On the Generation of Core Dynamo Action

CIDER Meeting, KITP 7/7/16

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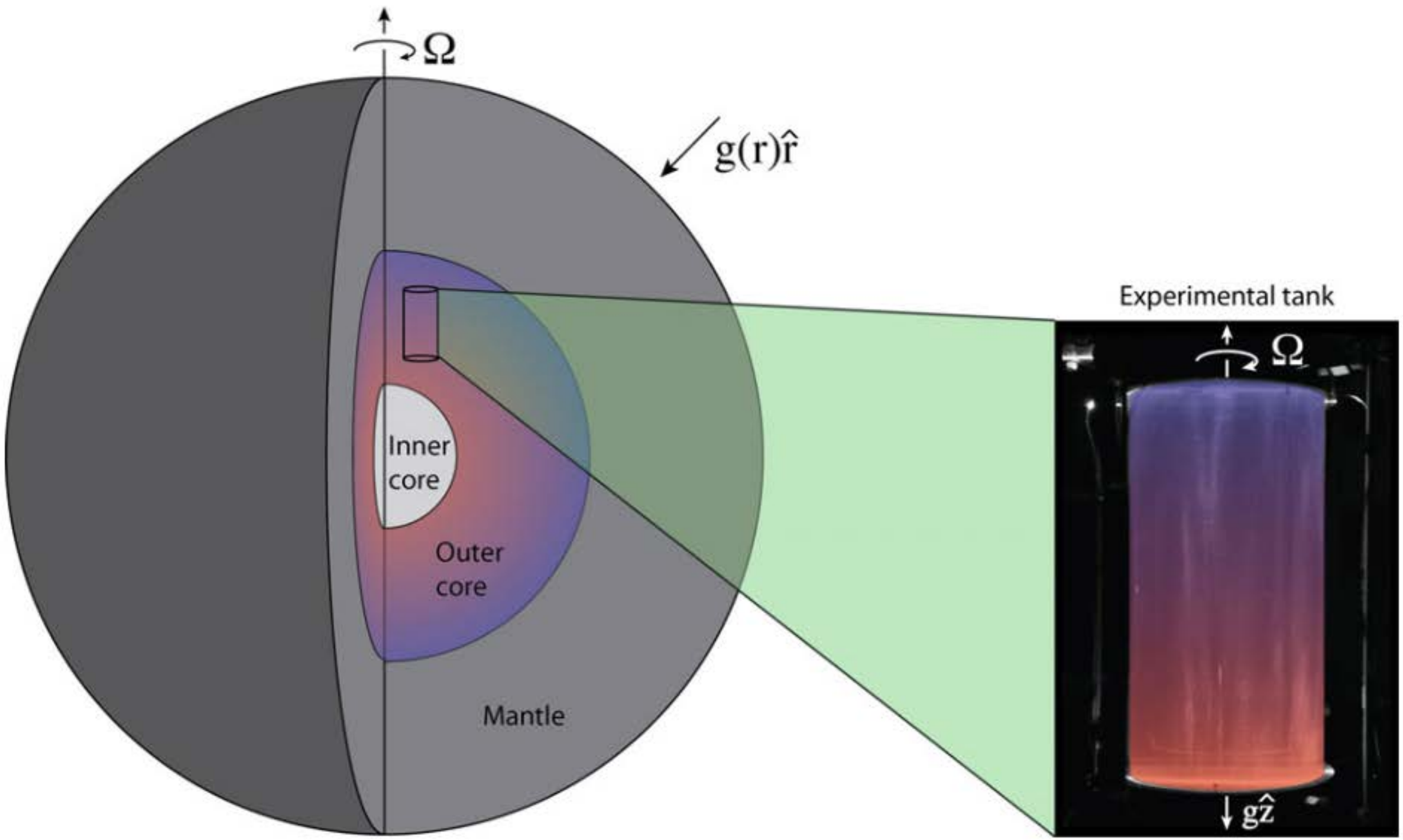
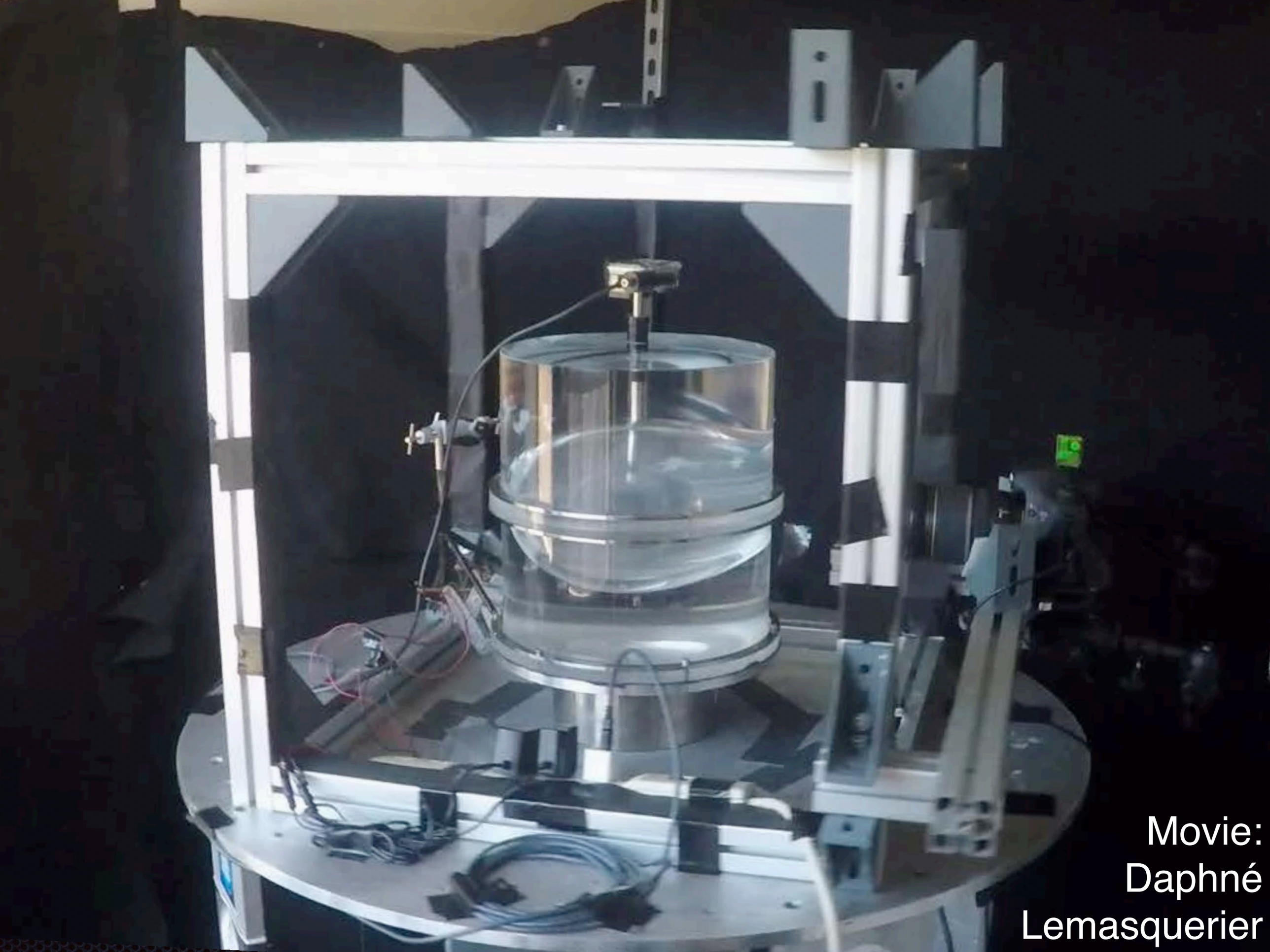
Beyond record players?...

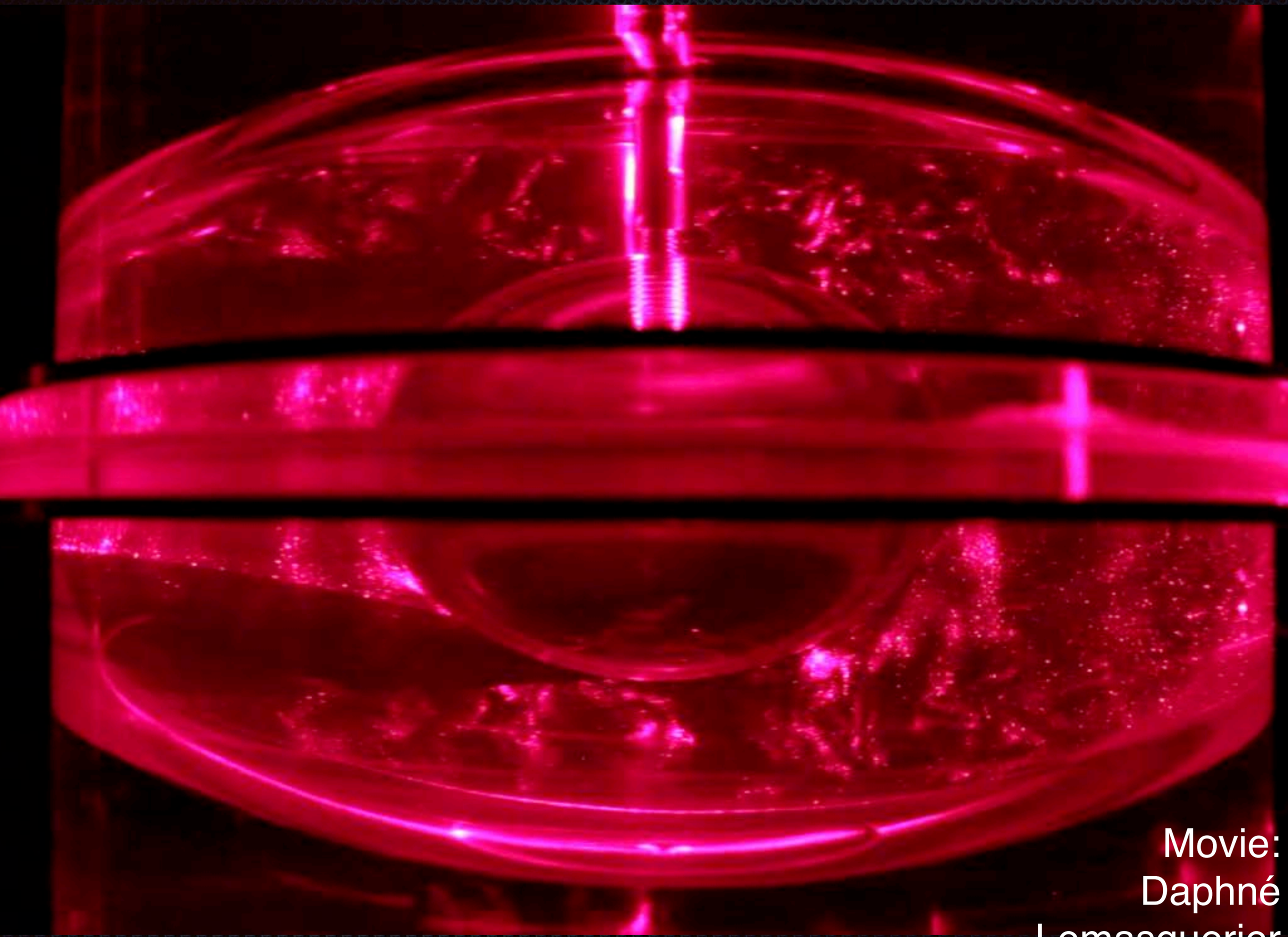


Cheng et al. *GJI* 15


Movie: J. Cheng



Movie:
Daphné
Lemasquerier


Movie:
Daphné
Lemasquerier

The Geodynamo

- Generates a magnetic fields and then continually regenerate that field
- Magnetohydrodynamic (MHD) process
  - ***Converts kinetic energy of flowing conductor into magnetic energy***

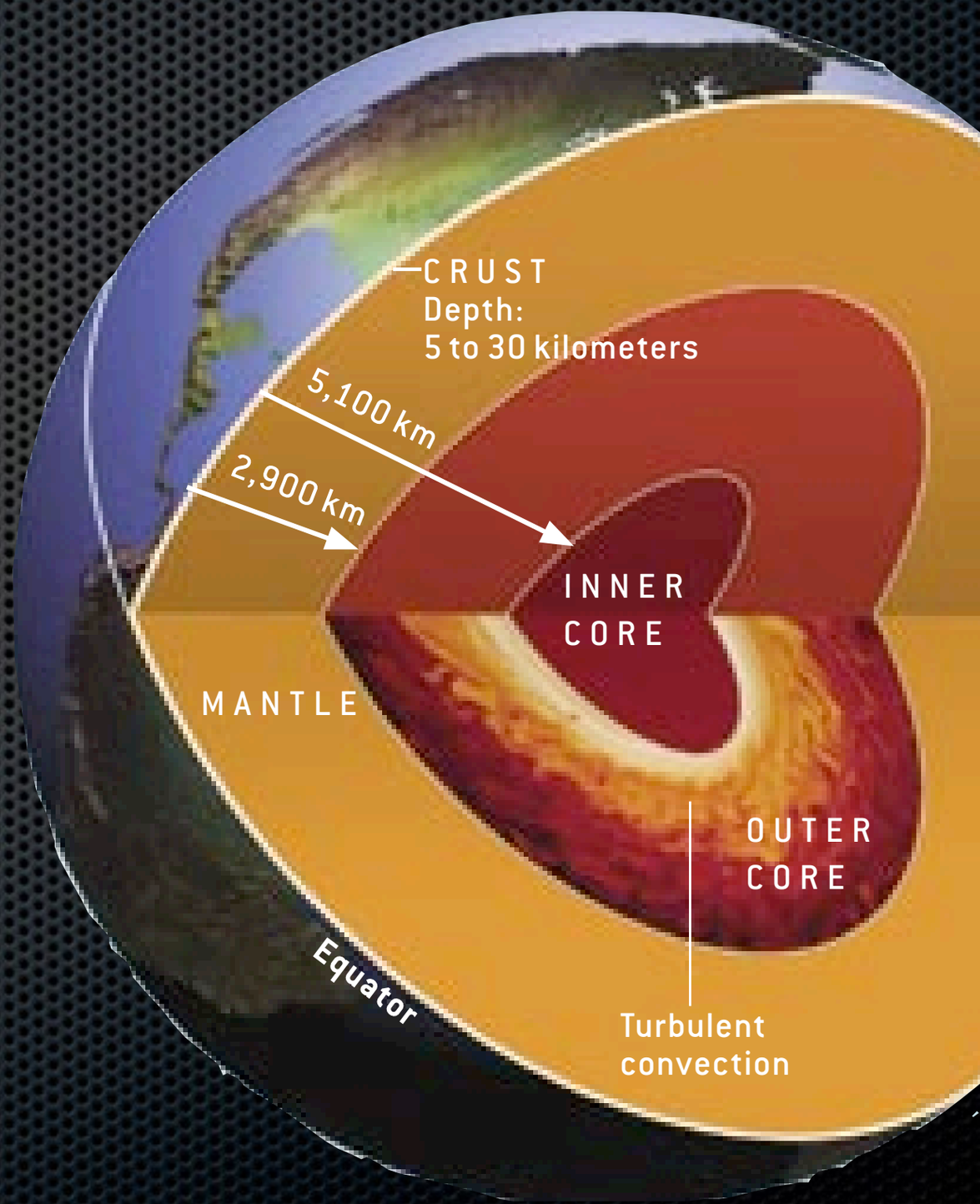


Image: Glatzmaier & Olson, SciAm 05

Successful MHD Modeling

- Navier-Stokes Equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -\frac{1}{\rho_o}\nabla p + \alpha T \mathbf{g} + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho_o}\mathbf{J} \times \mathbf{B}$$

- Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

- Energy Equation

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \nabla^2 T + S$$

- Current Density

Ampere's Law: $\mathbf{J} = \frac{1}{\mu_o}\nabla \times \mathbf{B}$

- Continuity

$$\nabla \cdot \mathbf{u} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

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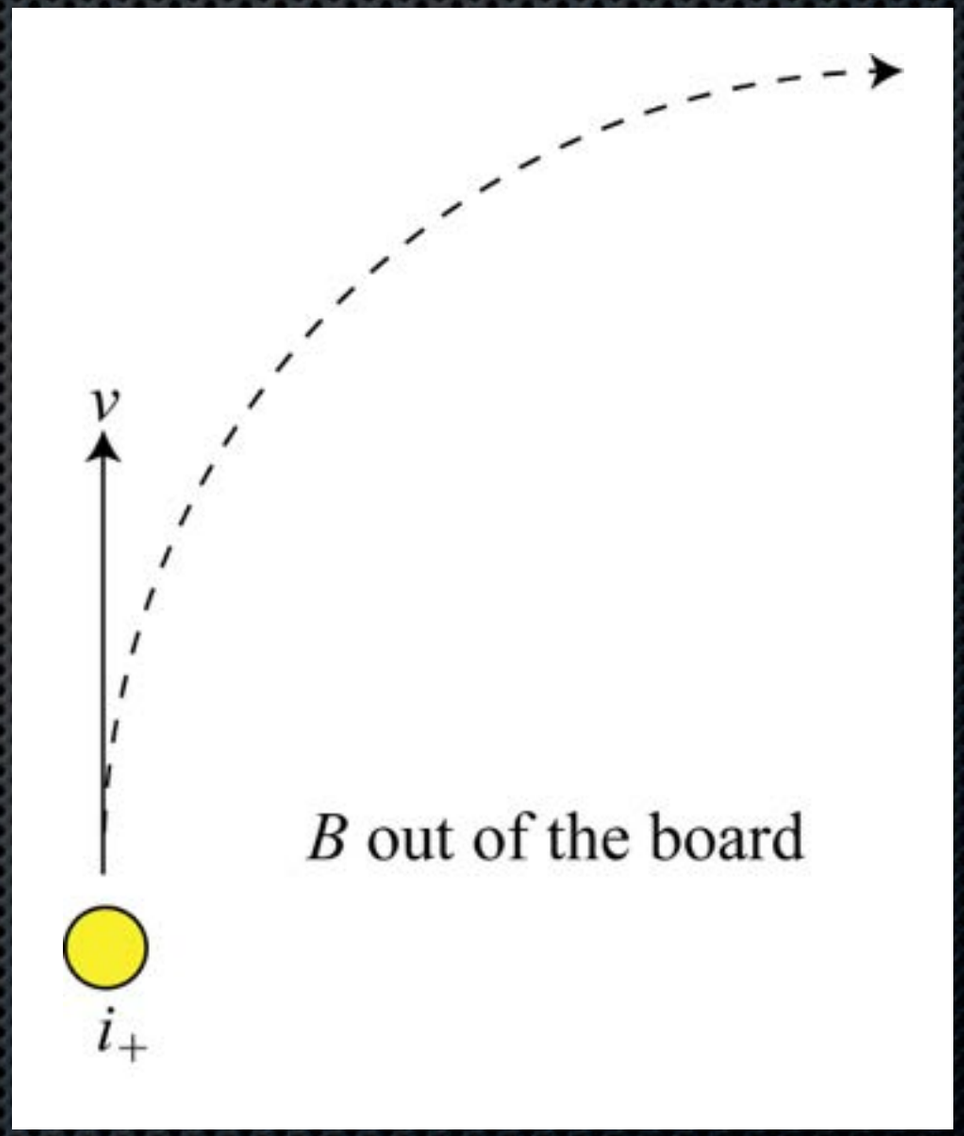
Lorentz Force on a Particle i

$$m_i \frac{d\vec{v}_i}{dt} = q_i \vec{v}_i \times \vec{B}$$

- Moving charged particle pushed off to its right



$$R_{gr} = \frac{m_i v_i}{q_i B}$$

Lorentz Force in a Continuum

$$m_i \frac{d\vec{v}_i}{dt} = q_i \vec{v}_i \times \vec{B}$$

- Using the current density J, where n is the charge density

$$\vec{J} = nq < \vec{v}_i >= \frac{I}{A}$$

- Gives:

$$\rho \frac{d\vec{u}}{dt} \delta V = nq\vec{v}_d \delta V \times \vec{B}$$

Lorentz Force in a Continuum

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- Gives:

$$\rho \frac{d\vec{u}}{dt} = \vec{J} \times \vec{B}$$

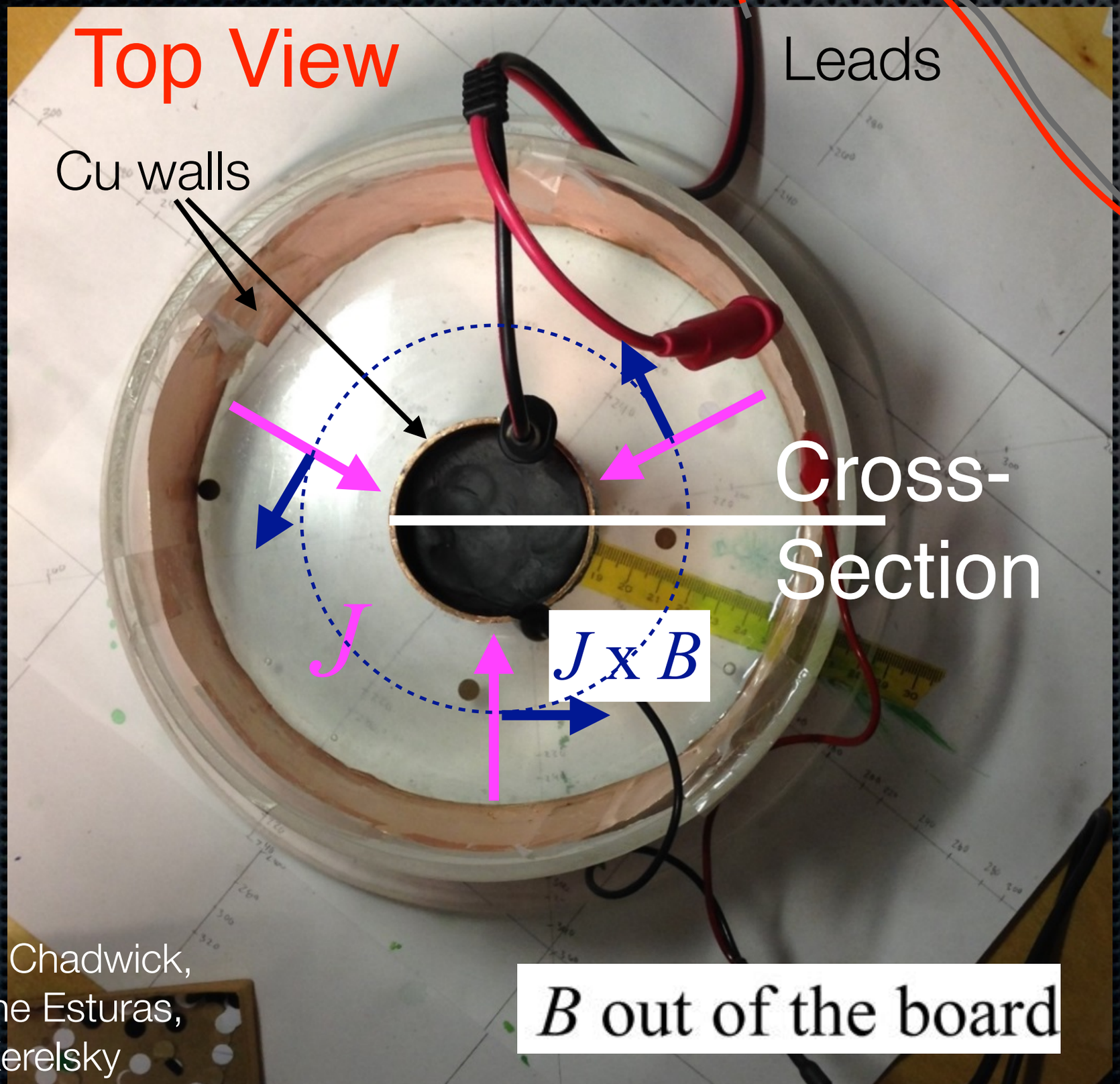
- *NB*: u is now the <u>material velocity</u>!

Lorentz Force in a Continuum

- An example maybe?
  - MHD Pump
    - "Opposite" of a dynamo: J and B drive u

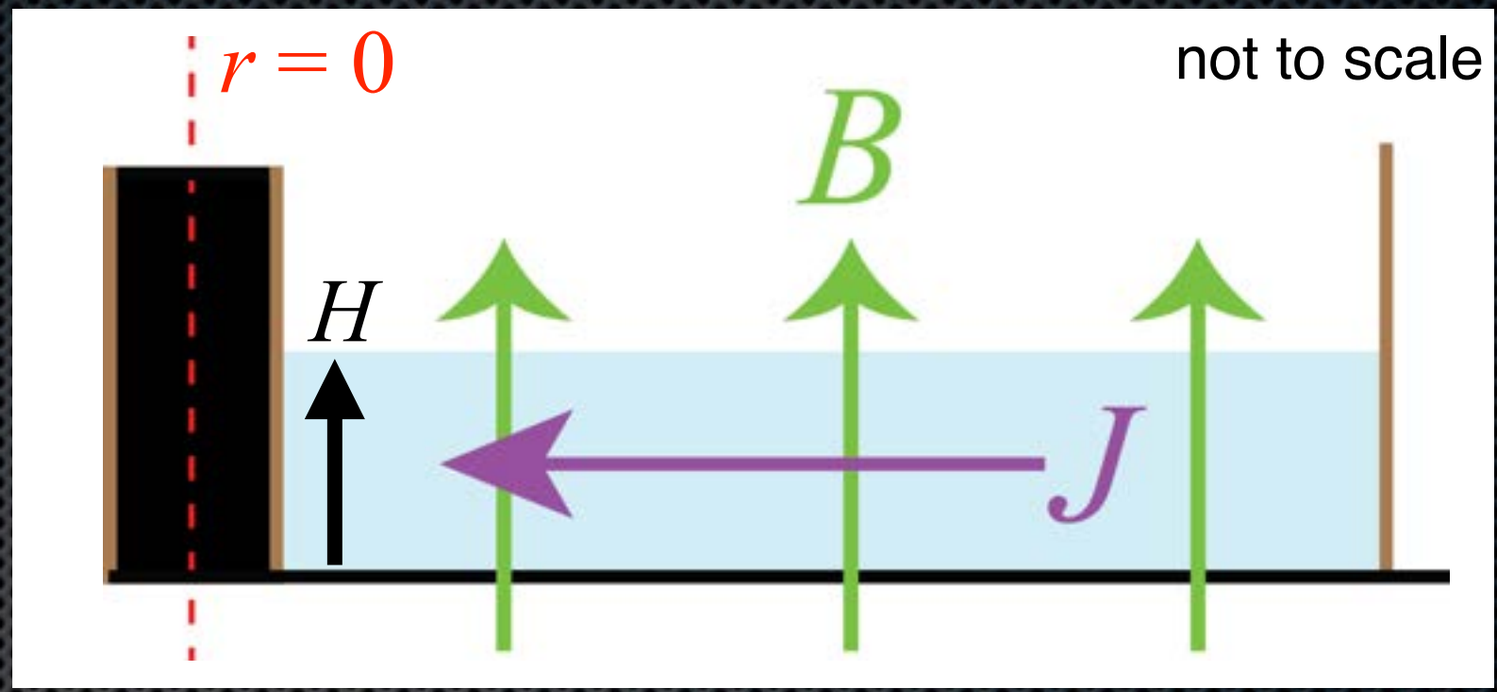
$$0 = \vec{J} \times \vec{B} + \mu \nabla^2 \vec{u}$$

MHD Pump



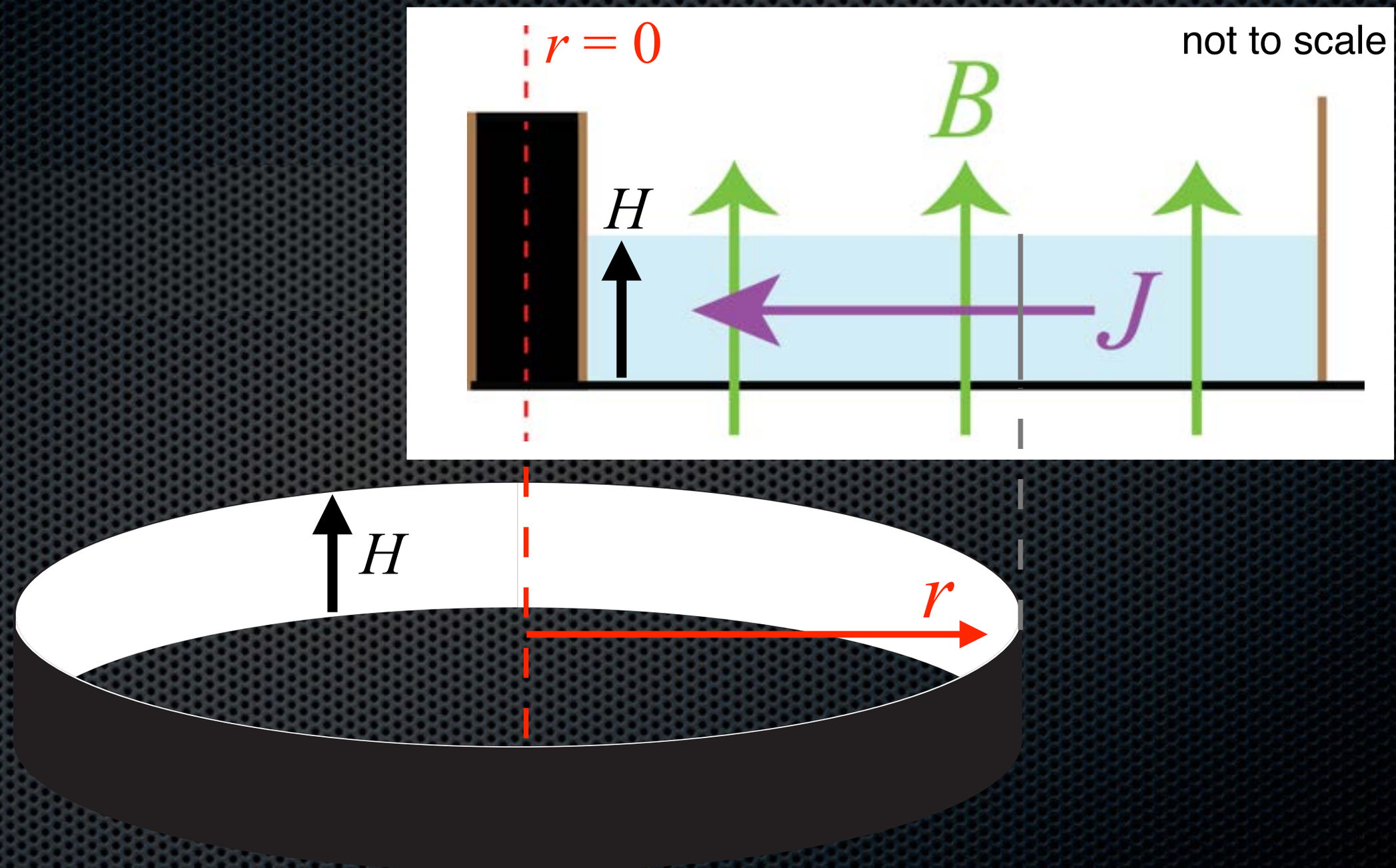
Austin Chadwick,
Lorraine Esturas,
Alex Kerelsky

