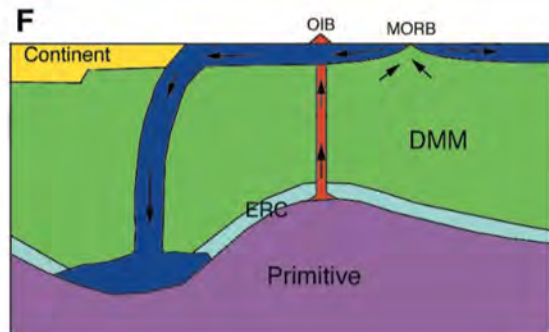
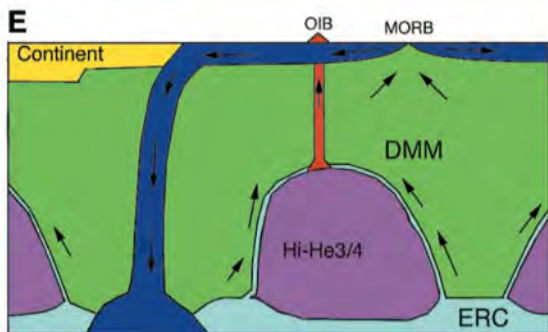
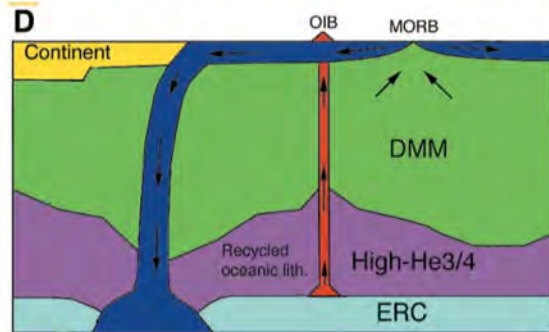
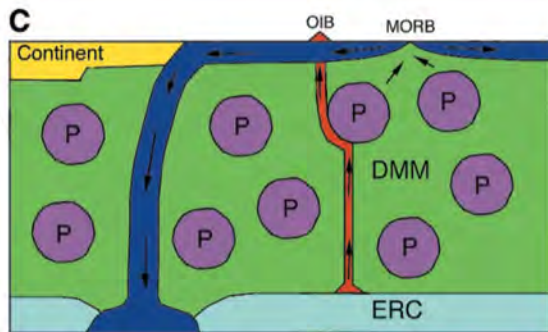
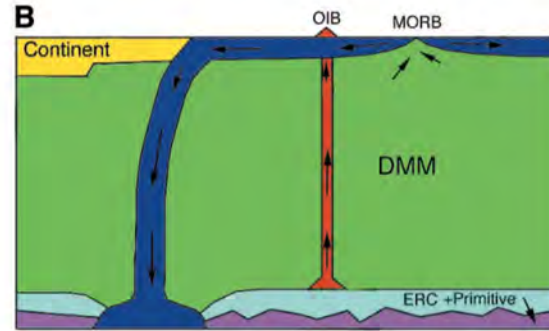
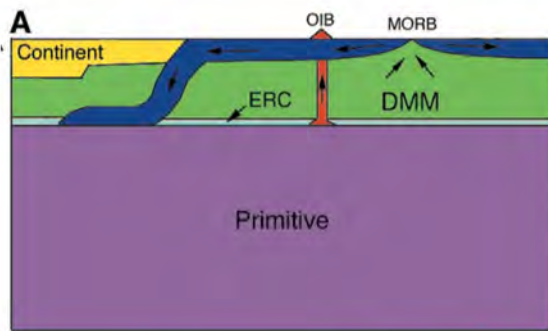


Geodynamics Lecture #3

Max Rudolph
Department of Earth and Planetary Sciences
University of California, Davis



To understand plausibility of models, we need

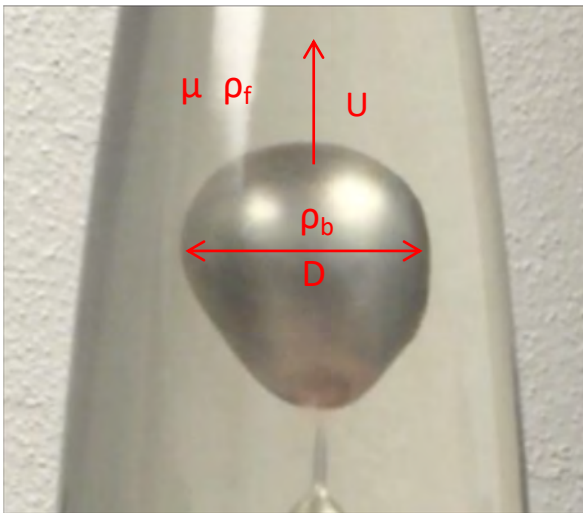
Seismic images (not the focus of this lecture!)

Material properties and behavior

Deformation of the surface

ERC = Enriched Recycled Crust

Thermochemical convection in a lava lamp



Ascent Timescale:

$$\tau_{\text{ascent}} = \frac{D}{U} = \frac{\frac{a}{2}}{\frac{a^2 g (\rho_f - \rho_b)}{3\mu_f}}$$

Cooling Timescale:

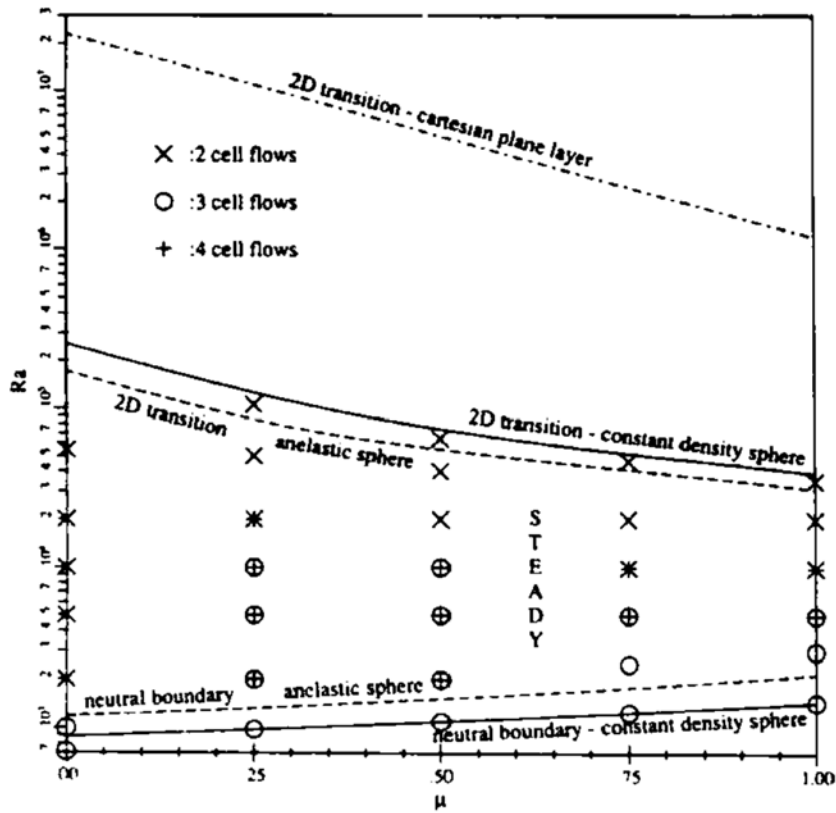
$$\tau_{\text{cooling}} = \frac{a^2}{\kappa}$$

Rayleigh number is related to ratio of time scales

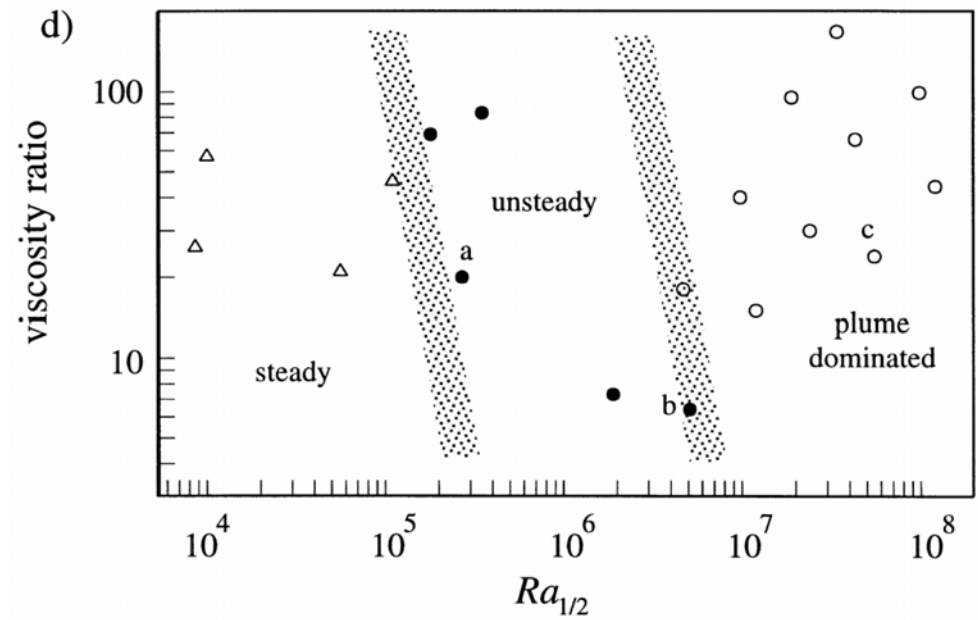
$$Ra = \frac{\tau_{\text{cooling}}}{\tau_{\text{ascent}}} \sim \frac{(\rho_f - \rho_b) g a^3}{\mu_f \kappa}$$

Faster ascent -> Higher Ra -> More vigorous convection
 Faster cooling -> Low Ra -> No (or less vigorous) convection

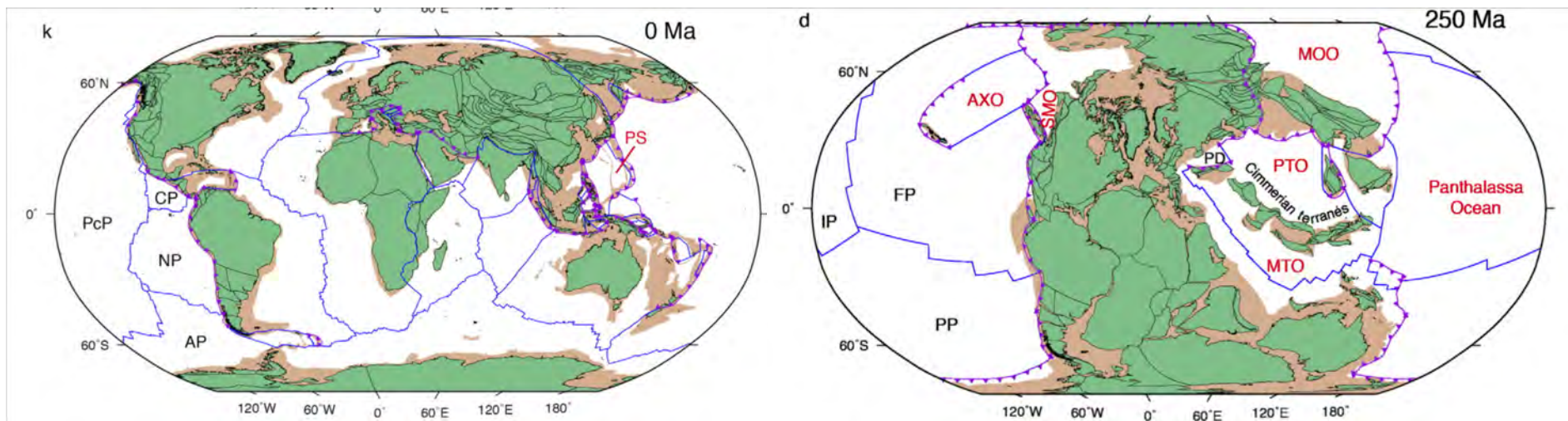
Steady to unsteady transition near $Ra\ 10^5$



Solheim and Peltier 1990



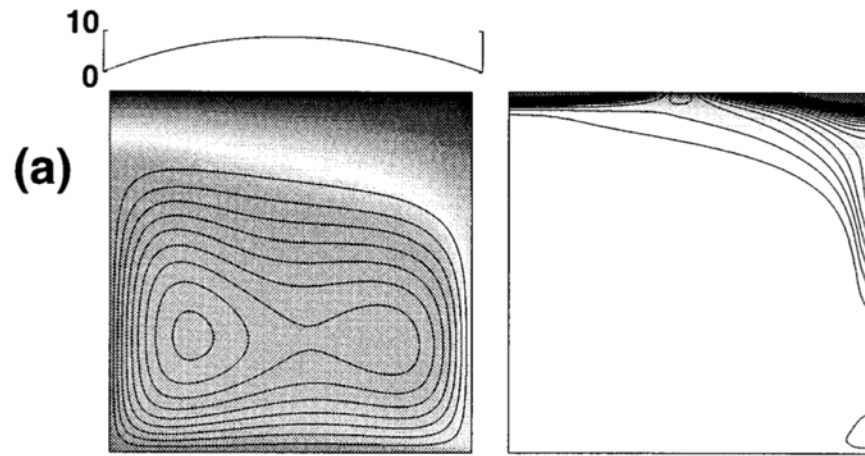
Weeraratne and Manga 1998



Changing configuration of plate boundaries

Surface motions are clearly unsteady, so flow in mantle must be as well.

Matthews et al. 2016

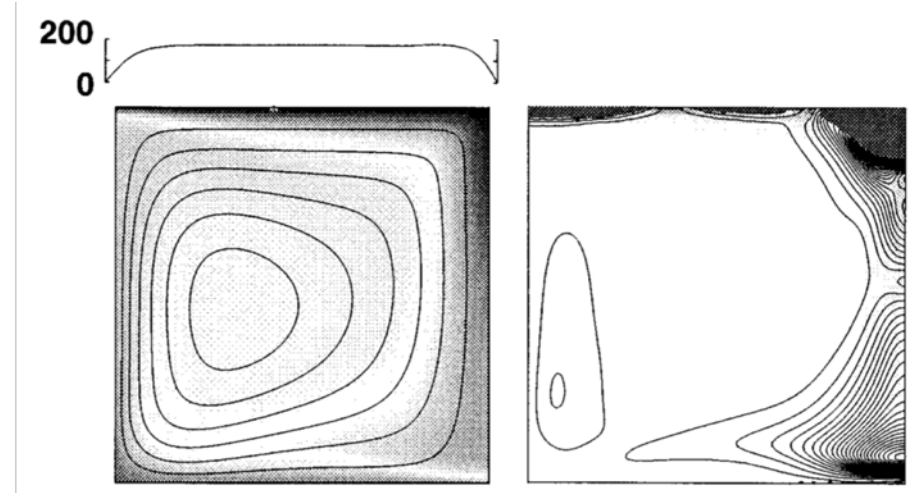


Stagnant lid regime
 $Ra=10^7$
 $\Delta\eta=3\times 10^4$
 $\tau_0=10^6$ ($c_0=120$ MPa)
 $\tau_1=10^7$ ($\mu=0.013$)

$$\tau'_{\text{yield}} = \tau_0 + \tau_1 z',$$

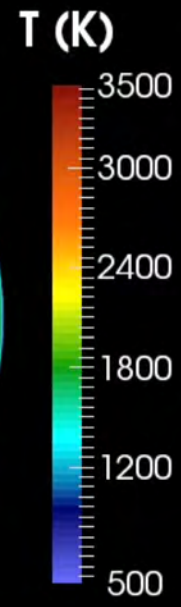
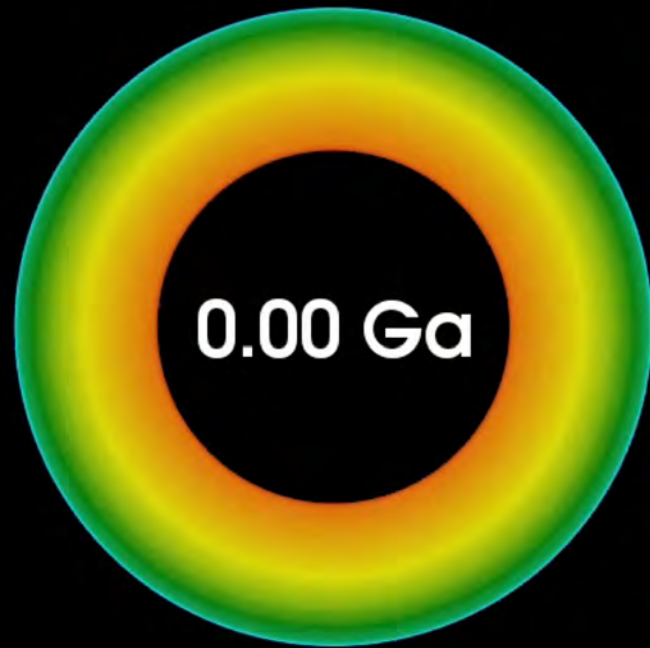
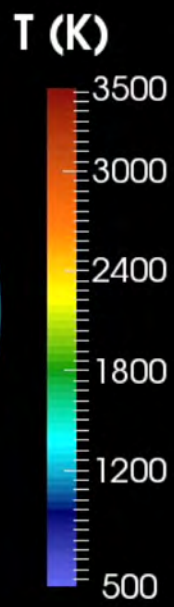
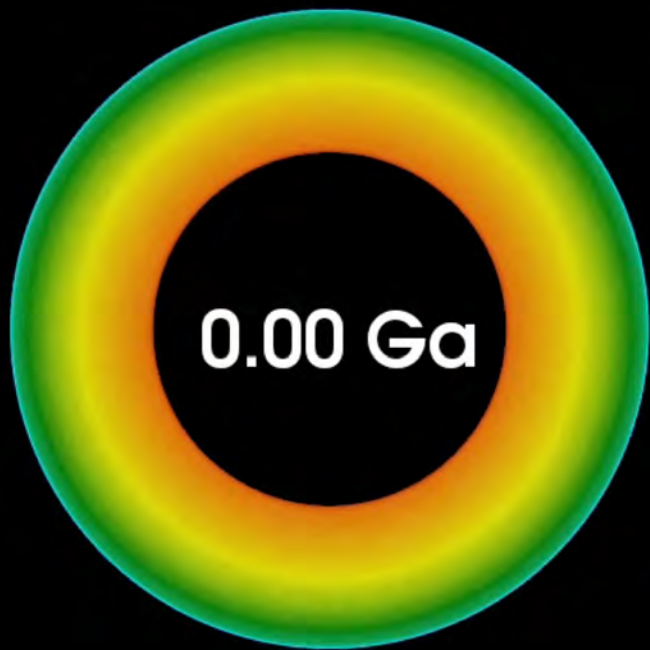
$$\tau_0 = \frac{d^2}{\kappa\eta_0} c_0,$$

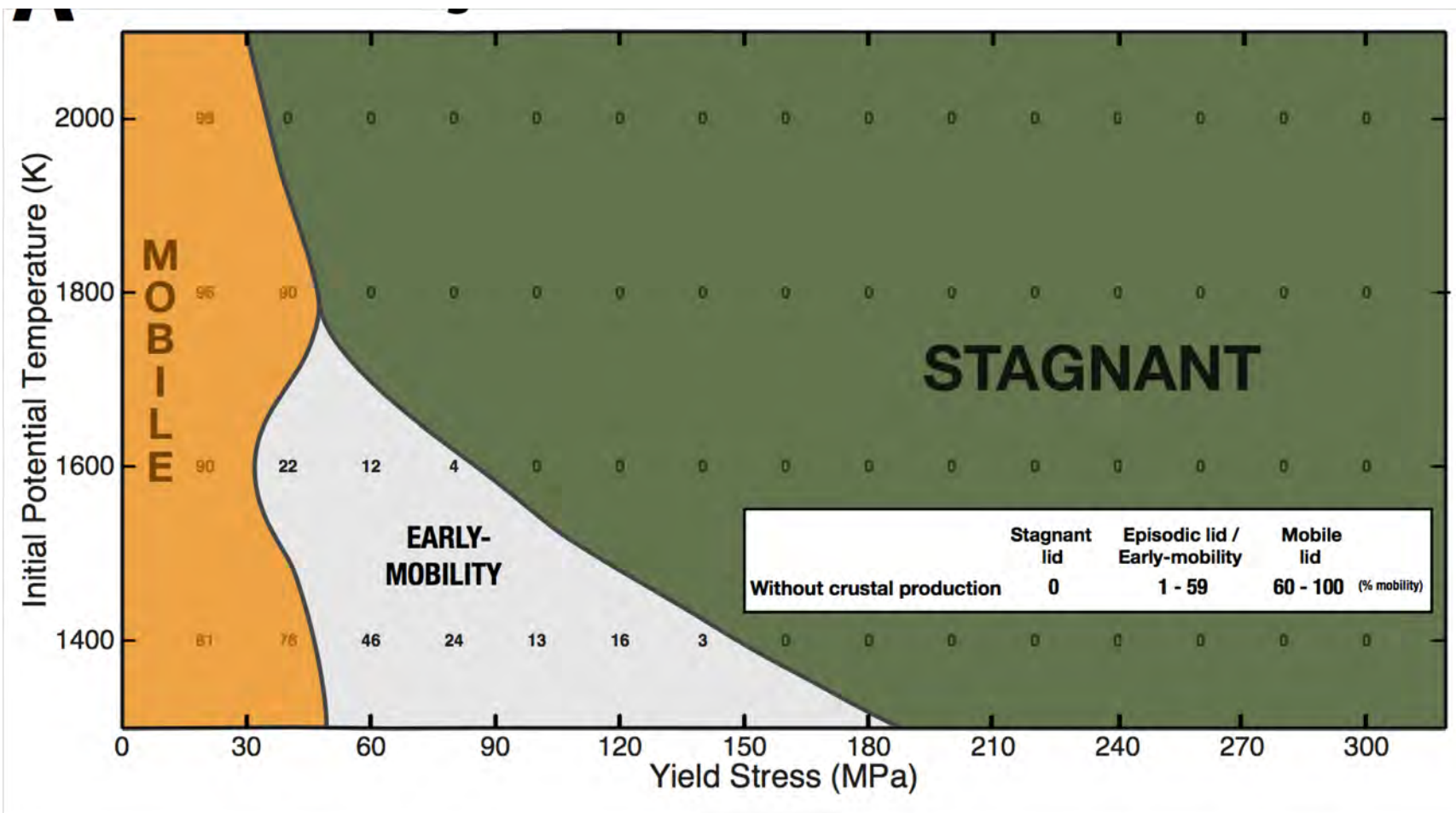
$$\tau_1 = \frac{\mu\rho g Ra_0}{\alpha\Delta T}$$



