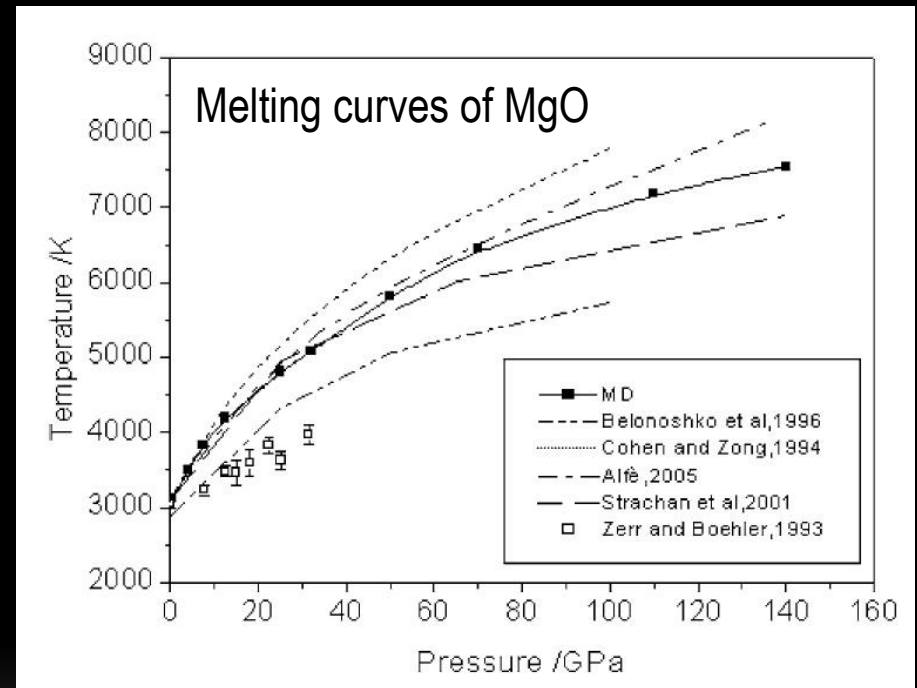
$$D(P,T) = D_0 \, exp\left(-\frac{gT_m(P)}{T}\right)$$

$$g = H^*(P)/RT_m(P)$$

Melting curves of MgO

Liu et al 2006

$T_m(P) \equiv$ Melting temperature at a given pressure

$g \equiv$ dimensionless constant determined empirically
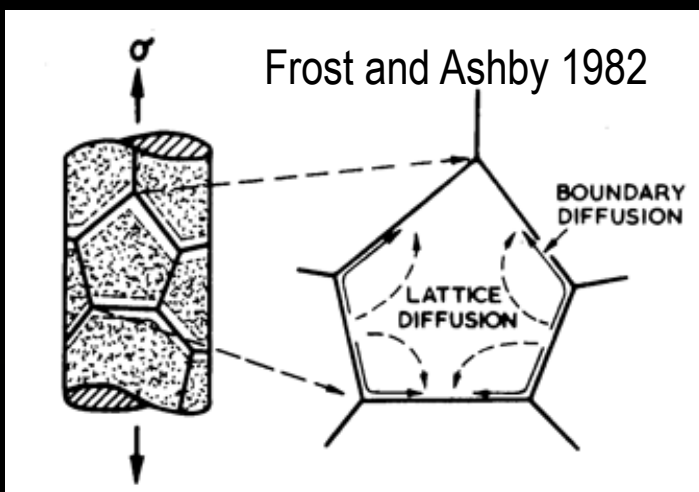
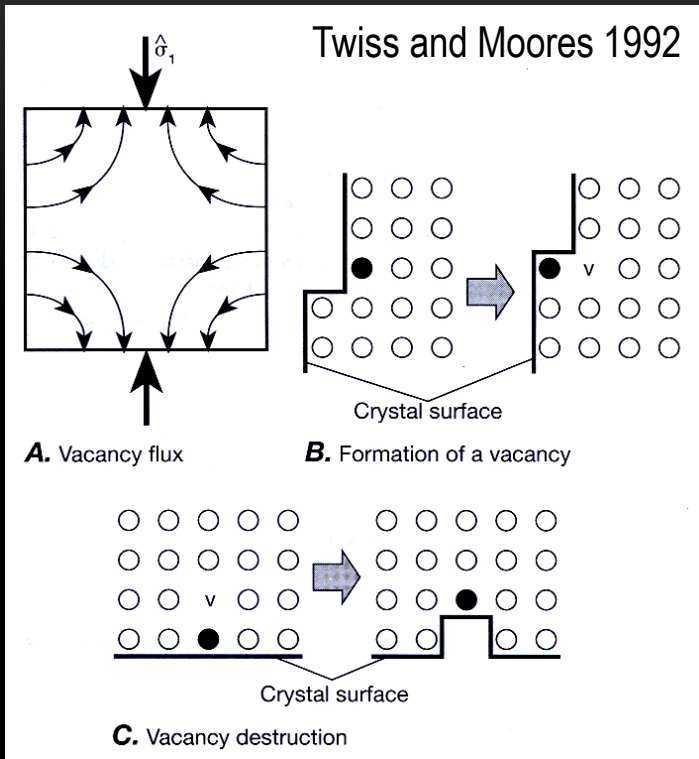$H^*(P) \equiv$ activation enthalpy at a given pressure

$R \equiv$ gas constant

FLOW LAWS 1 :DIFFUSION CREEP

DIFFUSION CREEP

Twiss and Moores 1992

A. Vacancy flux

B. Formation of a vacancy

Crystal surface

C. Vacancy destruction

Crystal surface

Frost and Ashby 1982

BOUNDARY DIFFUSION

LATTICE DIFFUSION

- Newtonian (linear) rheology - $\dot{\varepsilon} \propto \sigma$

- Intracrystalline (Nabbaro-Herring) or along grain boundaries (Coble)

- Efficiency depends on diffusion coefficients and grain size

- Dominates at low stress and small grain size

  - low strain rate and high temperature

NABARRO (1948), HERRING (1950): DIFFUSION CREEP

- Non-hydrostatic stress causes differences in vacancy concentrations of crystal surfaces

- Vacancies move from low compressive stress to high compressive stress and atoms move from high compressive stress to low
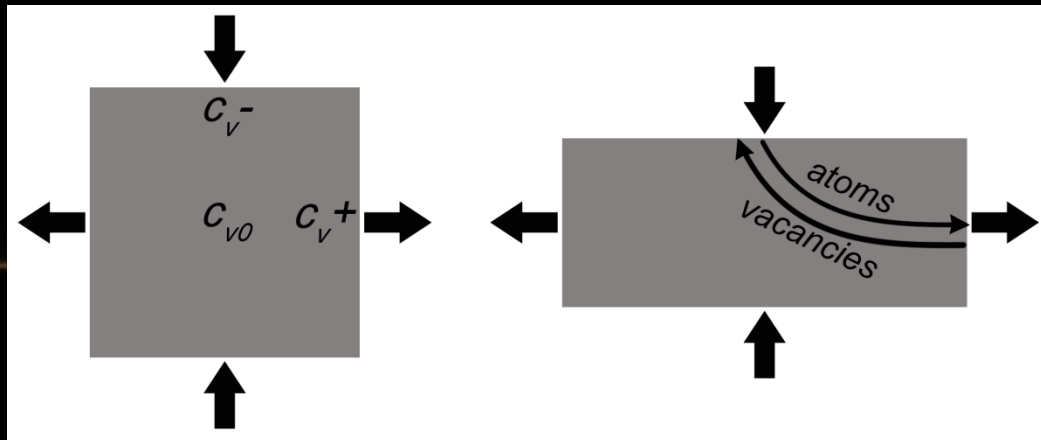
$$J = -D_v \nabla c$$

$$D_v \equiv diffusion\ coefficent\ of\ vacancies$$

$$J \approx \alpha D_v \frac{c_v^+ - c_v^-}{h}$$

$$\alpha = geometric\ factor - (grain\ shape\ and\ diffusion\ path)$$
$$h = diffusion\ length\ scale\ (grain\ size)$$

NABARRO (1948), HERRING (1950): DIFFUSION CREEP

So our change in length per second is

$$\frac{\Delta h}{dt} = \frac{\phi \Omega}{h^2}$$

$$\phi \equiv number\ of\ vacancies\ /sec = Jh^2$$

$$\Omega \equiv \text{atomic volume}$$

And strain rate ends up looking like

$$\dot{\varepsilon} = \alpha D_v \frac{c_v^+ - c_v^-}{h}\frac{\Omega}{h}$$

Since vacancy concentration has an Arrenius type relationship

$$\dot{\varepsilon} \approx \alpha \frac{D_v c_{v0} \Omega}{h^2}\left(\frac{\sigma \Omega}{RT}\right)$$

$$\sigma \equiv \sigma_1 - \sigma_3$$

Relating the diffusion coefficient for self-diffusion to vacancy diffusion we end up with

$$\dot{\varepsilon} = \frac{\alpha D \sigma \Omega}{h^2 RT}$$

If GBS→impossible $\alpha = 16$
If GBS→possible $\alpha = 40$

COBLE CREEP

Frost and Ashby 1982

- Assumes grains are spherical and GBS is possible

- Diffusion creep where diffusion occurs along grain boundaries
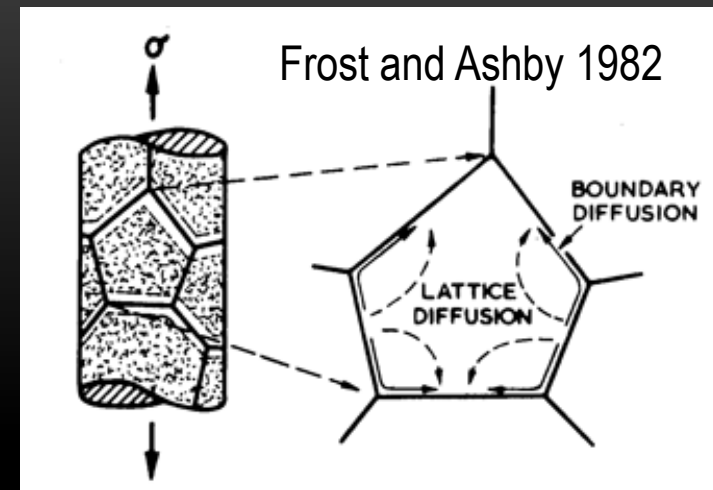
$$\dot{\varepsilon} = \frac{\alpha D_{gb} \delta \sigma \Omega}{\pi h^3 RT}$$

$$D_{gb} \equiv diffusion\ coefficient\ of\ grain\ boundary$$

$$\delta \equiv thichness\ of\ grain\ boundary, typically \sim atomic\ length\ scale$$
$$\alpha = 148$$

Note:
N-H and Coble really happen in tandem

MICROSCOPIC MECHANISMS OF DEFORMATION 2: DISLOCATIONS

THEORETICAL SHEAR STRENGTH VS EXPERIMENTAL

- Frenkel 1926

    – ideal shear strength of a crystal

$$\tau_m = \frac{G}{2\pi} \frac{b}{a}$$

$G \equiv shear\ modulus$

- if $b \sim a$ then

$$\tau_m = \frac{G}{2\pi}$$

For metals shear modulus G is in the range of 20-150 GPa so τ_m should be ~3-30 GPa but experiments show more like 0.5-10 MPa!
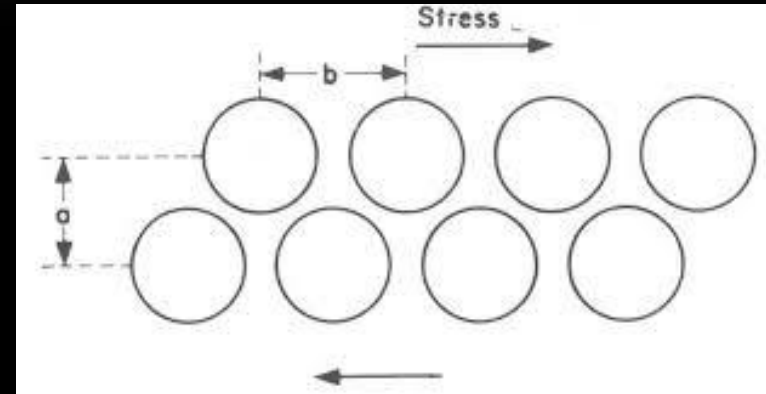
CONCEPT OF DISLOCATIONS
TAYLOR (1934), OROWAN (1934), POLANYI (1934)

Taylor (1934)

Positive Dislocation.
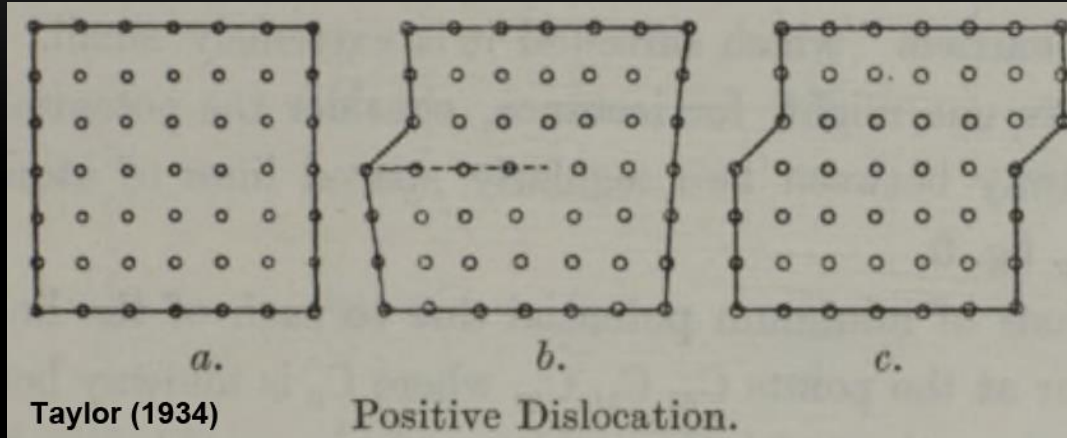
Dislocations in dolomite

http://faculty.wiu.edu/K-Hall/CM/Dislocations.htm
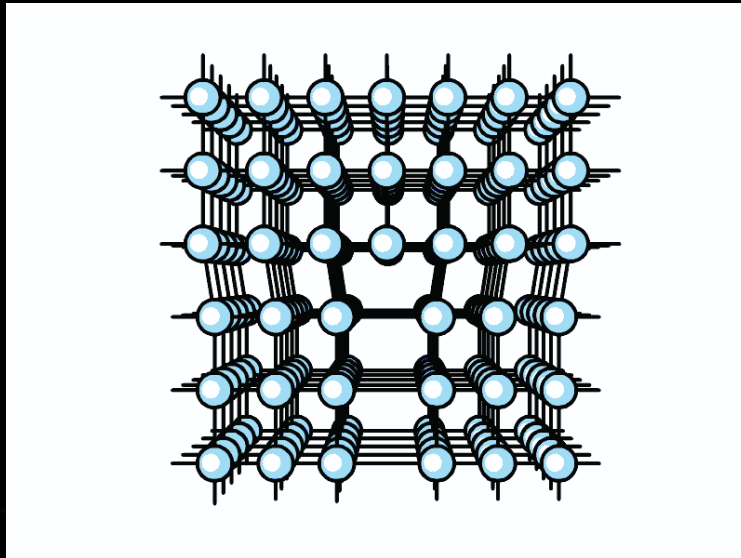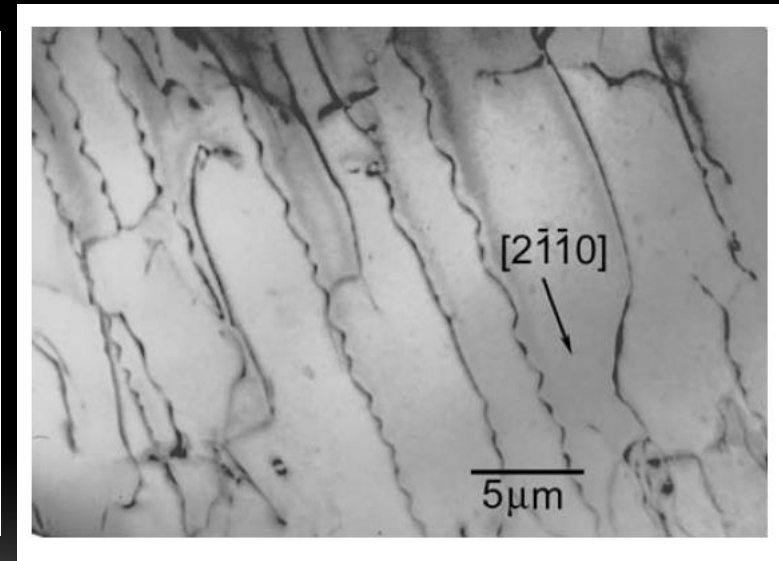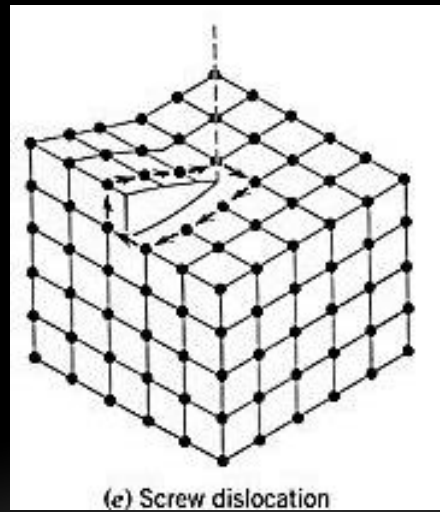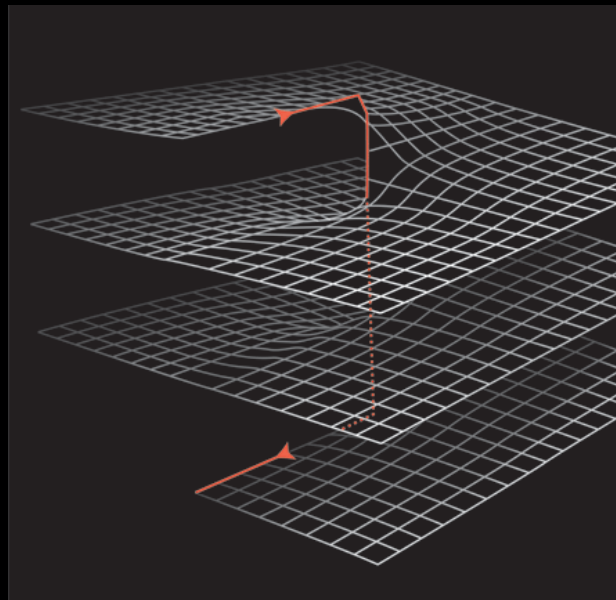
SCREW DISLOCATION

Barber et al 2007

MIXED DISLOCATIONS
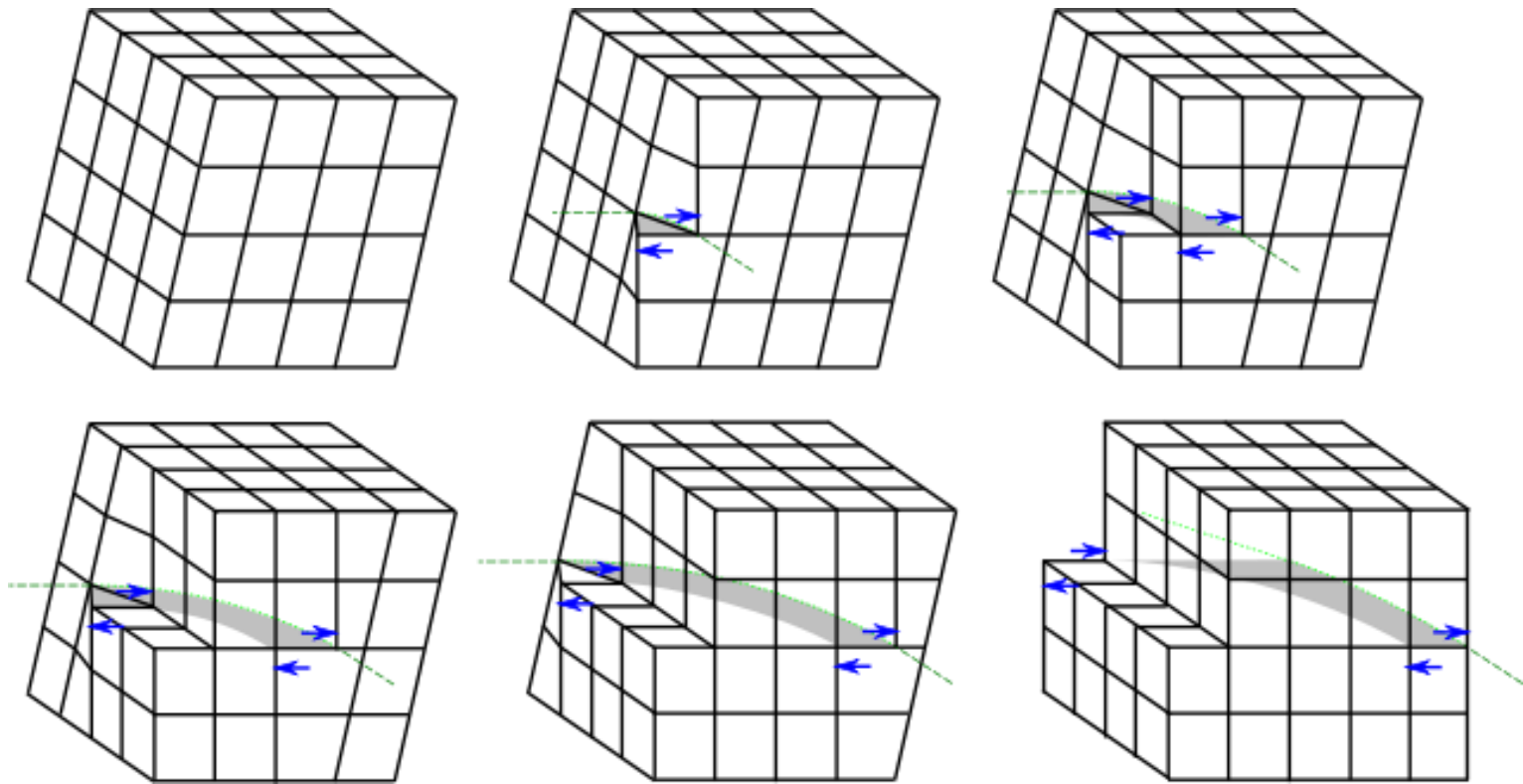
PEIERLS STRESS/LATTICE FRICTION, PEIERLS (1940)