MHD Pump



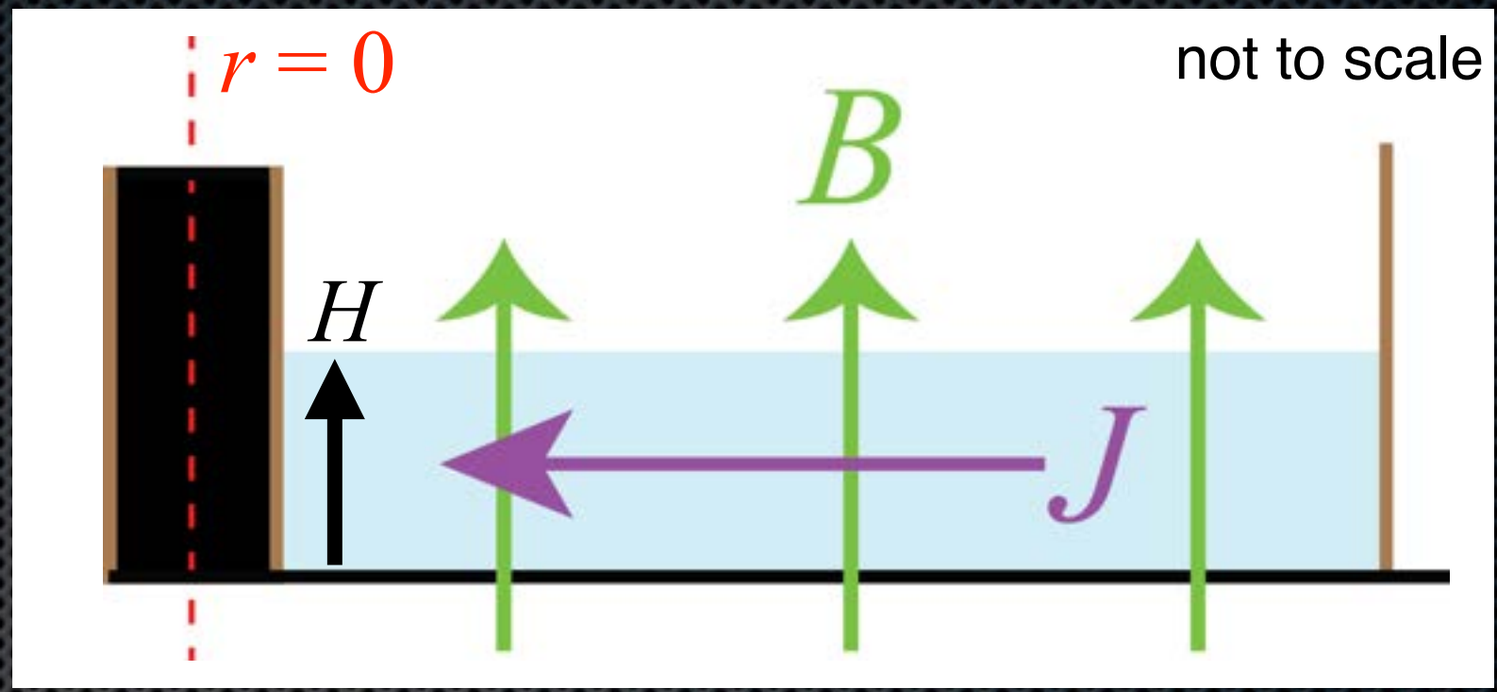
MHD Pump

$$J = \frac{I}{A}$$



MHD Pump

$$J = \frac{I}{A} = \frac{I}{2\pi r\, H} \propto \frac{1}{r}$$



- Thin layer: $H << r_{inner}$

$$0 = \vec{J} \times \vec{B} + \mu \nabla^2 \vec{u}$$

$$0 = \frac{IB}{2\pi r H} + \mu \frac{d^2 u_\phi}{dz^2}$$

$$u_\phi(z = 0) = 0$$

$$u_\phi(H) = \left(\frac{IBH}{4\pi\mu}\right)\frac{1}{r}$$

MHD Pump

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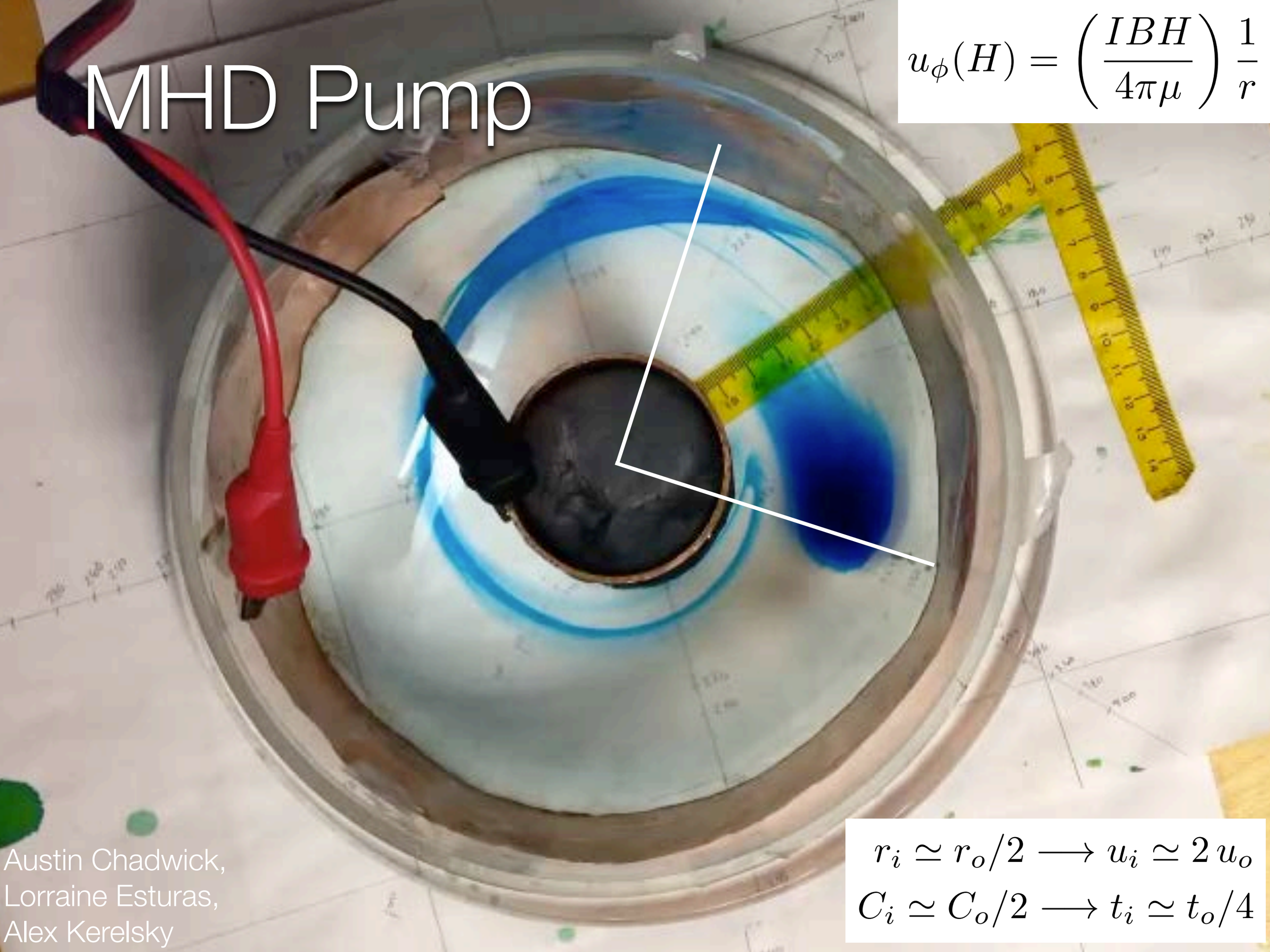

$$r_i \simeq r_o/2 \longrightarrow u_i \simeq 2\,u_o$$
$$C_i \simeq C_o/2 \longrightarrow t_i \simeq t_o/4$$

Austin Chadwick,
Lorraine Esturas,
Alex Kerelsky

MHD Pump

- "Opposite" of a dynamo
  - J x B drives flow, u, in an electrically conducting fluid
    - ***Now to dynamos where u generates (J, B)...***

Dynamo Essentials

- Navier-Stokes Equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + 2\mathbf{\Omega} \times \mathbf{u} = -\frac{1}{\rho_o}\nabla p + \alpha T \mathbf{g} + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho_o}\mathbf{J} \times \mathbf{B}$$

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The Induction Equation

$$\frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} + \nabla \times \vec{u} \times \vec{B}$$

- Compare to the (scalar) heat equation

$$\frac{dT}{dt} = \kappa \nabla^2 T + \epsilon$$

The Induction Equation

$$\frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} \quad \longrightarrow \tau \sim \frac{L^2}{\eta}$$

- Compare to the (scalar) heat equation

$$\frac{dT}{dt} = \kappa \nabla^2 T \quad \longrightarrow \tau \sim \frac{L^2}{\kappa}$$

The Induction Equation

$$\frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} \quad \longrightarrow \tau \sim \frac{L^2}{\eta}$$

- Magnetic diffusivity ~ 1 m^2/s in core fluids
- Magnetic diffusion time ~ 15 kyr or so
  - Geologically rapid decay

The Induction Equation

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The Induction Equation

$$\frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} + \nabla \times \vec{u} \times \vec{B}$$

- For a self-sustaining dynamo, the inductive source term must be able to exceed the diffusion term

The Induction Equation

$$\frac{\partial \vec{B}}{\partial}$$

- Fo m
  m

- Magnetic Reynolds number:

$$Rm = \frac{UL}{\eta} \gtrsim 40$$ for dynamo action

Lab vs. Numerical Dynamos?

The Induction Equation

$$\frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} + \nabla \times \vec{u} \times \vec{B}$$

- For a self-sustaining dynamo, the inductive source term must be able to exceed the diffusion term
- Magnetic Reynolds number:

$$Rm = \frac{UL}{\eta}$$

- *NB*: magnetic field lines are carried along with the fluid at large Rm

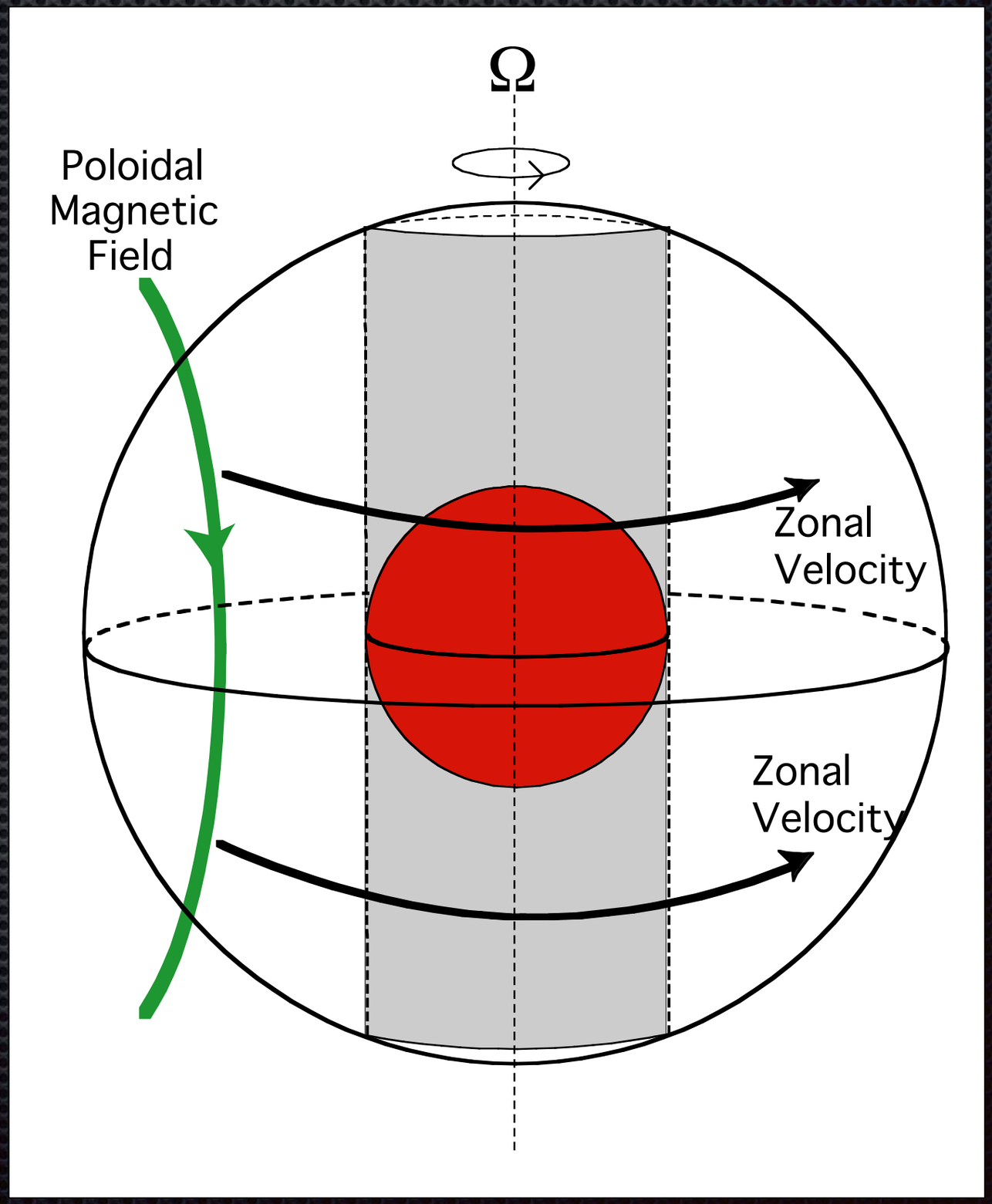
The Omega-Effect

- Shearing out of magnetic field orthogonal to its direction (**axisymmetric**)
- Inductive stretching term:

$$\nabla \wedge (u\hat{\phi}) \wedge (B_o \hat{s})$$

$$= B_o \hat{s} \cdot \nabla u \hat{\phi}$$

$$= B_o \frac{\partial u}{\partial s} \hat{\phi}$$



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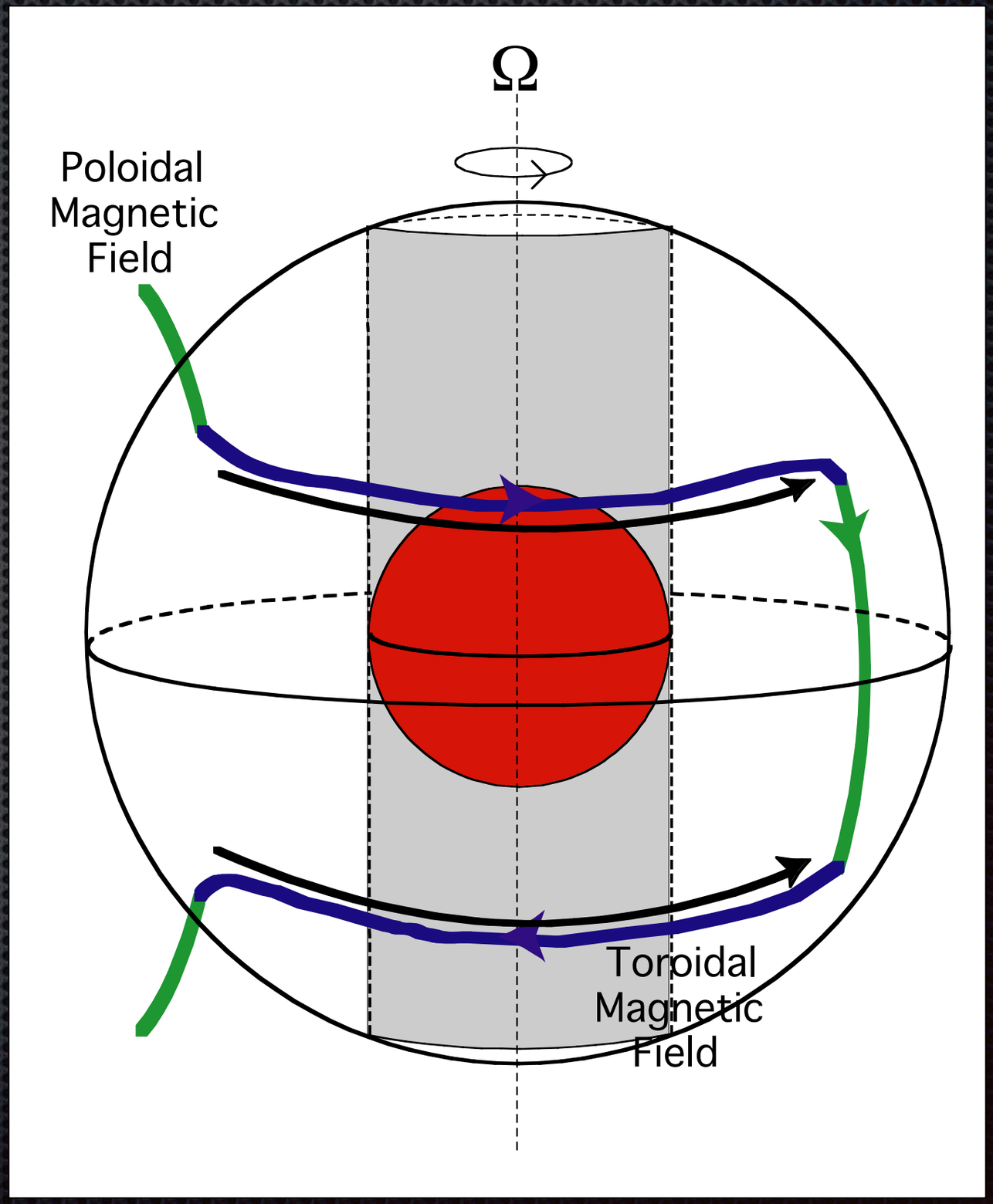

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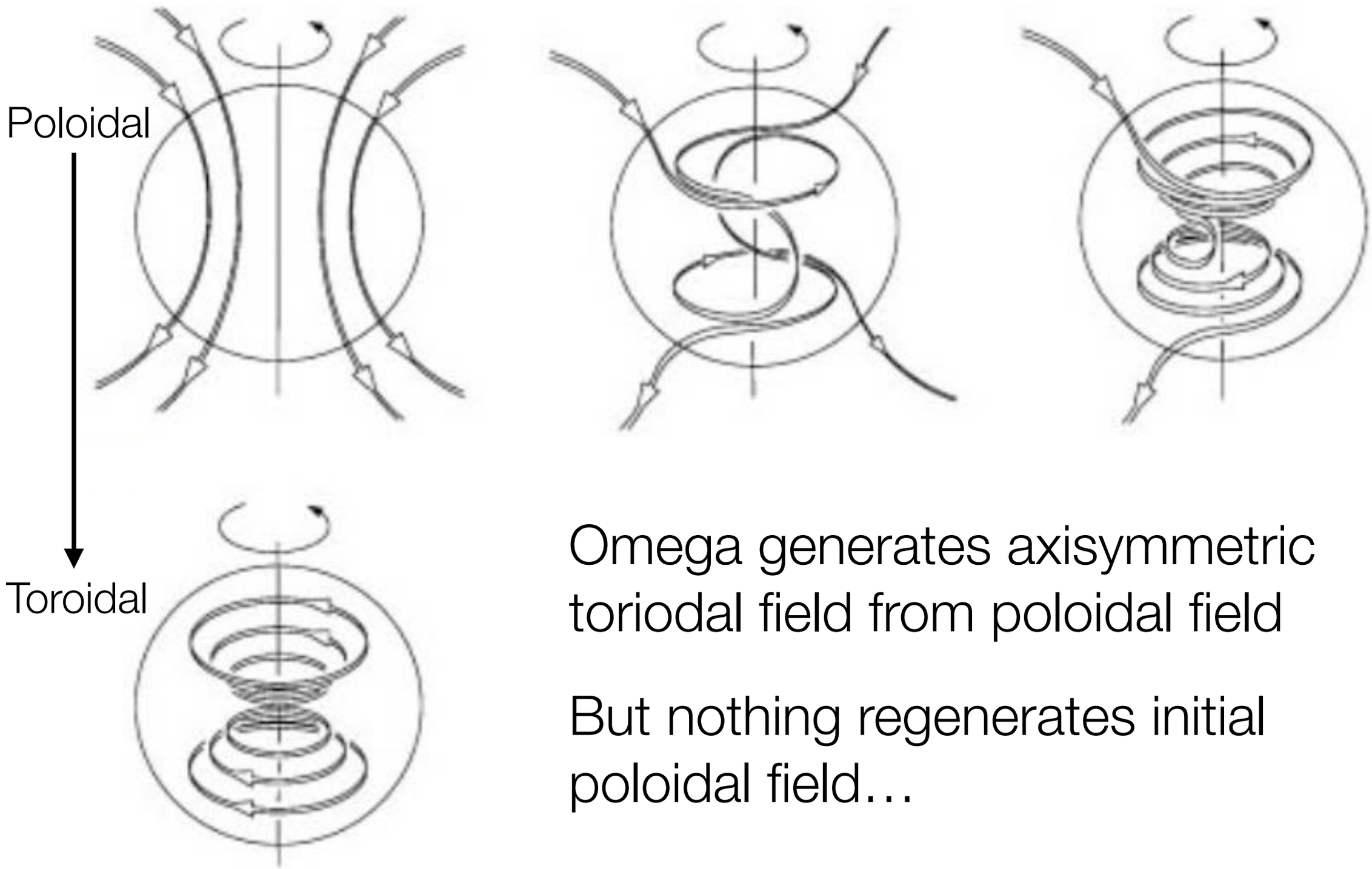


Omega generates axisymmetric toriodal field from poloidal field

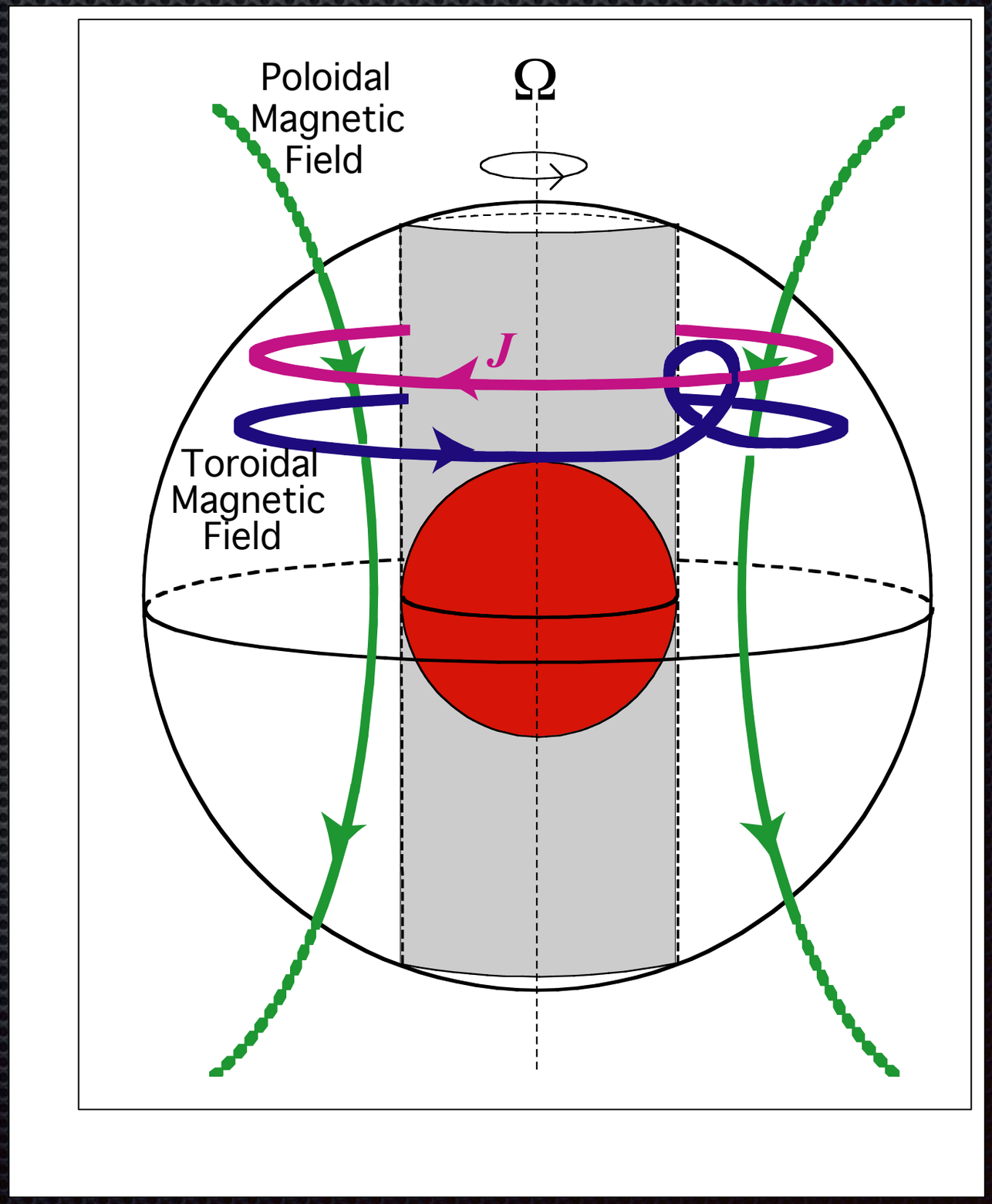
But nothing regenerates initial poloidal field…

citation?

The Alpha-Effect

$$\frac{1}{\mu_o}\nabla \times \vec{B} = \vec{J}$$

- ***Helical*** twisting of magnetic field lines (**Non-axisymmetric!!!**)
- Can generate J parallel to toroidal B
  - Ensemble parallel J generates toloidal B
  - Now Omega-effect can re-generate toroidal field… **DYNAMO!**



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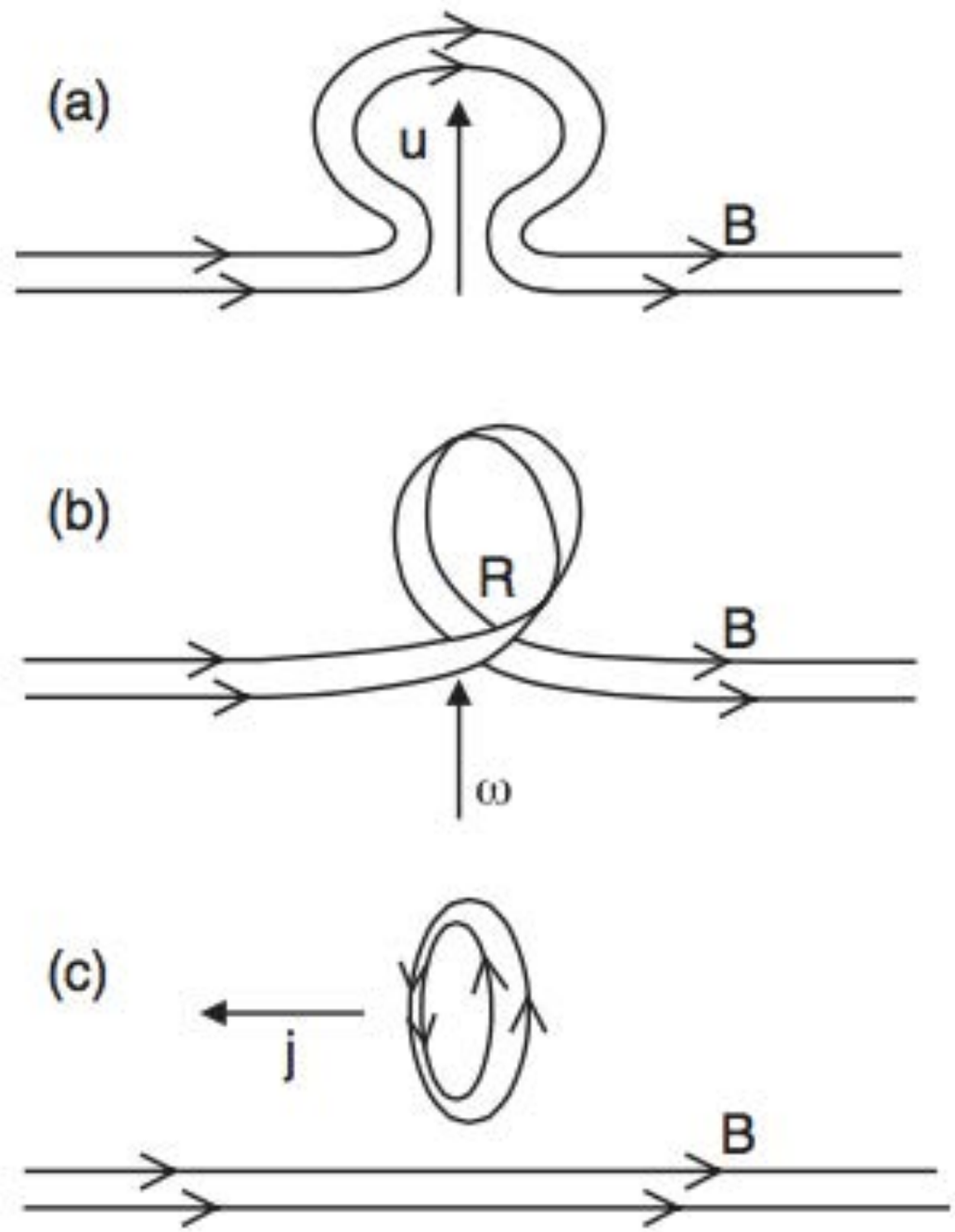
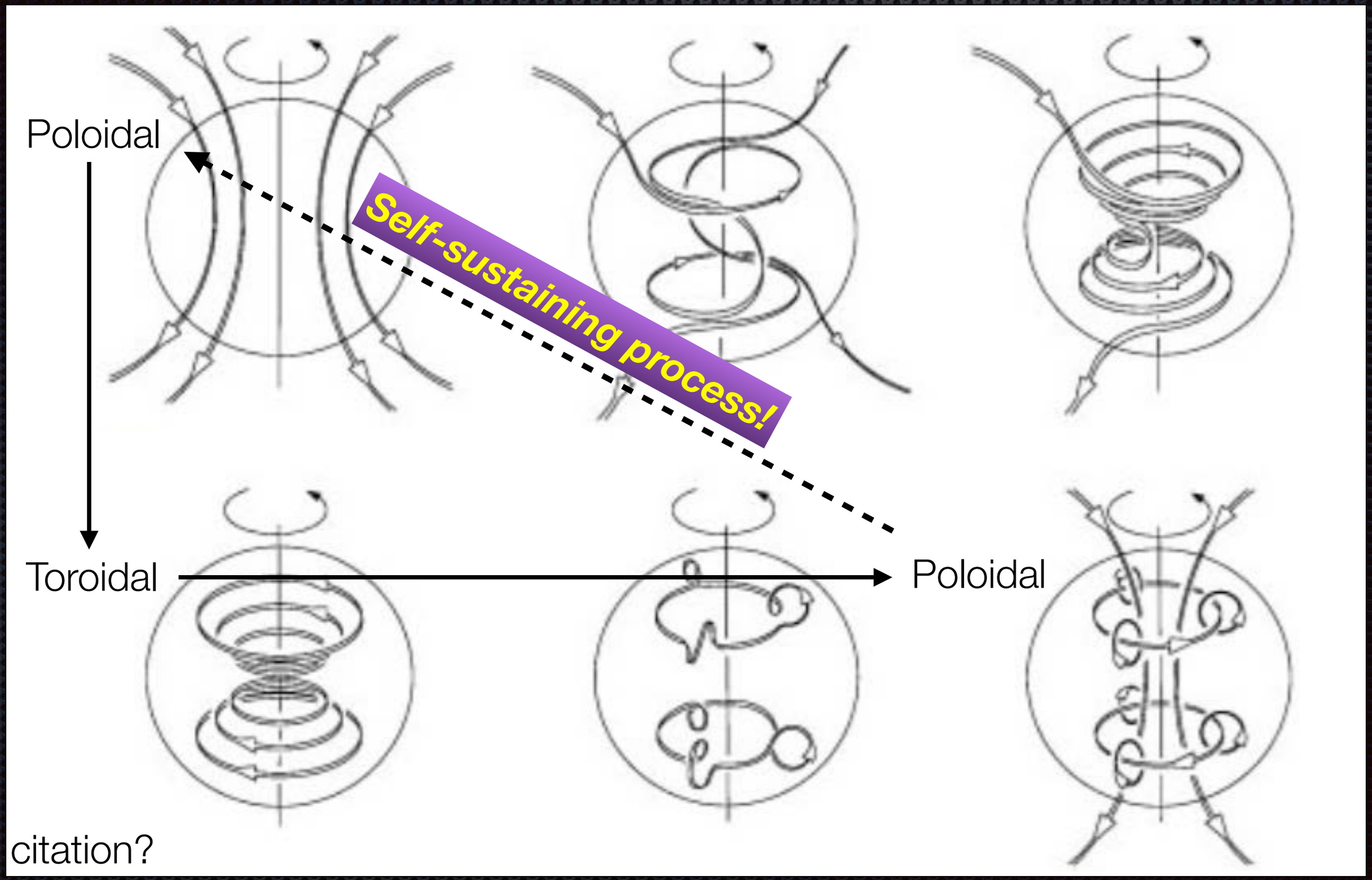