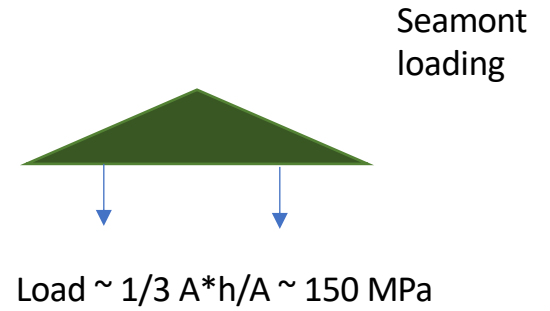
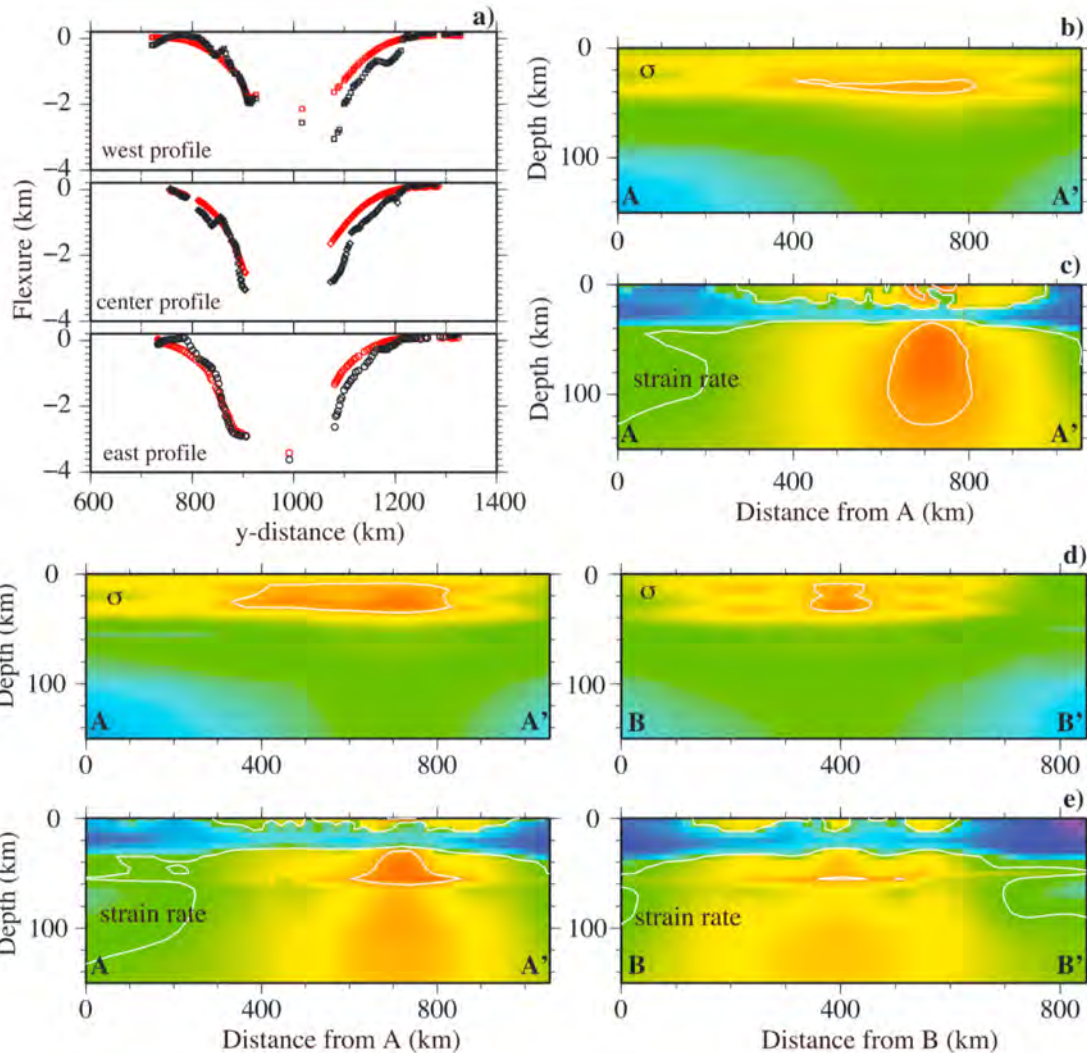
Mobile lid regime
 $\tau_0=10^5$ ($c_0=12$ MPa)

(Moresi and Solomatov 1998)





Lourenço et al. 2016 *EPSL*



Fit:
 Flexural topography
 Depth distribution of seismicity
 Include elasticity, plastic deformation

Result:
 Lithospheric stresses are 100-200 MPa

Zhong and Watts 2012 *JGR*

Plate-like flow and mixing

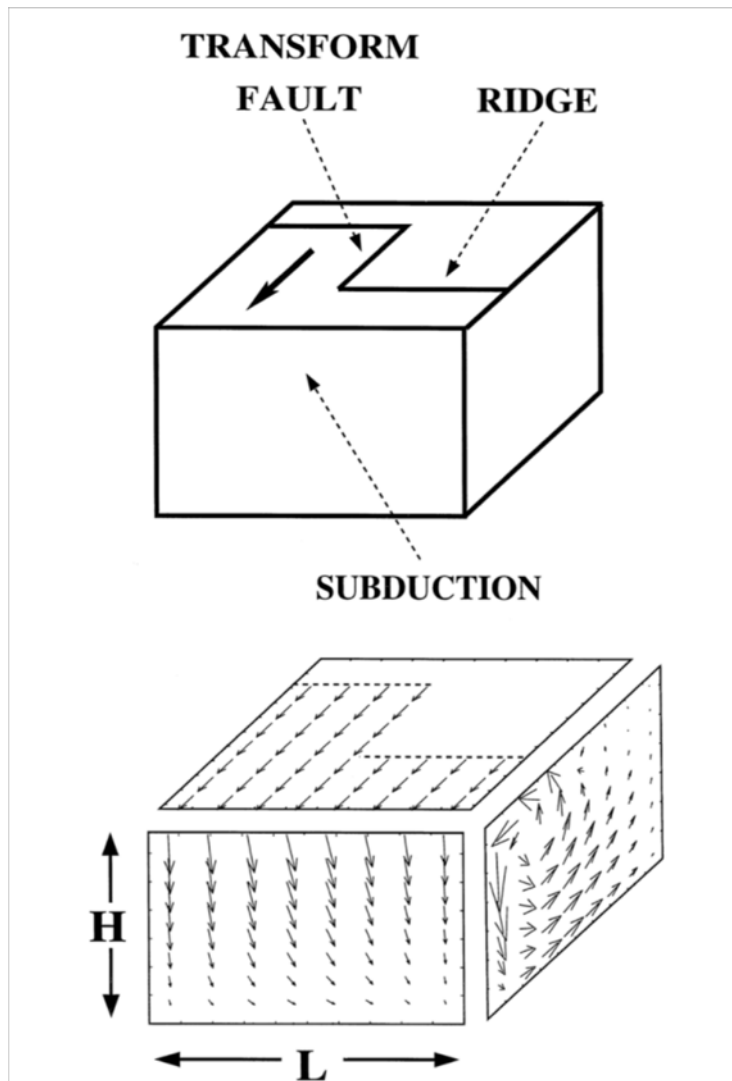
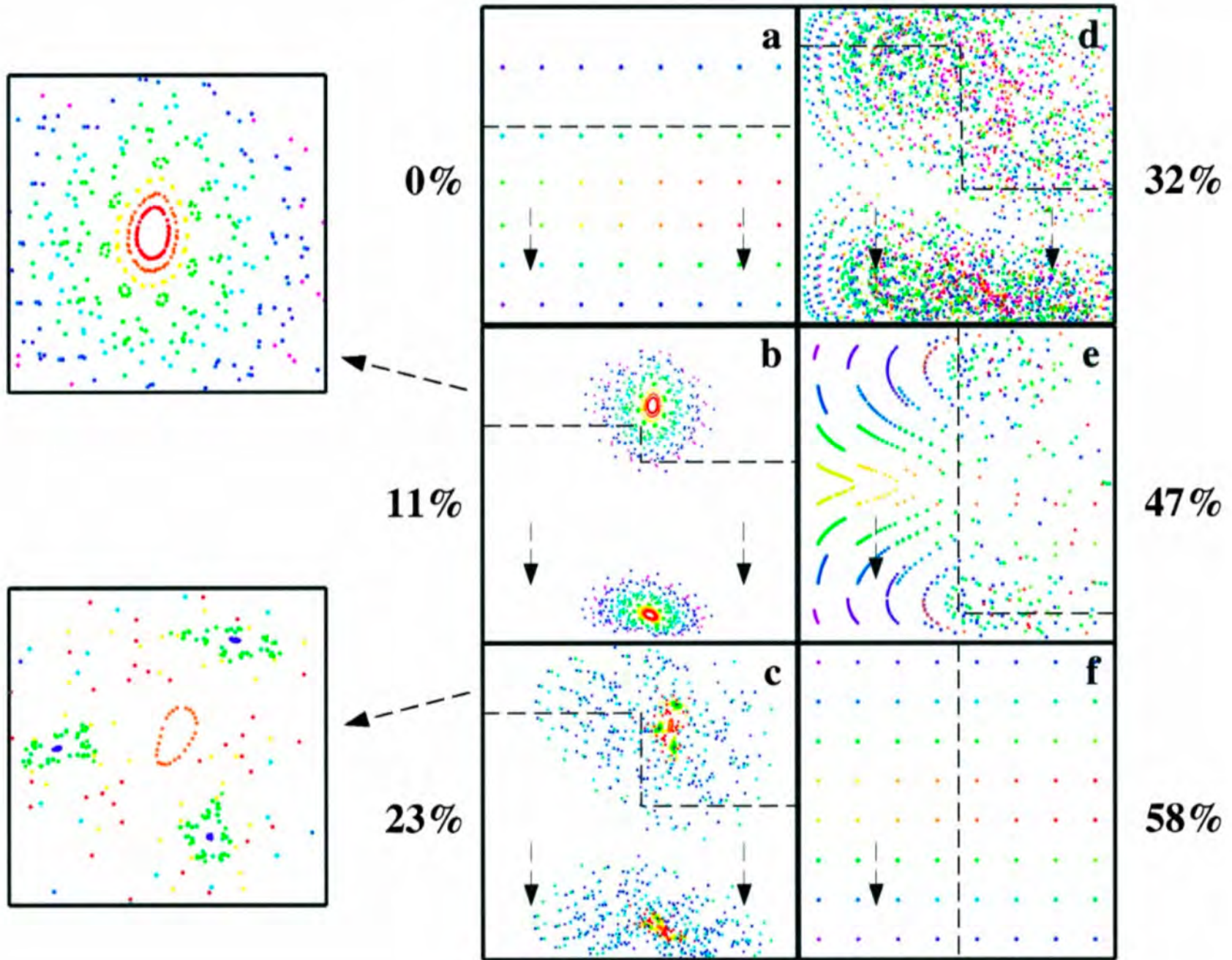
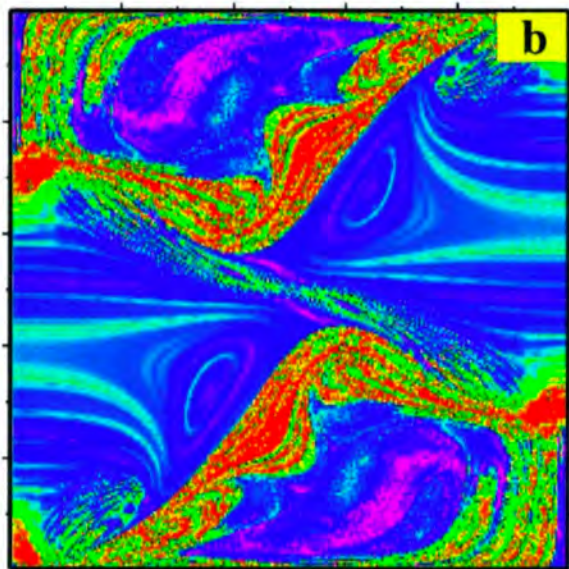
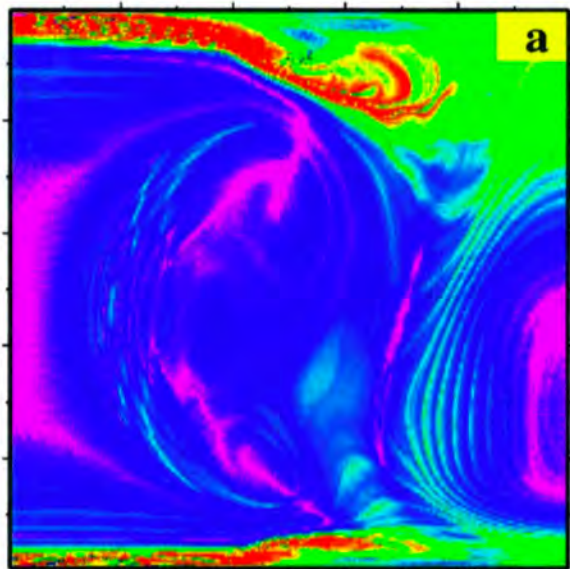


Fig. 1. Box of aspect ratio $2/3$ in which the computation is performed. The flow is only driven by a surface motion mimicking a real plate with a segmented ridge, a transform fault and a subduction zone (*top*). The resulting flows at the surface, the front and the right-hand faces are depicted at the bottom.





Poloidal only

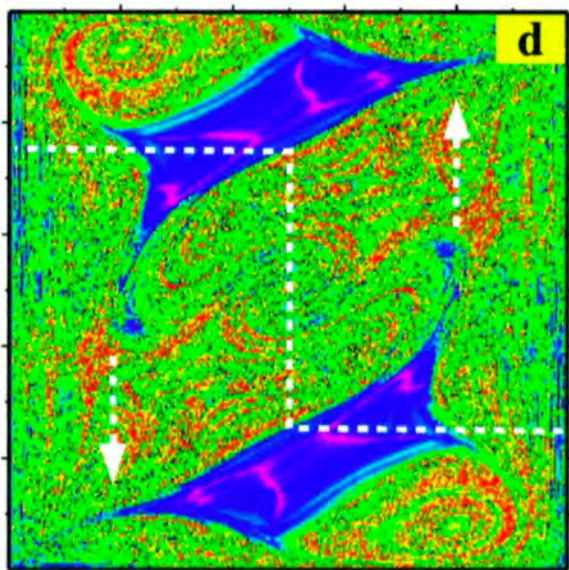
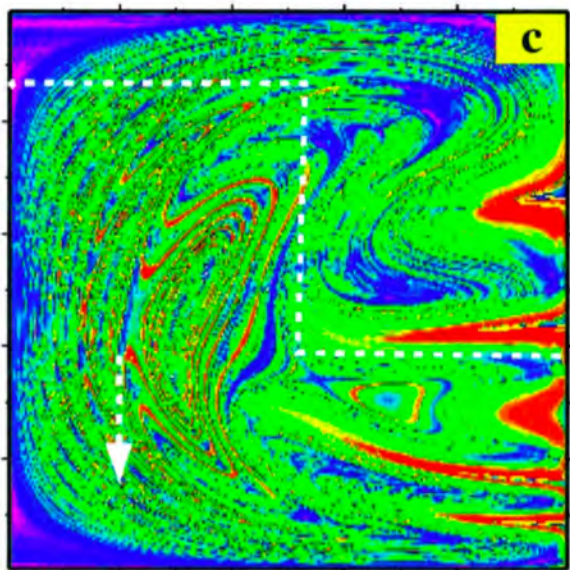
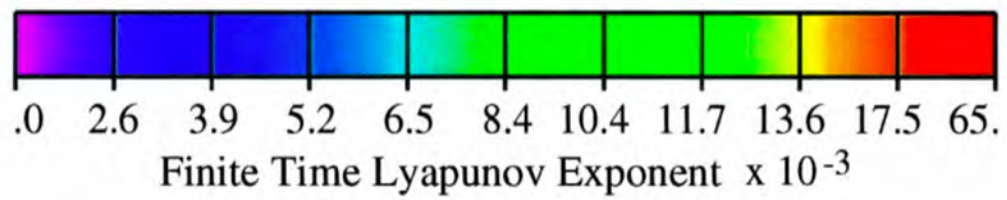
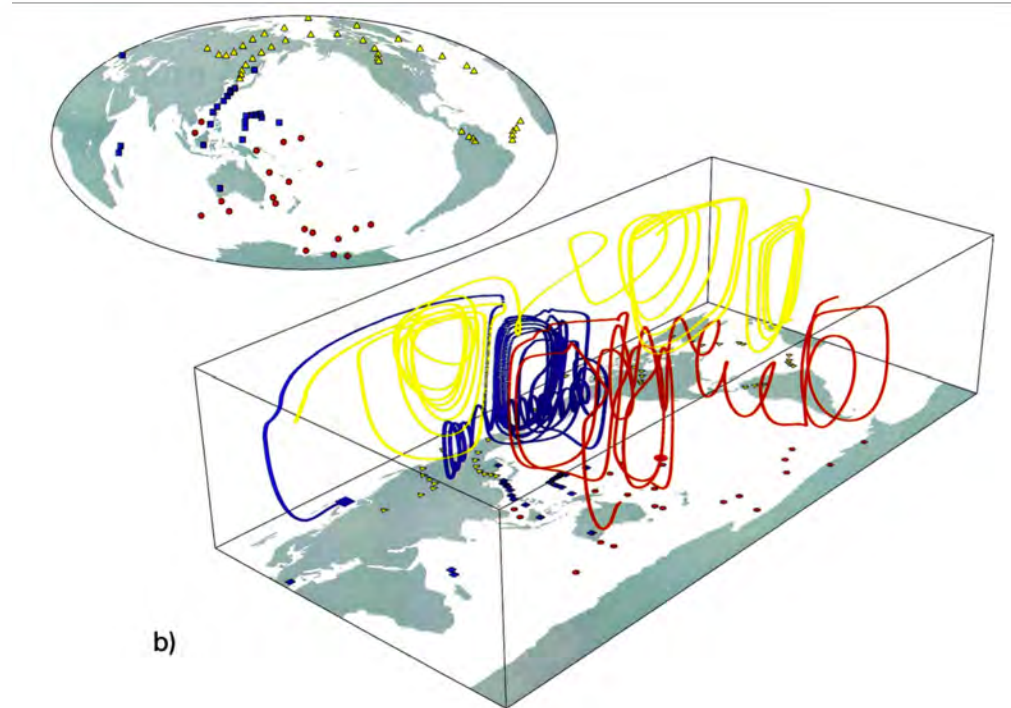
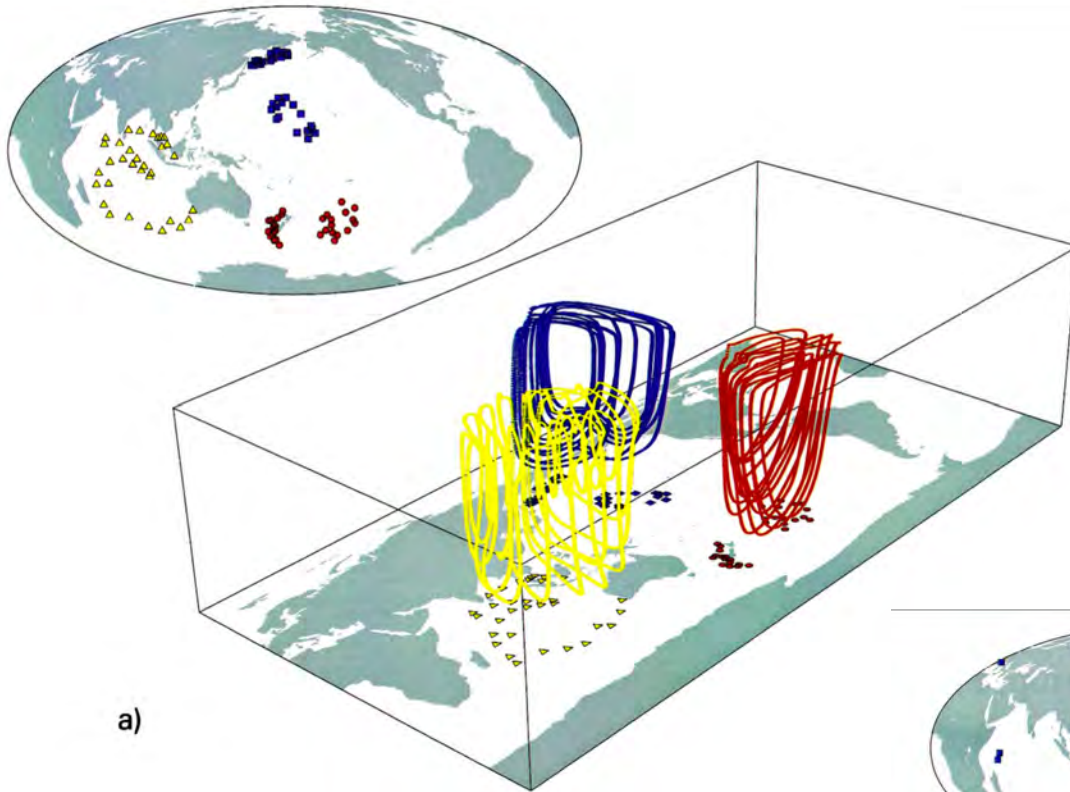