- Frequently called Peierls-Nabarro model

  - Peierls made algebraic error and had an extra 2 in the exponent

  - Nabarro corrected this in 1947

*"All I did was formulate and solve the equations representing his [Orowan] model.
I had wanted this to be a joint paper with Orowan, but he refused
The matter did not seem important at the time and I did not take more interest in
the problem of dislocations"*

$$\sigma_p = \frac{2G}{w} exp\left(- \frac{2\pi}{w} \frac{d_{hkl}}{\overline{b}} \right)$$

$w \rightarrow depends\ on\ dislocation\ character$

$w = 1\ for\ screw\ dislocations$

$w = 1 - v\ for\ edge\ dislocations, where\ v \equiv Poisson\ ratio$

$G \equiv shear\ modulus$

$d_{hkl} \equiv spacing\ of\ the\ glide\ plane\ (lattice\ plane\ spacing$
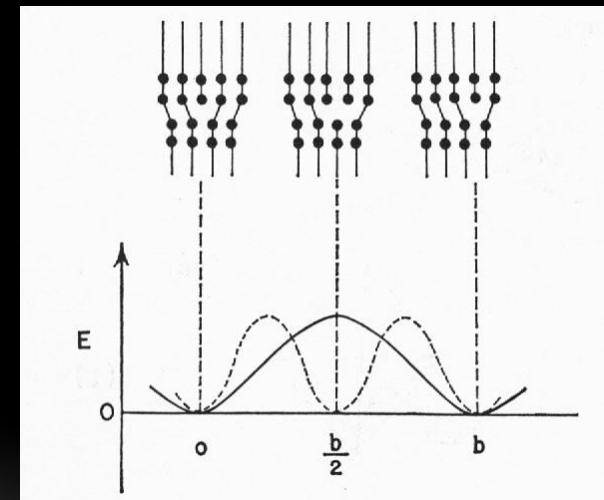
FLOW LAWS 2 :DISLOCATION GLIDE AND CREEP

DEFORMATION MECHANISMS AT HIGH P-T DISLOCATION GLIDE AND CREEP

http://faculty.wiu.edu/K-Hall/CM/Dislocations.htm
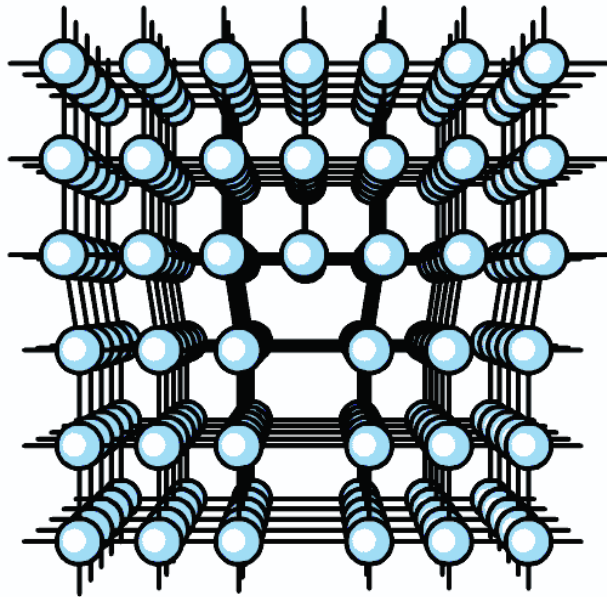
- Glide is governed by "lattice friction" Peierls Stress – stress needed to move a dislocation without thermal activation

- Slip system {hkl}<uvw>

- Dislocation creep – glide + climb (diffusion)

- Non linear - $\dot{\varepsilon} \propto \sigma^n$

  - For silicates and oxides n ~3-5

DISLOCATION GLIDE

- Average shear strain in the crystal multiple (N#) of parallel dislocations

- $\varepsilon_{avg} = N \dfrac{\overline{b}}{d} \dfrac{\Delta L}{L}$

- And if we deal with one unit of depth we get

  - $\varepsilon_{avg} = \dfrac{N}{V} \, \overline{b} \Delta L$

- Define dislocation density as the number of dislocations per volume $\rho = \dfrac{N}{V}$

  - $\varepsilon_{avg} = \overline{b} \rho \Delta L$

- Take the time derivative to get strain rate

- $\dot{\varepsilon} = \overline{b} \rho \dfrac{d\Delta L}{dt}$

- Since $\dfrac{d\Delta L}{dt}$ is velocity
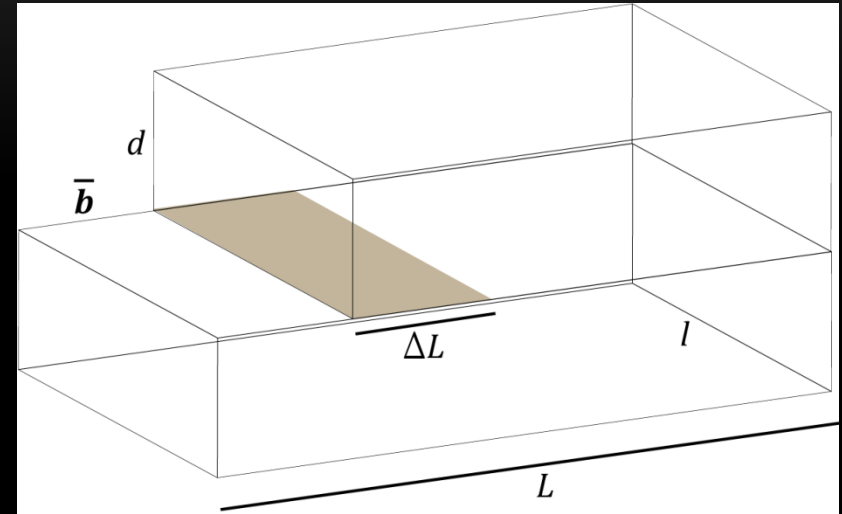
Orowan equation
$$\dot{\varepsilon} = \rho \overline{b} v$$

(a) Dislocation

(b) Pinning

2 μm

DISLOCATION CREEP

Poirier 1985
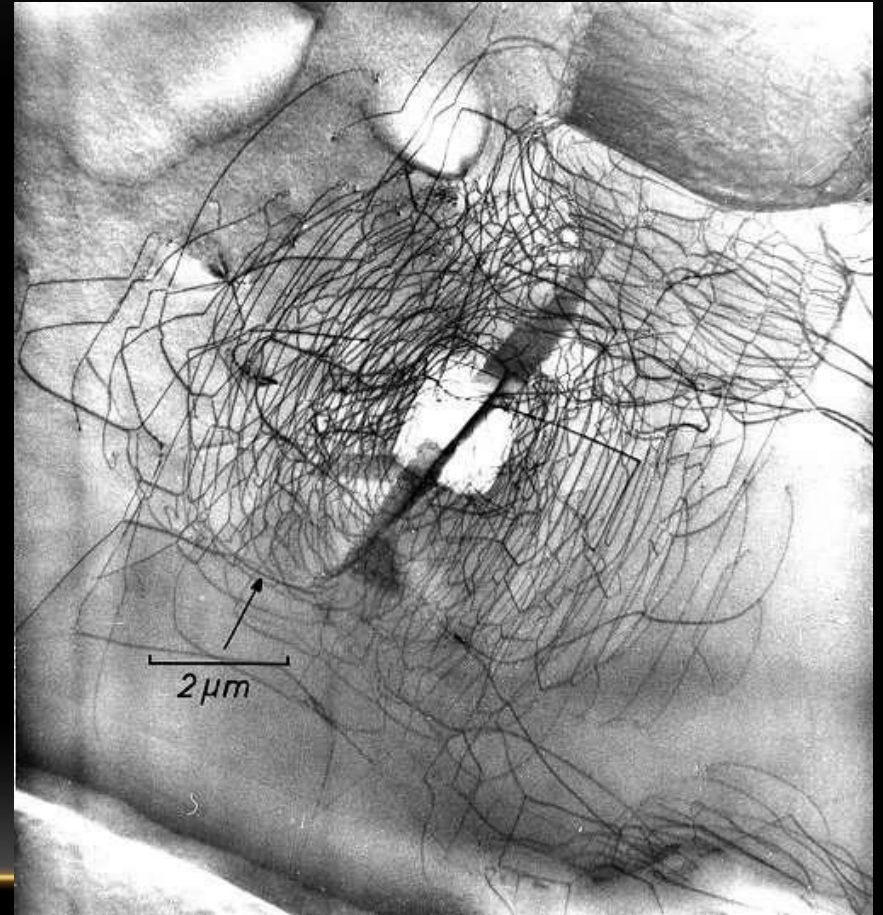
- Dislocation creep – glide + climb (diffusion)

- At high temperature climb allows dislocations to climb around obstacles by absorbing or emitting vacancies

Rewrite velocity in terms of time to glide t_g and time to climb t_c

$$v = \frac{\Delta L}{t_g + t_c}$$

For climb controlled

$$v = \frac{\Delta L}{t_g + t_c} \approx \frac{\Delta L}{t_c} = \Delta L \left(\frac{v_c}{d_{hkl}} \right)$$

Then the Orowan looks like

$$\dot{\varepsilon} = \rho \overline{b} v_c \left(\frac{\Delta L}{d_{hkl}} \right)$$

WEERTMAN MODEL(1968, 1972)

Modified from Karato 2008
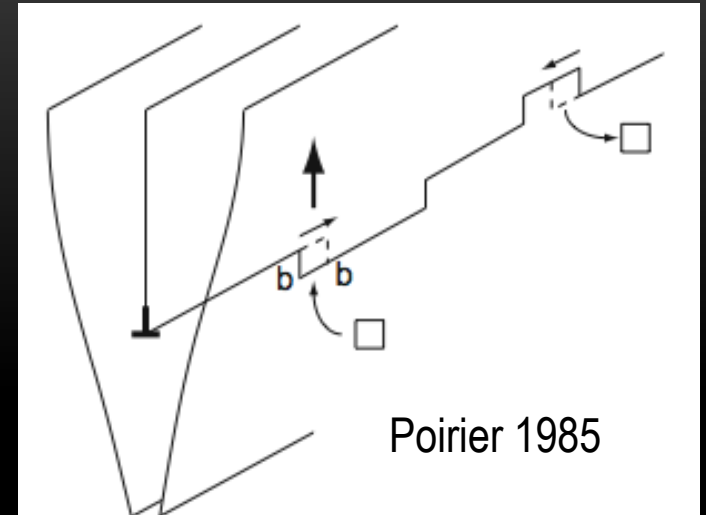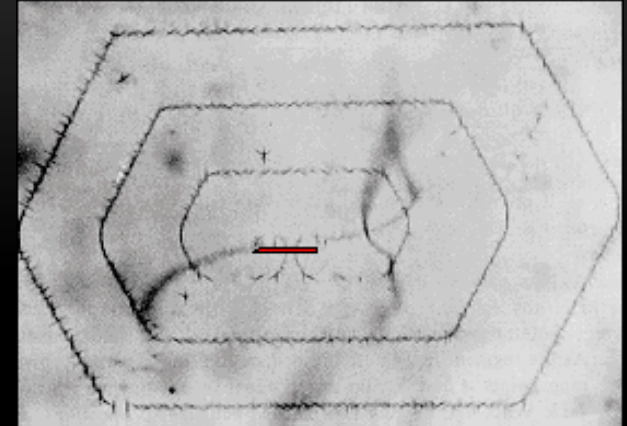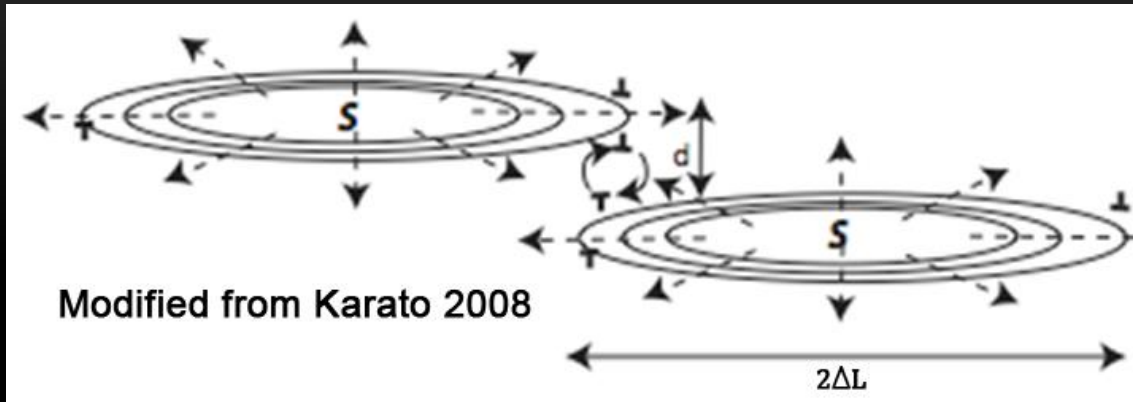
Frank Reed Source in Si
Barrett, Nix, and Tetelman 1973

- Treat dislocations as loops that propagate out (like Frank Read source)

- Loops that are a distance of d apart can climb into each other and annihilate (recovery) – dipole

- Assumes balance of dislocation formation and recovery – steady state.

WEERTMAN MODEL(1968, 1972)

Modified from Karato 2008
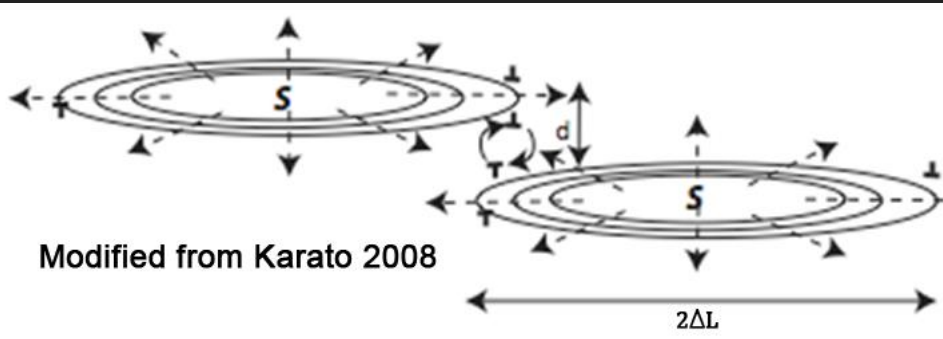
the density of dislocation sources (M) is

$$M = \frac{1}{\pi \Delta L^2 d_{hkl}}$$

assuming that dislocations become mobile only when a dipole annihilates (steady state)

$$\rho = M\left(\frac{4\pi \Delta L^2}{d_{hkl}}\right) = \left(\frac{4}{d_{hkl}^2}\right)$$

Putting this into climb controlled Orowan gives

$$\dot{\varepsilon} = \left(\frac{4}{d_{hkl}^2}\right)\overline{b}\frac{\Delta L}{d_{hkl}}\nu_c = 4\overline{b}\frac{\Delta L}{d_{hkl}^3}\nu_c$$

Simplifying

$$\dot{\varepsilon} = \frac{4\overline{b}}{(\pi M)^{1/2}(d_{hkl})^{7/2}}\nu_c$$

Climb velocity is to the diffusion coefficient and stress so

$$\nu_c \propto D\sigma$$

and

$$\sigma = G\frac{\overline{b}}{d_{hkl}}$$

$$\dot{\varepsilon} = \alpha\frac{D}{\overline{b}^{3.5}M^{0.5}}\left(\frac{\sigma}{G}\right)^{4.5}\frac{G\Omega}{RT}$$

**Note: No grain size dependence
No single viscosity**

GENERAL FLOW LAW: DISLOCATION AND DIFFUSION CREEP

$$\dot{\varepsilon} = A \left(\frac{\sigma}{G}\right)^n \left(\frac{b}{h}\right)^m exp\left(-\frac{E_a + PV_a}{RT}\right)$$

$A \equiv \text{Pre exponential factor} - \text{determined empirically}$

$b \equiv \text{lattice spacing}$

$h \equiv \text{grain size}$

- Diffusion creep n=1 and m=2.5

- For dislocation creep n=3-5 and m=0

- We have $\dot{\varepsilon} \propto \sigma^n \; vs \; \dot{\varepsilon} \propto \sigma$

- If stress is small then diffusion dominates because $\dot{\varepsilon}$ becomes v.v. small if n>1

- Diffusion time scale increase as the square of length/grain size ($D \approx \frac{l^2}{t}$) so need small grain size

- If temperature is high diffusion become more efficient because the diffusion coefficient increases exponentially