Roberts 2008

Alpha-Omega Dynamo

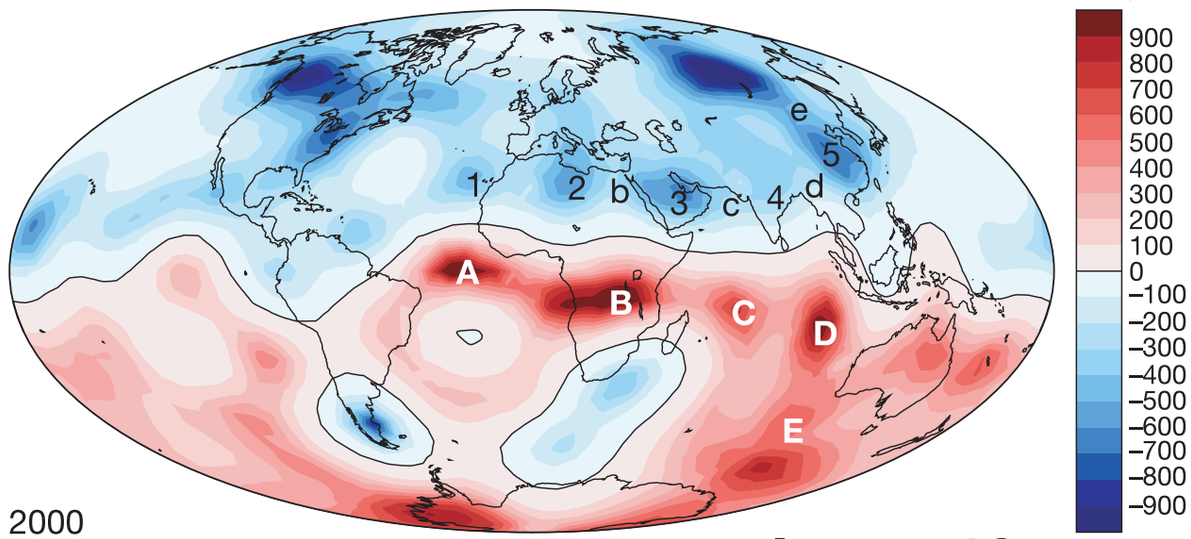


citation?

Observations

Jackson, *Nature* 2003

B_r CMB

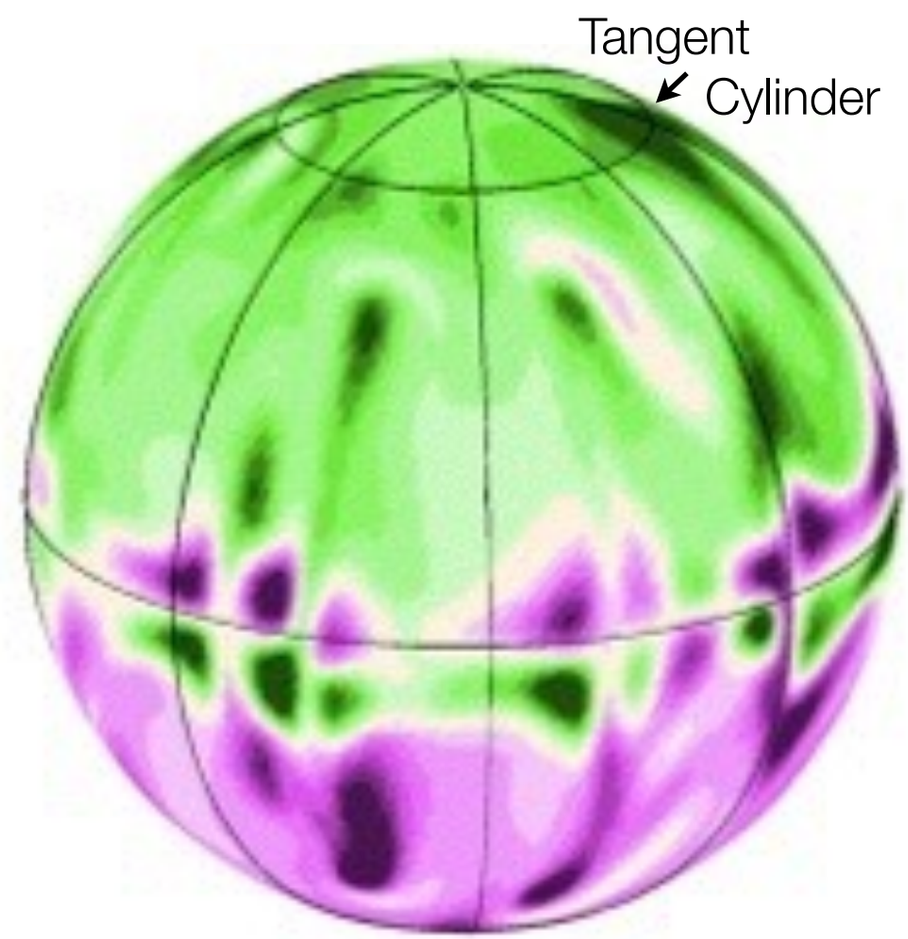


L_{max} ~ 13

Models

Soderlund et al. *EPSL* 2012

B_r CMB



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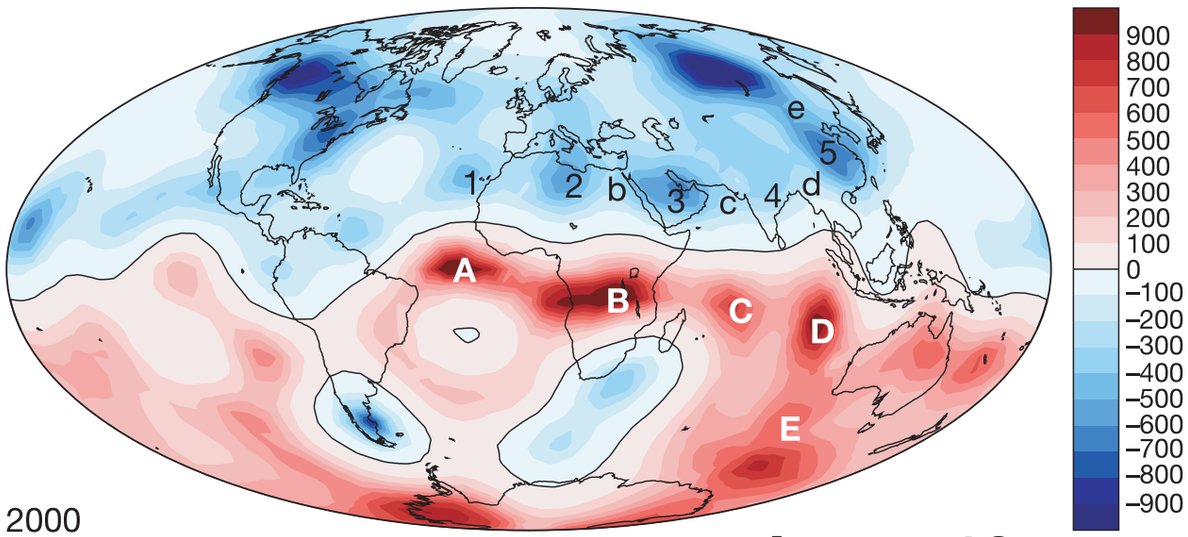


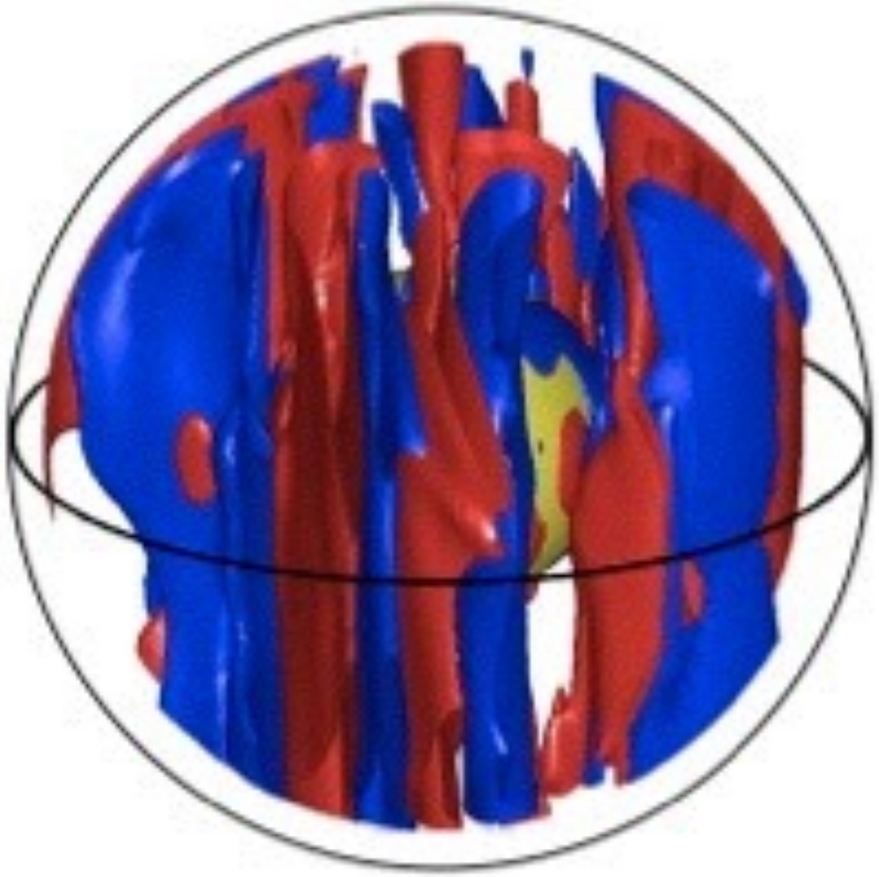
2000

L_{max} ~ 13

Models

Soderlund et al. *EPSL* 2012

***z*-vorticity**



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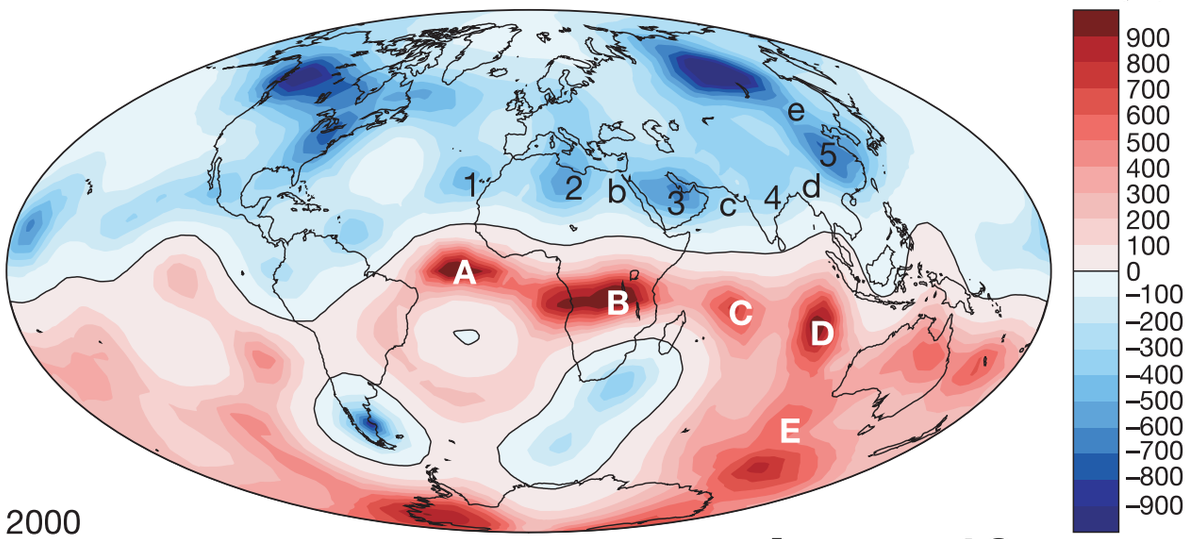