Plate-driven flow



Mixing in present-day flowfield



van Keken and Zhong 1999



Wave cut terraces in Nunavut record post-glacial rebound

The Motion of a Viscous Fluid Under a Surface Load

N. A. HASKELL, *Massachusetts Institute of Technology*

(Received April 25, 1935)

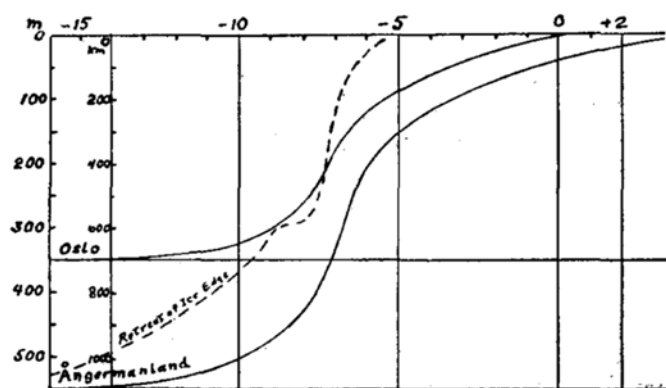


TABLE I. Values of the quantities entering in Eq. (5.42) for various times taken as the initial instant.

| $t=0$ | a | $b \times 10^3$ | $\zeta(0, 0)$ | $\nu \times 10^{-21}$ |
|---|-----|-----------------|---------------|-----------------------|
| 5,000 B.C. | 3.9 | $1.27 \pm .07$ | 147 | 2.6 ± 0.2 |
| 4,000 " | 2.7 | $1.19 \pm .09$ | 118 | 3.2 ± 0.3 |
| 3,000 " | 2.2 | $1.24 \pm .11$ | 94 | 3.0 ± 0.4 |
| 2,000 " | 1.8 | $1.25 \pm .15$ | 74 | 2.9 ± 0.5 |
| Mean value: $\nu = 2.9 \times 10^{21} \text{ cm}^2 \text{ sec.}^{-1}$ | | | | |

TABLE III. Values taken for the coefficient of viscosity of various substances.

| MATERIAL | TEMPERATURE | η (g. cm ⁻¹ sec. ⁻¹) |
|----------------|-------------|--|
| Shoemakers wax | 15°C | 2×10^8 |
| Asphalt | 15 | 3×10^{10} |
| Ice | 0 | 5×10^{12} |
| Glass | 575 | 1×10^{13} |
| Lead | 20 | 1×10^{16} |
| Calcite | 18 | 1.5×10^{16} |
| Rocksalt | 18 | 2×10^{18} |

Very close to 10^{21} Pa-s

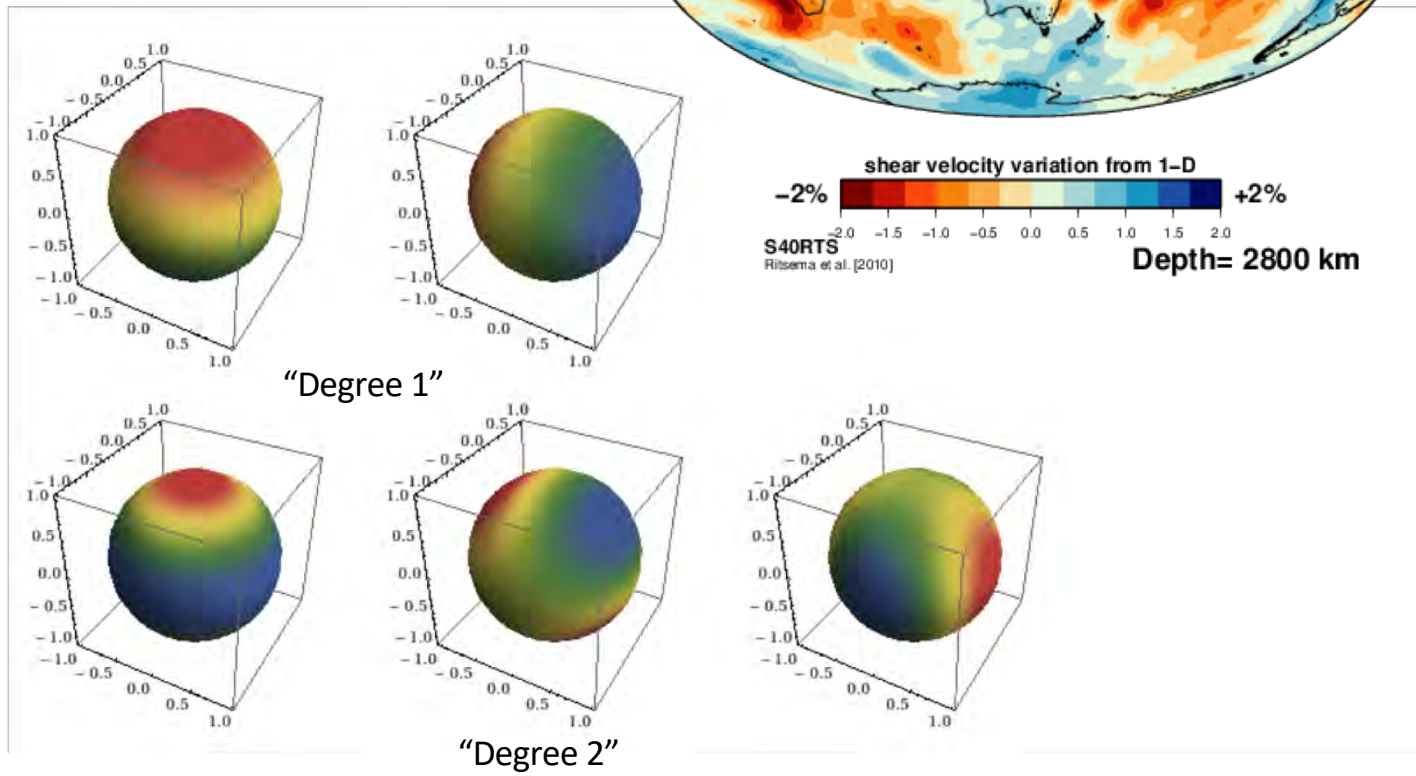
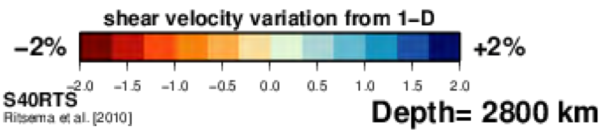
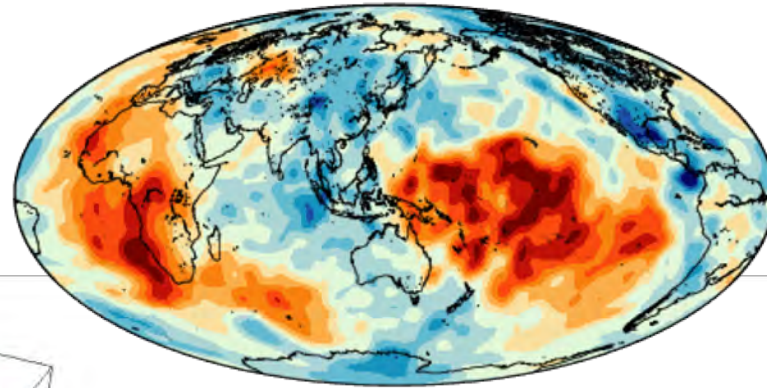
GOCE geoid



GOCE measurements from ESA

https://dlmultimedia.esa.int/download/public/videos/2016/01/027/1601_027_AR_EN.mp4

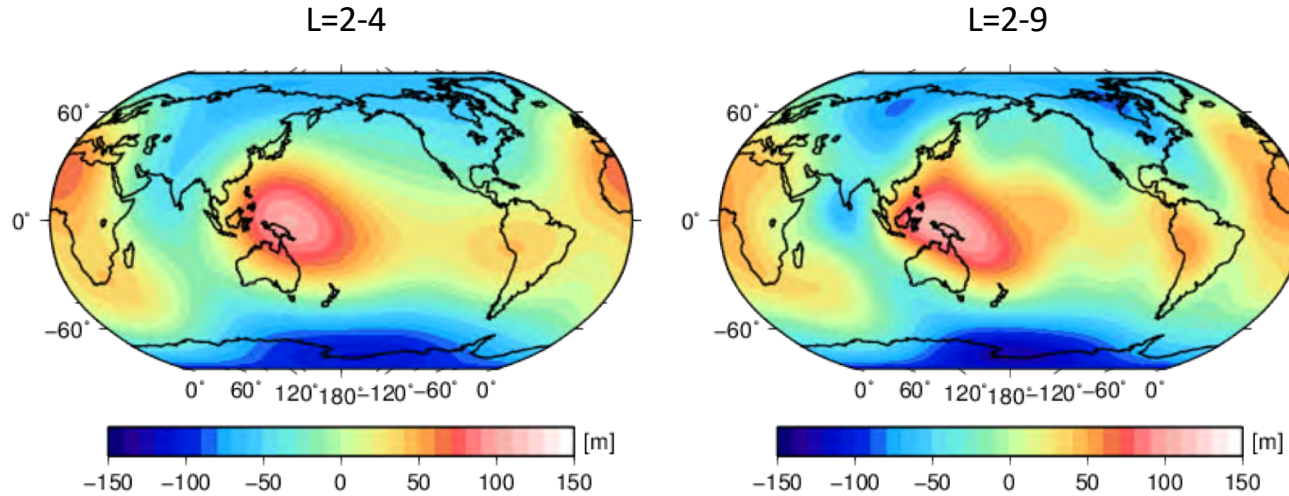
Spherical harmonics



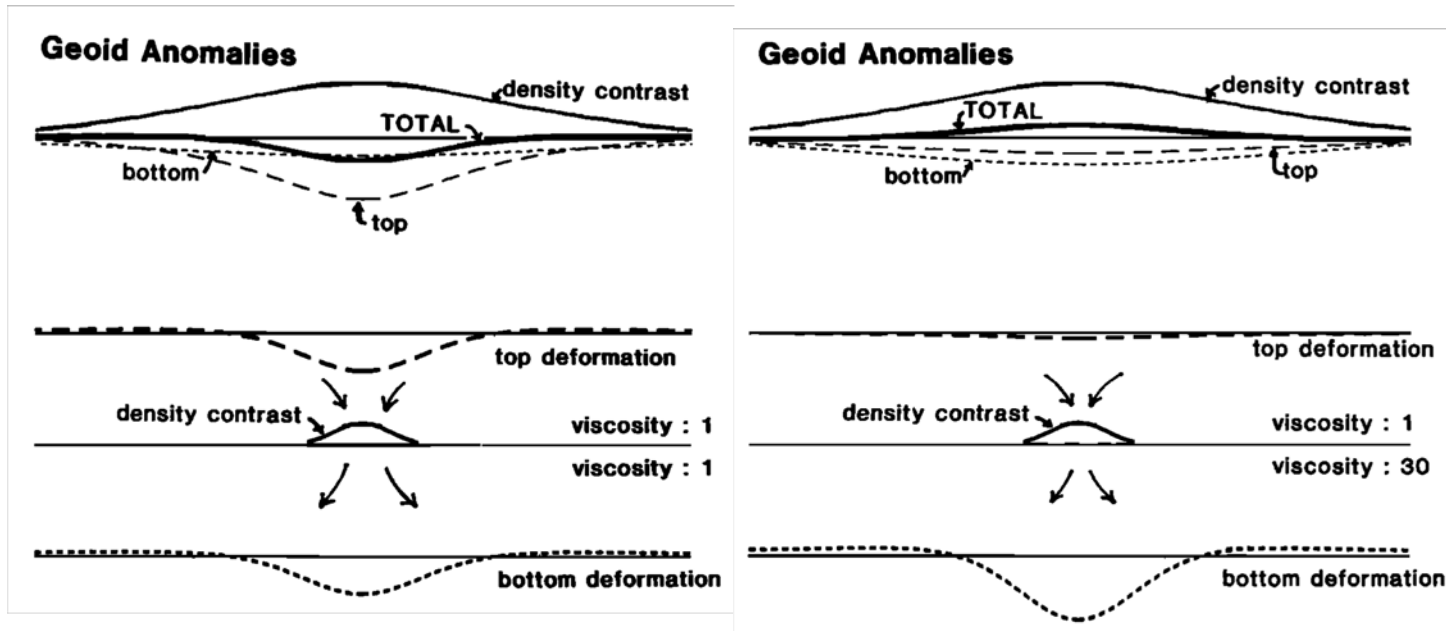
"Degree 1"

"Degree 2"

Long wavelength geoid: key constraint on mantle viscosity



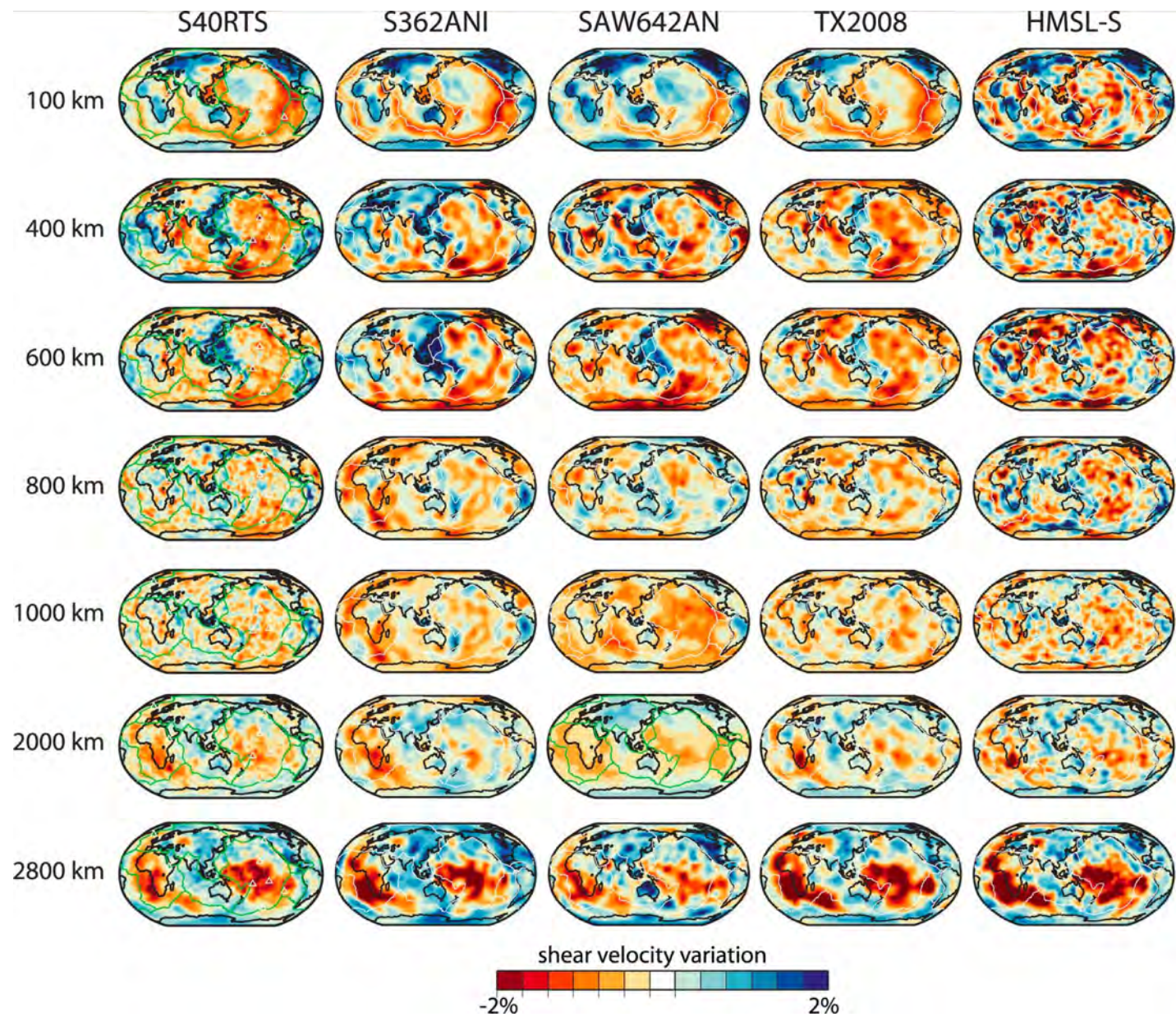
Observed long-wavelength non-hydrostatic geoid