WHERE DO YOU EXPECT DISLOCATION CREEP VS DIFFUSION CREEP
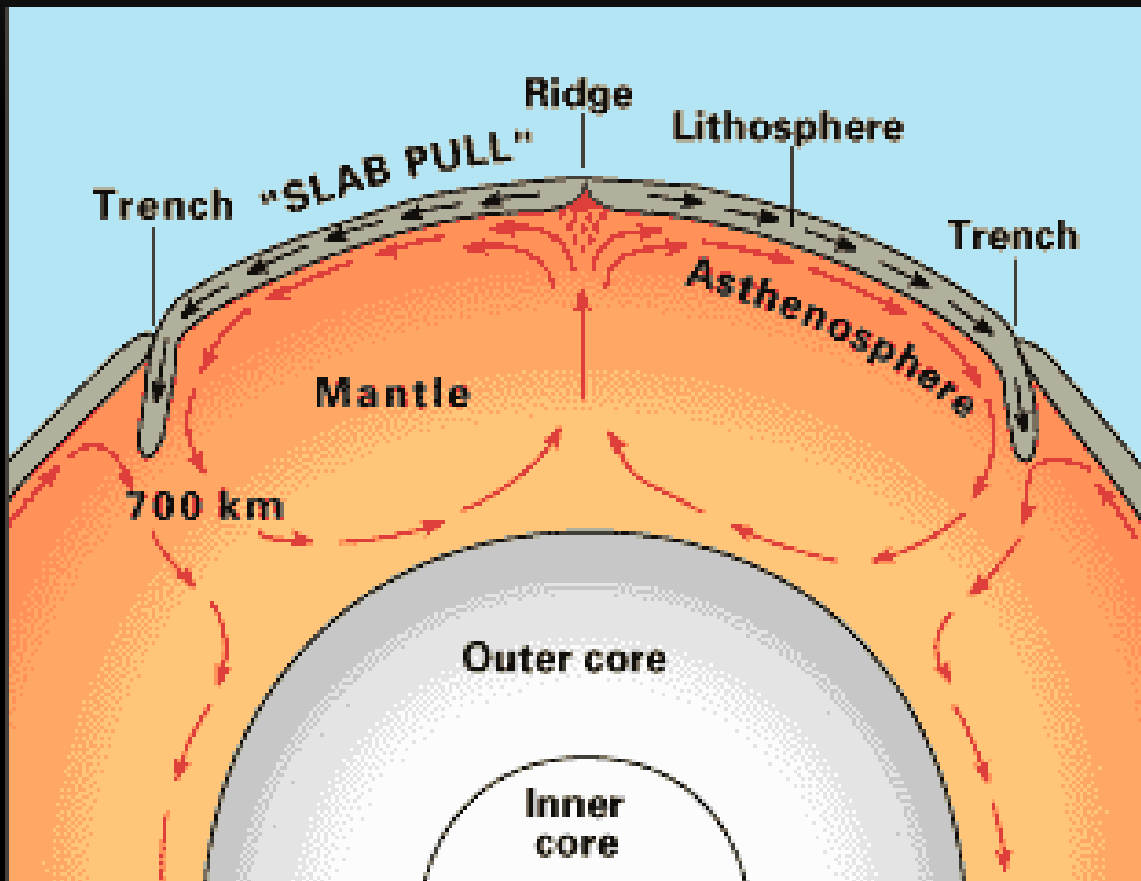
USGS

EFFECTS OF PARAMETERS IN THE FLOW LAW

King 2017
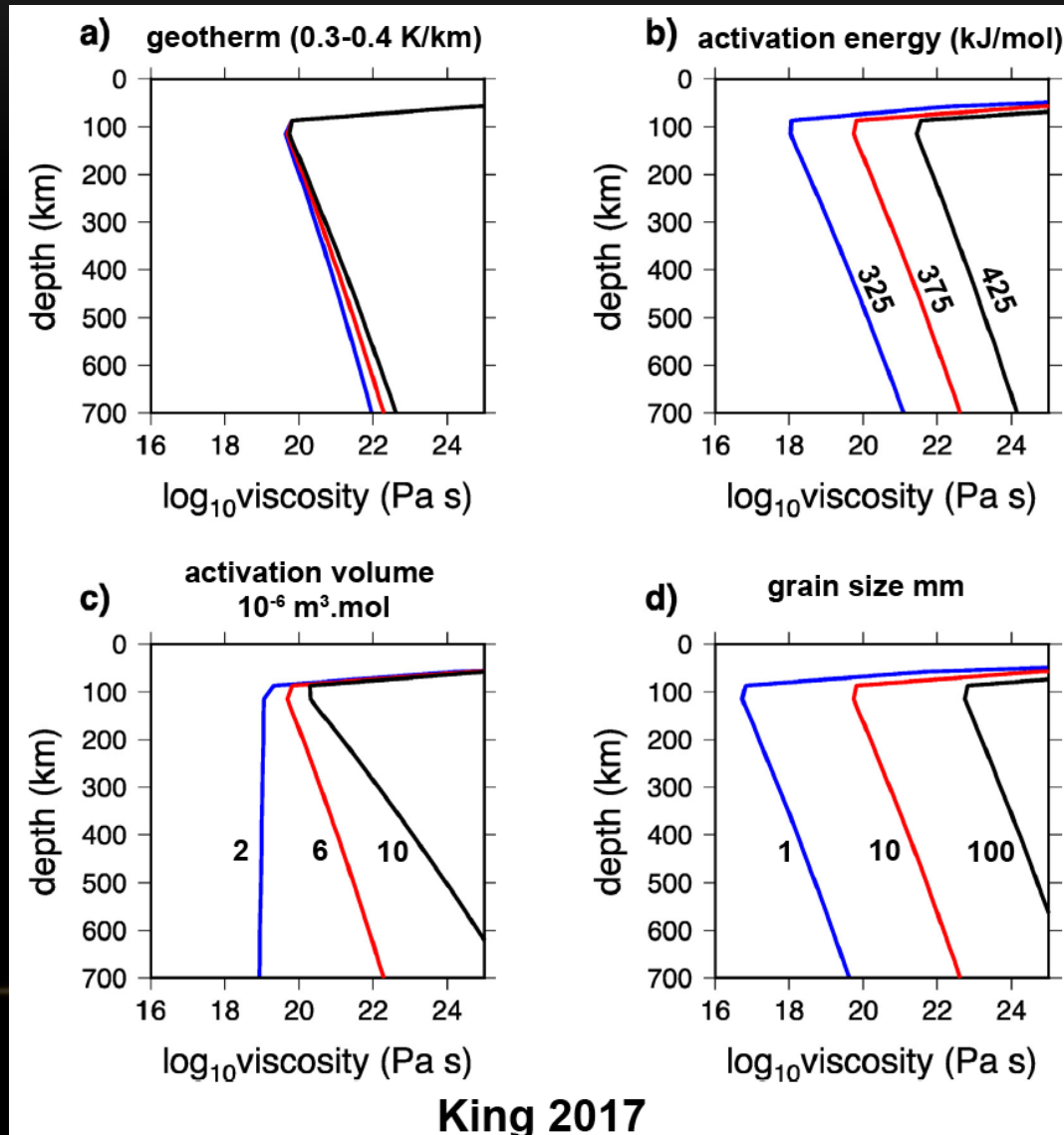
FLOW LAWS 3: DEFORMATION MECHANISM MAPS

CONSTRUCTION OF DEFORMATION MECHANISMS MAPS
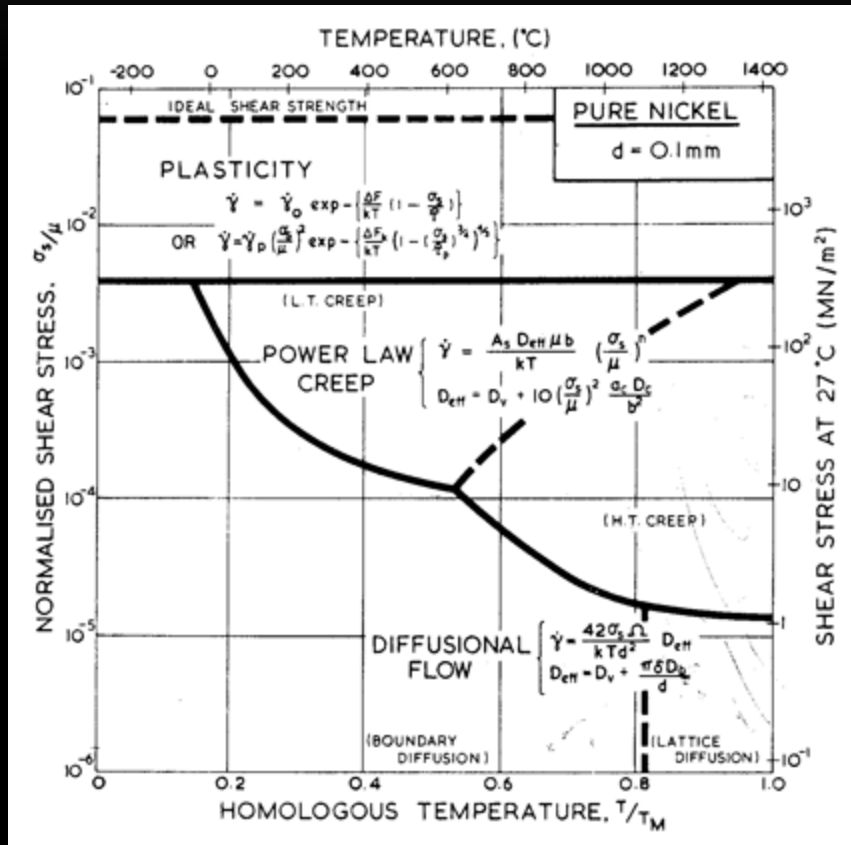
- Semi empirical – compile material properties and fit data to flow laws

- In general the mechanisms that yields the fastest strain rate for a given set of conditions dominates

- Field boundaries are where two mechanisms have equal rates

Frost and Ashby 1982

DEFORMATION MECHANISMS MAPS FOR OLIVINE

Frost and Ashby 1982
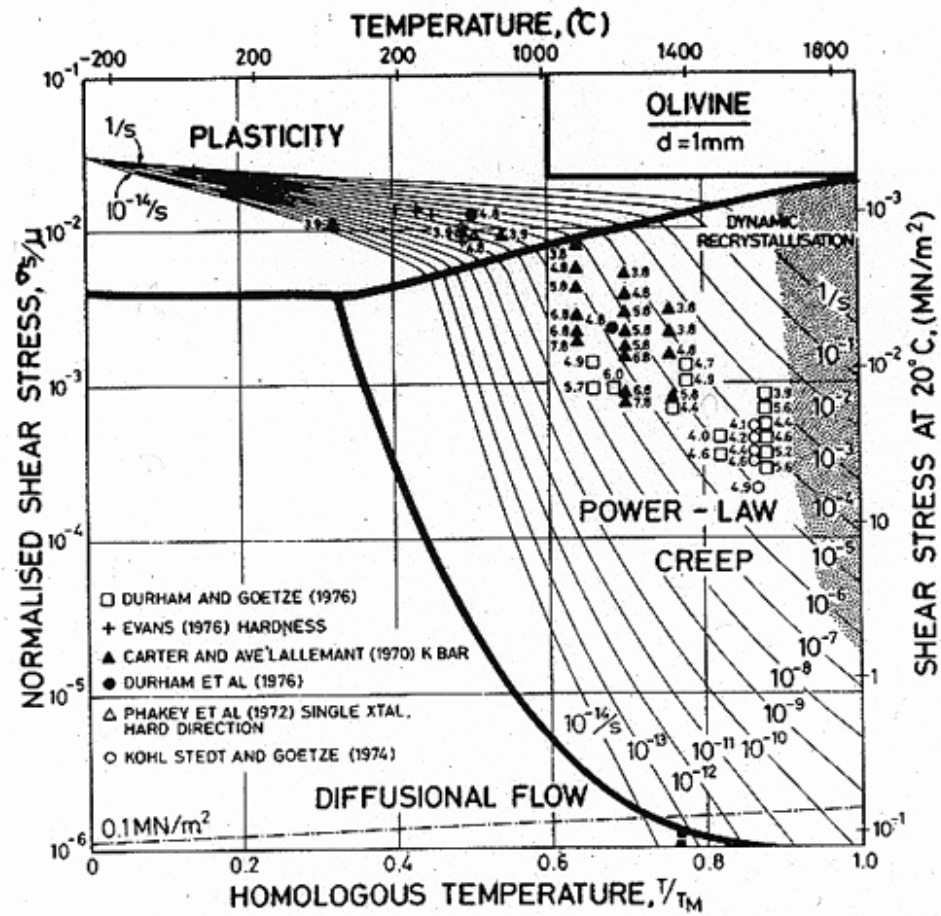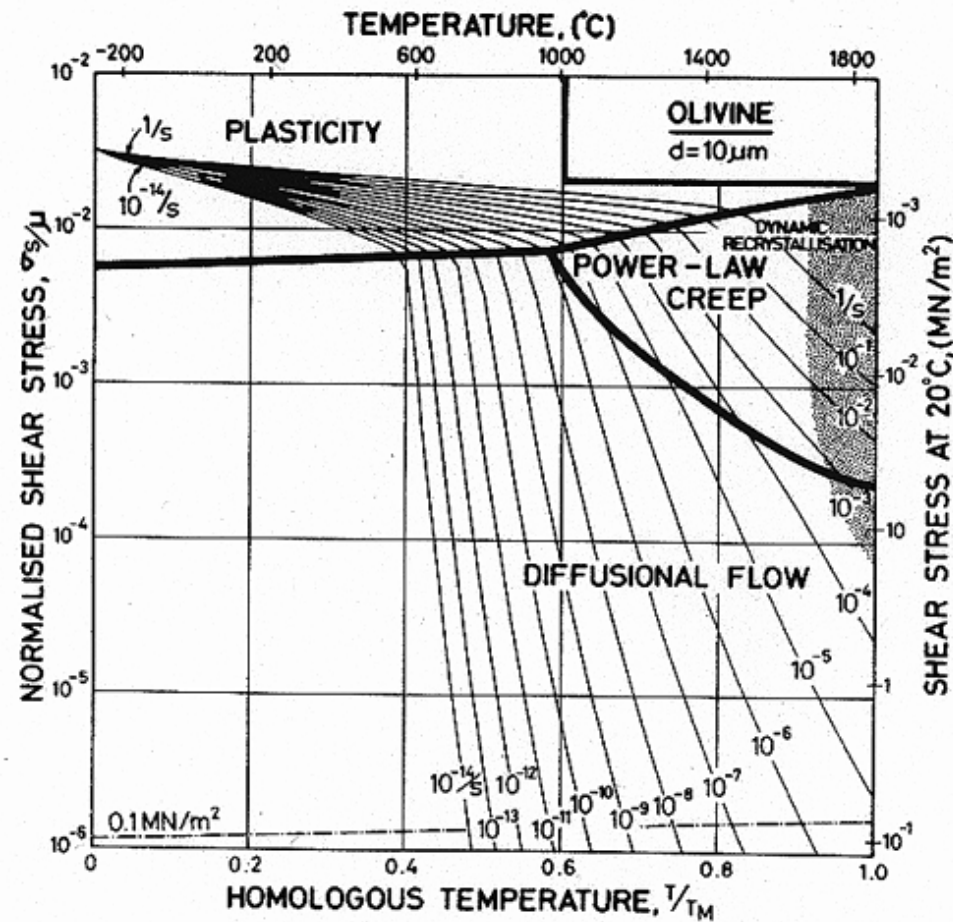
DISLOCATIONS: TEXTURE AND ANISOTROPY

TEXTURE IN GLACIER SHEETS

University of Copenhagen Center for Ice and Climate, Niels Bohr Institute
http://www.iceandclimate.nbi.ku.dk/research/flowofice/ice_crystal_structure/

SCHMID'S LAW

Schmid and Siebel, "Mg" (1931)

Zn single crystal
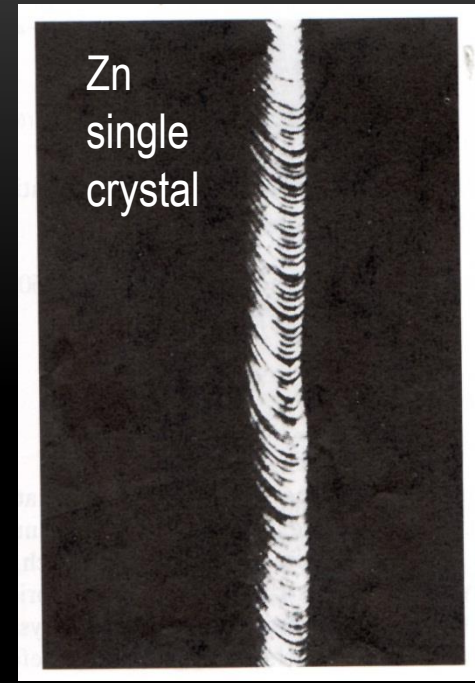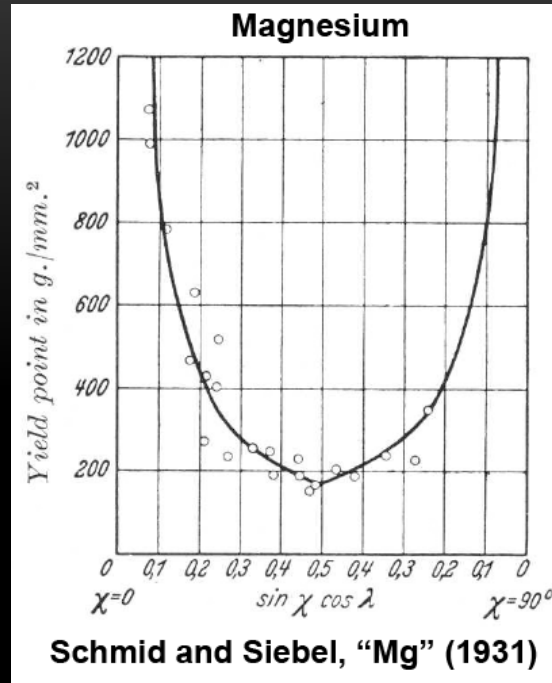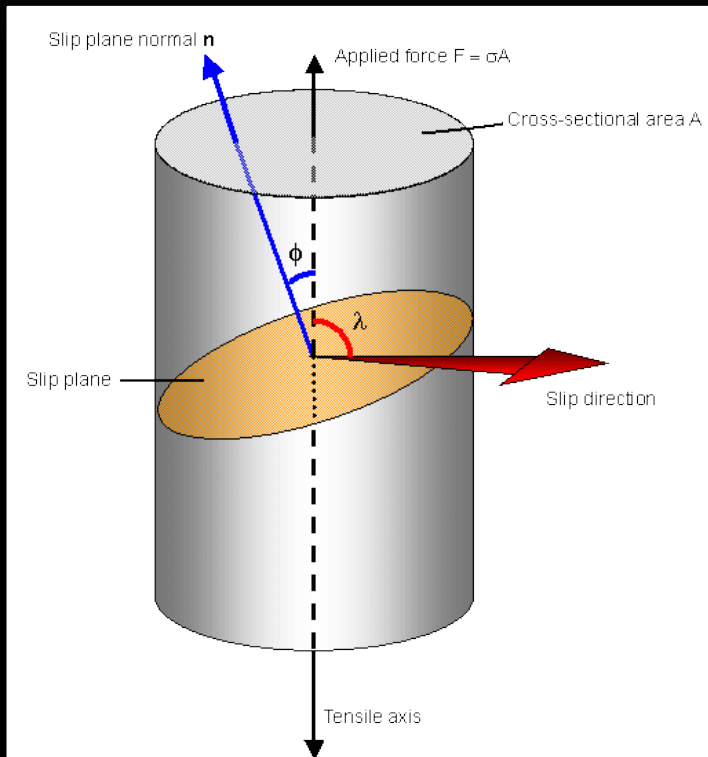
Rosbaud and Schmid 1925

- Resolved shear stress

  - $\tau_{RSS} = \sigma(\cos\phi\cos\lambda)$

- Schmid factor $\equiv \cos\phi\cos\lambda$

- Glide occurs when $\tau_{RSS} \geq \tau_{CRSS}$

MECHANISMS OF TEXTURE DEVELOPMENT
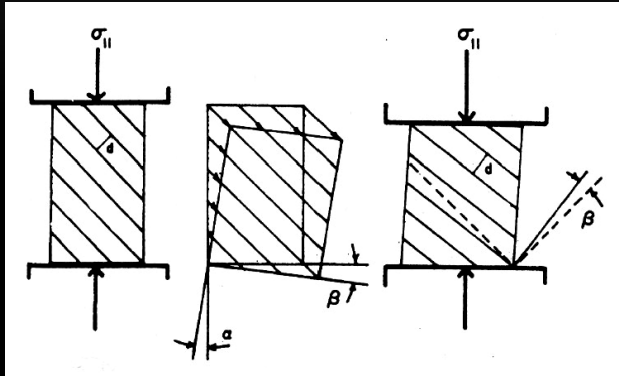
- Texture development
  - Twinning
  - Dislocation glide (hkl)[uvw]

Taylor : iso-strain
- Stresses too high
- Compatibility
  - Need five slip systems

Sachs : iso-stress
- Stresses too low
- Only favorably oriented slip systems deform

ANISOTROPY IN THE UPPER MANTLE

- Seismic waves (P) travel faster along shear zones than across them

  - Raitt 1963: Shor 1964

- Hess 1964 - anisotropy due to texture (CPO)

- Raleigh 1969

  - Deformation of olivine is controlled by slip on {0kl}[100]