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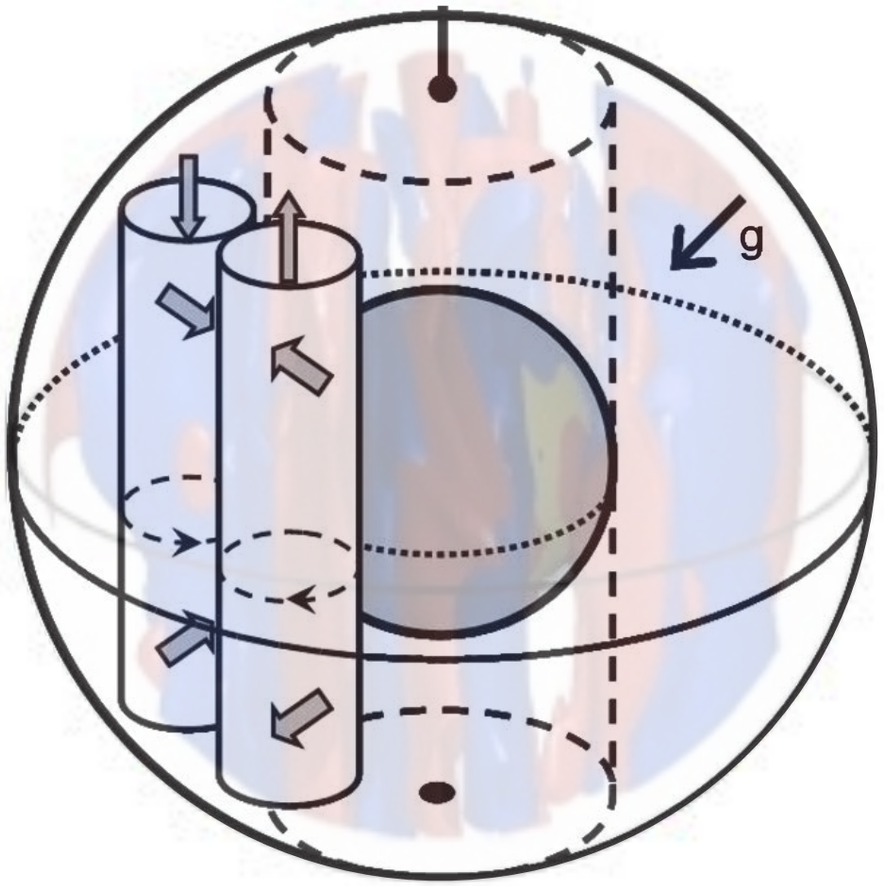
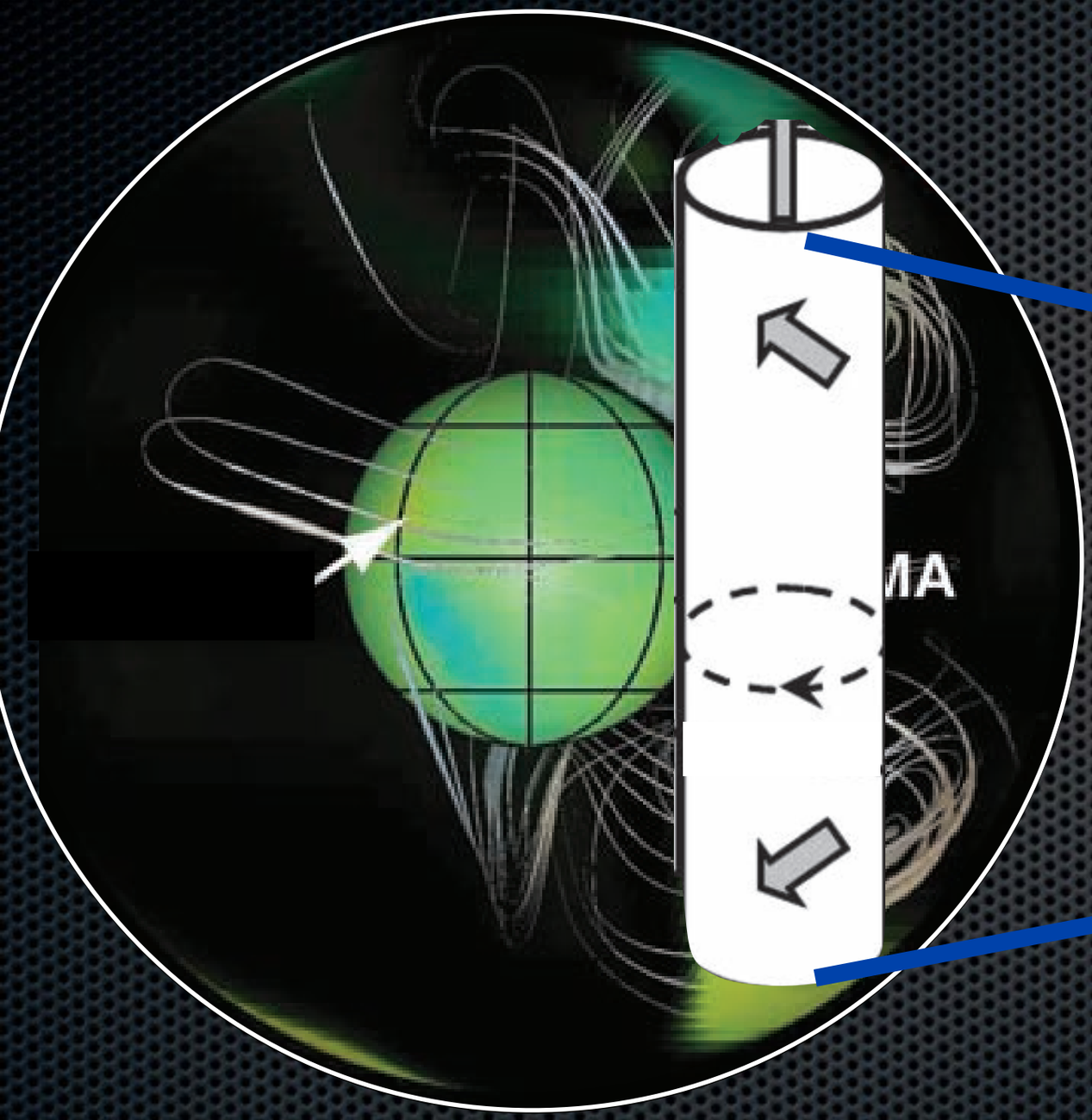
Christensen, *Enc. Solid Earth Geophys.* 2011

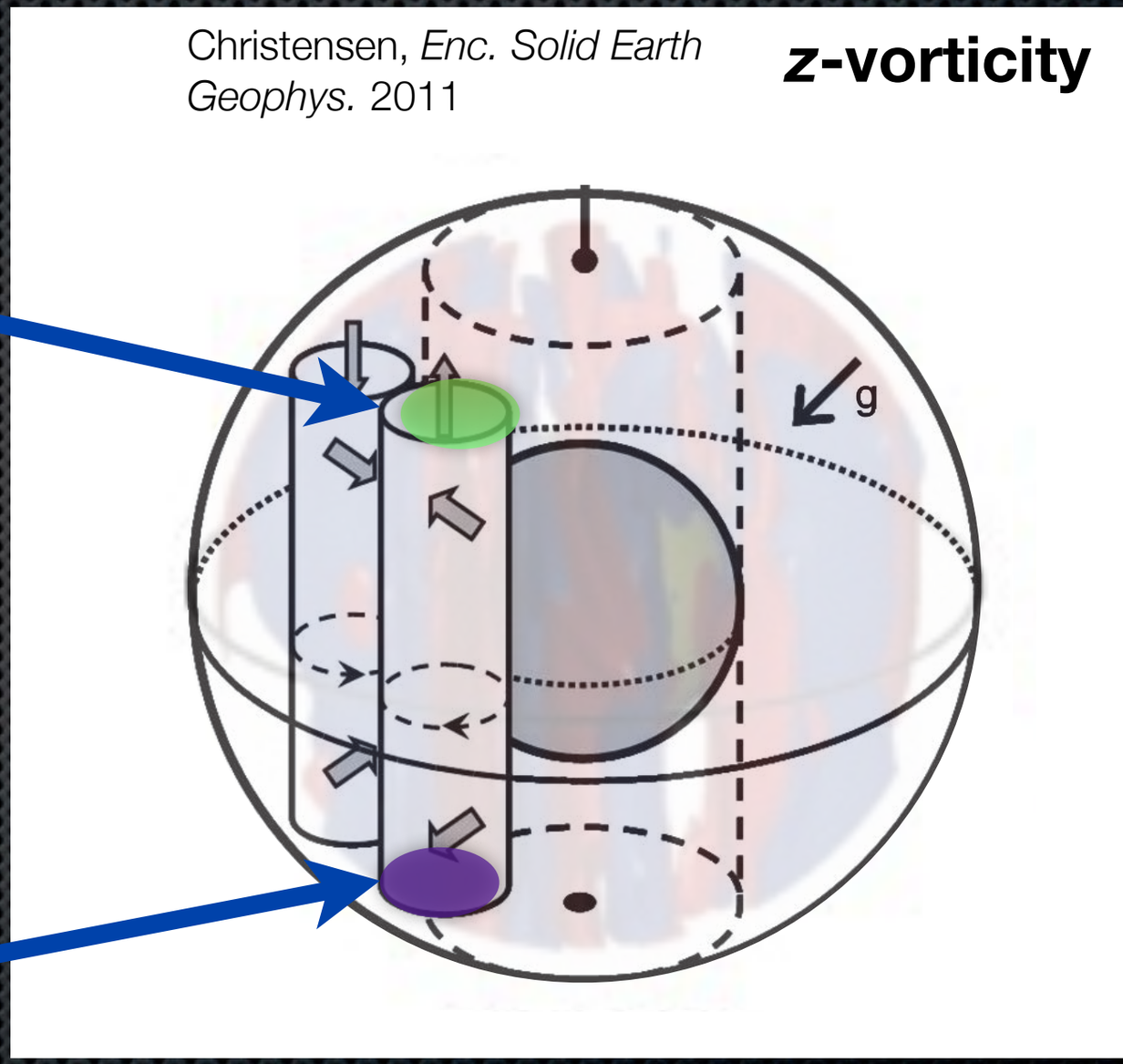
z-vorticity



Models



Christensen, *Enc. Solid Earth Geophys.* 2011



Aubert et al. *GJI* 2008

Observations

Models

Jackson, *Nature* 2003

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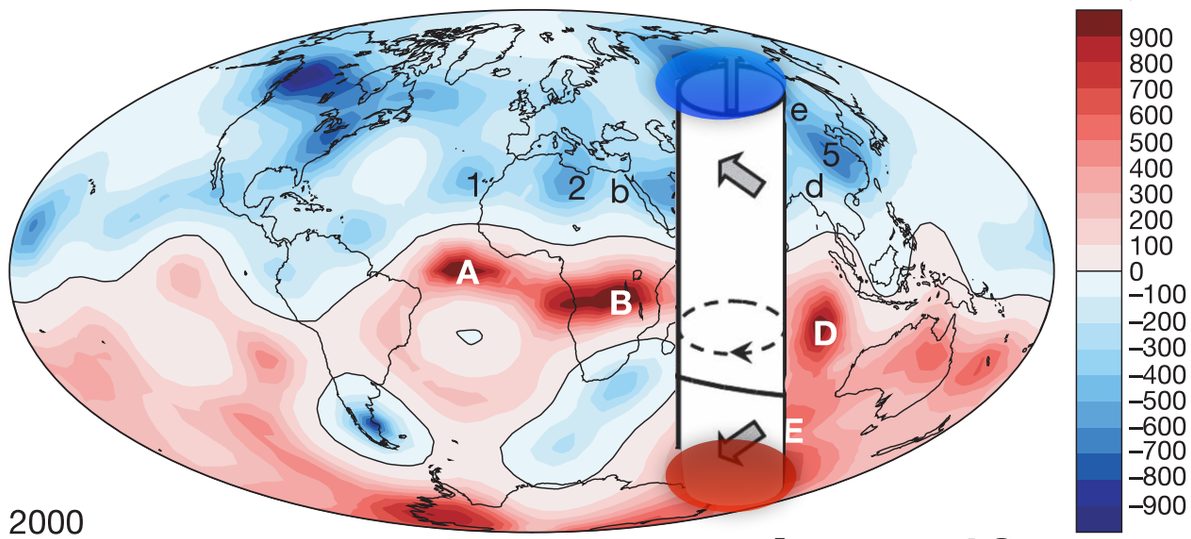

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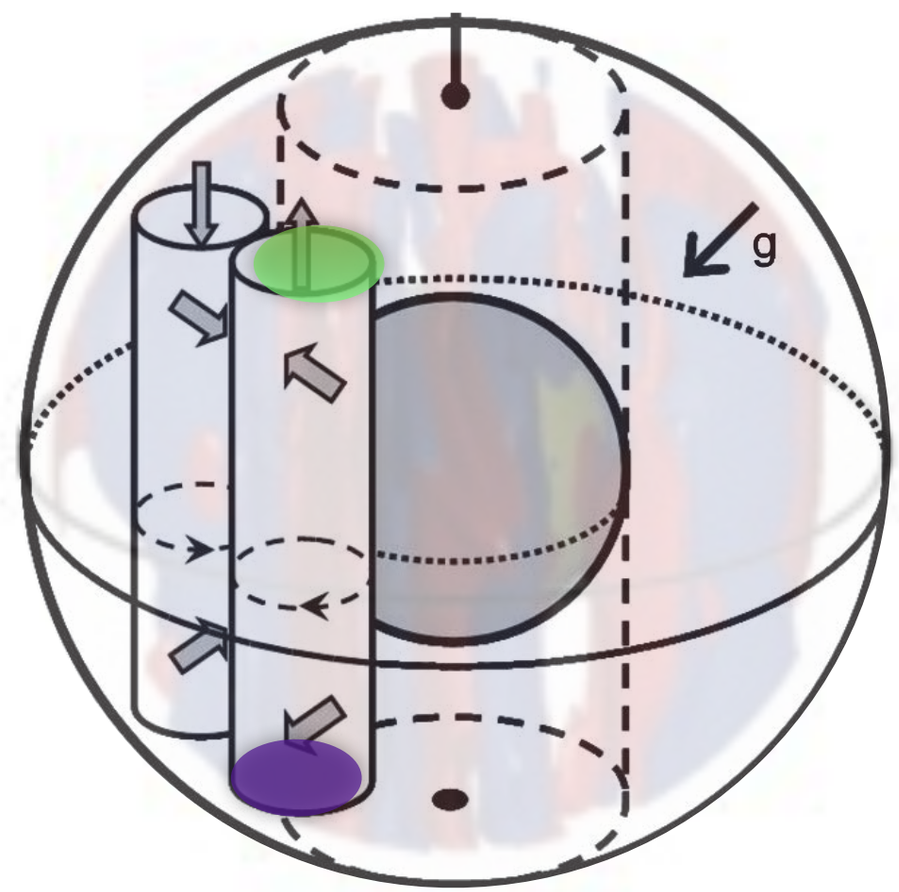


- **"Present" paradigm:** helical columns extrapolate to planetary cores; continually regenerate field