Geoid is sensitive to relative, not absolute, viscosities

From Hager 1984

Predicting geophysical observables using simulations of mantle flow



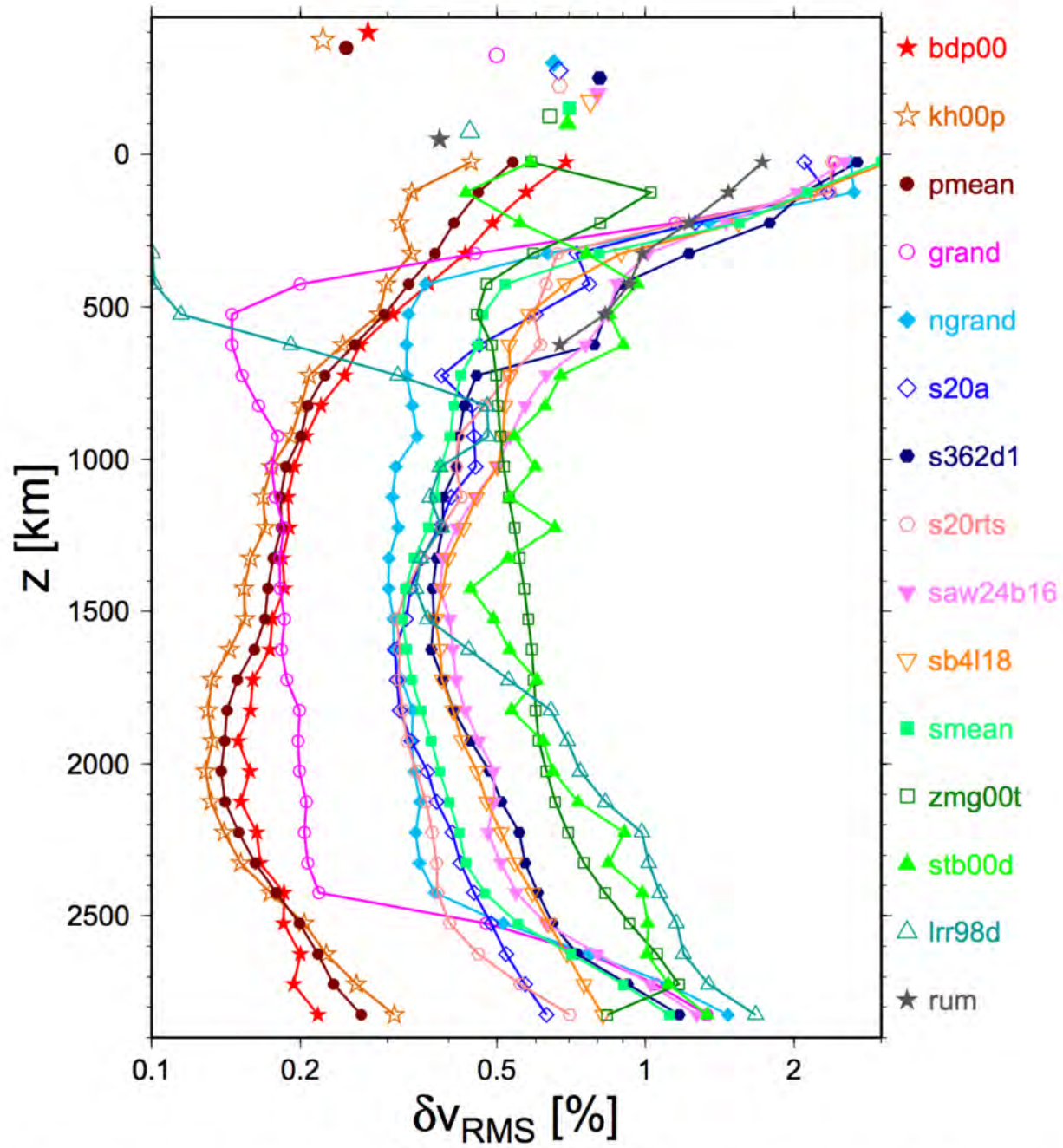
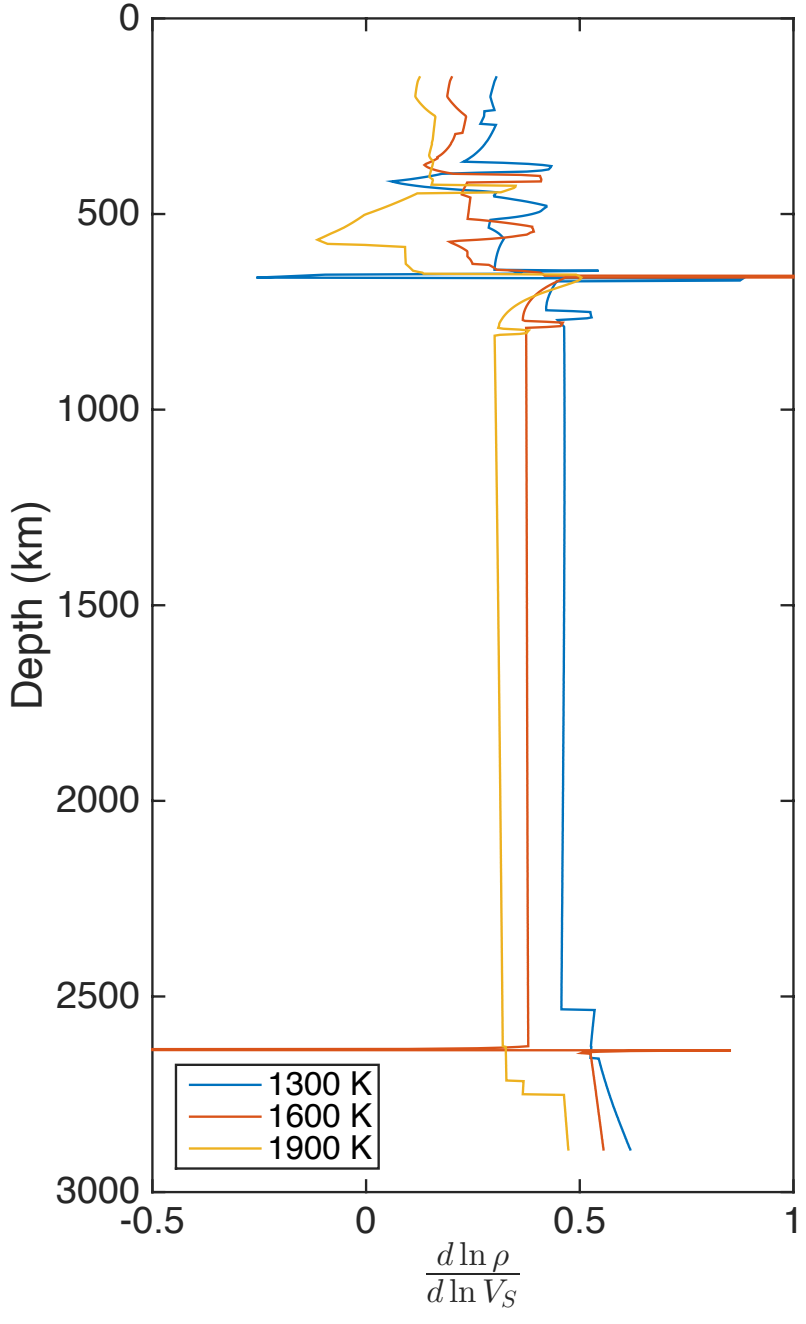
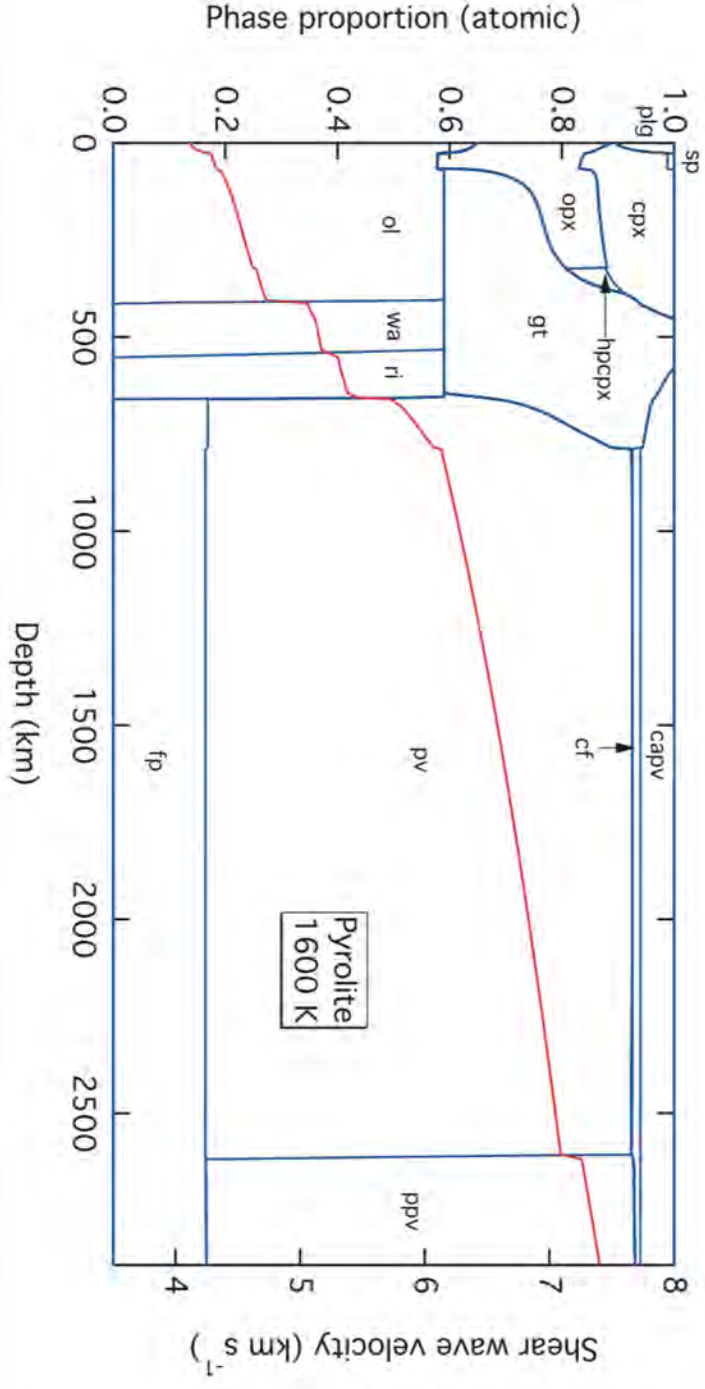
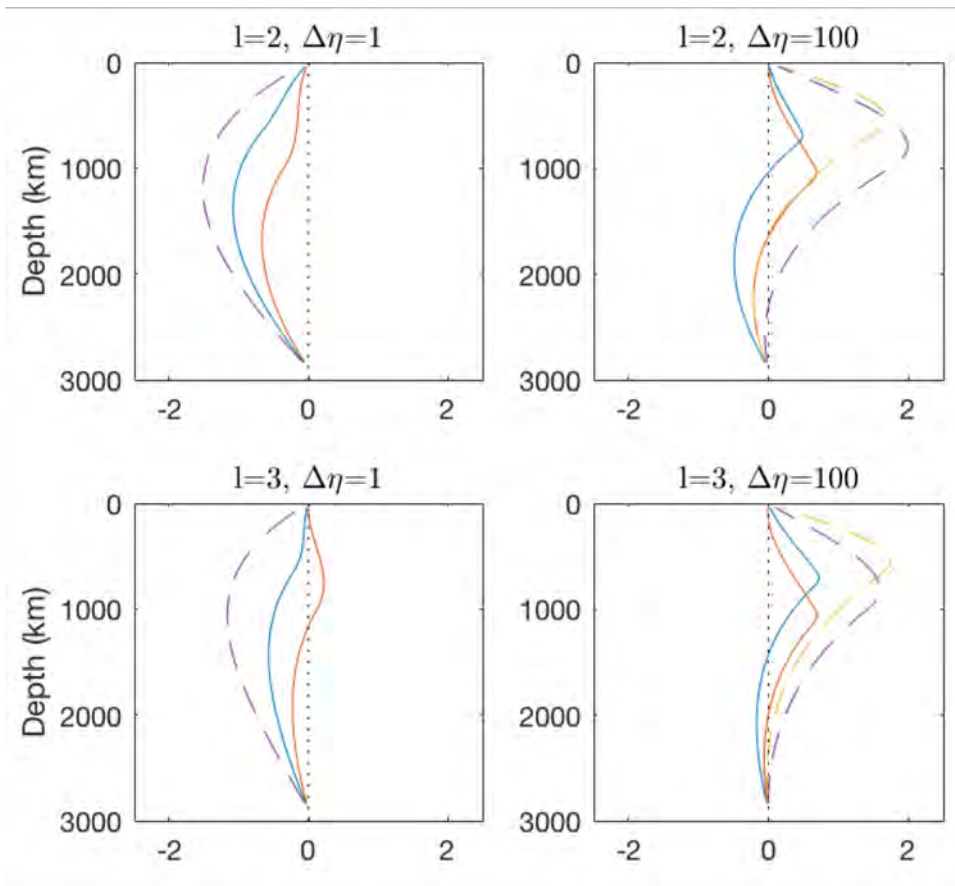


Figure 3. The δv_{RMS} versus z , $\ell_{\text{max}} = 31$. Symbols at $z < 0$ denote depth averaged $\langle \delta v_{\text{RMS}} \rangle$.

Stixrude and Lithgow-Bertelloni, 2011



(Mantle flow models are then calculated using propagator matrix technique or finite elements)

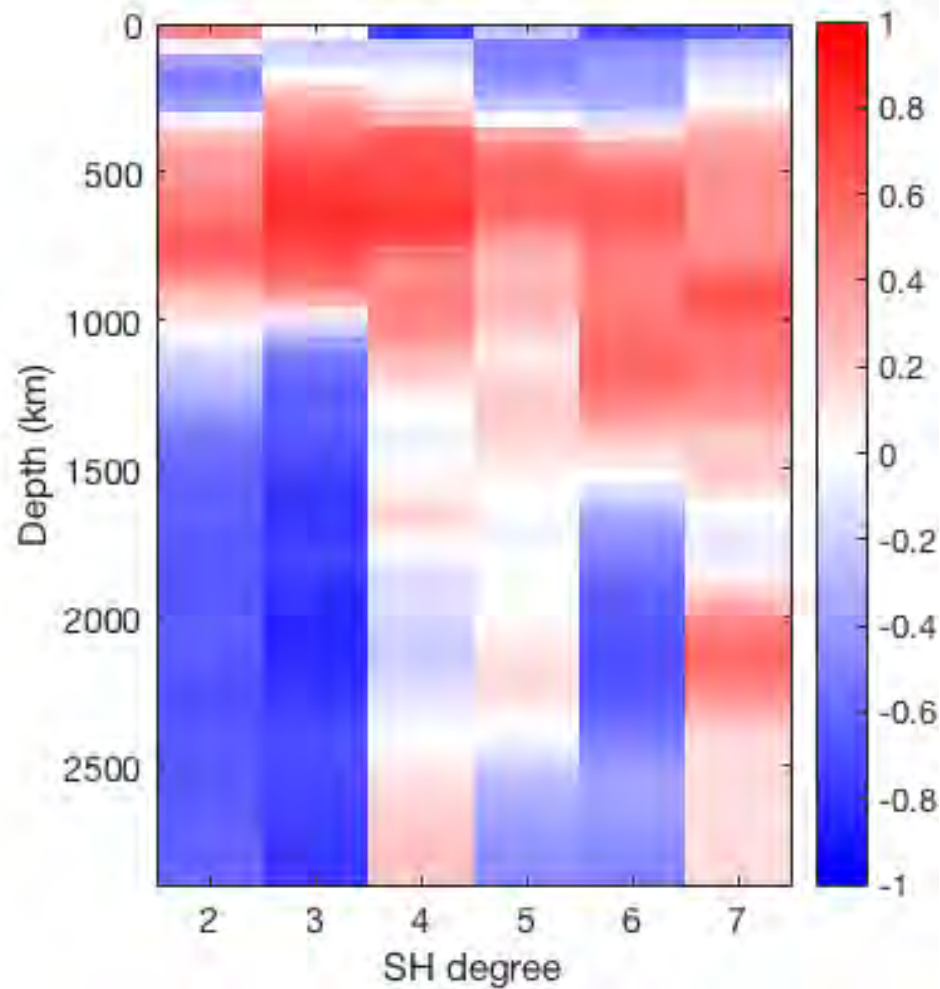
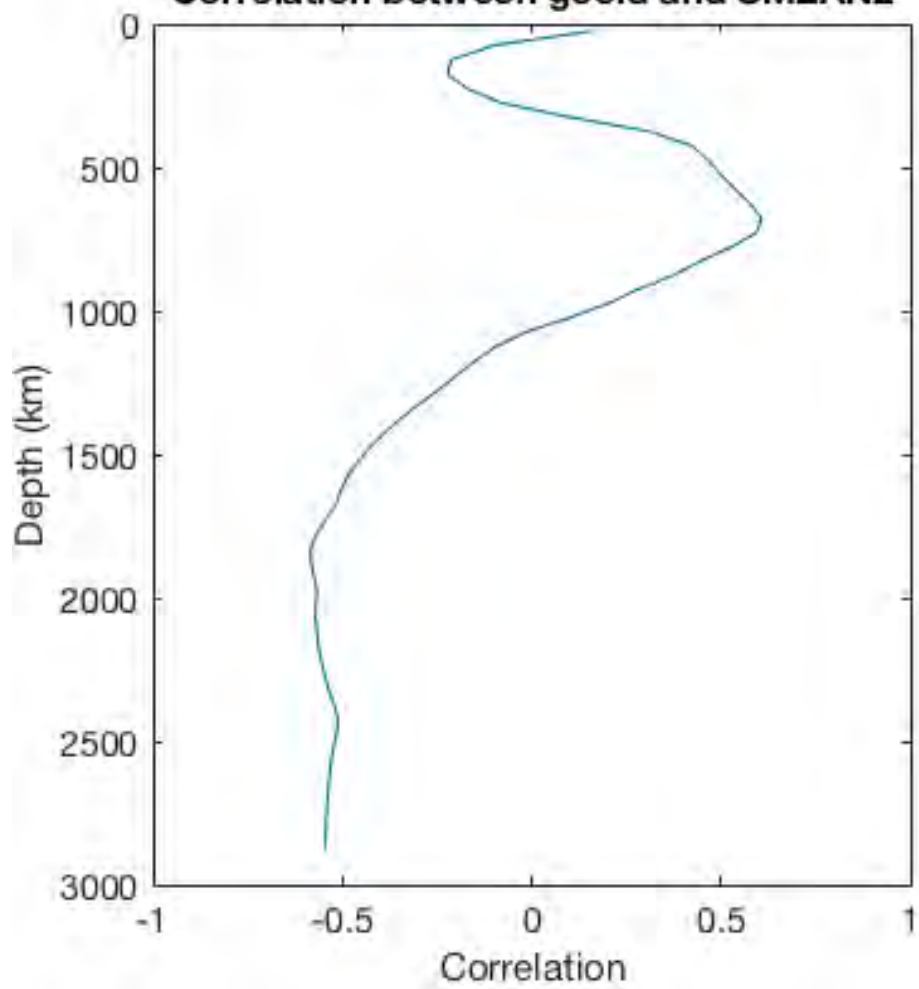


Geoid Kernels tell you a lot, but not 'the whole story'

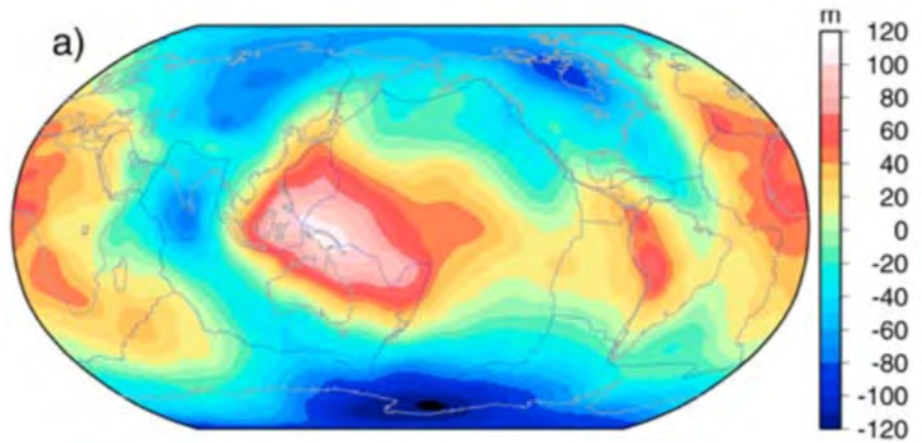
Kernel is a non-linear function of the viscosity structure.

We must explore parameter space to find the best kernel, i.e. the kernel that allows you to minimize misfit for a given density structure.

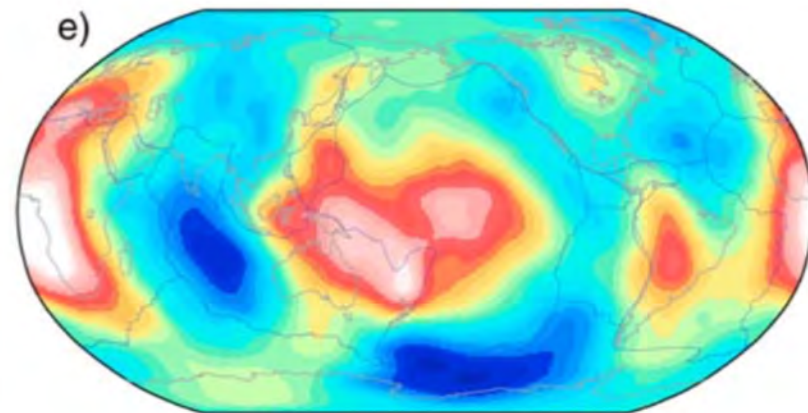
Correlation between geoid and SMEAN2



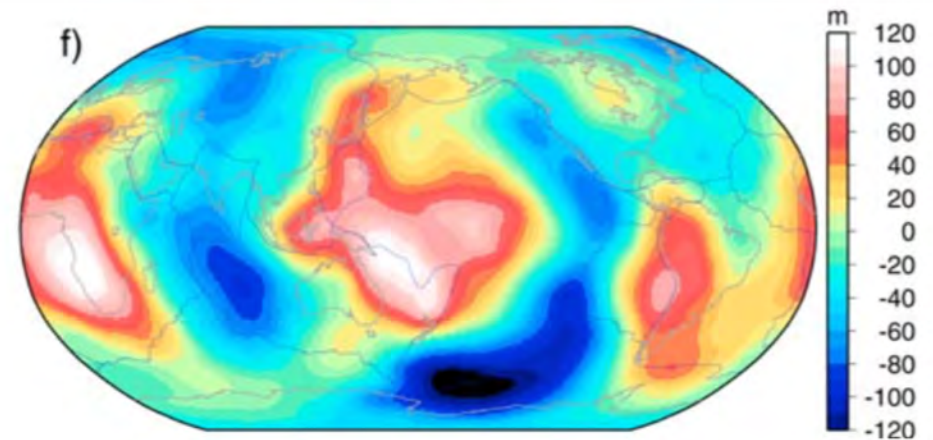
Observed geoid



No LVVs



With LVVs

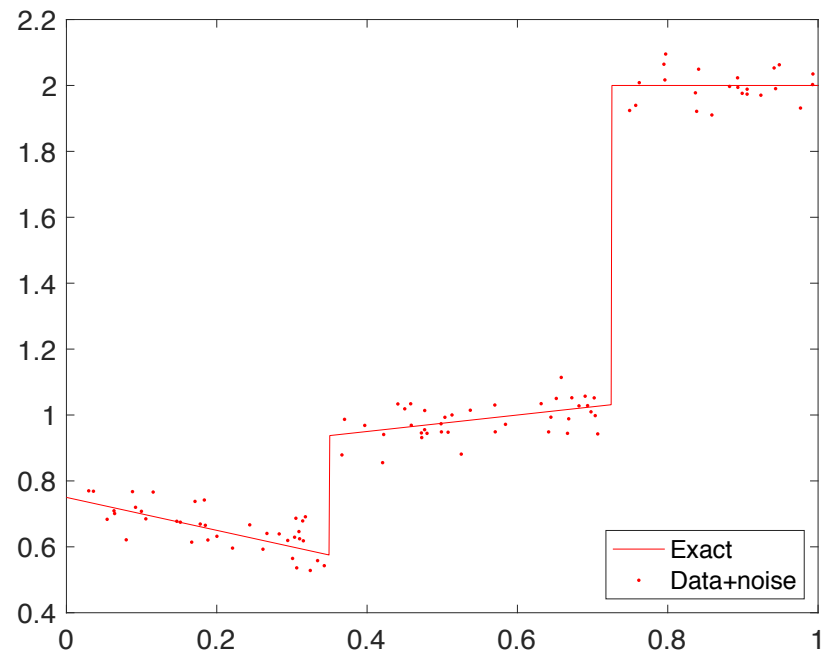


Transdimensional Inversion

Example: Find the optimal representation of some data using piecewise linear functions.