- Francis 1969

  - [100] aligns with flow direction

  - Fast P-wave propagation is flow direction

Francis 1969

Wenk and Bulakh 2004
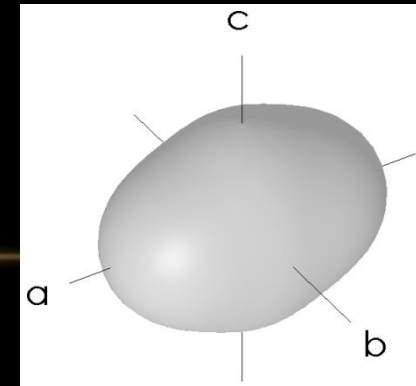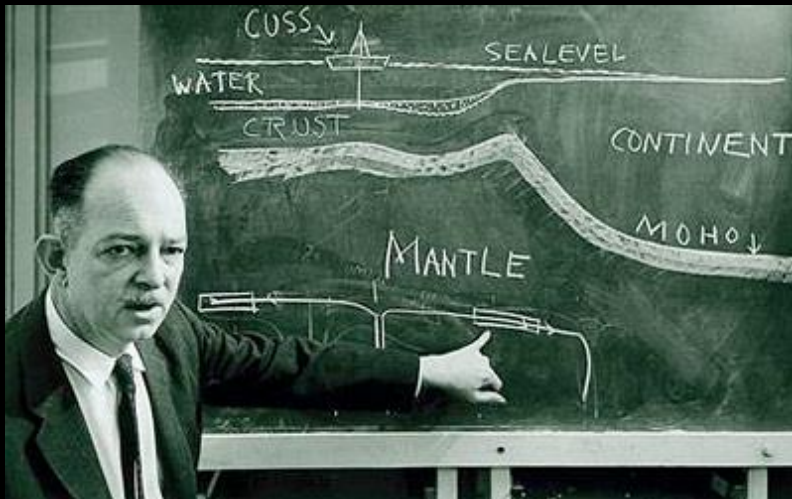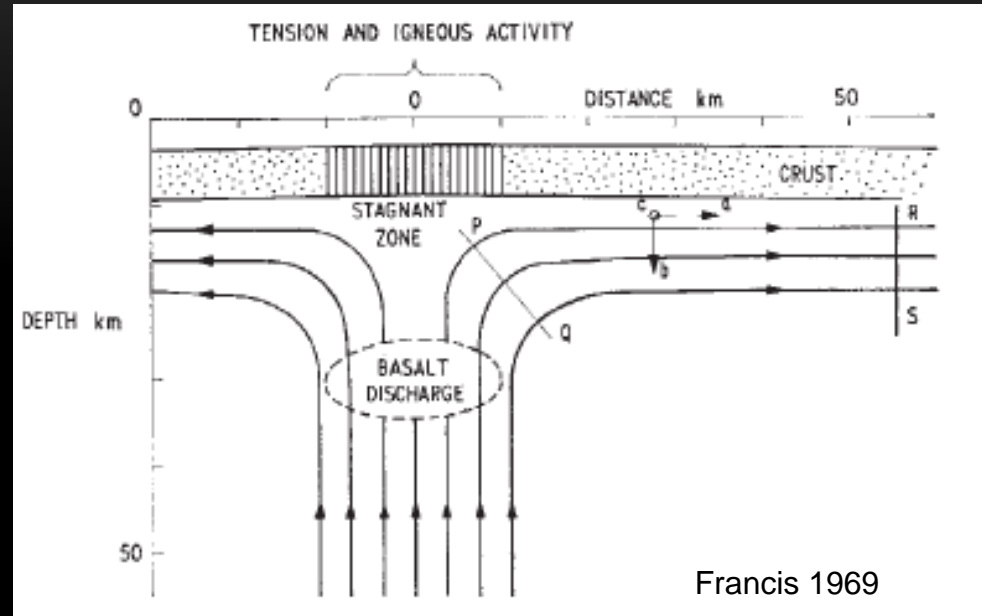
ACTIVITY OF MULTIPLE SLIP SYSTEMS
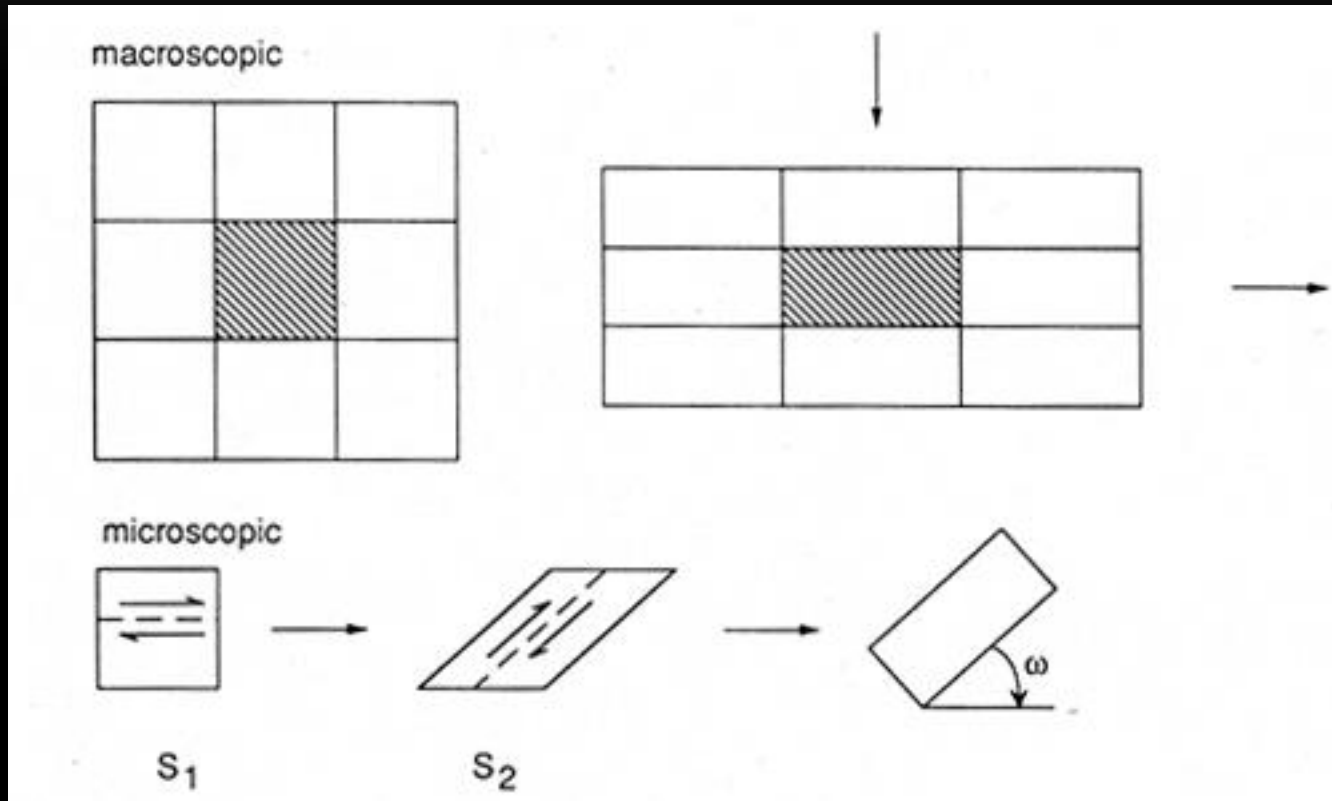
It is very difficult to activate only one slip system

REPRESENTATION OF TEXTURE: POLE FIGURES
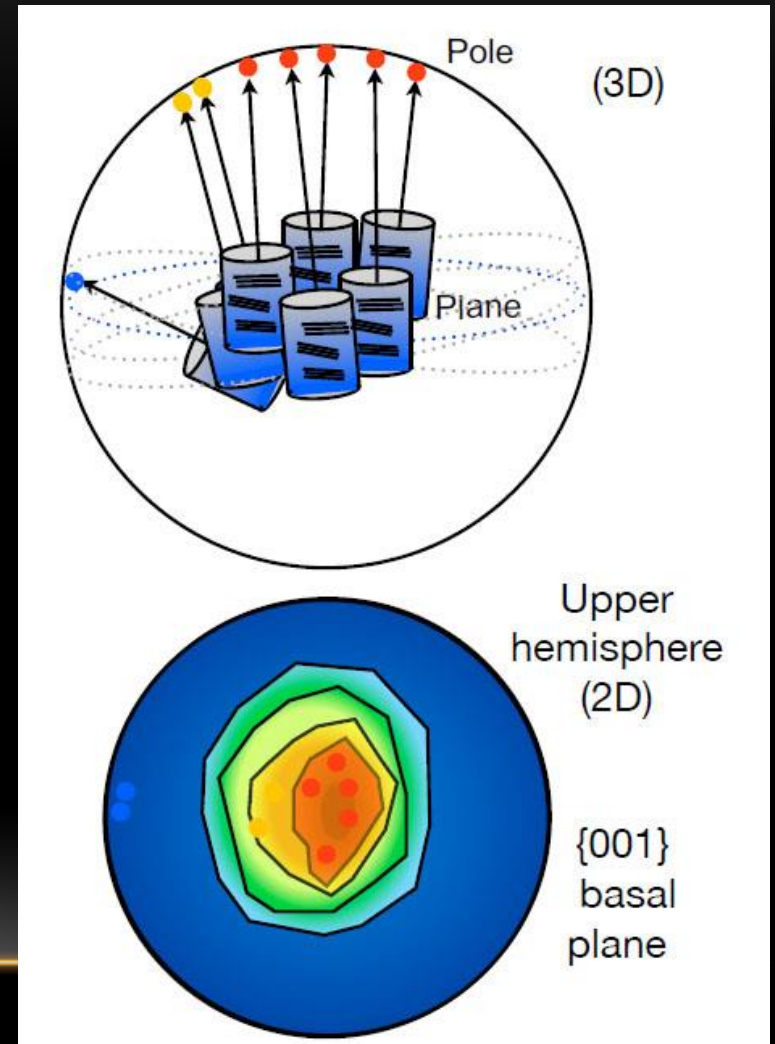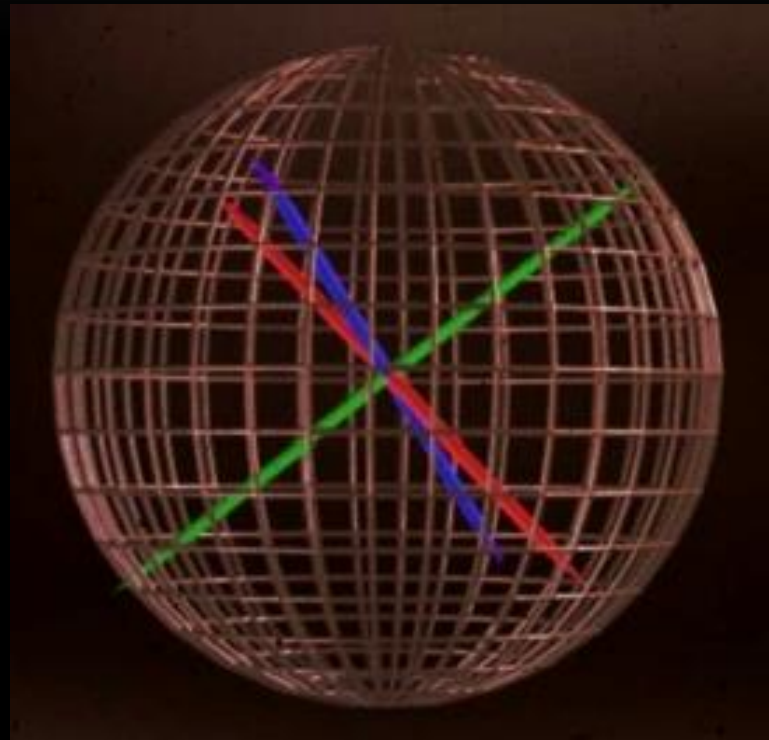
TEXTURES: INVERSE POLE FIGURES

MgSiO$_3$ pPv   MgGeO$_3$ pPv

Miyagi et al. 2010   Miyagi et al. 2011
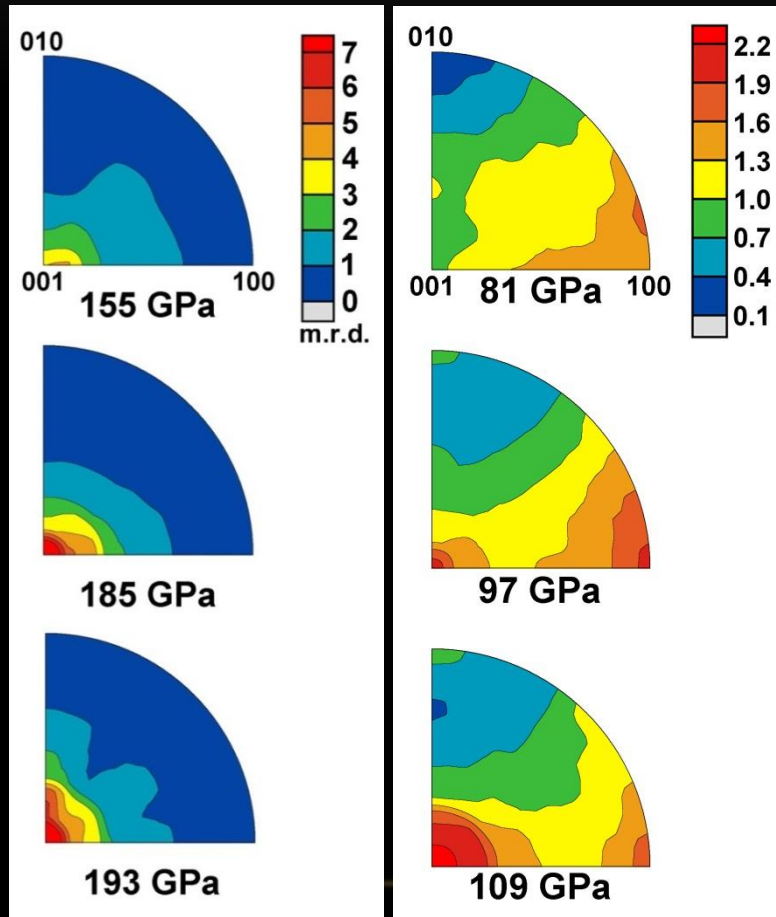
- Probability of finding a pole to a lattice plane in the compression direction

- multiples of random distribution (m.r.d.)

- IPF has symmetry of mineral

A CAUTION ON INTERPRETATIONS OF TEXTURES

Ferropericlase {110}<1-10> slip

Halite {110}<1-10> slip (Wenk et al. 2009)
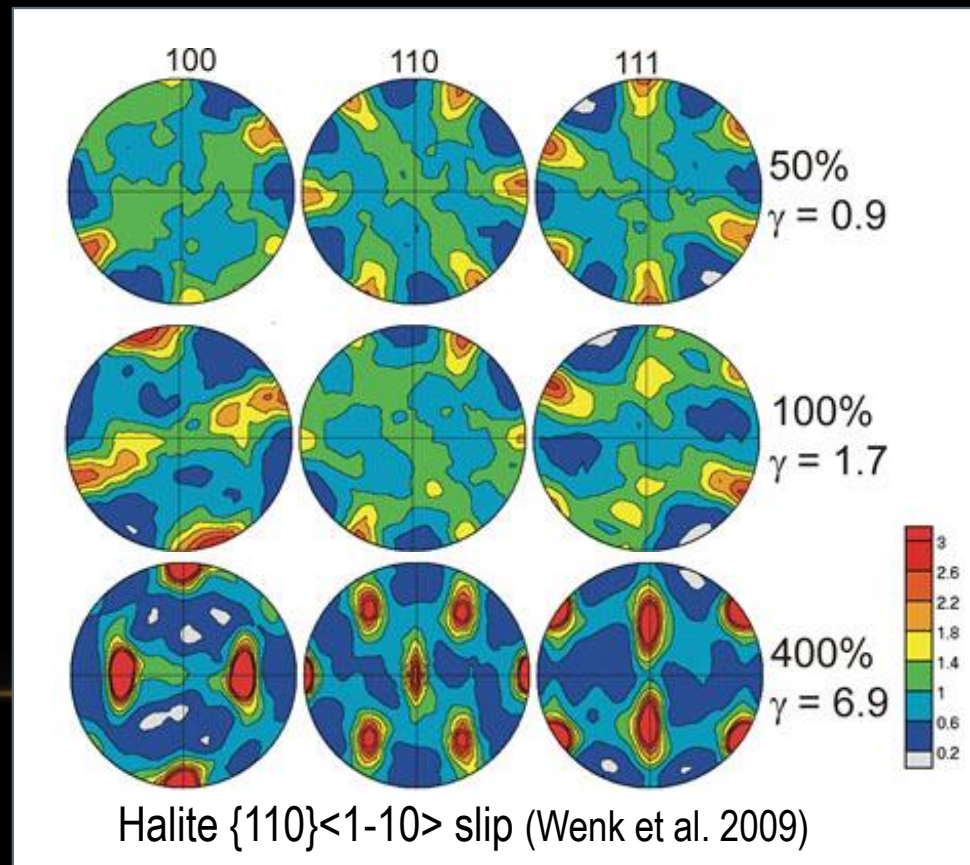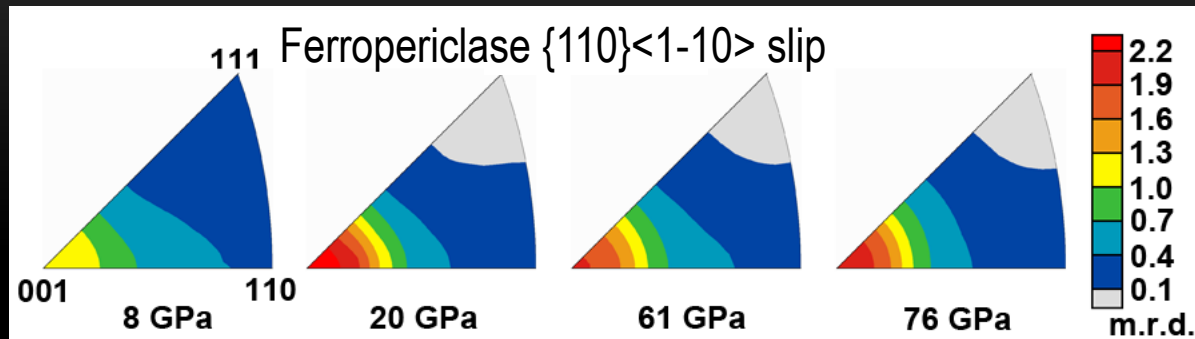
TECHNIQUES TO STUDY MANTLE RHEOLOGY (TRANSITION ZONE AND LOWER MANTLE)

WHAT DO WE WANT?

- Controlled rheology experiments
  - At P-T conditions of the core mantle boundary

  - Over a range of compositions

  - And a range of stresses/strain rates

Something like this that goes to ~130 GPa and ~3000K…

WHAT CAN WE DO?

Controlled rheology devices

- RDA (Yamazaki and Karato 2001), D-DIA (Wang et al 2003), DT-Cup (Hunt et al 2014)

  - Controlled strain rates

    - $\dot{\varepsilon} \sim 10^{-5}$ s^{-1}

  - Relatively large samples

  - Better control of sample environment

- P-T conditions of transition zone and top of the lower mantle

Diamond anvil cell

- Radial diffraction in the Diamond anvil cell

  - Poorly constrained strain/strain rate

    - $\dot{\varepsilon} \sim 10^{-3}$ s^{-1} to 10^{-4} s^{-1}

    - But typically discontinuous

  - Small samples

  - Relatively poor control of sample environment

- Pressures of inner core but ambient T

- ~60 GPa and 1500-2000 K (Miyagi et al 2013)

DEFORMATION-DIA (D-DIA)
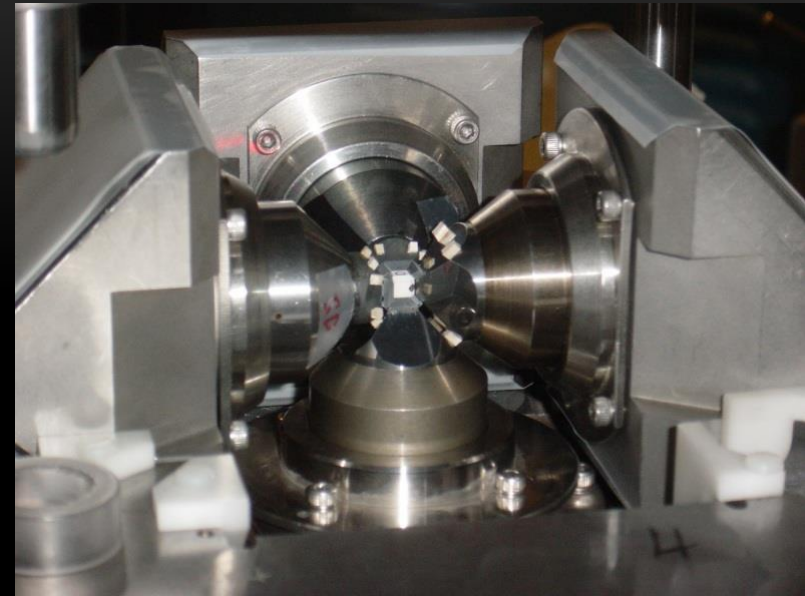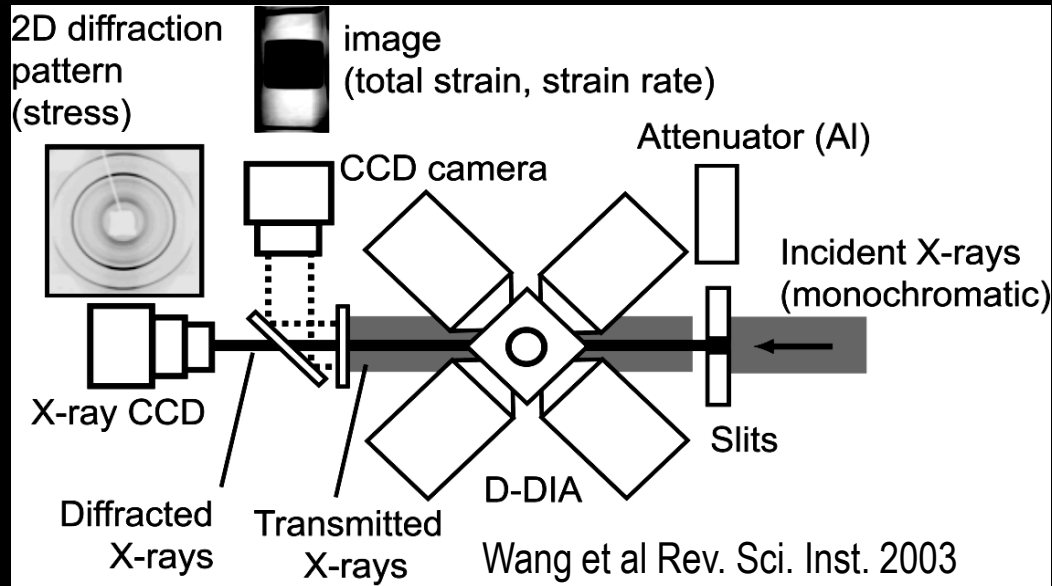
- P-T condition equivalent transition zone and top of lower mantle