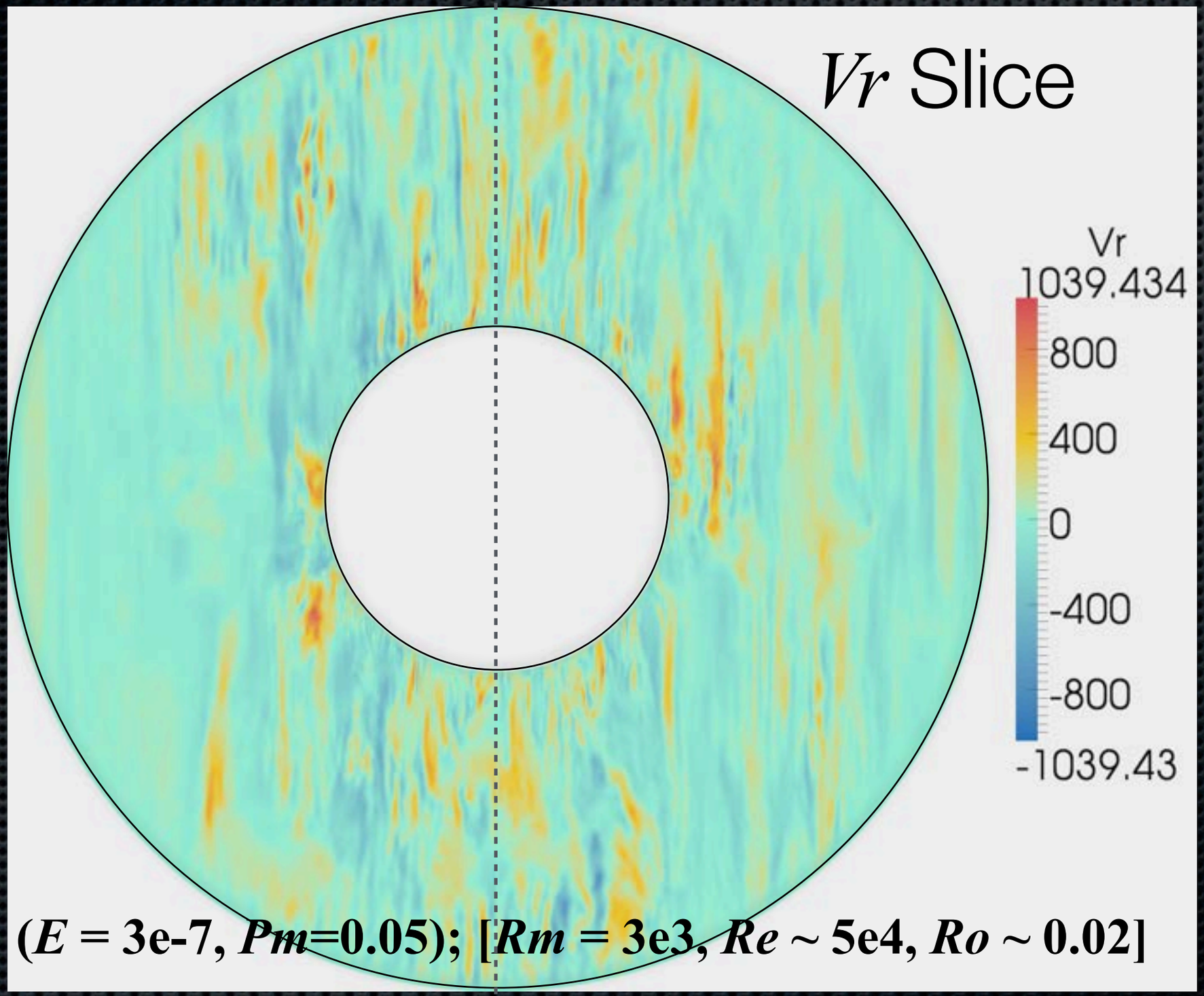
Summary

- Dynamos require:
  - Energy source
    - Organized flow
  - Conducting fluid
  - Large length scales

$$Rm = \frac{UL}{\eta}$$

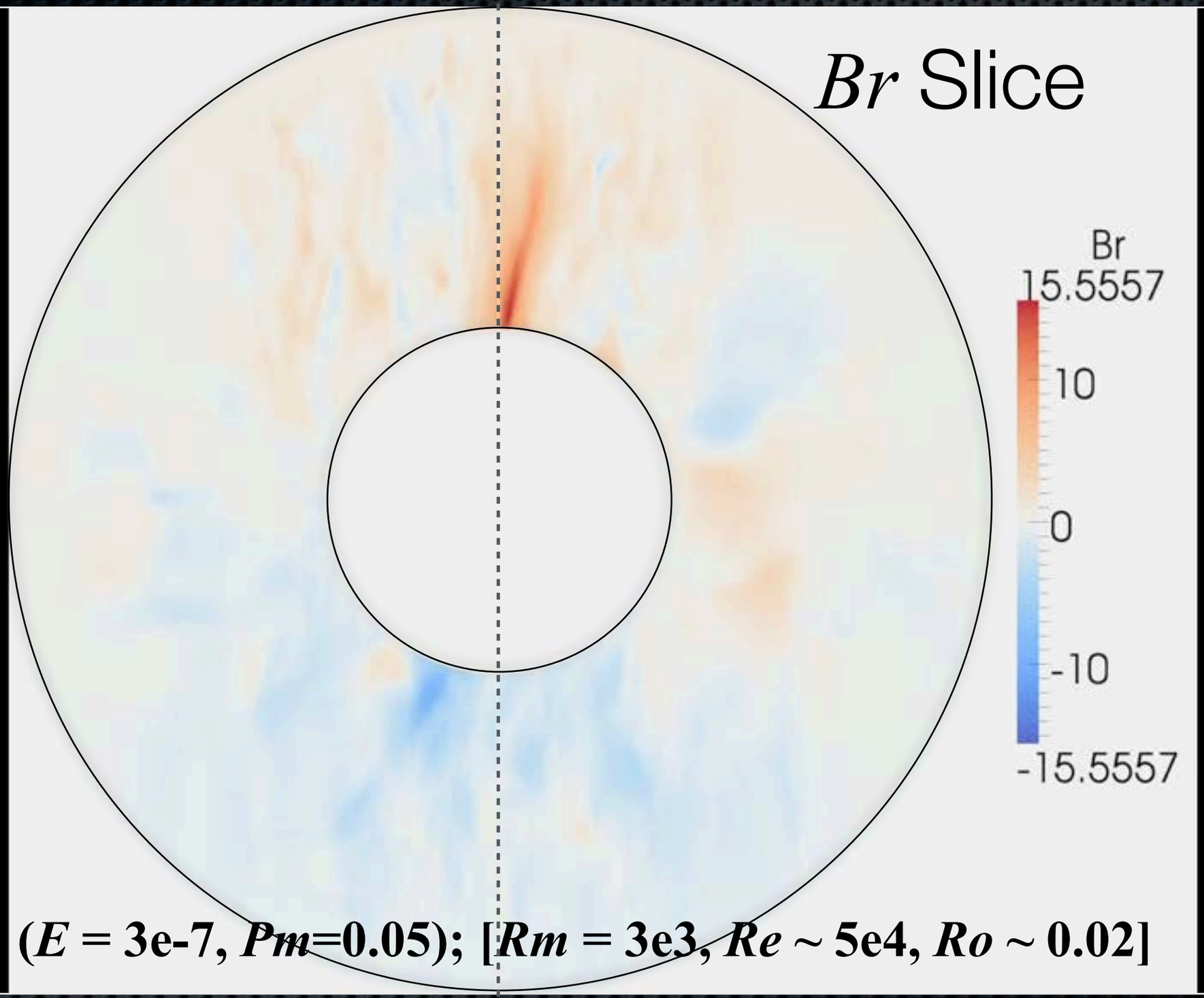
- Two stage regeneration process
  - Non-axisymmetric flow in at least one stage
- Paradigm: Large-scale helical columns

Sheyko's Turbulent Dynamo



Andrey Sheyko (2014)

Sheyko's Turbulent Dynamo



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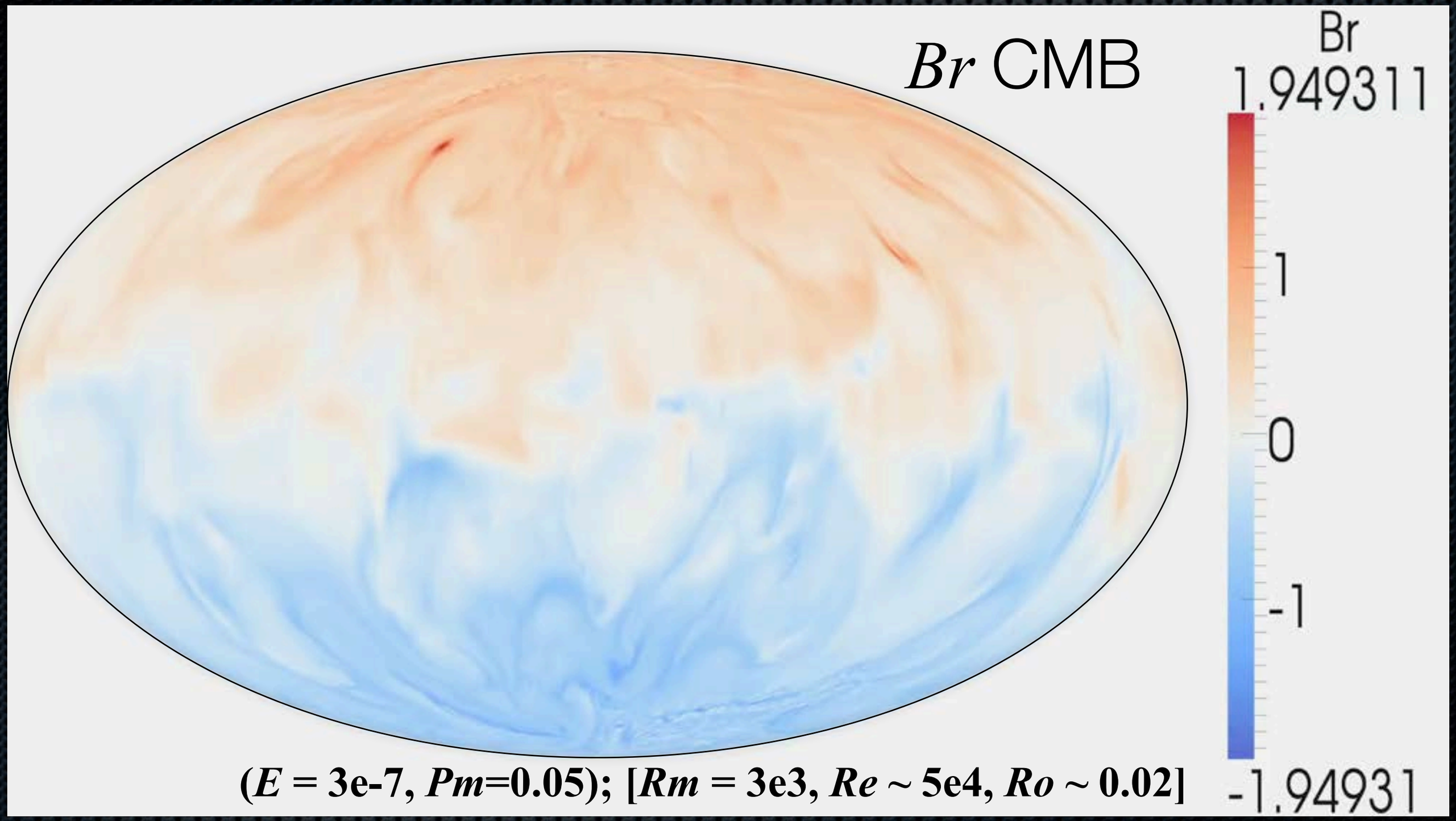

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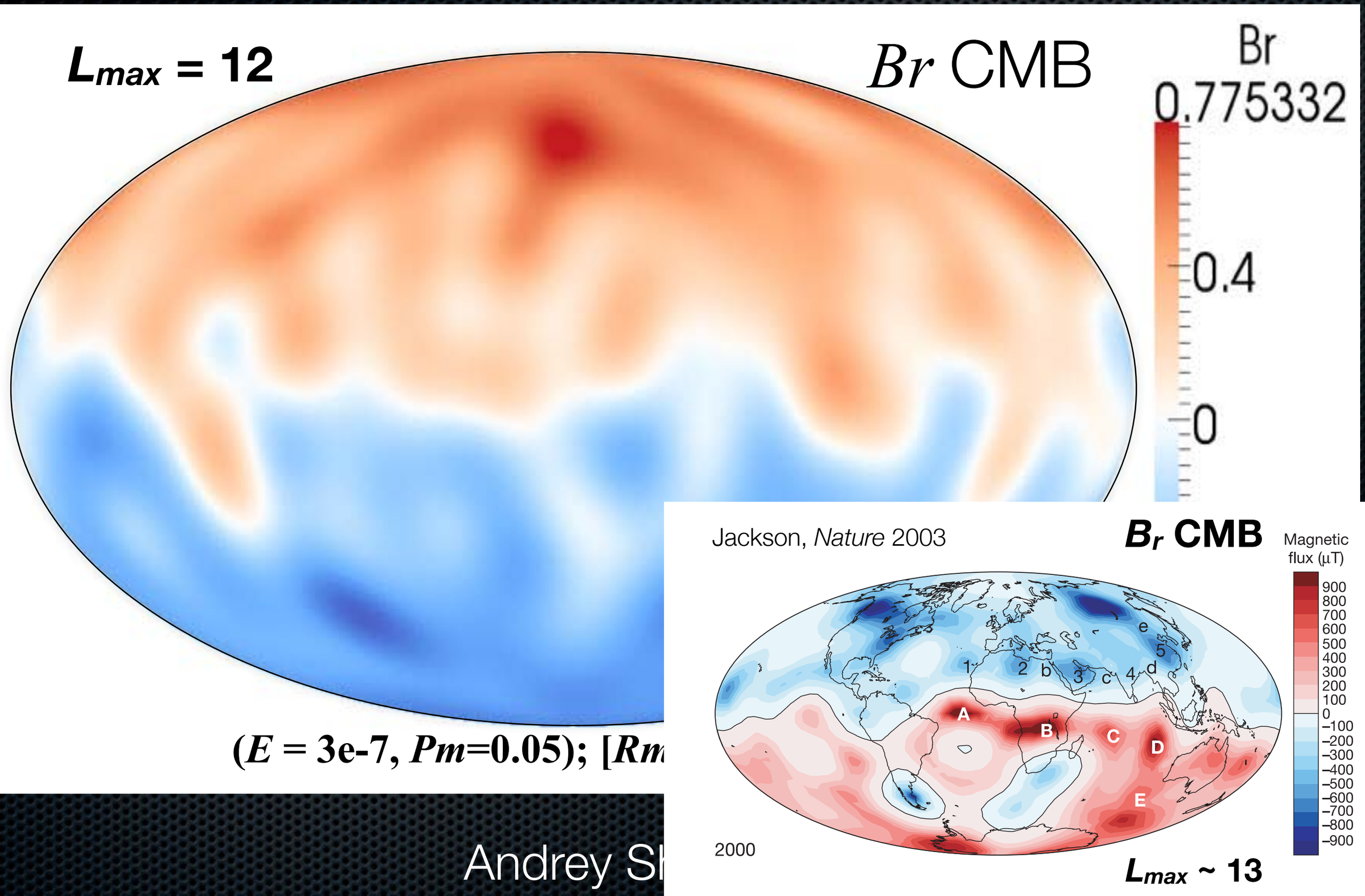
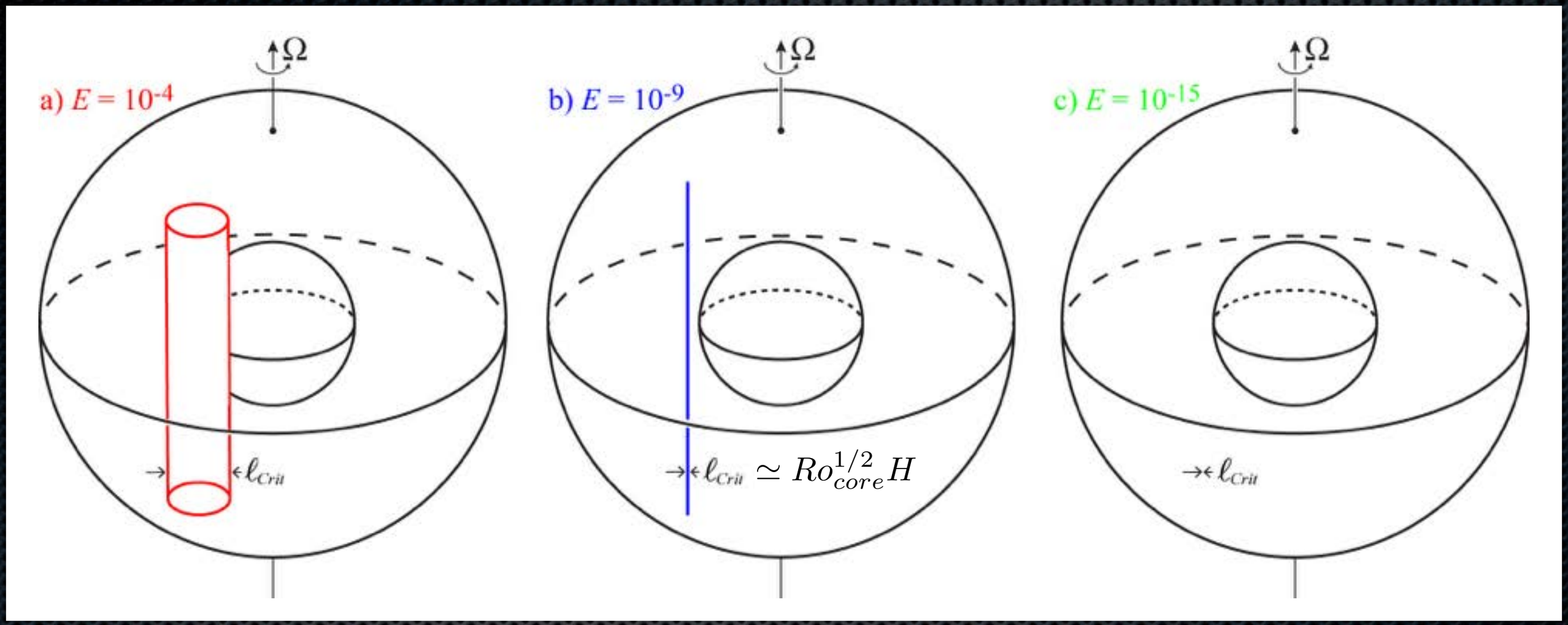