The number of discontinuities is not known *a priori*

Noise is added to the data



$$P(D|G) = \frac{1}{\sqrt{(2\pi)^{n_{lm}} |C_D|}} \exp\left(-\frac{\Phi(G)}{2}\right)$$

$$\Phi(G) = \underline{R}^T \underline{C}_D^{-1} \underline{R}$$

Accept new model with k' parameters over old with k parameters
With probability:

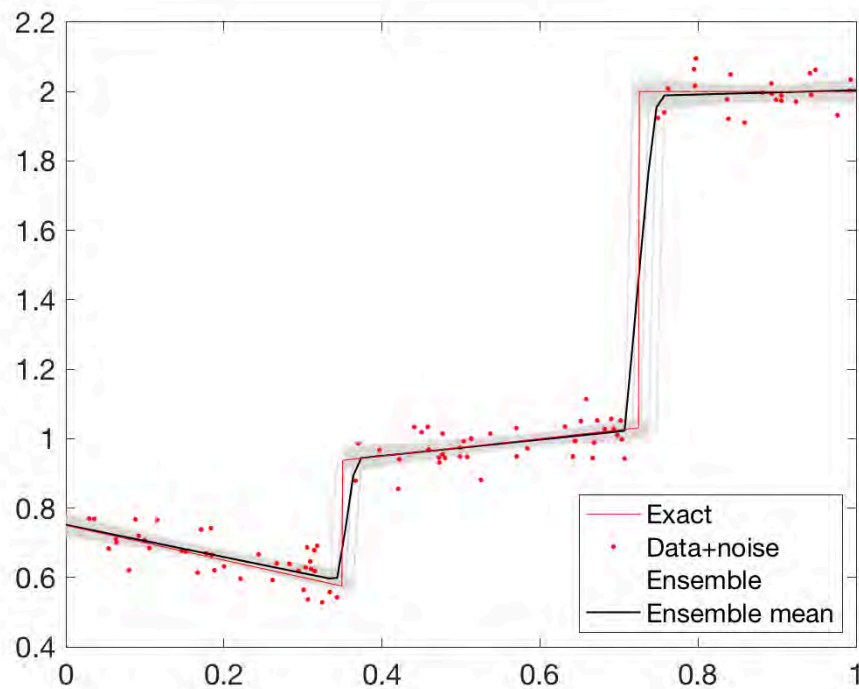
$$\min\left(1, \frac{P(D|G')}{P(D|G)} \frac{k+1}{k'+1}\right)$$

Transdimensional Inversion

Example: Find the optimal representation of some data using piecewise linear functions.

The number of discontinuities/cells is not known *a priori*

Noise is added to the data



$$P(D|G) = \frac{1}{\sqrt{(2\pi)^{n_{lm}} |C_D|}} \exp\left(-\frac{\Phi(G)}{2}\right)$$

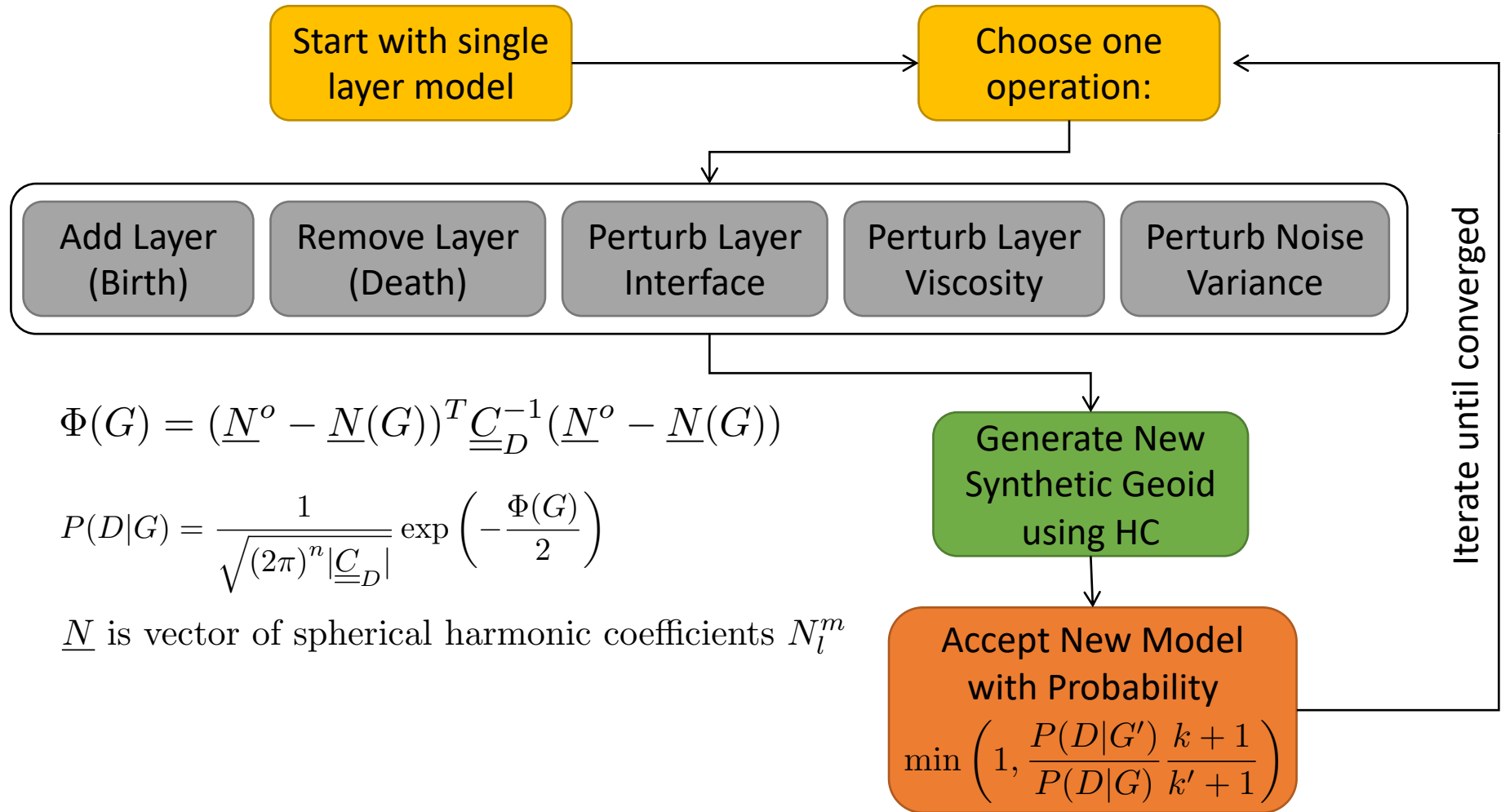
$$\Phi(G) = \underline{R}^T \underline{C}_D^{-1} \underline{R}$$

Accept new model with k' parameters over old with k
With probability:

$$\min\left(1, \frac{P(D|G')}{P(D|G)} \frac{k+1}{k'+1}\right)$$

Penalizes over-parameterization

Transdimensional Hierarchical Inversion



$$\Phi(G) = (\underline{N}^o - \underline{N}(G))^T \underline{\underline{C}}_D^{-1} (\underline{N}^o - \underline{N}(G))$$

$$P(D|G) = \frac{1}{\sqrt{(2\pi)^n |\underline{\underline{C}}_D|}} \exp \left(-\frac{\Phi(G)}{2} \right)$$

\underline{N} is vector of spherical harmonic coefficients N_l^m

(Similar to Kolb and Lekic, 2013)

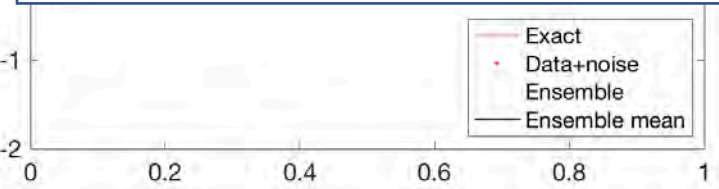
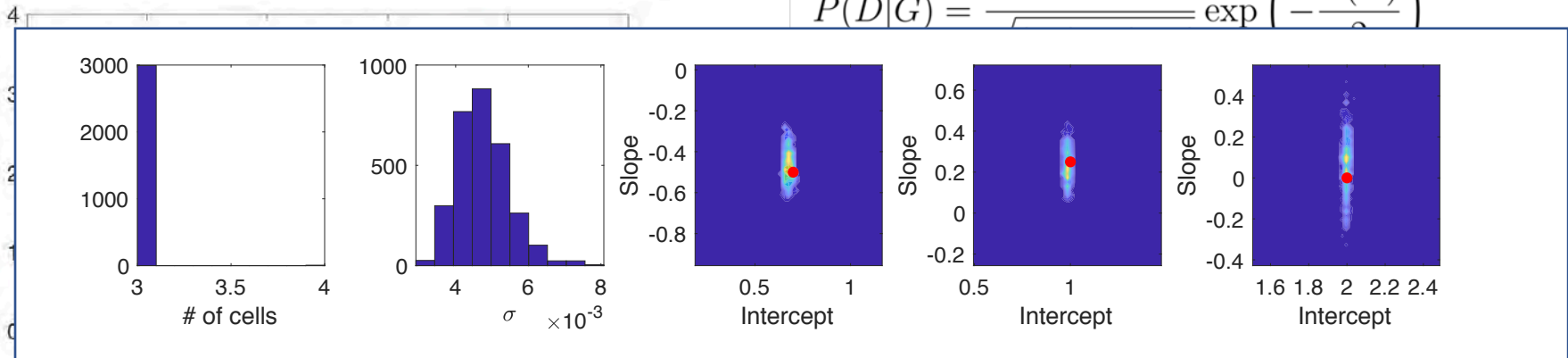
Transdimensional Inversion

Example: Find the optimal representation of some data using piecewise linear functions.

The number of discontinuities is not known *a priori*

Noise is added to the data

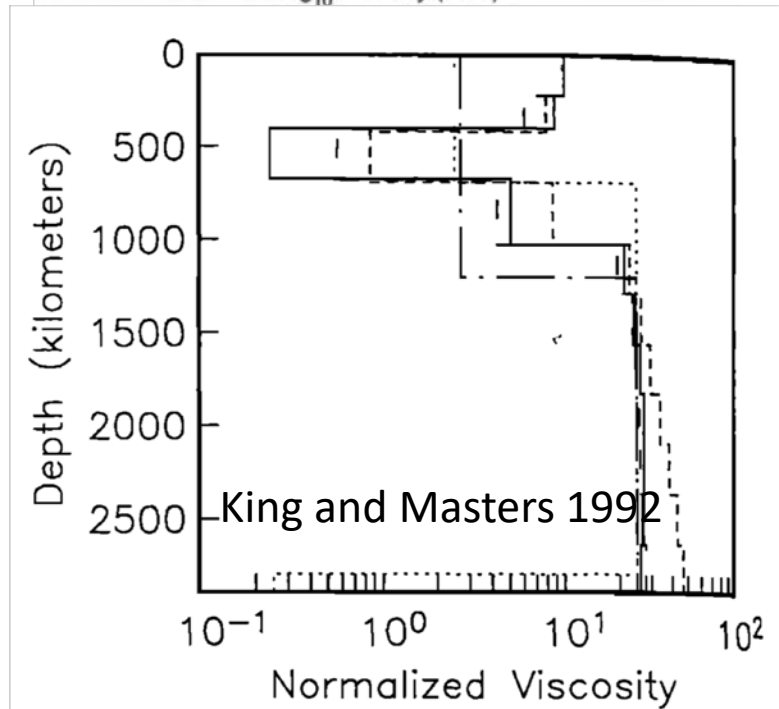
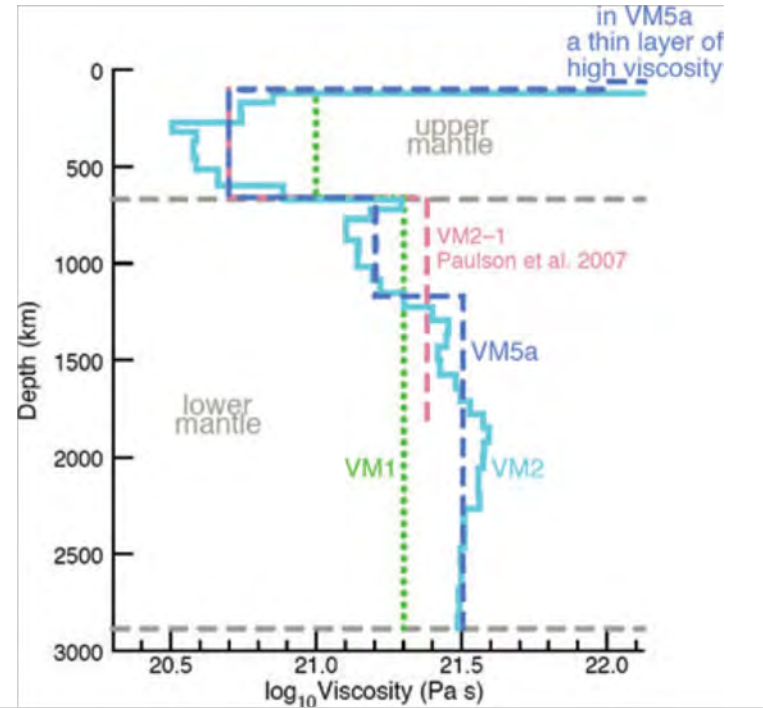
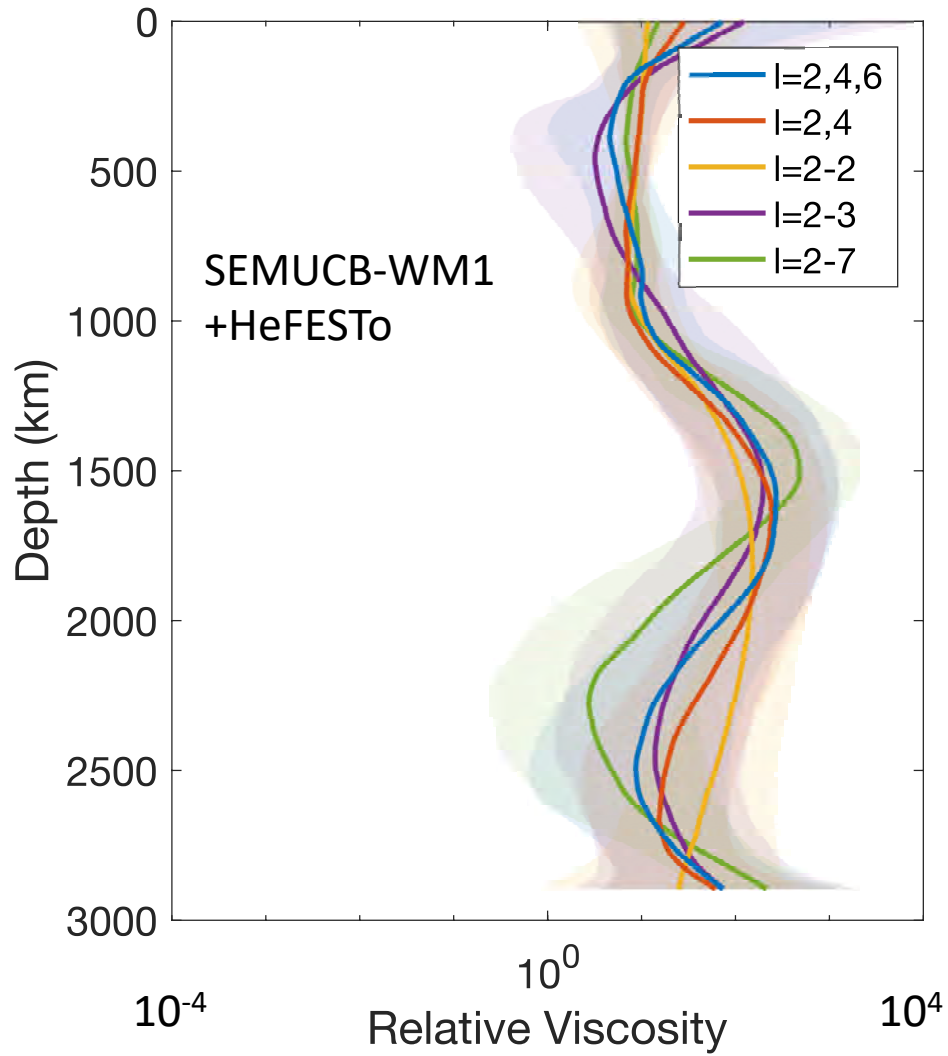
$$P(D|G) = \frac{1}{Z} \exp\left(-\frac{\Phi(G)}{\sigma^2}\right)$$



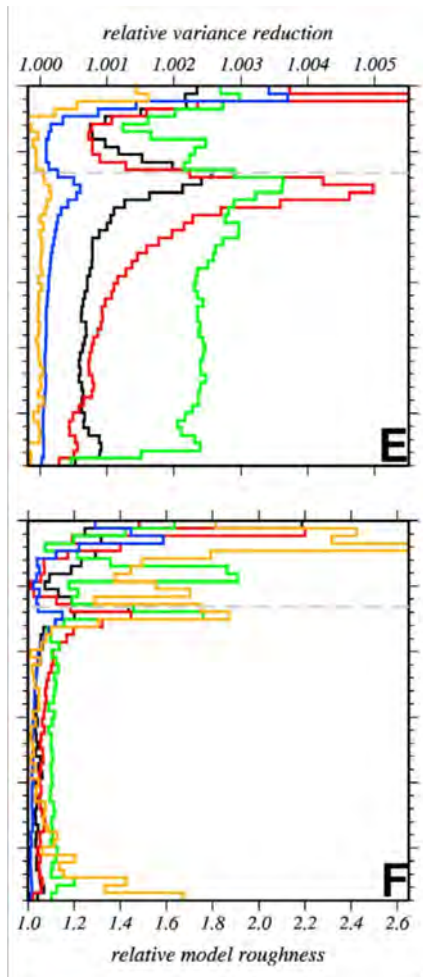
$$\min\left(1, \frac{P(D|G')}{P(D|G)} \frac{k+1}{k'+1}\right)$$

Penalizes over-parameterization

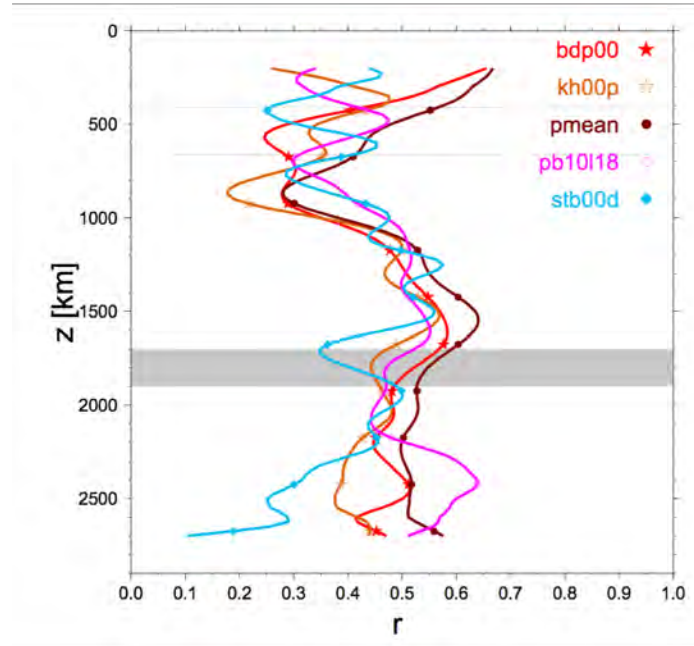
Viscosity profiles and 95% confidence intervals