- 1-2mm samples

http://www.compres.stonybrook.edu

RADIOGRAPHY-MEASURE STRAIN

0 % strain

30 % strain

ROTATIONAL DRICKAMER APPARATUS –SHEAR STRAIN

Karato and Weidner 2008
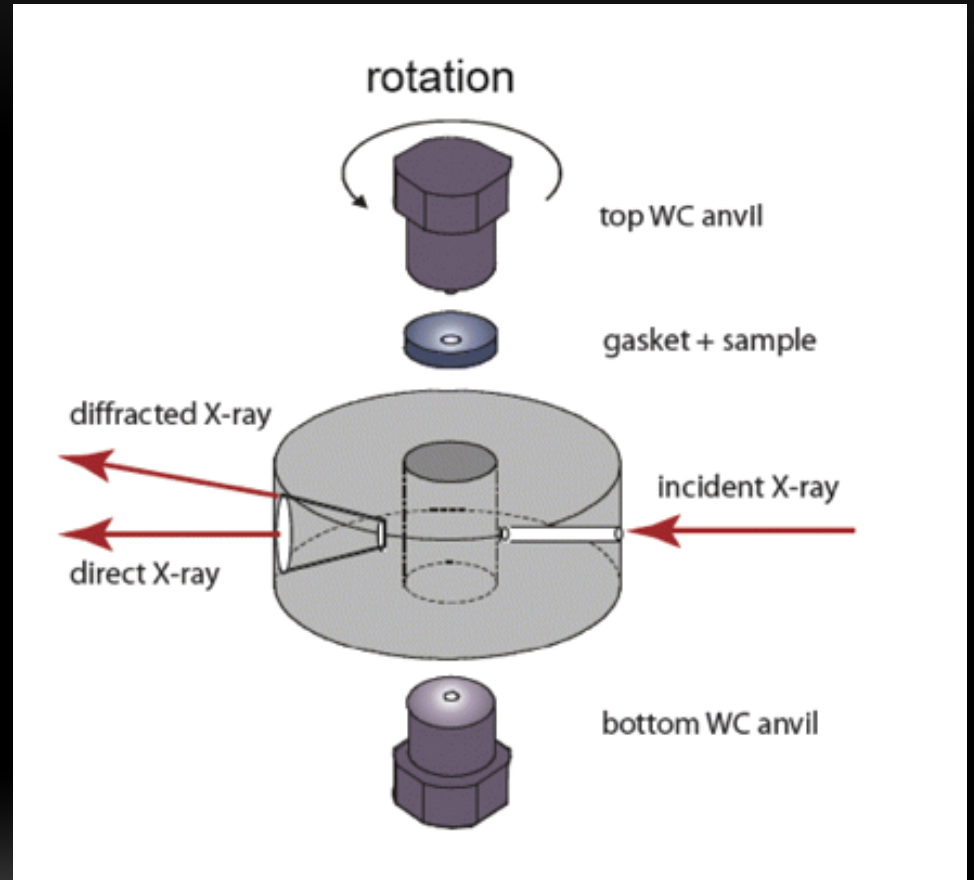
P-T conditions of base of transition zone/
top of lower mantle ~660 km

STRAIN RATE AND STRESS STRAIN

Ringwoodite at 23 GPa and 1800 K
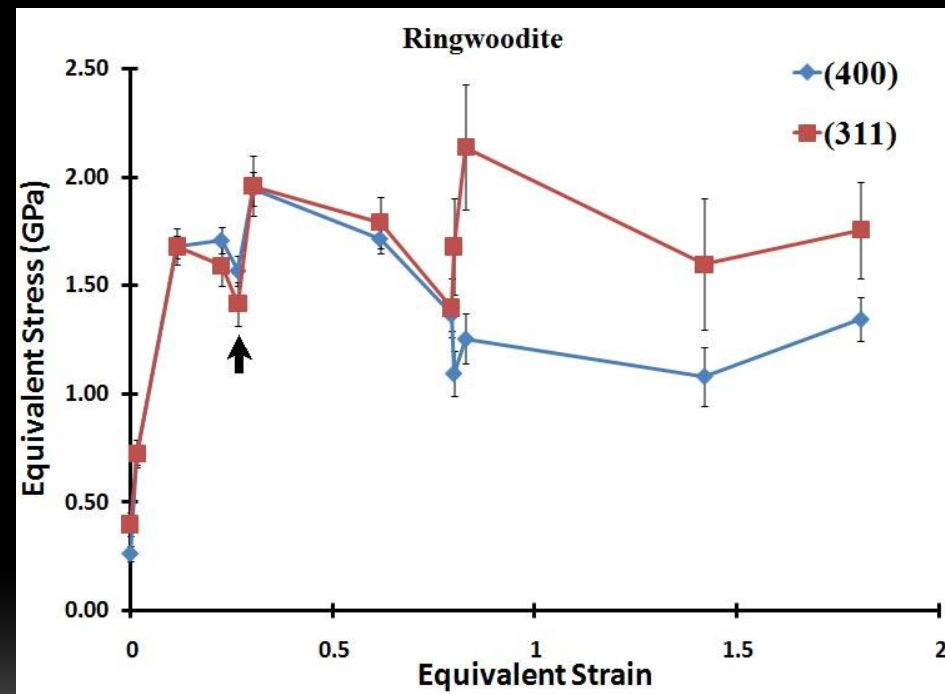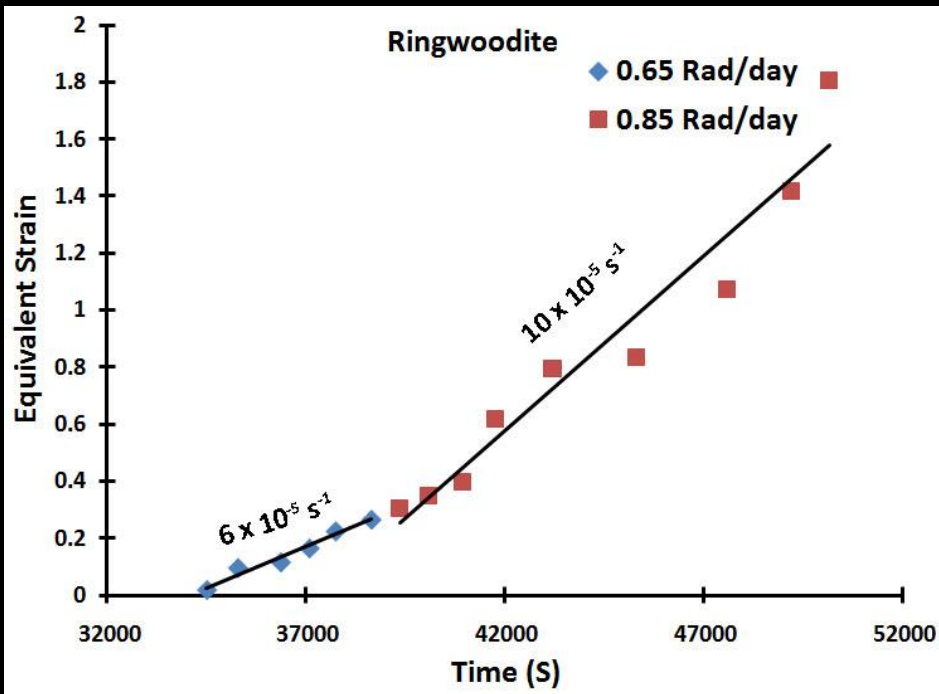
Miyagi et al. 2014

DIAMOND ANVIL CELL (DAC)

150 μm
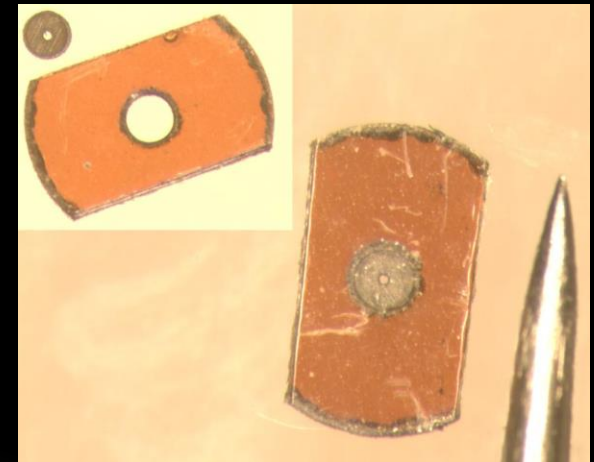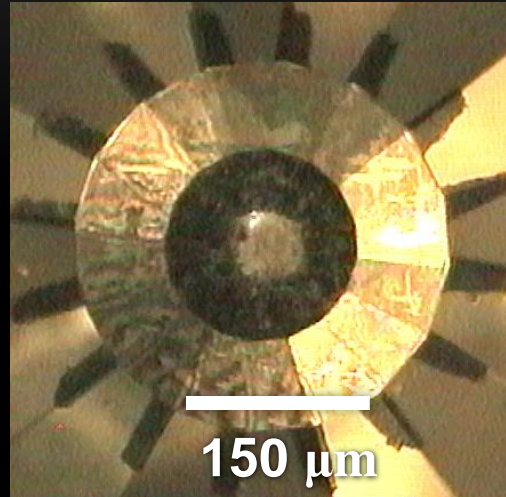
- Trade off between sample size and maximum pressure
- Pressure > than center of the earth (365 GPa)
- No good way to measure strain

RESISTIVE HEATING
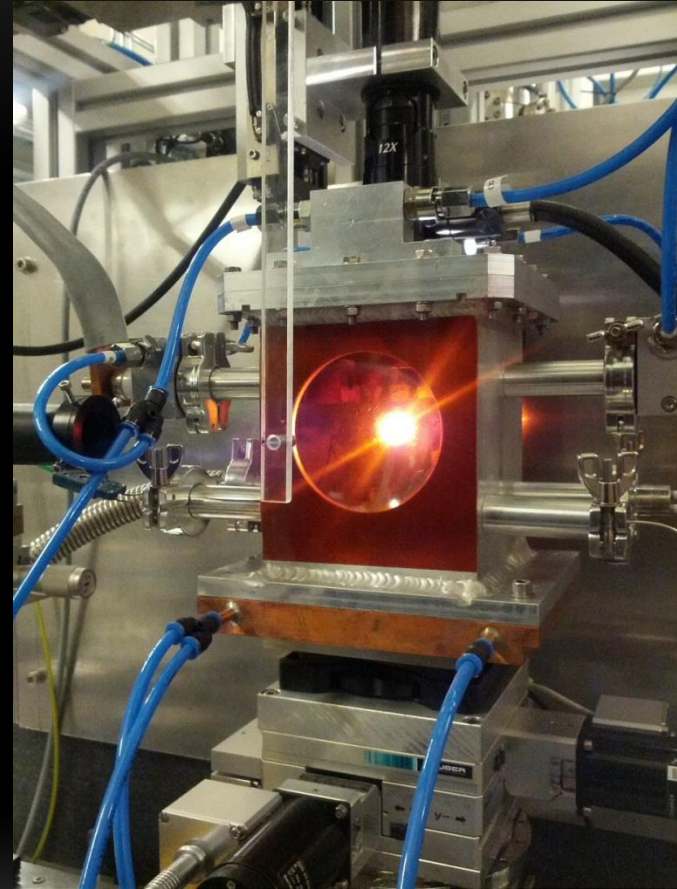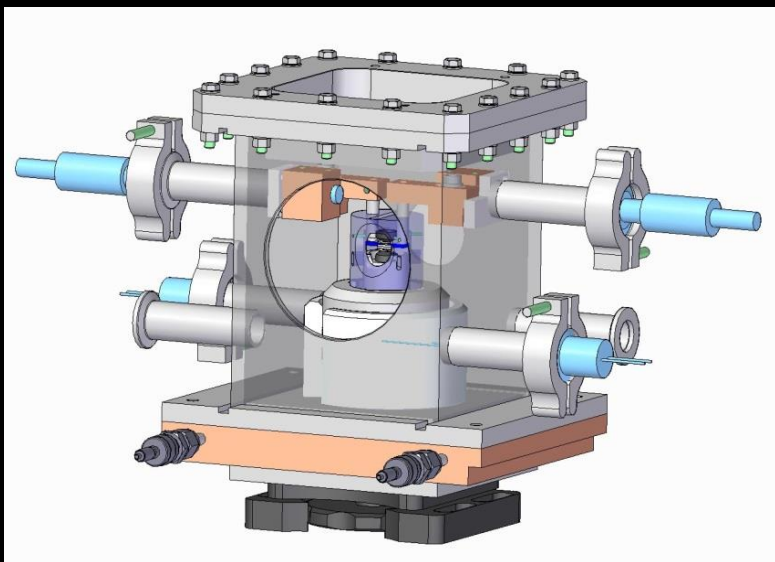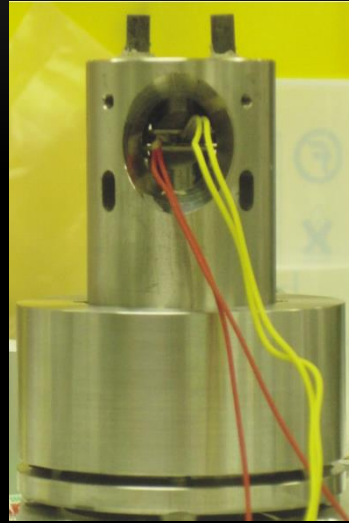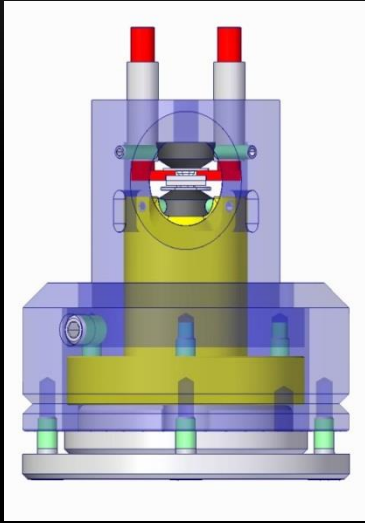
P-T condition ~1500km depth

THEORETICAL STUDIES ON MANTLE RHEOLOGY

Kraych et al 2016

Karato (2010)
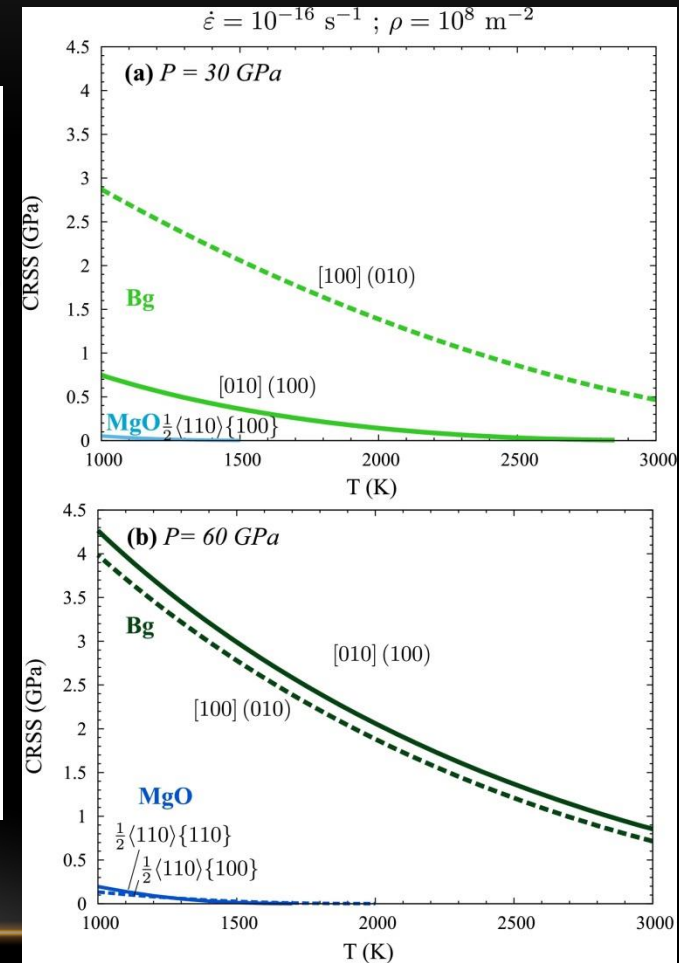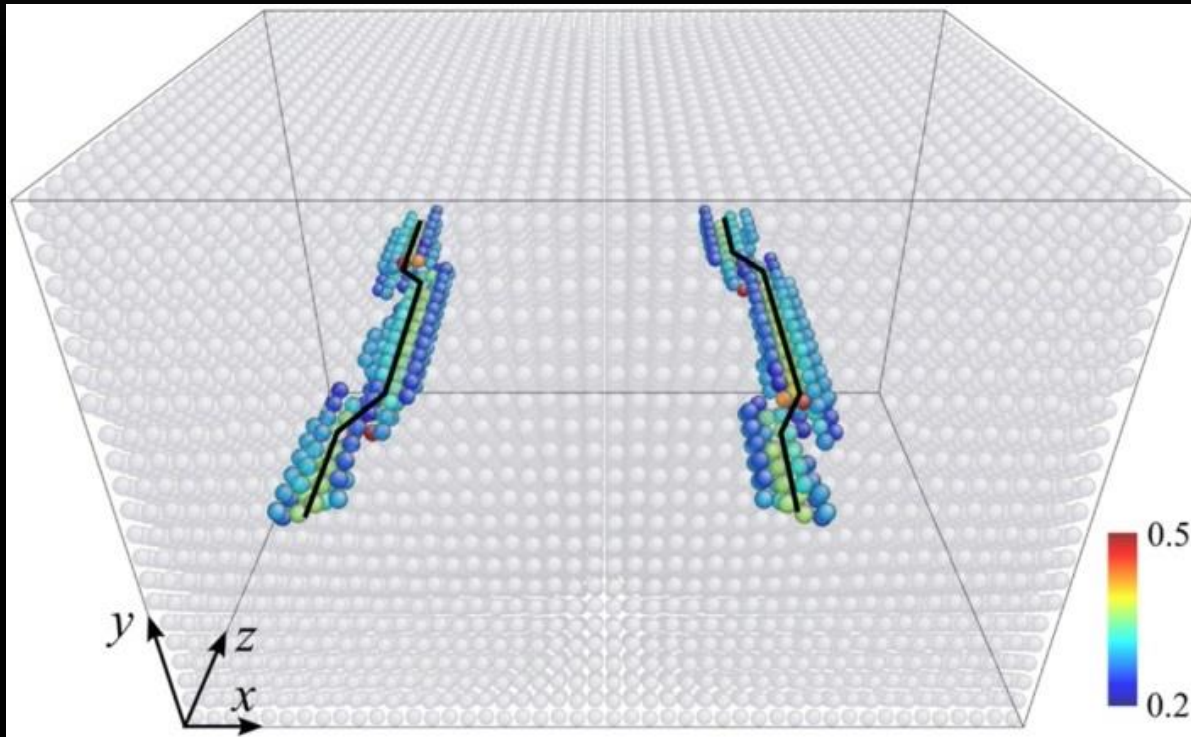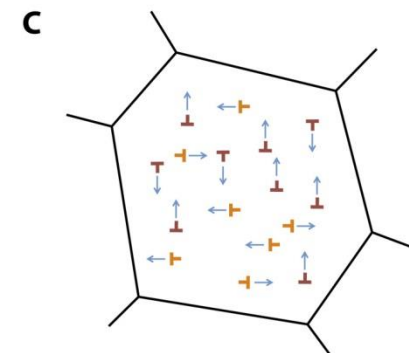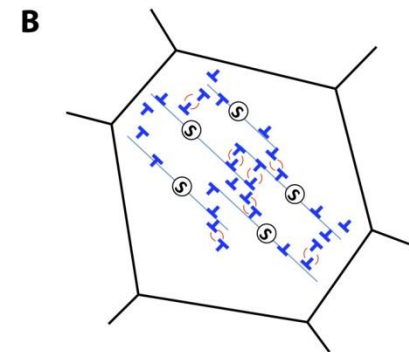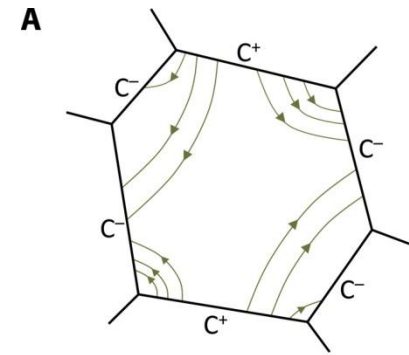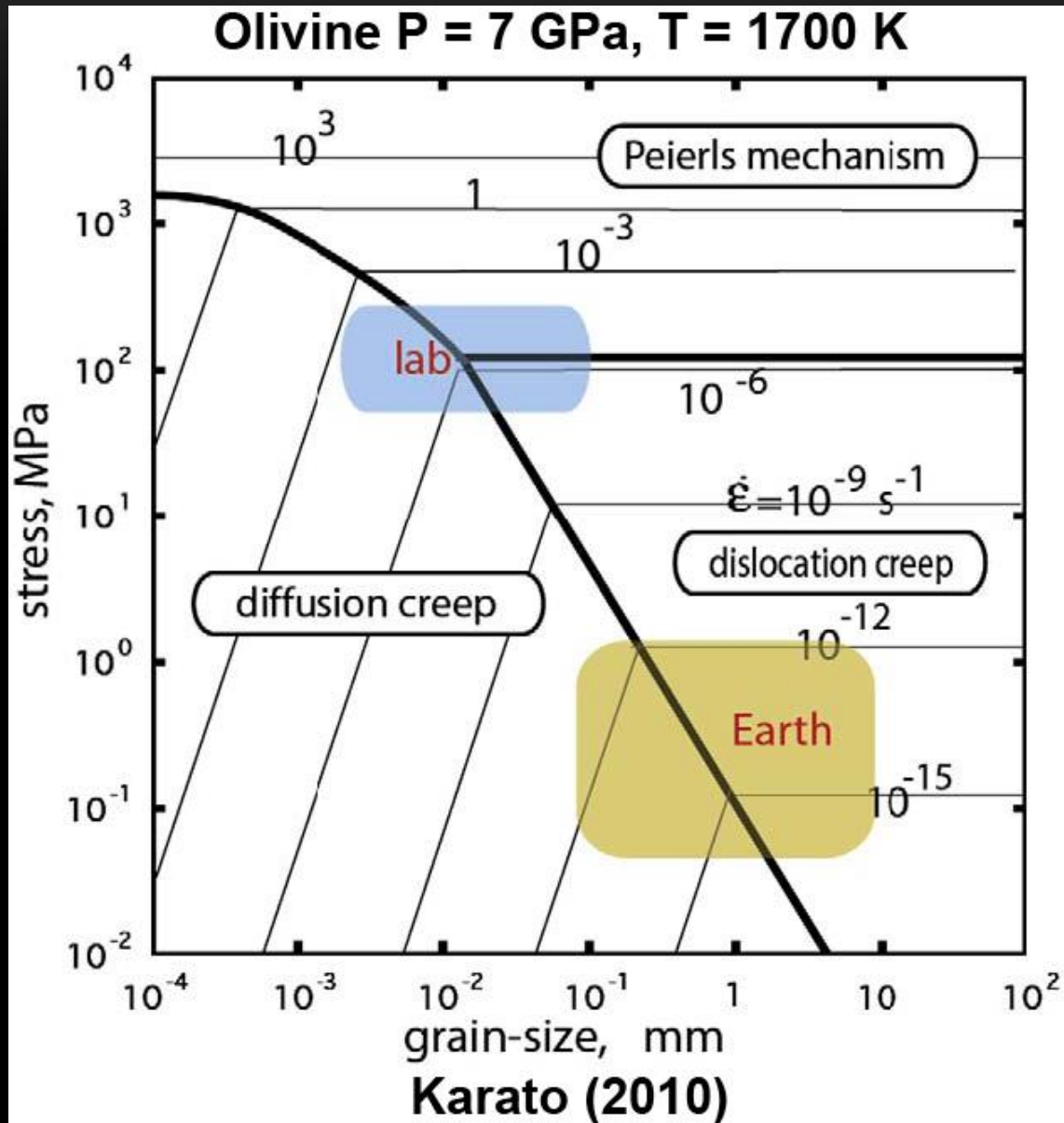
Boiole et al 2017