Andrey Sh

Sheyko's Turbulent Dynamo

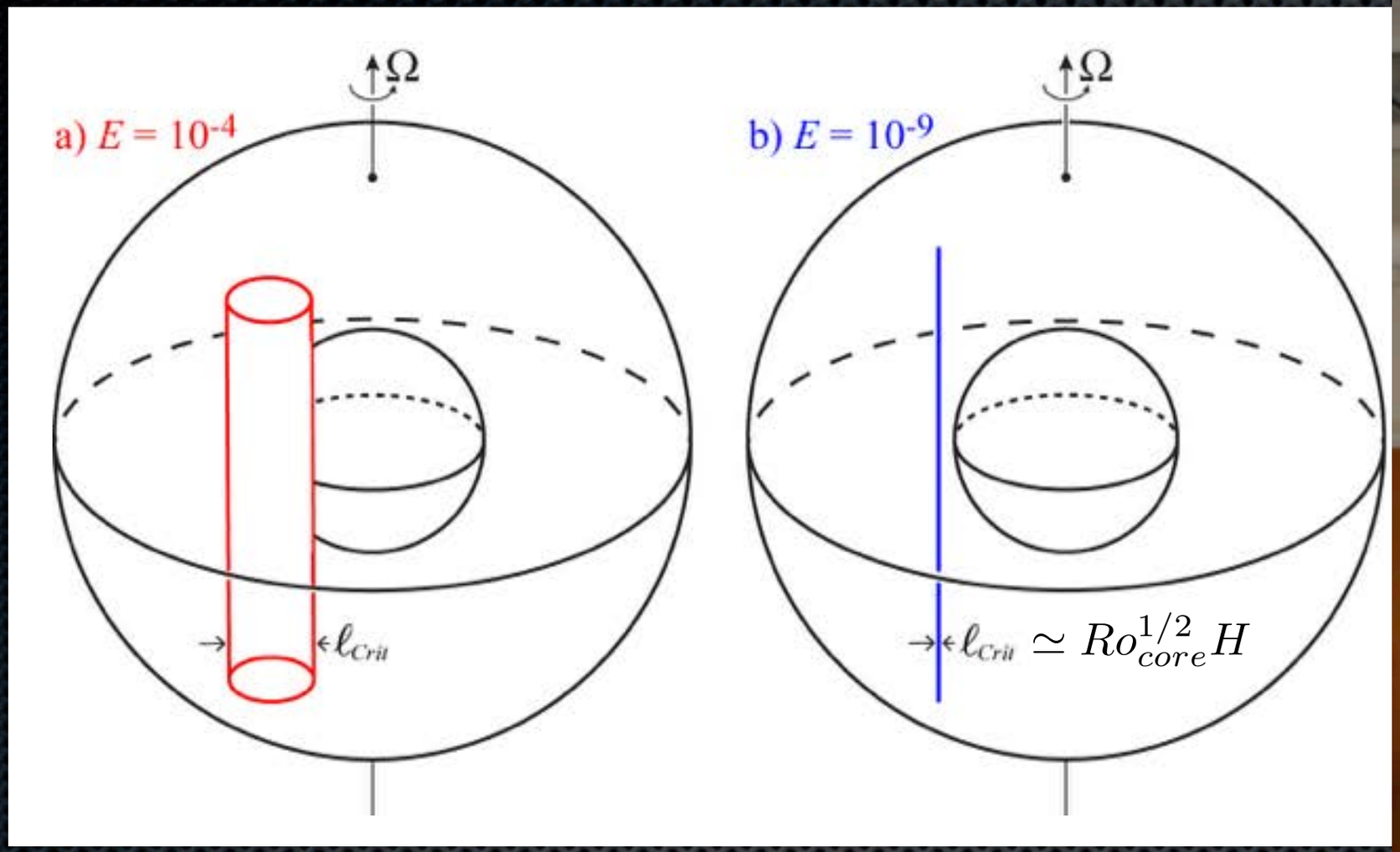
- Extreme model: magnetic flux patches no longer directly related to columns (scale separation)

Length Scale Questions



- Linear HD onset scale: $\ell \sim E^{1/3} H$
- In a turbulent HD core, $\ell \sim Ro^{1/2} H$

Length Scale Questions



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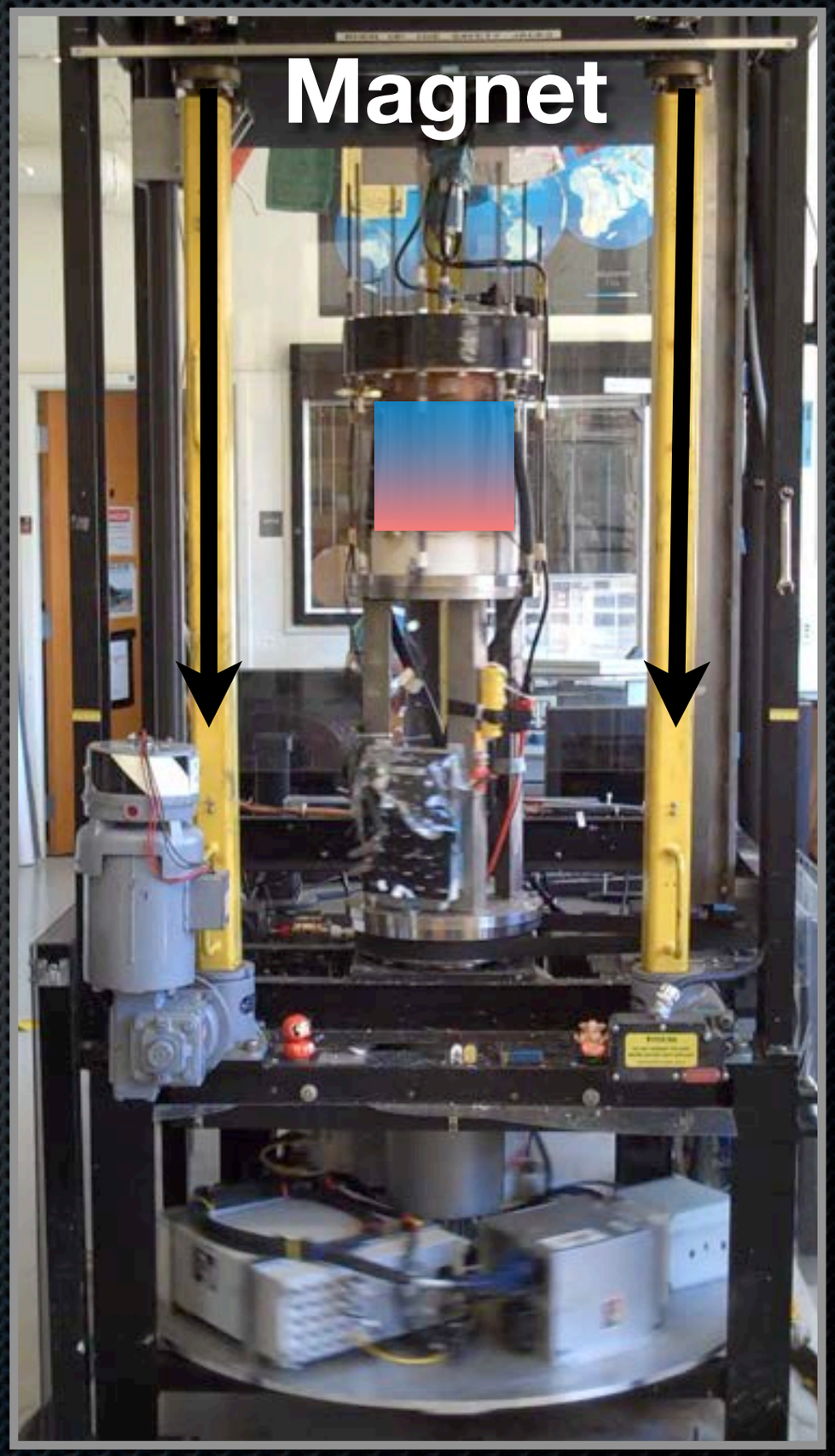
MHD Experiments

- Rotating magneto-convection in liquid metal
- Strong magnetic field can balance Coriolis
  - Different behaviors predicted



Top View:
Axial
Velocity

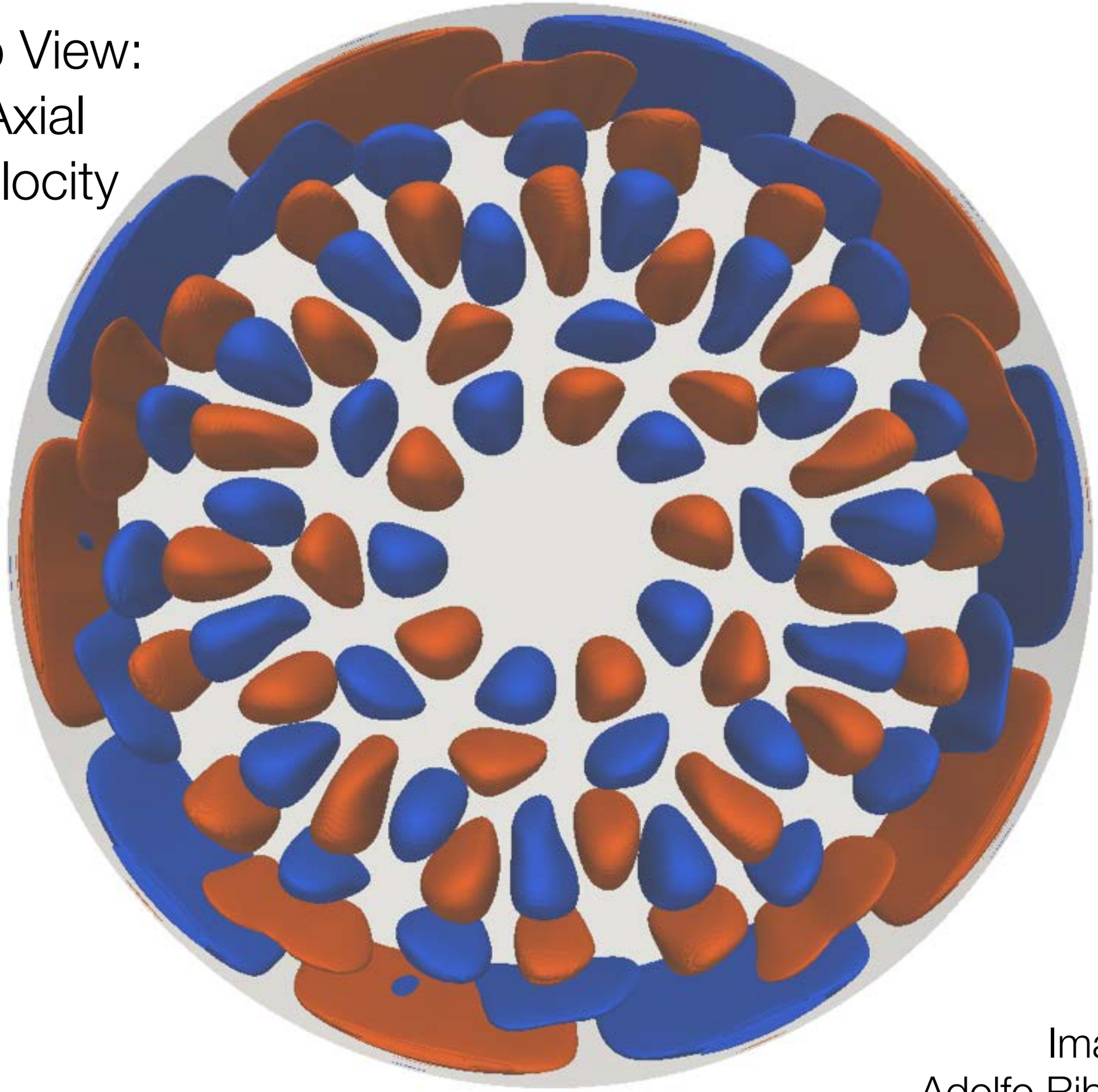


Image:
Adolfo Ribeiro

Lab MHD Experiments

- In **liquid metals**, fast *oscillatory* columns and slow wall modes
- *Wall modes* related to low latitude waves in observations
  - Missing in present dynamo models

Summary II

- "2000's" model: large, helical columns associated with B-flux patches
- State-of-the-art: helical flow, but flux patches no longer ***directly*** related to columns
- **Many open questions**
  - e.g., Dynamo action driven by rotating turbulence in liquid metals (Featherstone CIG/ALCF)
    - ***Paradigm shift? Or waiting for Godot?***

Thanks...

Thomas
Gastine
(IPGP)