Changes in radial correlation / vertical coherence

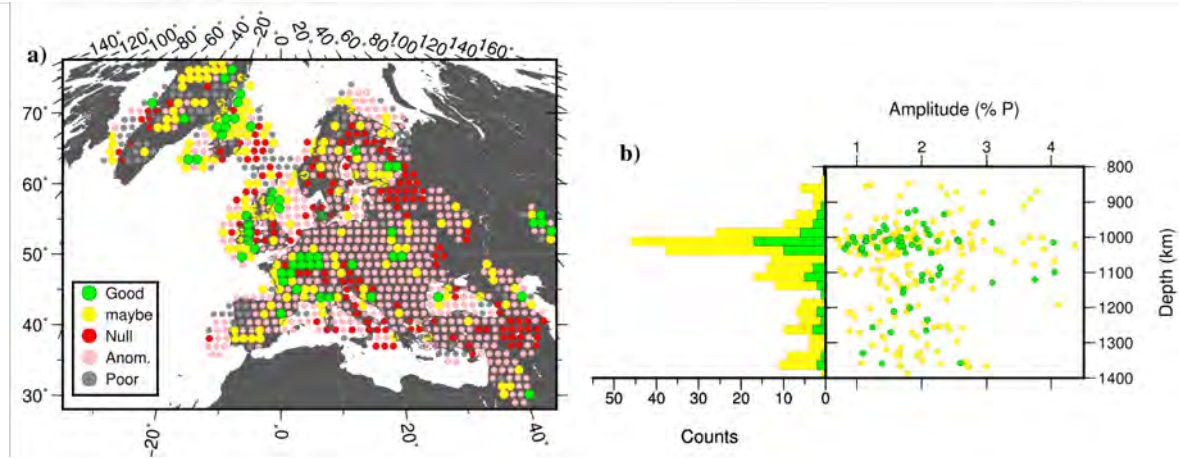
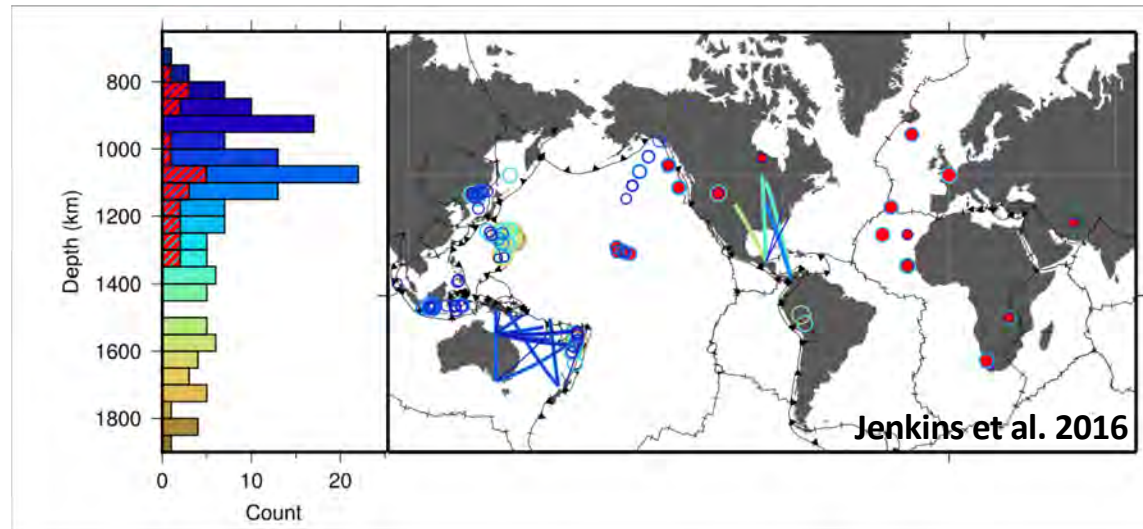


Boschi and Becker 2011



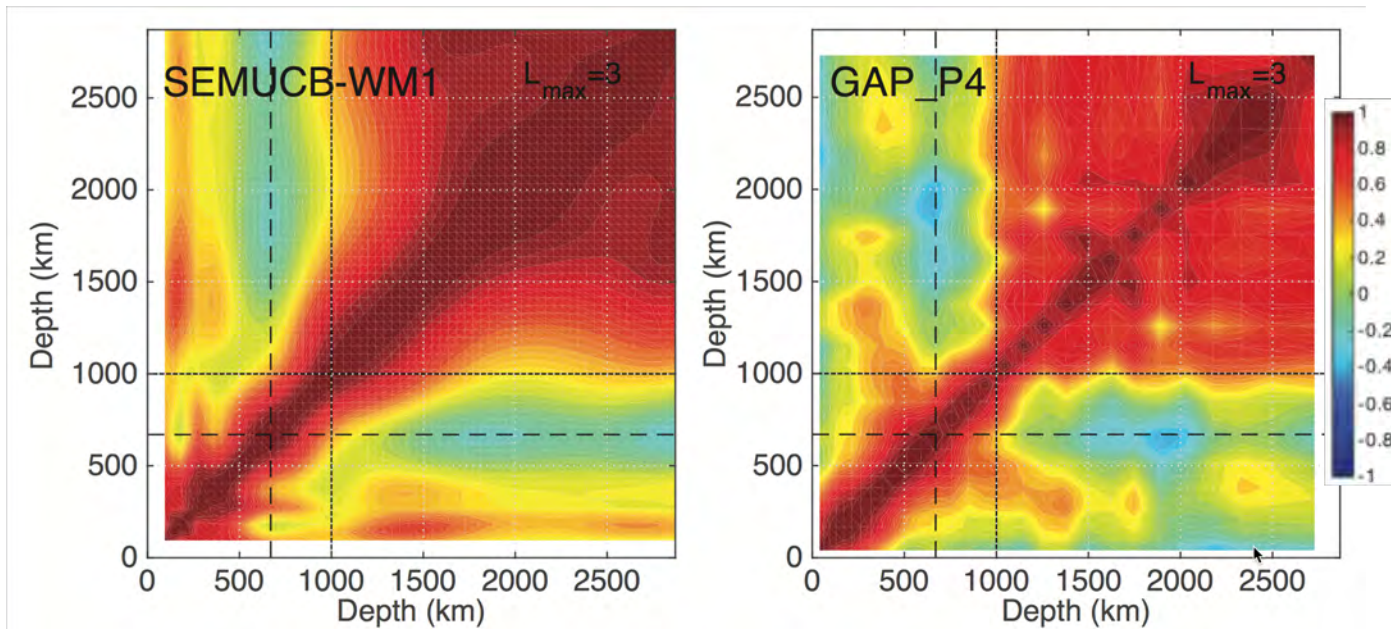
Boschi and Becker 2002

Seismic scatterers in the shallow lower mantle



Changes in radial correlation / vertical coherence

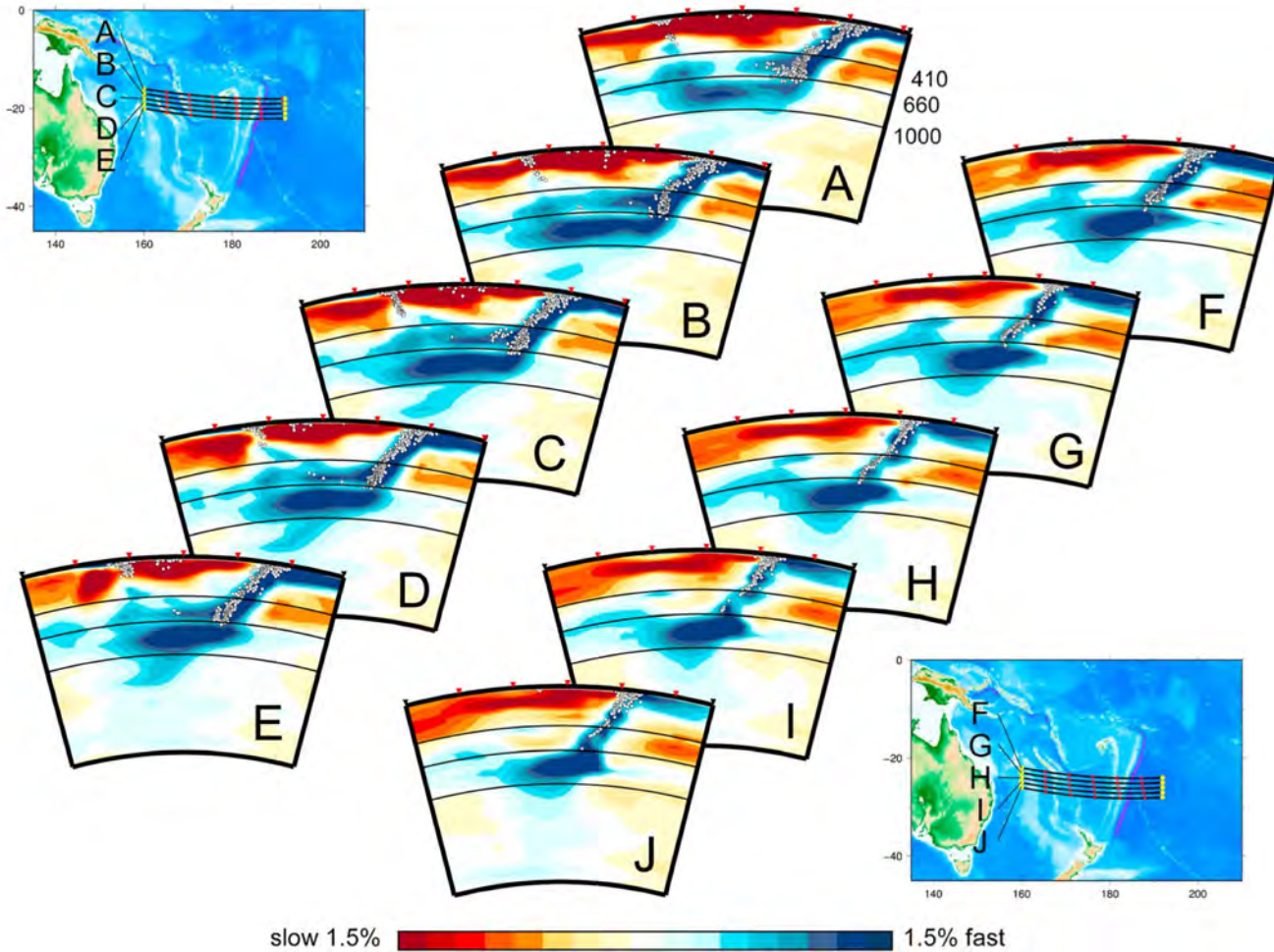
$$R_{\beta}(r,r') = \frac{1}{4\pi\sigma_{\beta}(r)\sigma_{\beta}(r')} \int_{S_1} \delta\beta(r,\Omega)\delta\beta(r',\Omega)d\Omega \quad (\text{Jordan et al. 1993})$$



Very long wavelength radial correlation

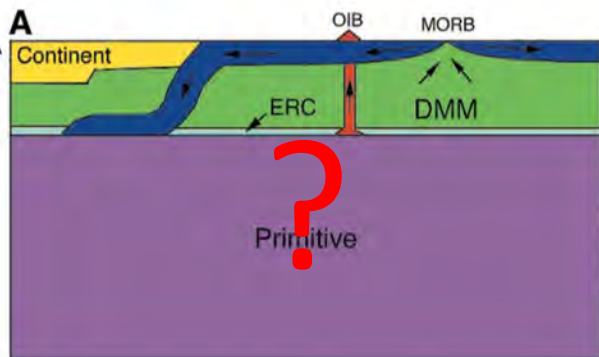
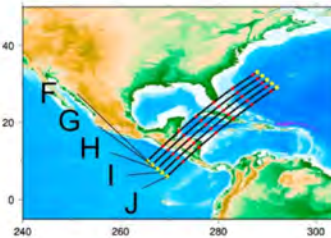
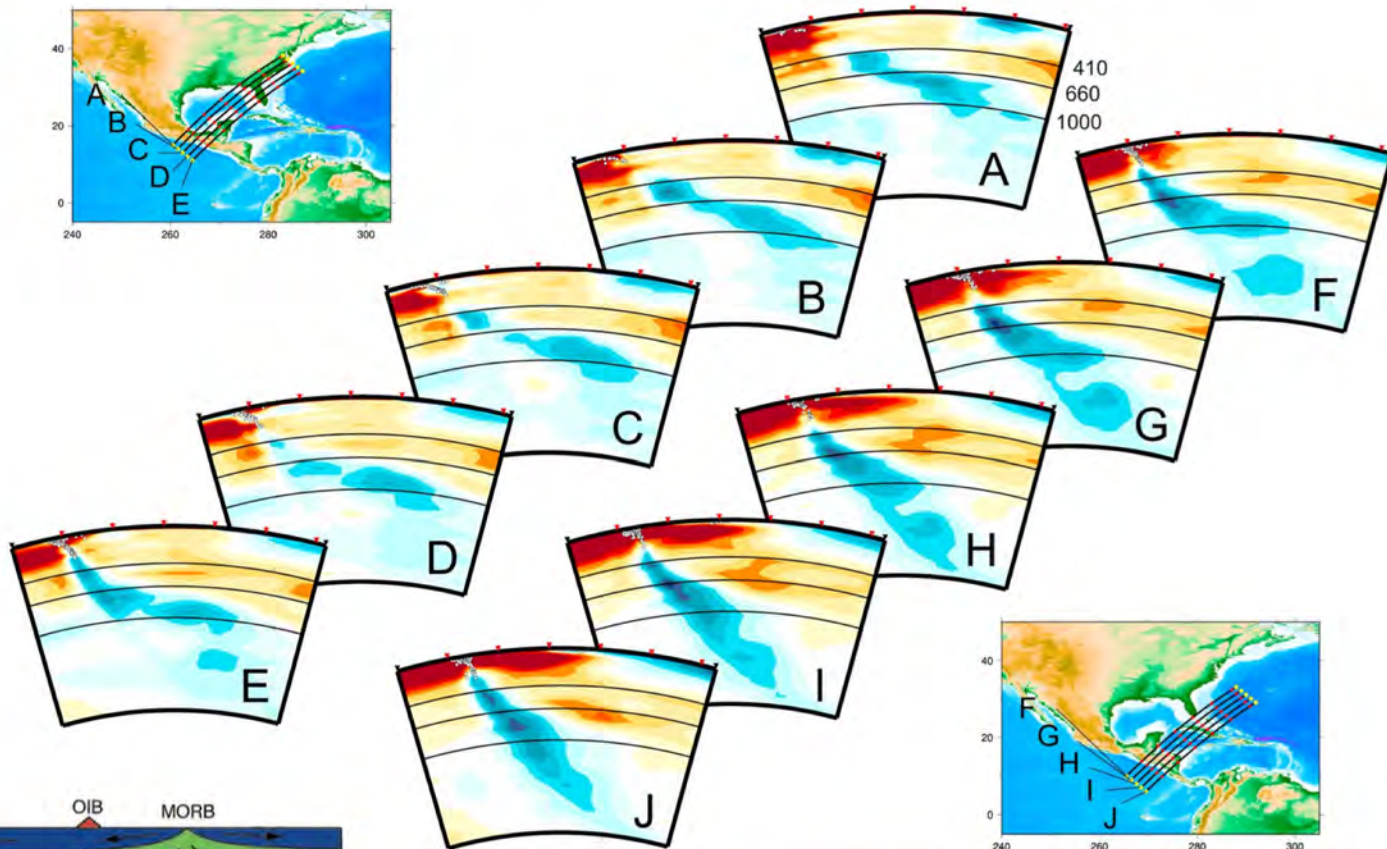
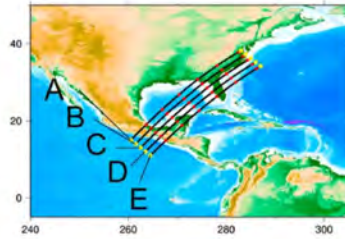
Slab stagnation below the transition zone

Tonga and Kermadec



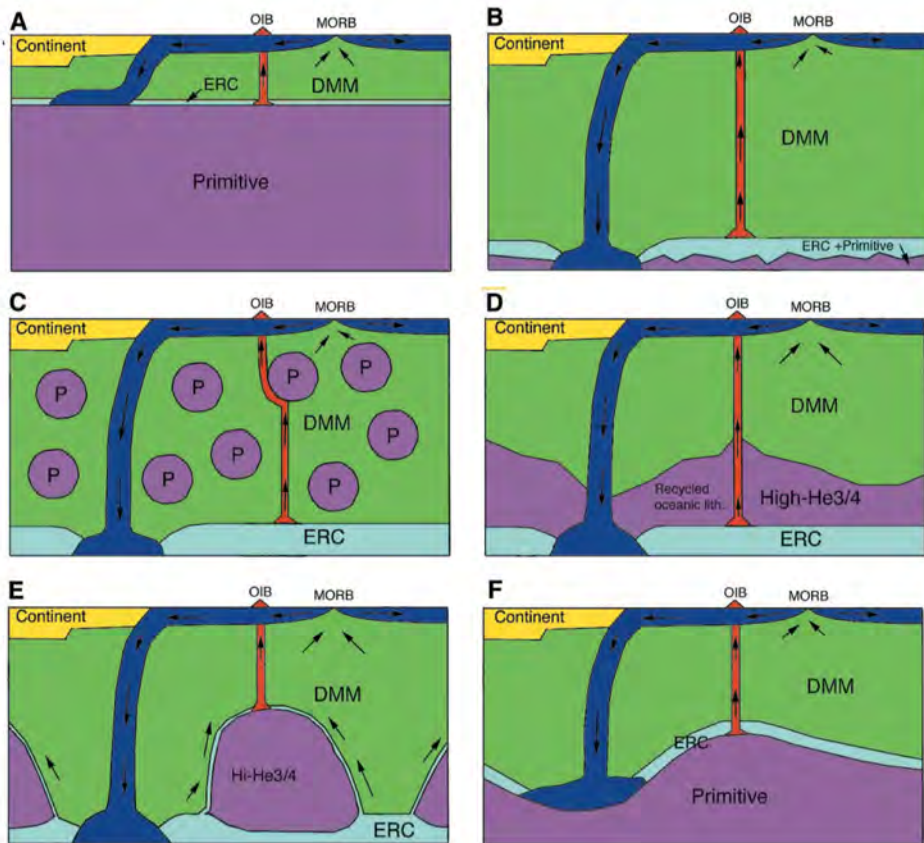
(not ubiquitous)

Central American Arc



1.5% slow 1.5% fast

How do variations in viscosity affect mixing?



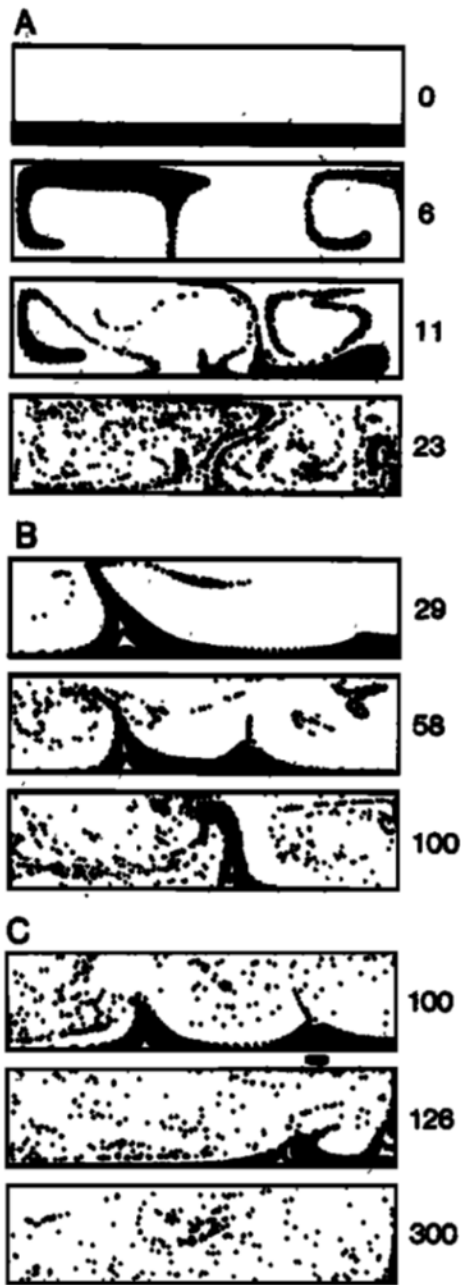


Fig. 2. Positions of tracers at different instants in the model runs A. Case 1 $\beta=0$, B. Case 3 $\beta=2.5$ and C. Case 4 $\beta=3.5$.