DEFORMATION MECHANISMS IN THE EARTH

SO HOW DO WE KNOW WHAT MECHANISMS OPERATE IN THE EARTH?

Upper Mantle

- Microstructural observations of rocks brought to the surface

  - Texture (crystallographic preferred orientation) indicates dislocation creep (glide)

  - Large strain but no texture – most likely diffusion creep

  - Note rocks that are brought to the surface have complex deformation histories

- Seismic anisotropy – most likely due to texture from dislocation creep
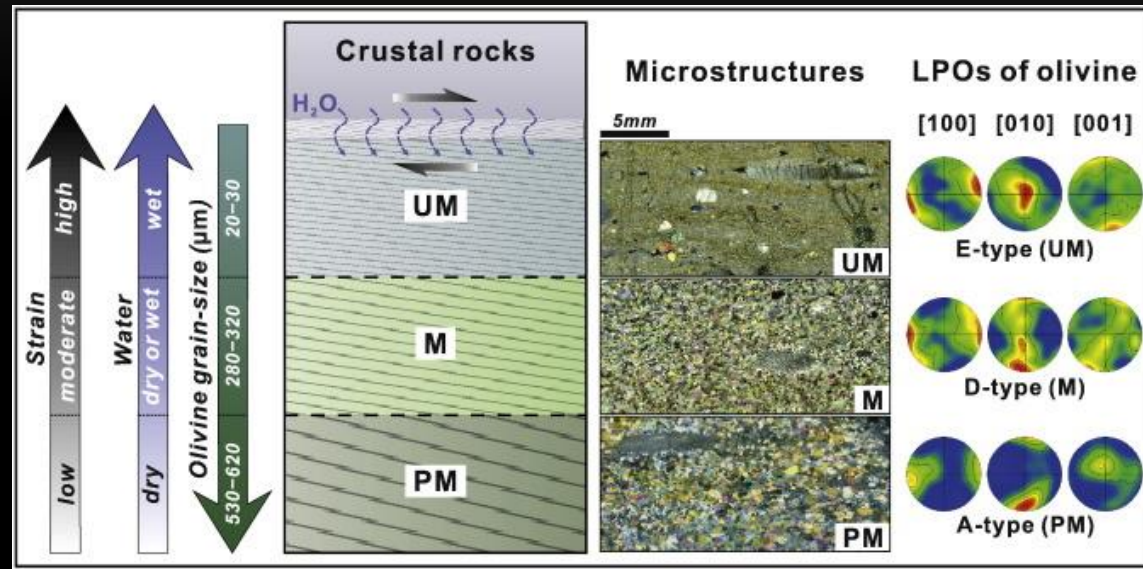
Park and Jung 2017

Transition Zone and Lower Mantle

- No samples for microstructure

- Rely on seismic observations
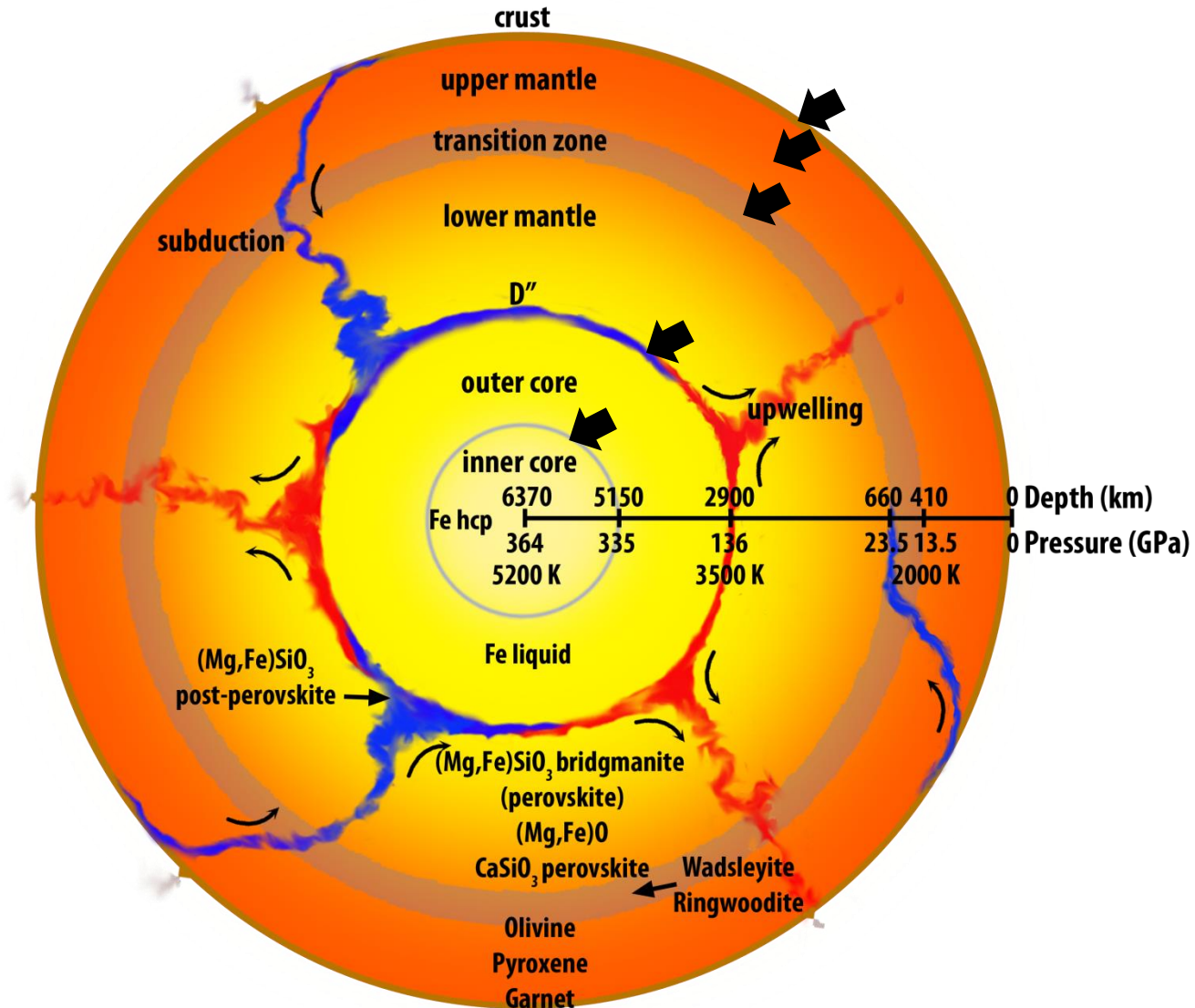
STRENGTH OF MANTLE MINERALS

WADSLEYITE: EXPERIMENTS AND THEORY
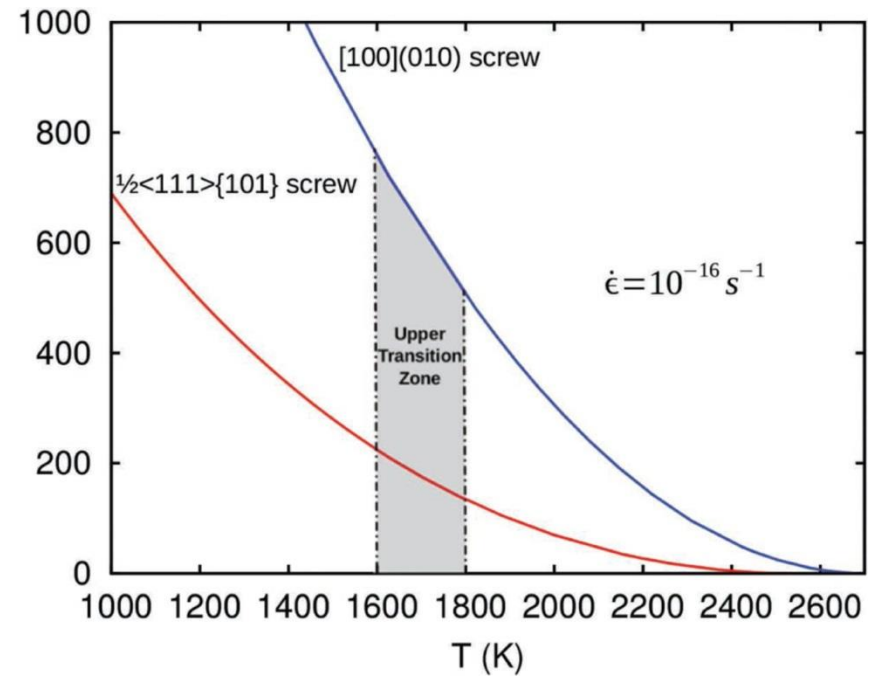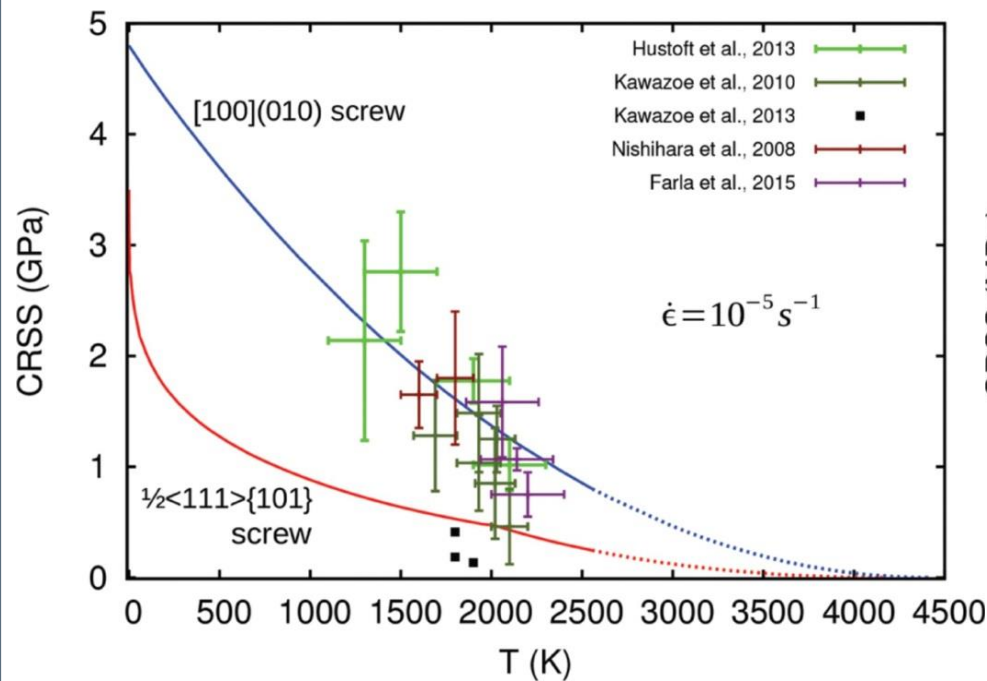
Ritterbex et al 2016
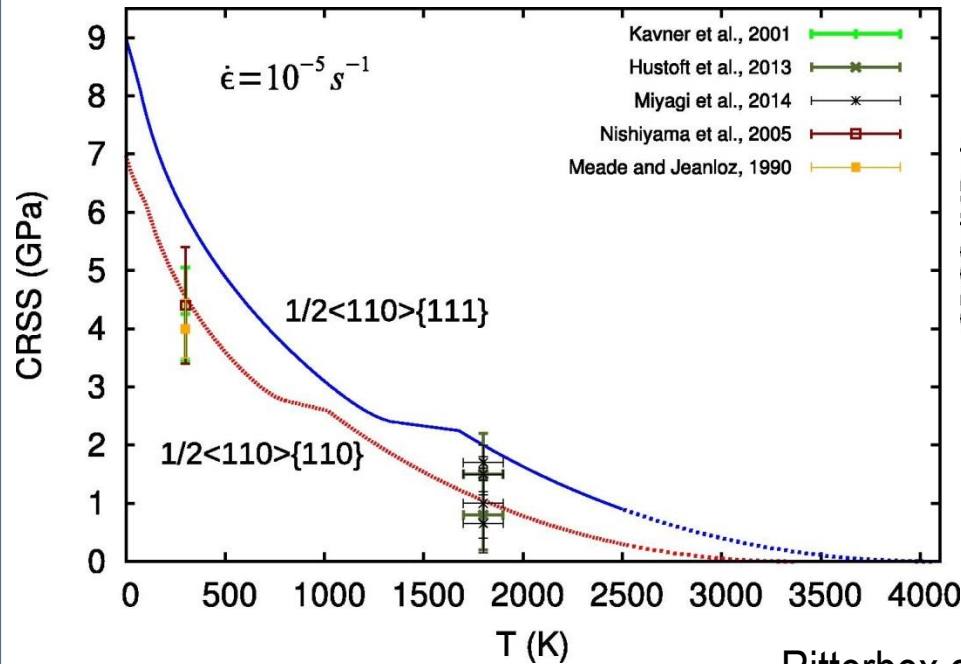
Ritterbex et al 2015
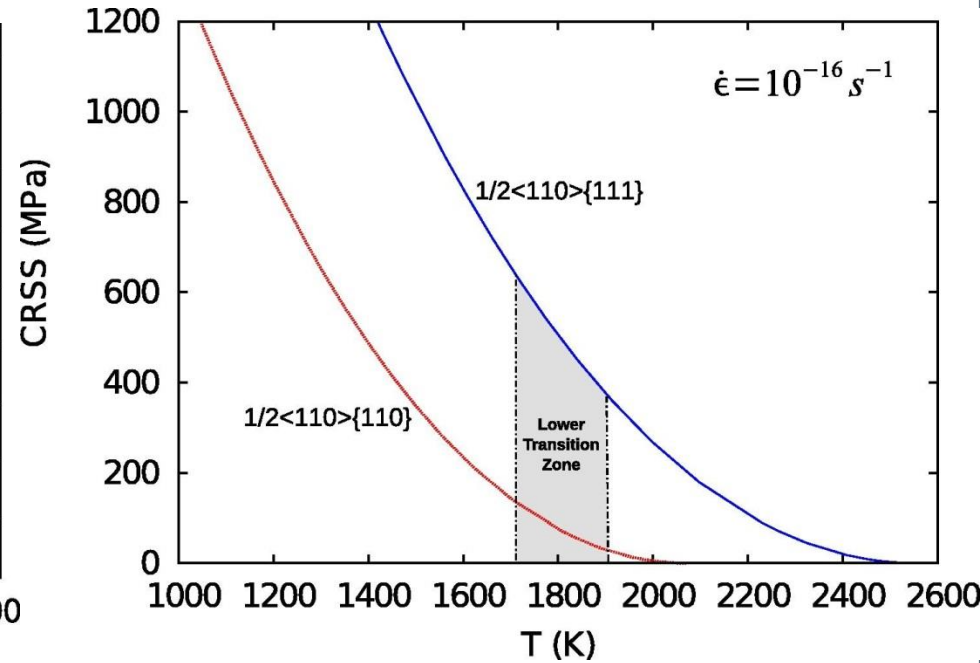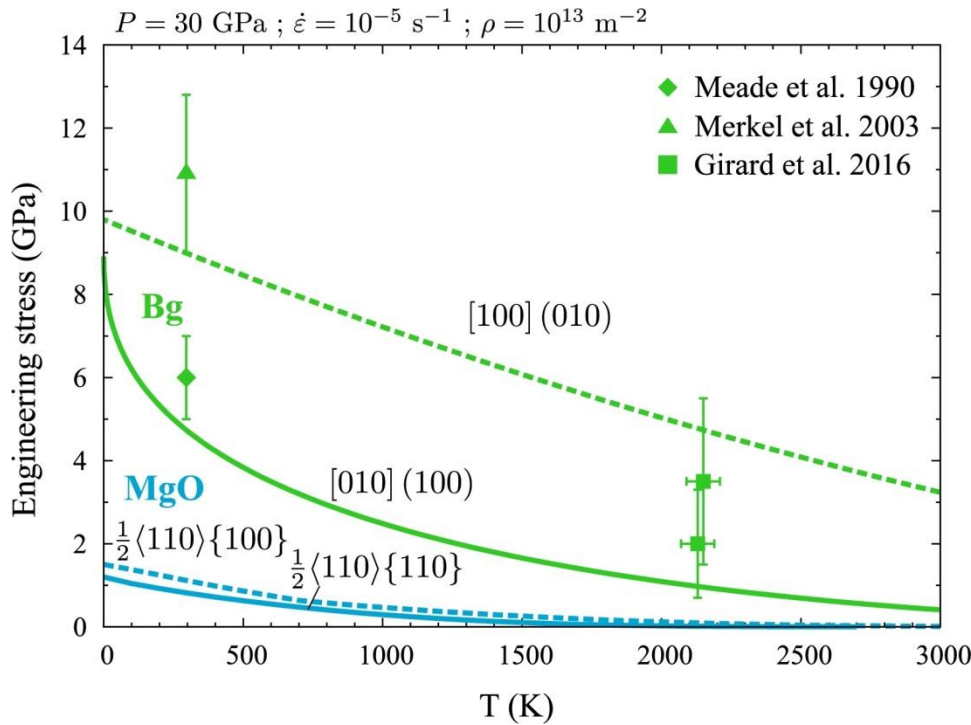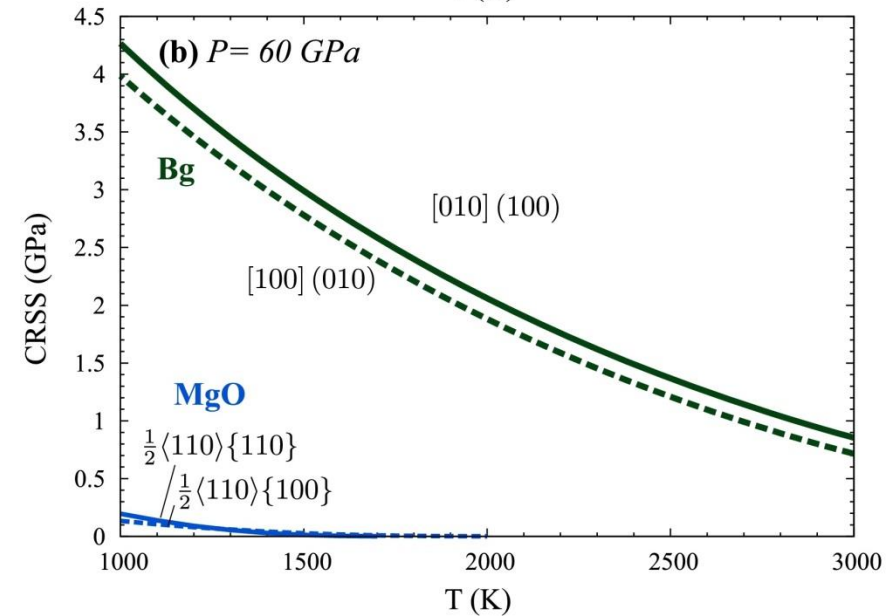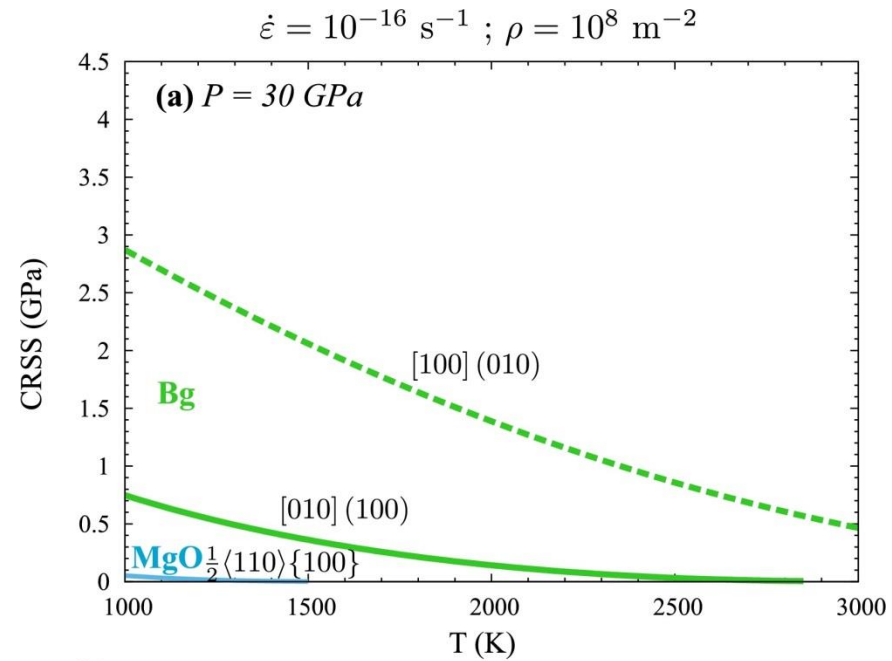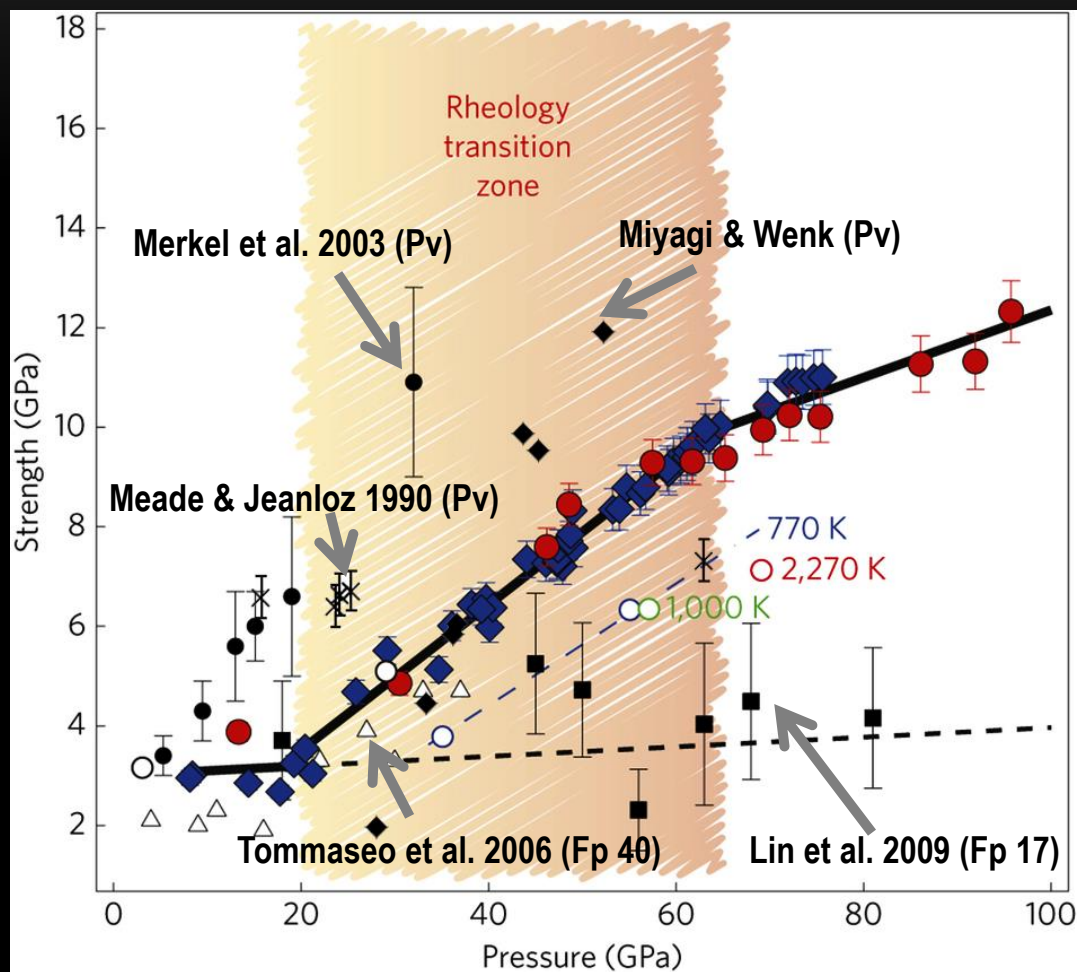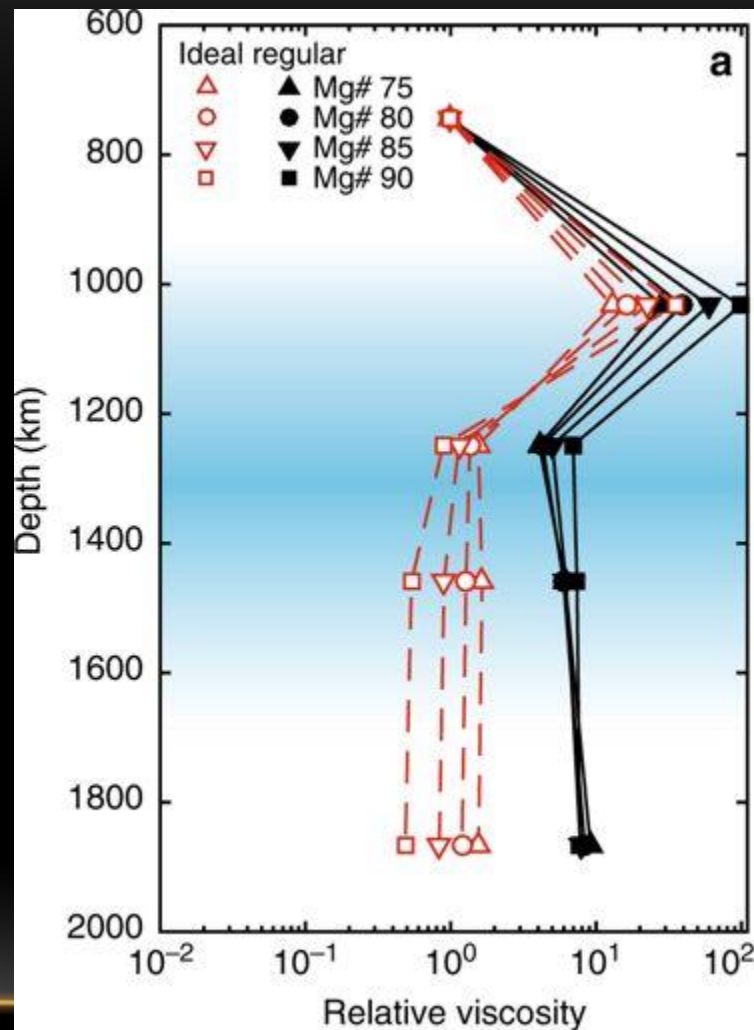
BRIDGMANITE: EXPERIMENTS AND THEORY

Kraych et al 2017

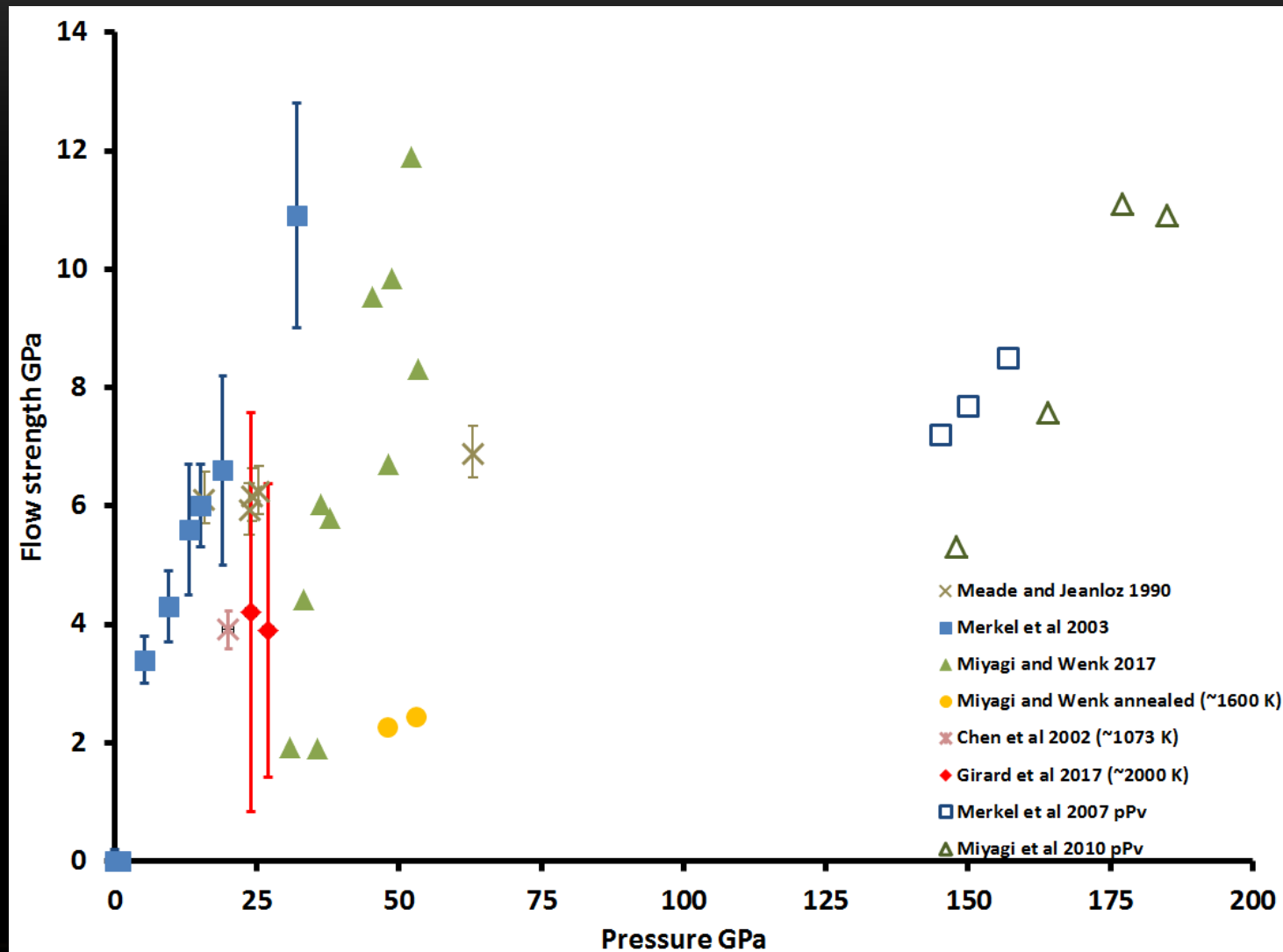# BRIDGMANITE AND FERROPERICLASE

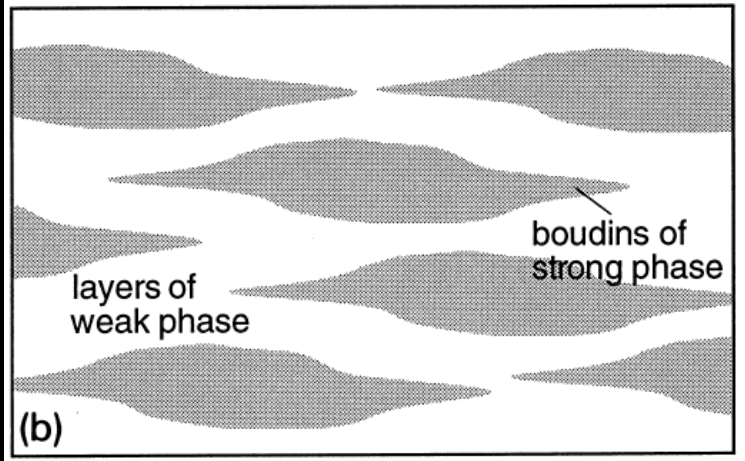Modified from Marquardt and Miyagi, 2015
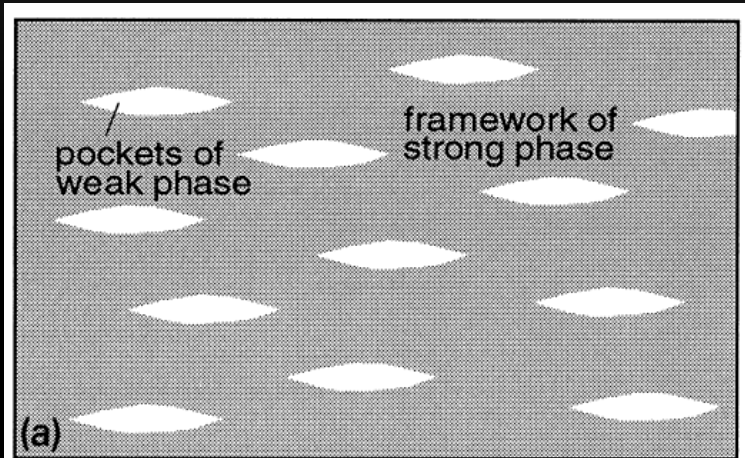
Deng and Lee 2017

BRIDGMANITE AND POST-PEROVSKITE

- Studies of low pressure analog materials seem to indicate that pPv is much weaker than Brg

- Hunt et al (2009) $CaIrO_3$; Dobson et al (2012) $NaCoF_3$

COMPLICATIONS: MULTIPLE PHASES

VISCOSITY OF 2-PHASE COMPOSITES

pockets of weak phase

framework of strong phase

(a)

boudins of strong phase
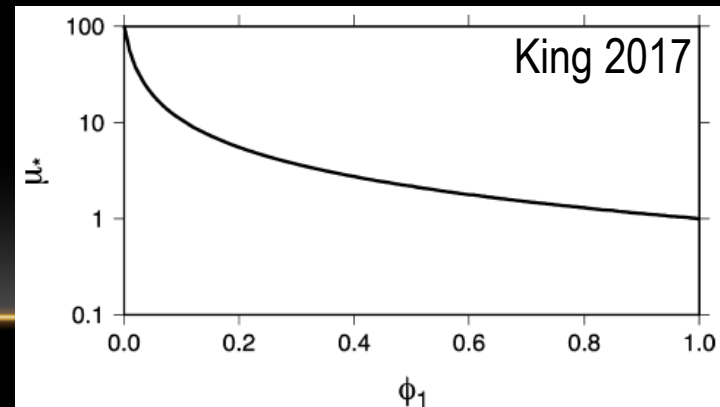
layers of weak phase

(b)

Handy 1999

Single phase

- $\mu = \dfrac{\sigma}{2\dot{\varepsilon}}$

2 phase viscosity

- LBF vs IWL microstructure

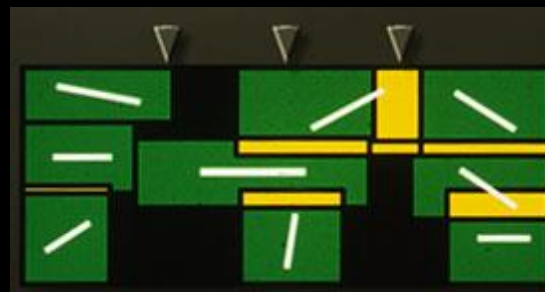- Takeda 1998 –formulae for viscosity of LBF and IWL (Newtonian only)

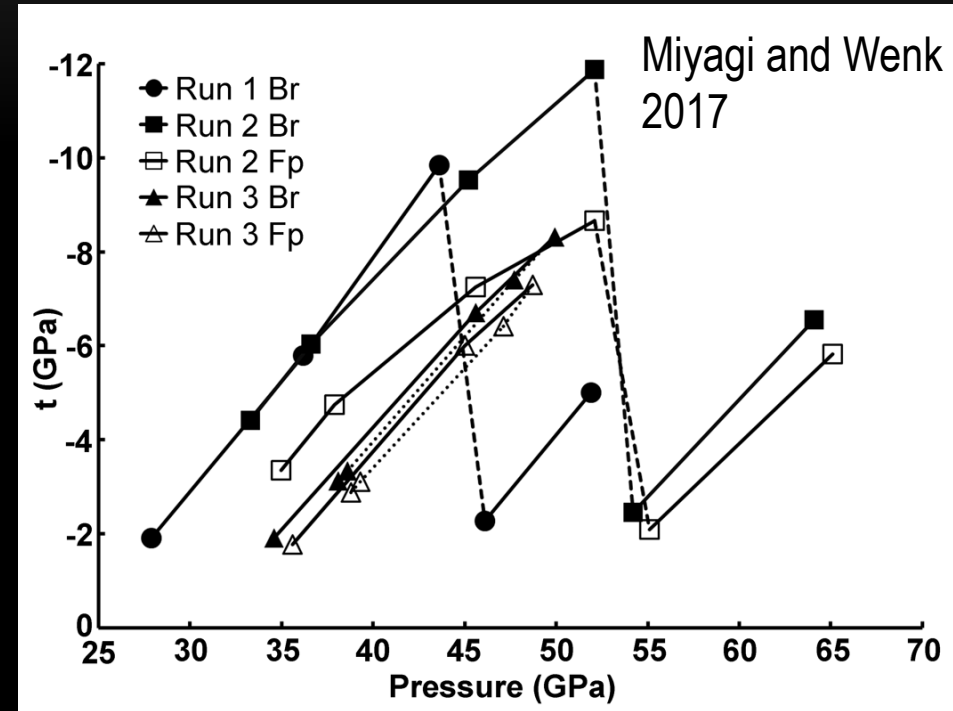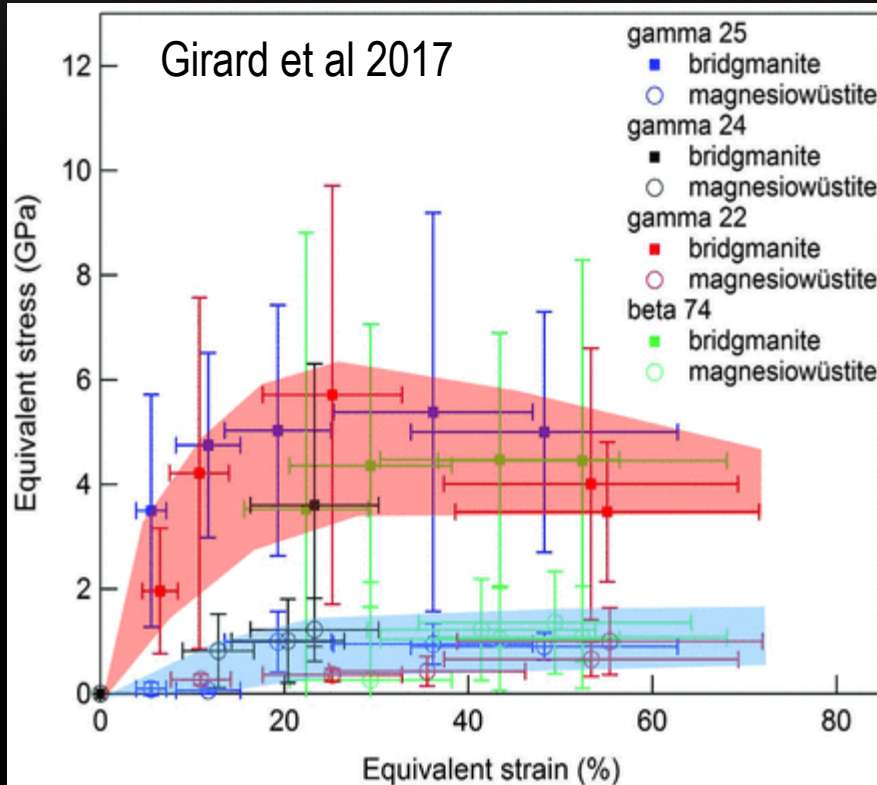King 2017