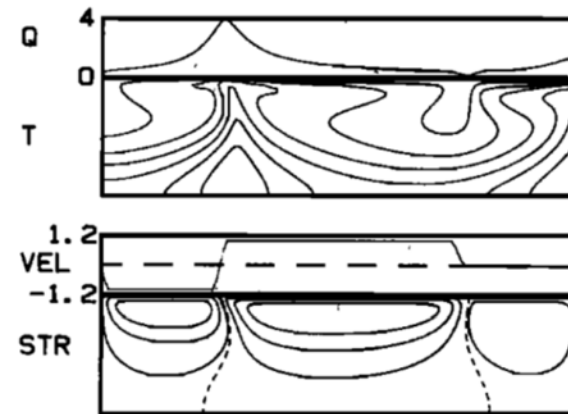
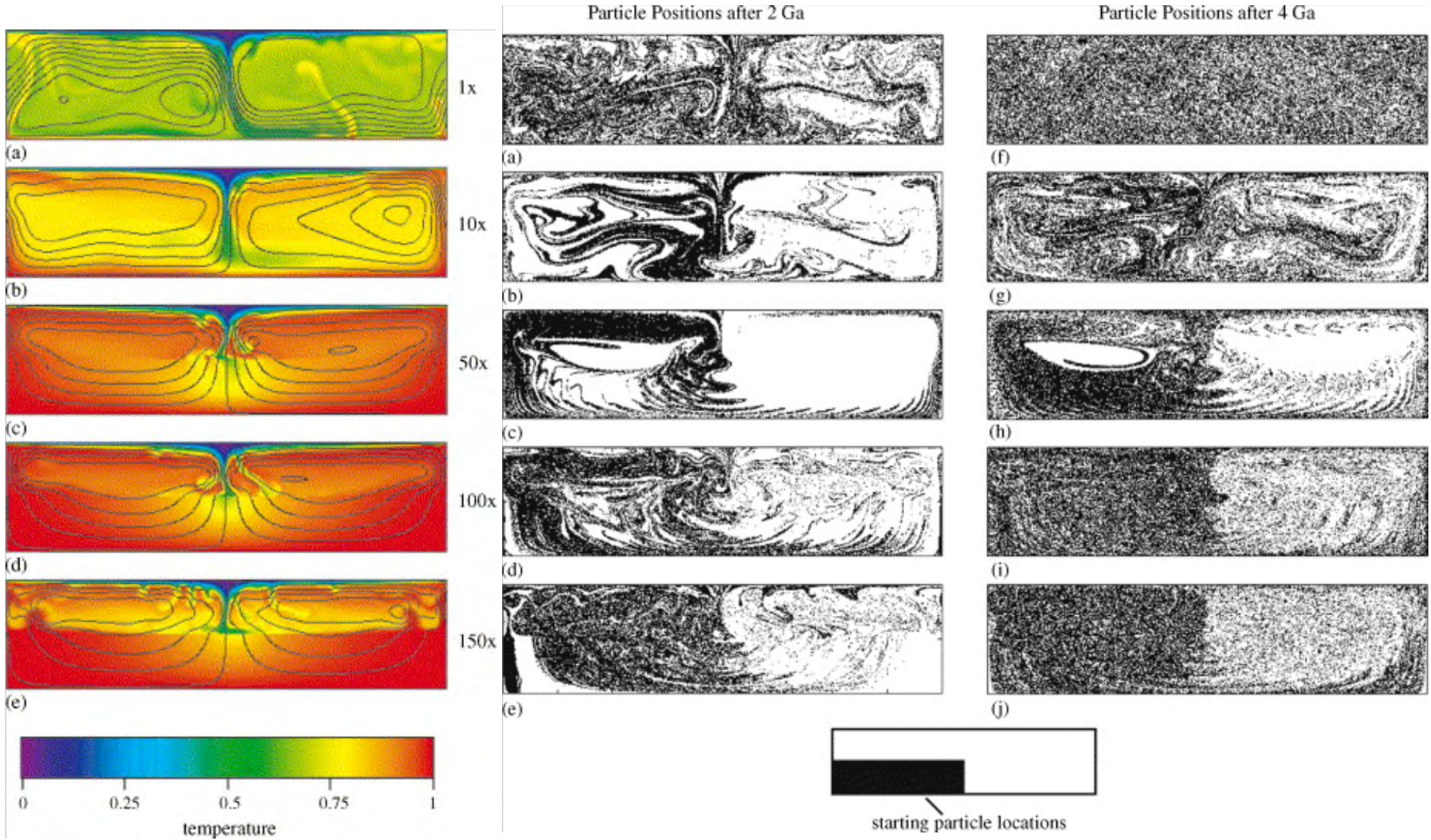
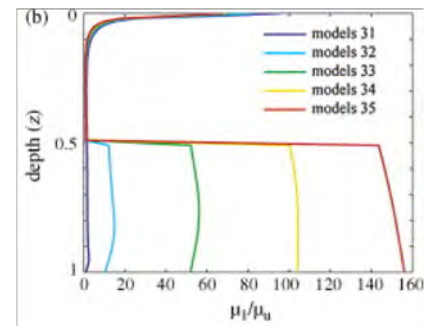


Fig. 1. Initial state for Case 4, with $Ra=5 \times 10^6$, $Pe=700$, $\beta=3.5$. Q is the top surface heat flux, T are the isotherms, VEL is the imposed top velocity (positive to the right), and STR are the stream lines.

$$\eta' = \frac{\eta}{\eta_0} 10^{\beta(1-y')}$$



Naliboff and Kellogg 2007



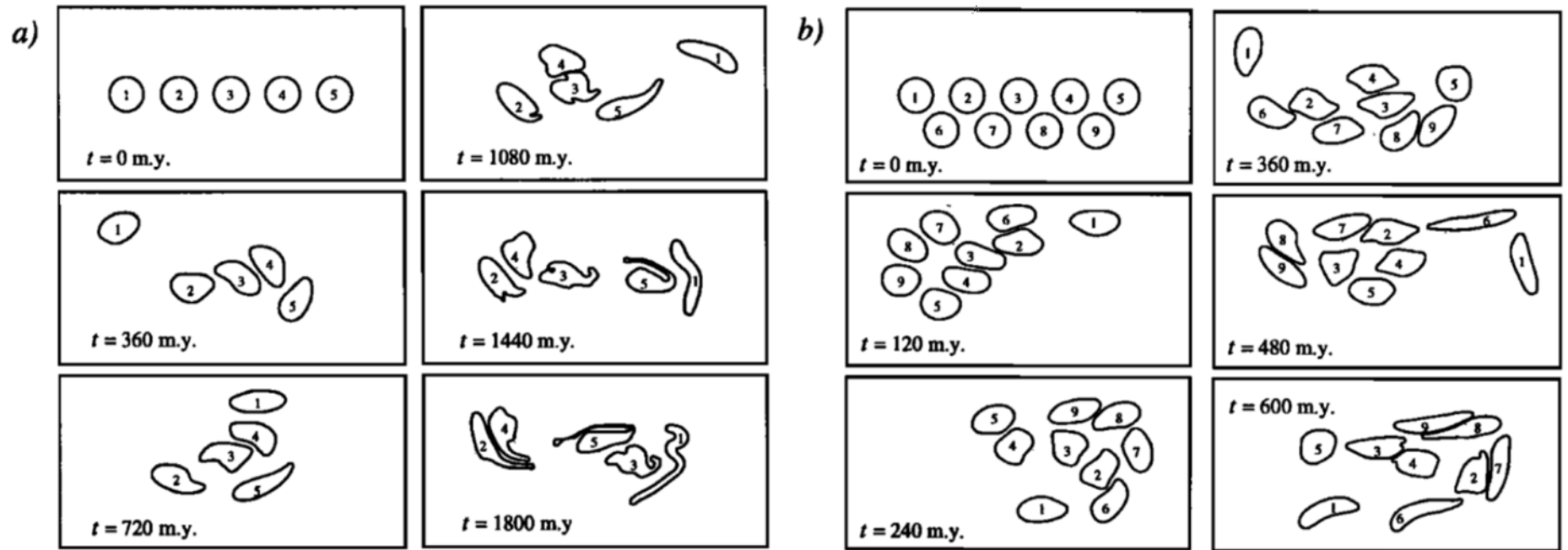


Figure 4. Stretching of blobs in simulations with a) five and b) nine blobs for $\lambda = 10$. Times are based on a mantle thickness of 3000 km and a plate velocity of 10 cm/year.

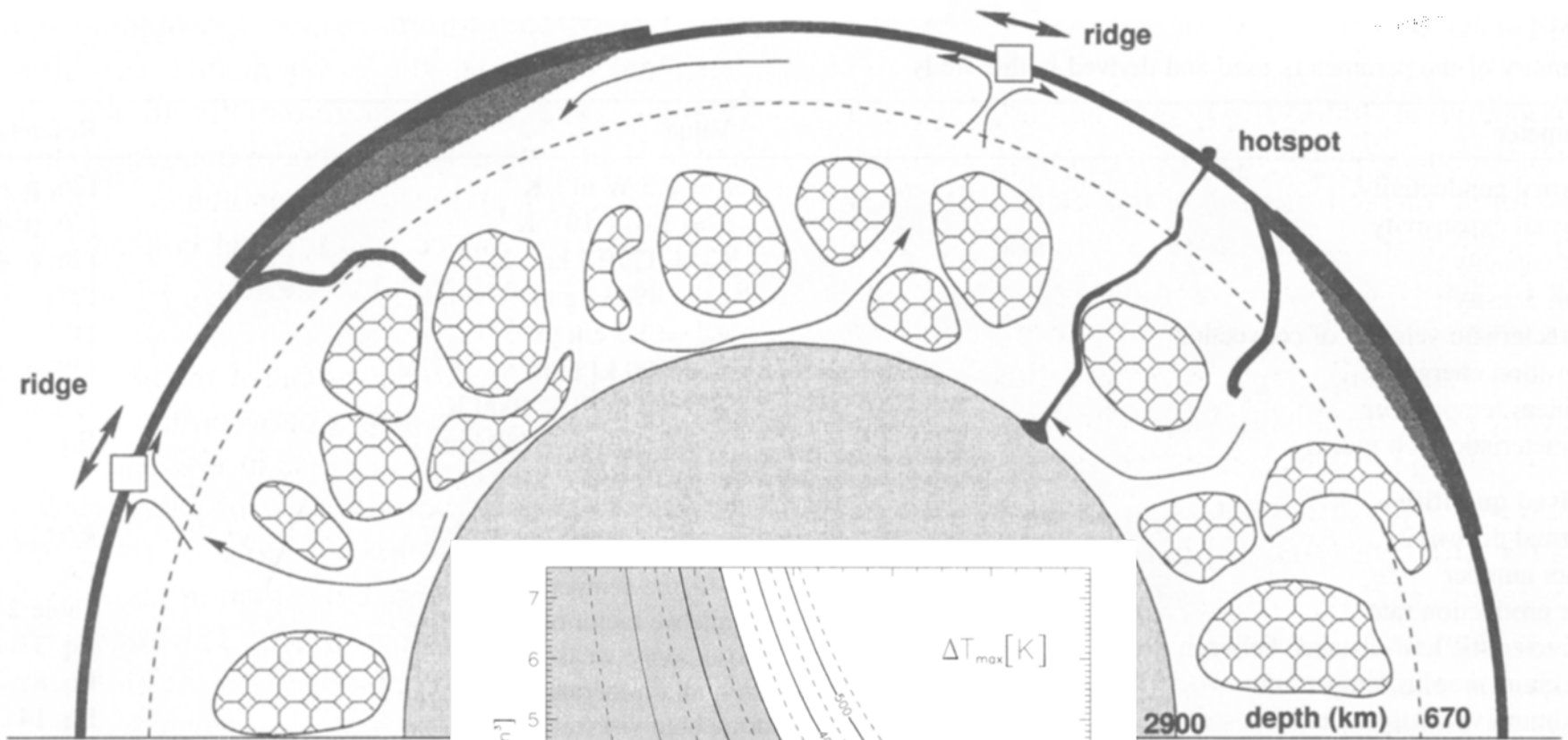
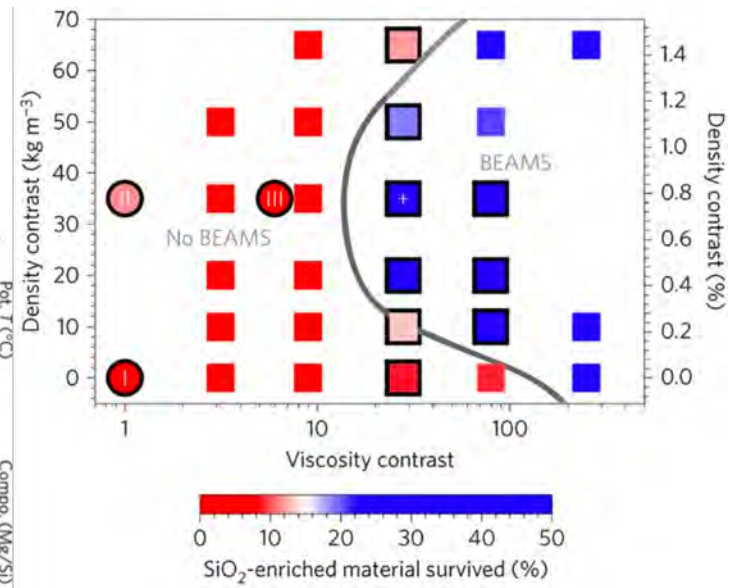
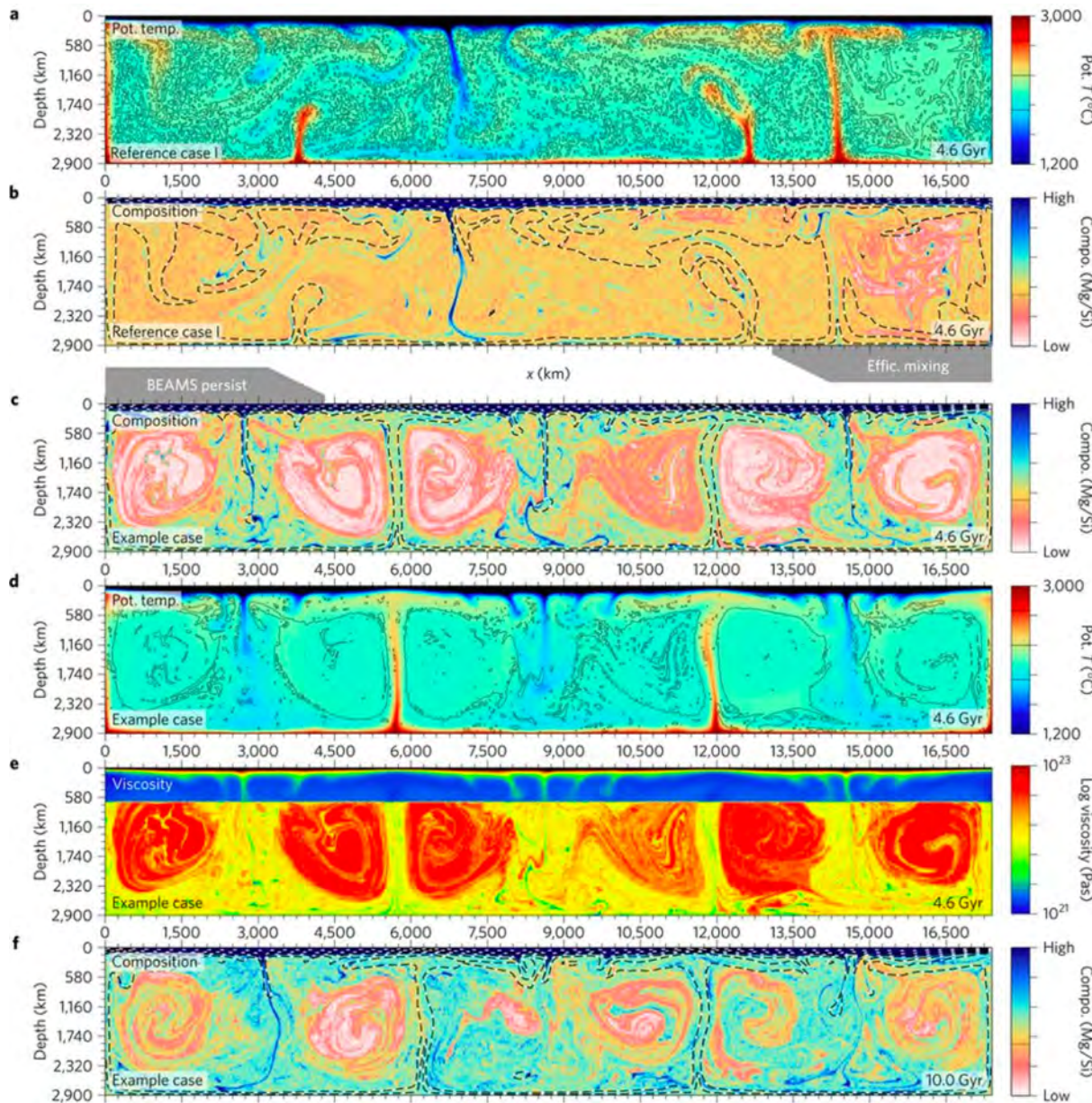
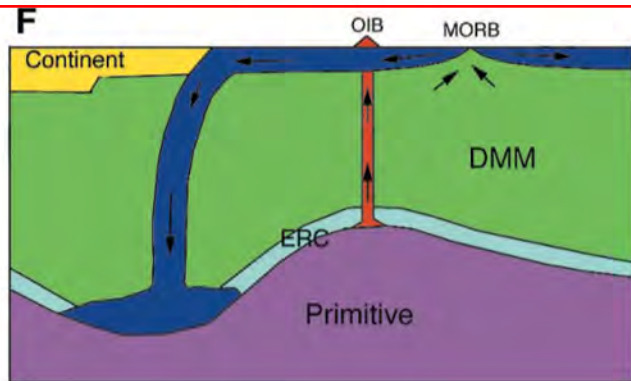
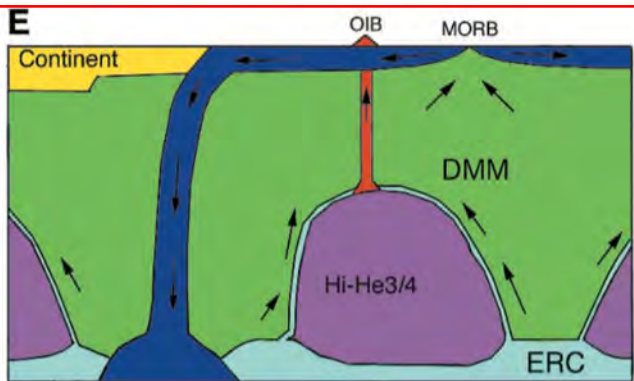
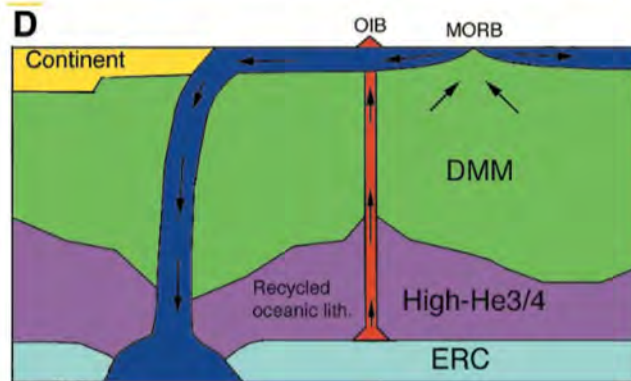
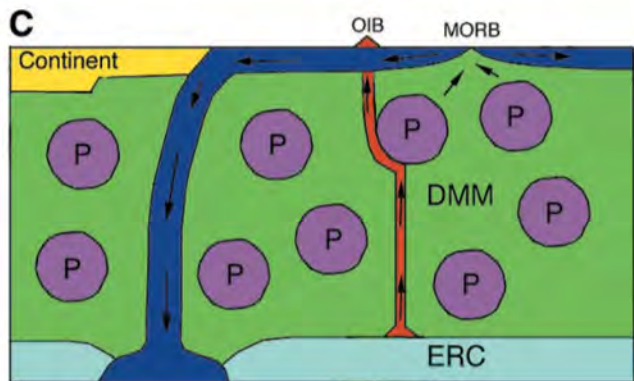
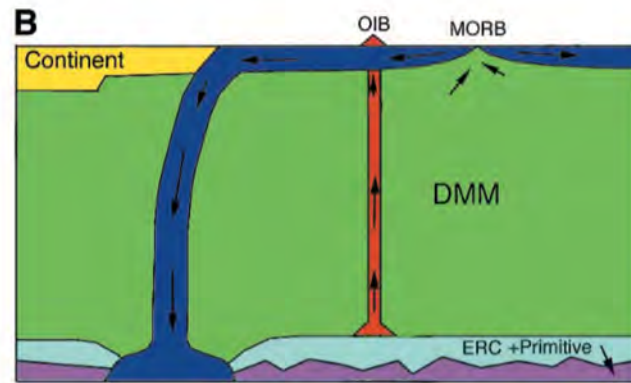
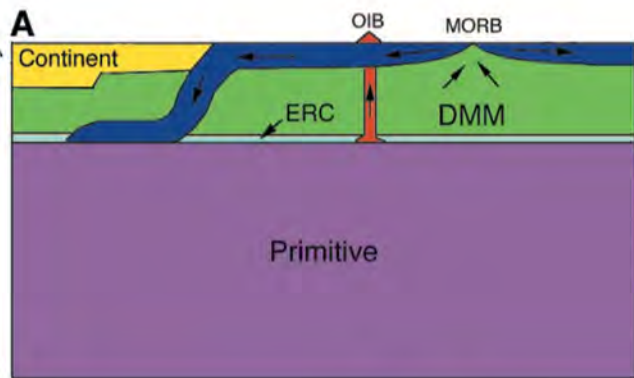