POLY PHASE PLASTICITY

iso-strain (strain rate)
- Stresses too high

- More like LBF

iso-stress
- Stresses too low

- More like IWL

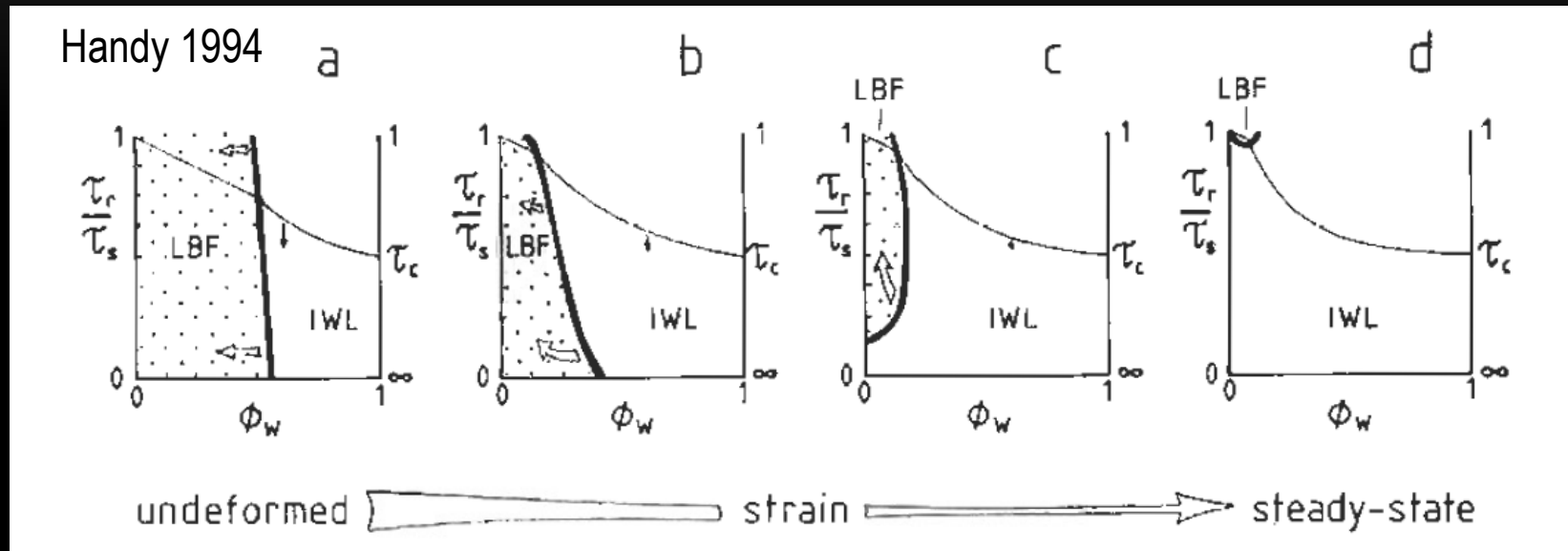# BRIDGMANITE AND FERROPERICLASE

Girard et al 2017

Miyagi and Wenk 2017

Do these show the same or different strength ratios?
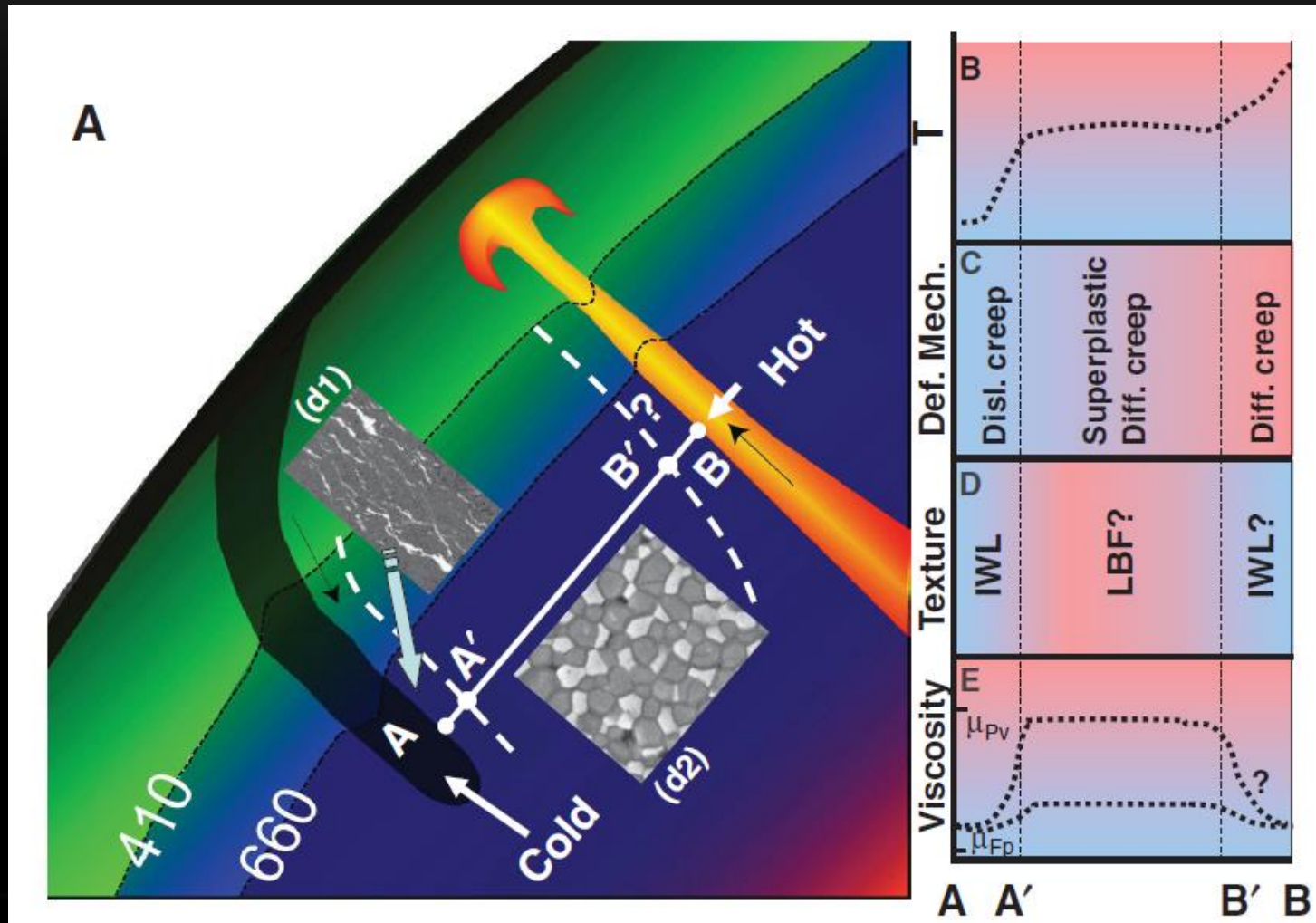
What is the stress-strain compatibility state?

WHERE DO WE EXPECT LBF VS IWL

Handy 1994

- LBF
    - Large volume fraction of strong phase
    - Low strain
- IWL
    - Large volume fraction of weak phase
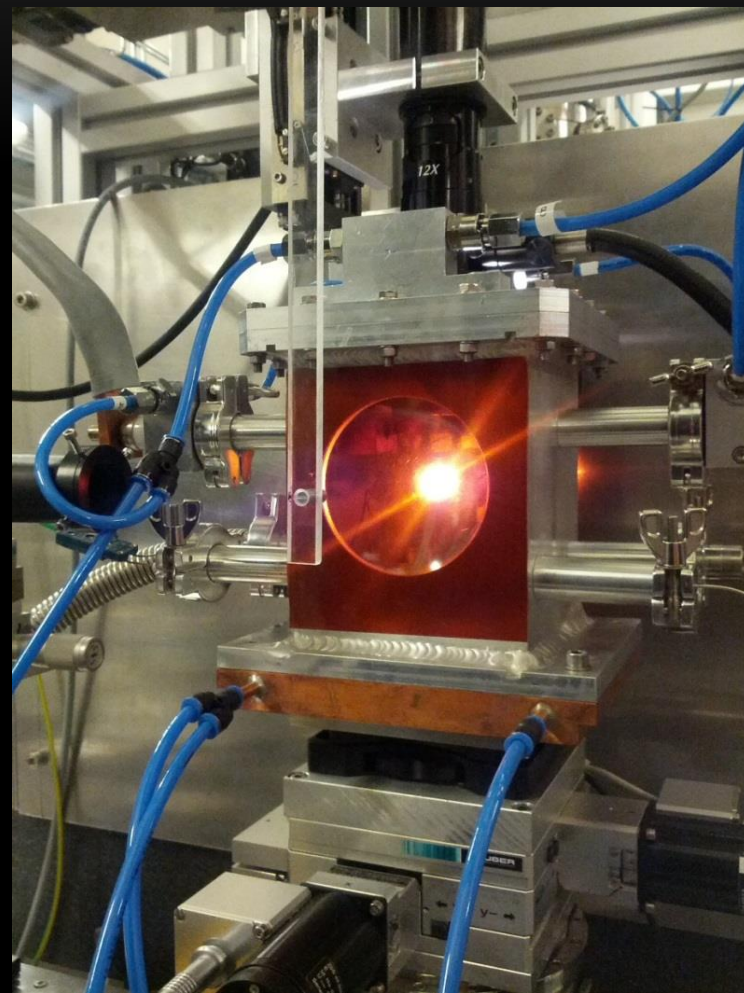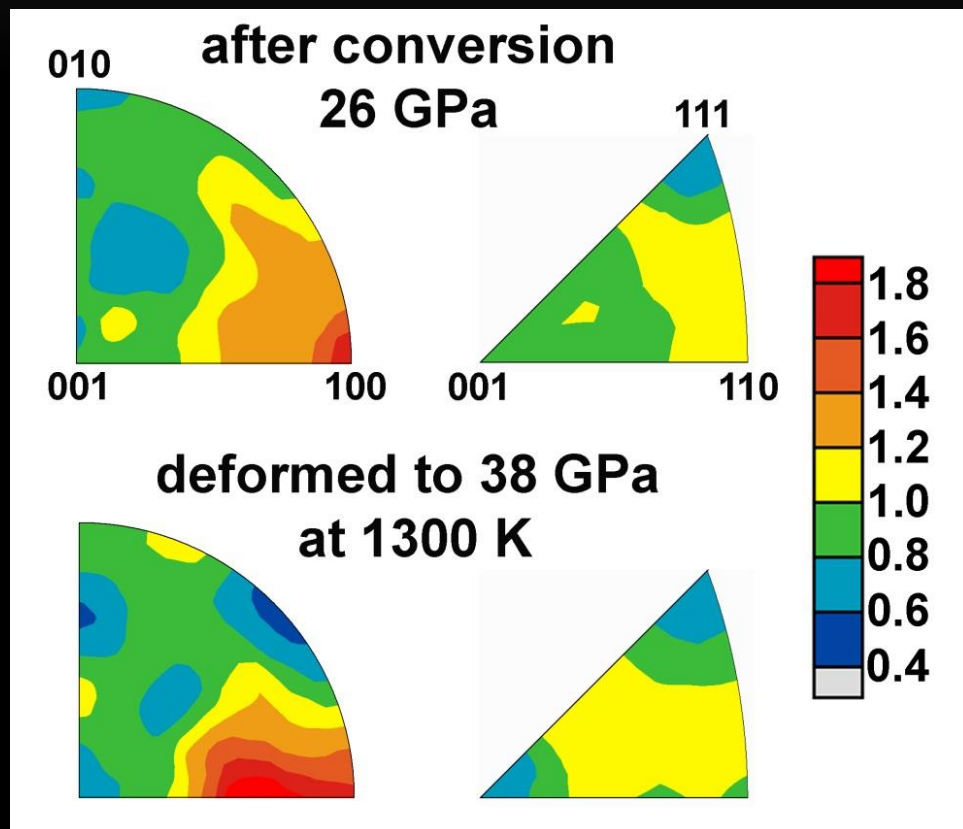   - Large strain

IMPLICATIONS FOR DEEP EARTH CONVECTION

Wang et al 2011- shear deformation of Olv+Fe,Ni,S alloy
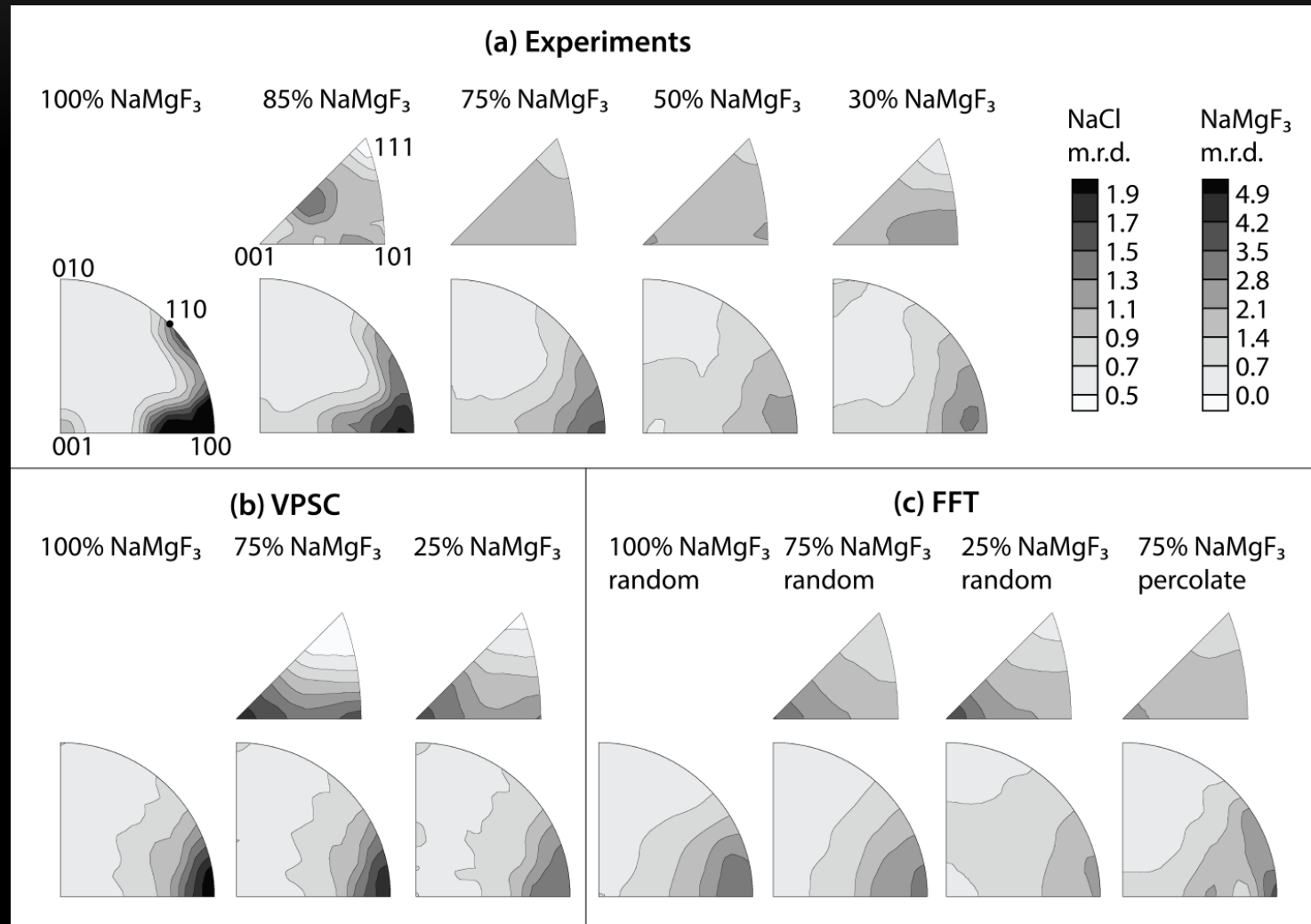Inset d2 - Pv+Fp from Yamazaki et al. 2009

TEXTURES OF TWO PHASE MATERIALS
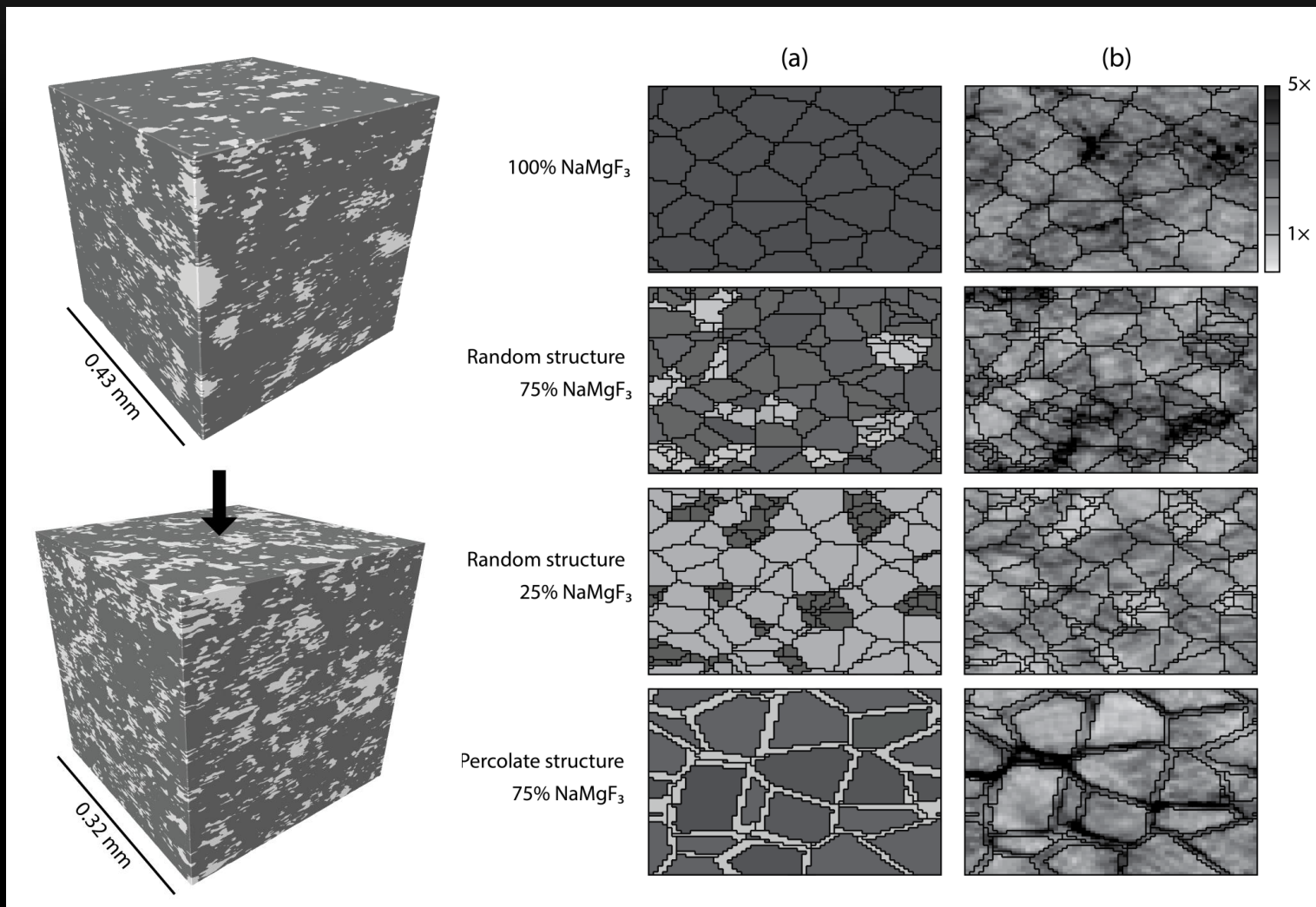
MgSiO$_3$ BRIDGMANITE IN THE HIGH-T DAC

Couper et al in prep

TEXTURE AND MICROSTRUCTURE

Kaercher et al. 2016

MICROTOMOGRAPHY AND FFT-VPSC

Kaercher et al. 2016

SOME CONCLUSIONS AND THOUGHTS

- High stress regions should favor dislocation creep

  - Colder, higher strain rates –slab and edges of convection cells

  - Expect seismic anisotropy

  - $\dot{\varepsilon} \propto \sigma^n$

- Low stress regions should favor diffusion creep

  - "stagnant" convection cores, possibly plumes

  - Expect seismic isotropy

  - $\dot{\varepsilon} \propto \sigma$

- High strains tend to favor interconnected weak layer so softer phase (Fp) should dominate viscosity

- Low strains favor load bearing framework so harder phase (Brg) tends to dominate

- Texture in poly-phase aggregates is not straight forward