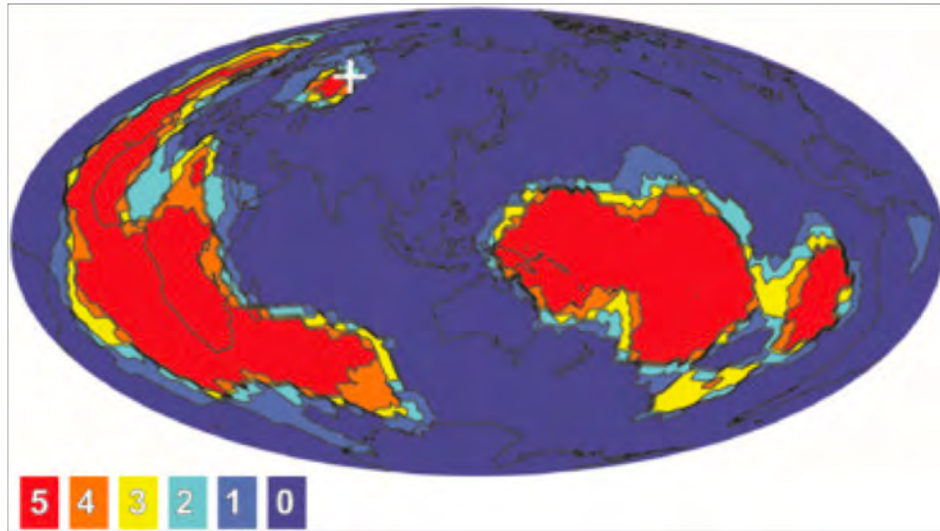
Fig. 1. The blob model of convection (morphology of slab penetration through represent the primitive reservoir. Surround the ridges and earlier continent-forming isotope source.

the whole mantle mode with varying in the cores of the convective cells and re depleted and degassed by melting at terial and lead to a heterogeneous OIB

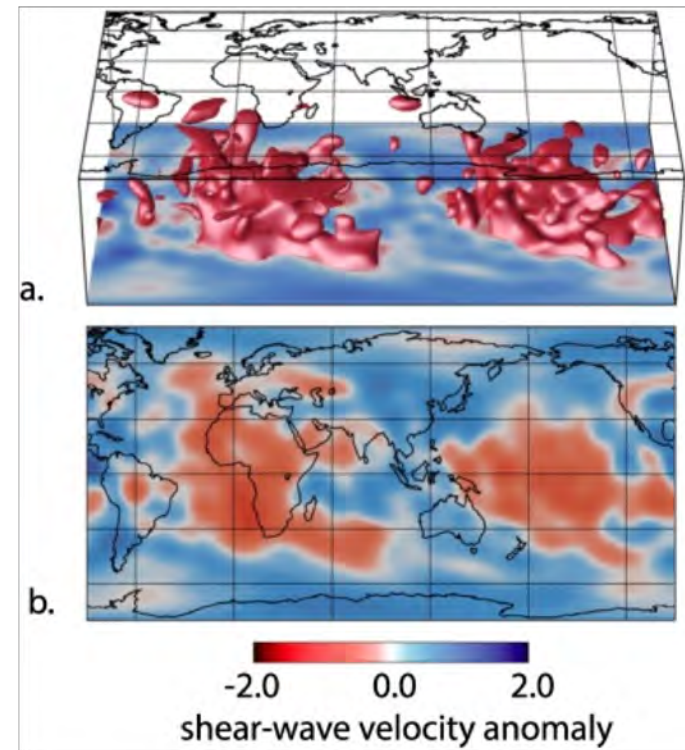


BEAMS
Ballmer et al. 2016

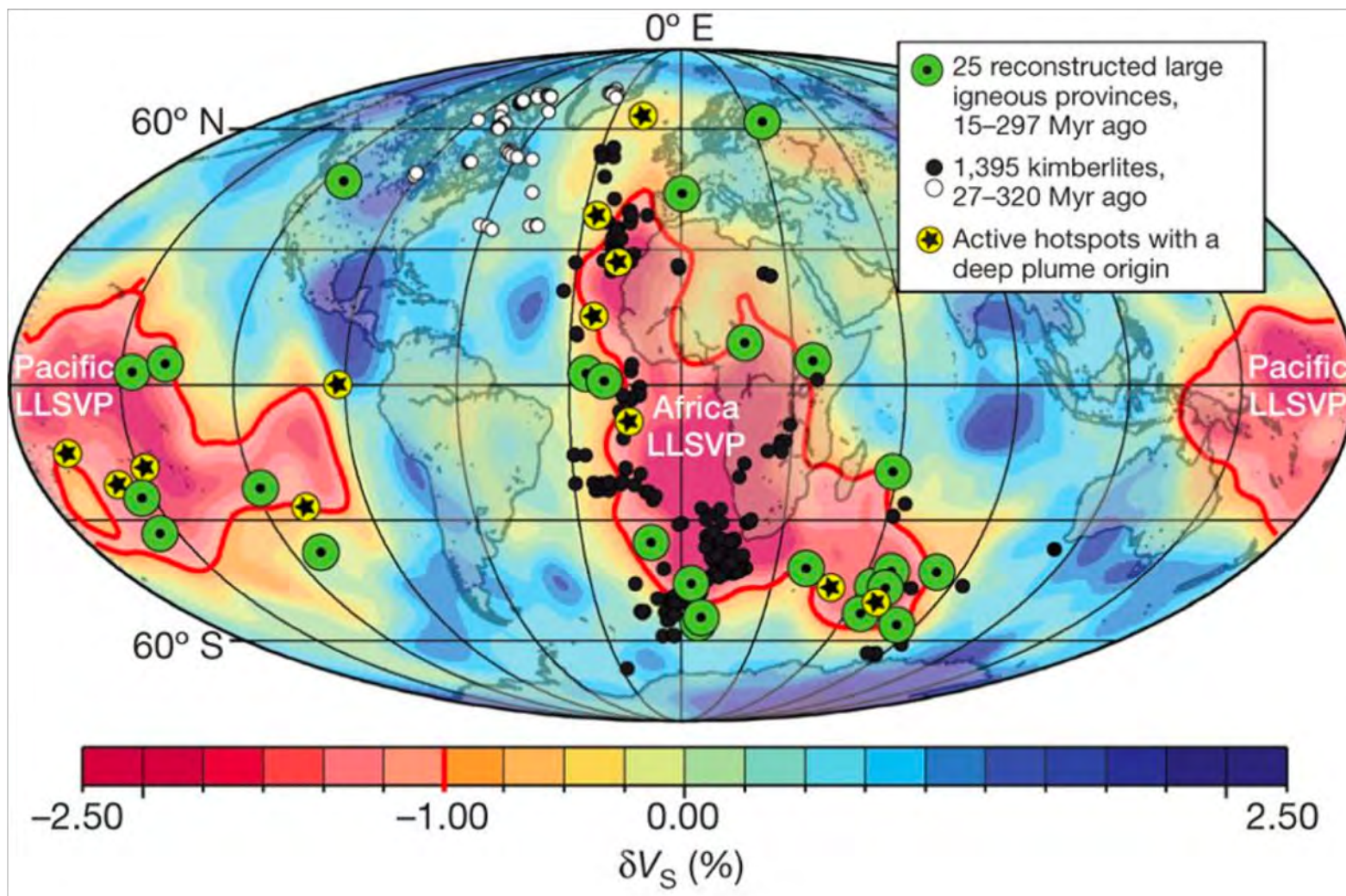




Lekic et al. 2012

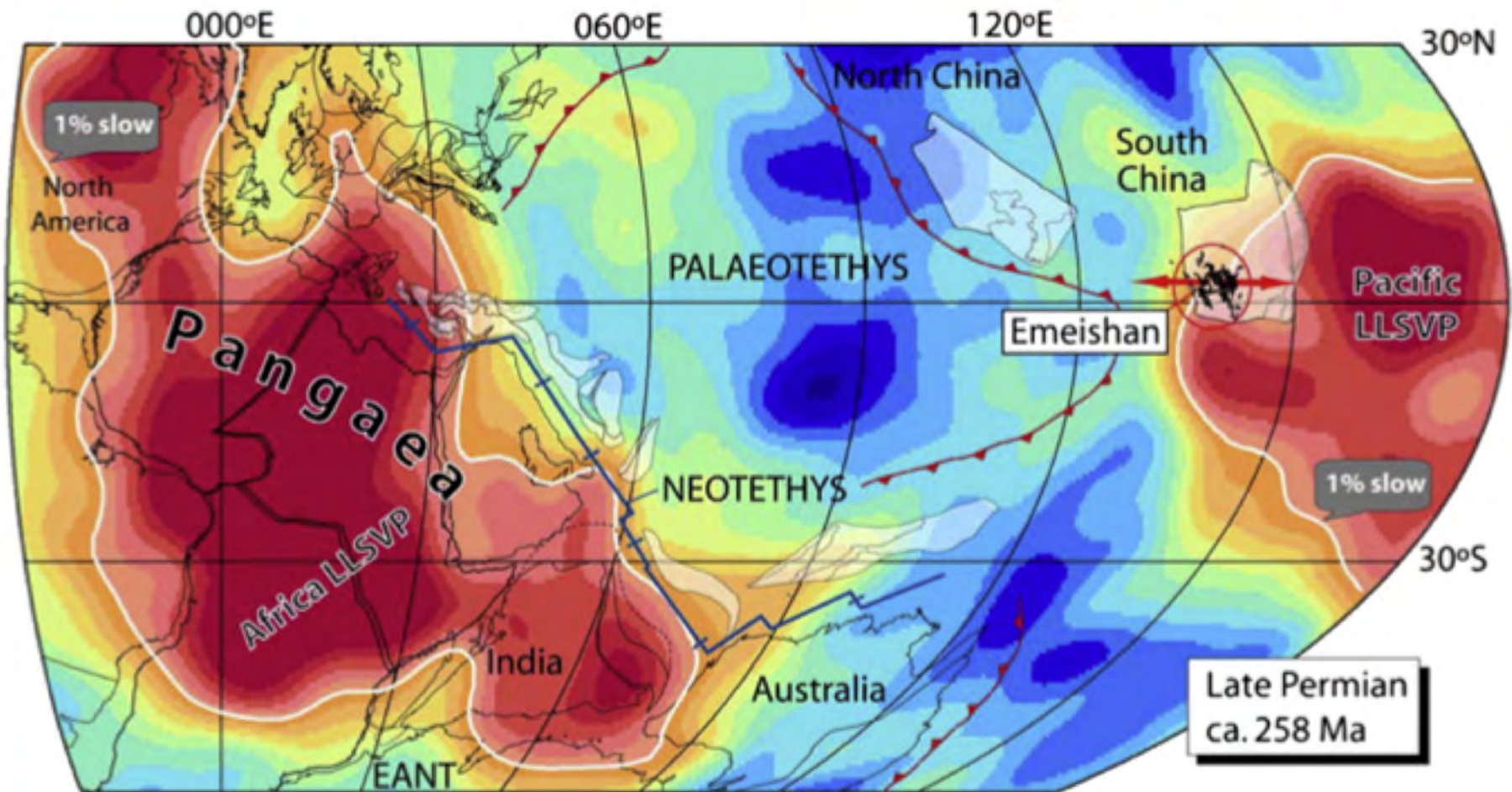


McNamara 2018 Tectonophysics
(tomography is s20rts, Ritsema et al. 1999)



Torsvik et al. 2010

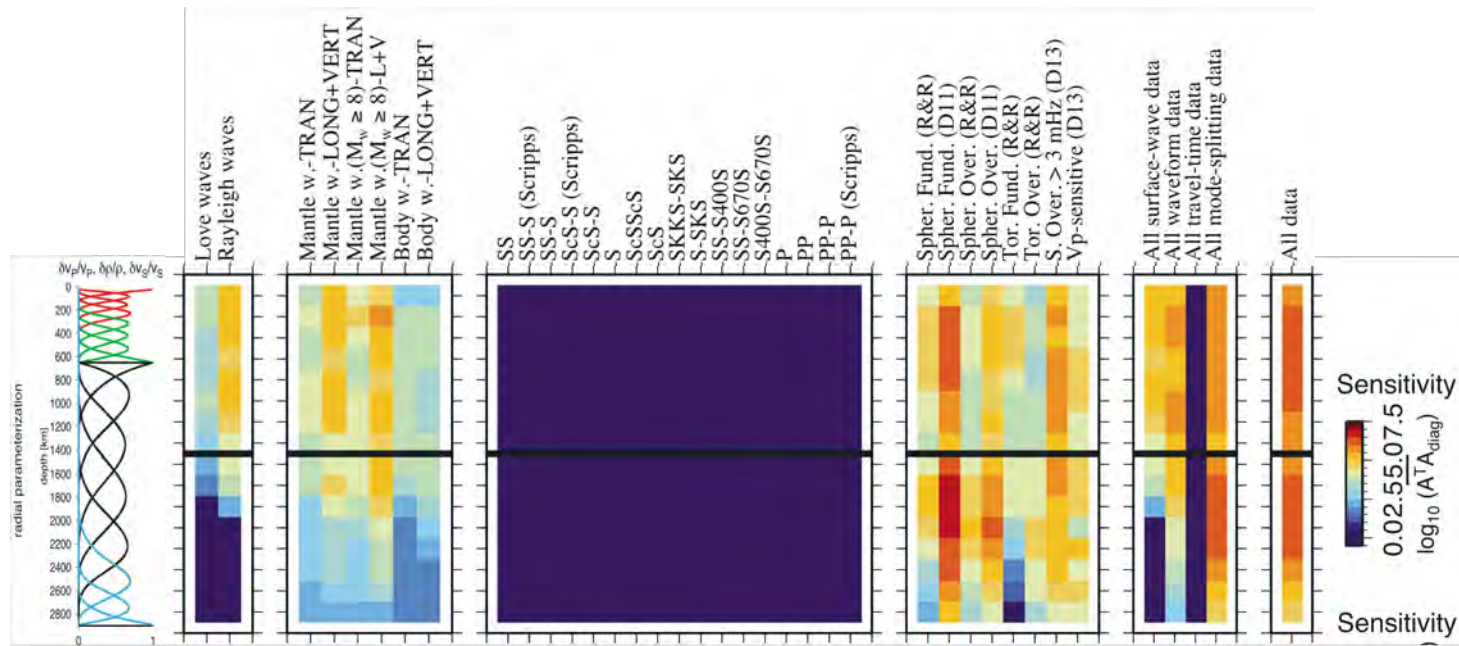
Fixed LLSVPs are being assumed to extend plate reconstructions further back in geologic time



Emeishan LIP – paleolongitude uncertain

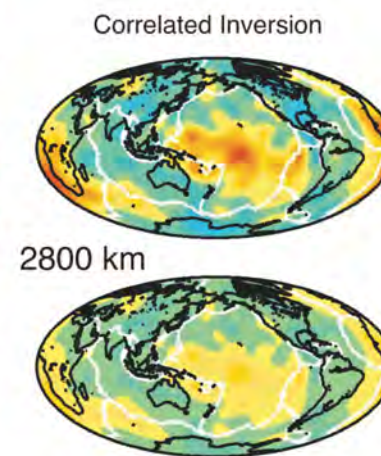
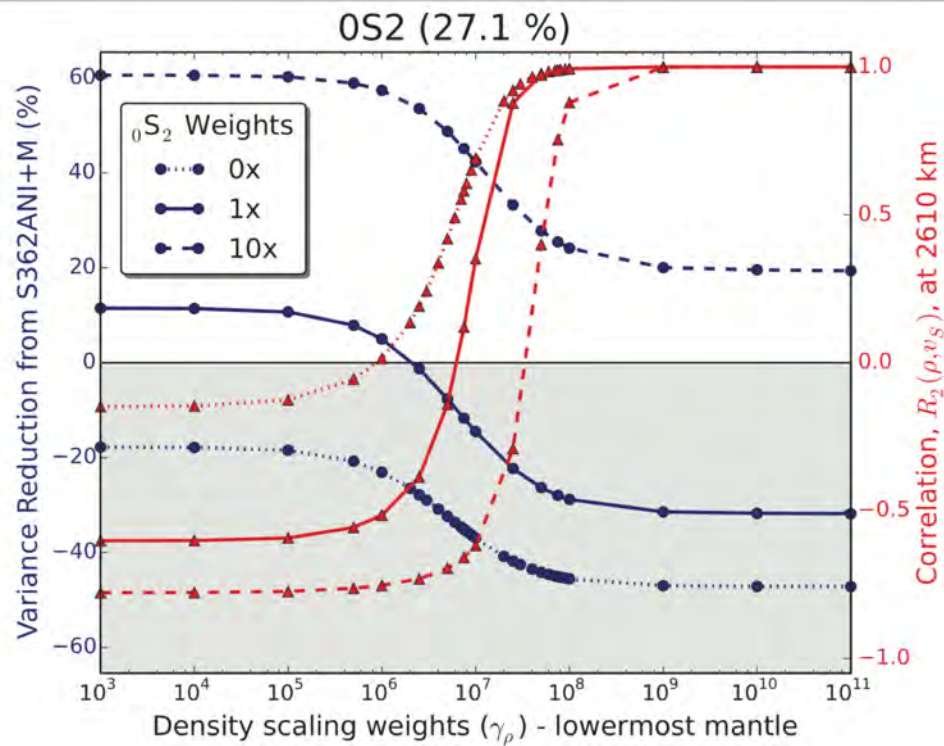
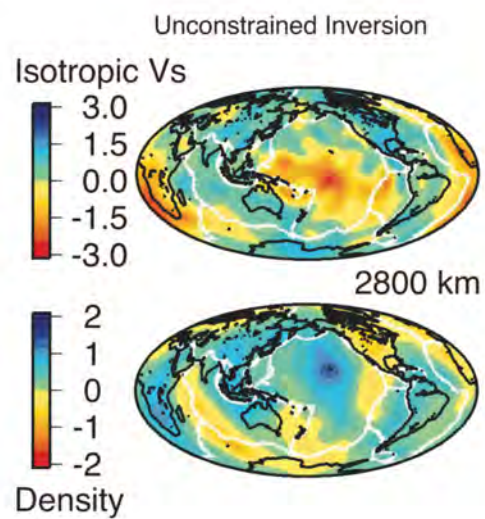
Torsvik et al. 2008

Joint Vp, Vs, ρ model

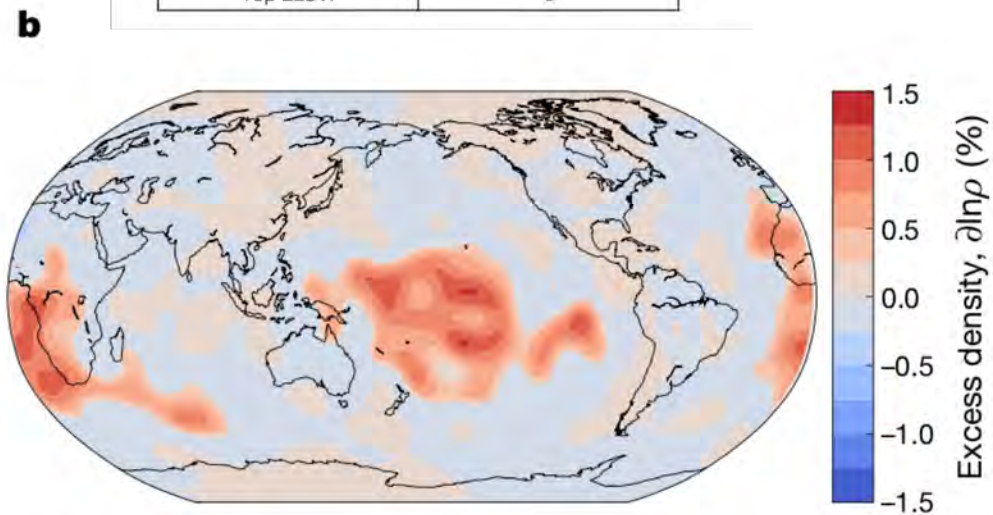
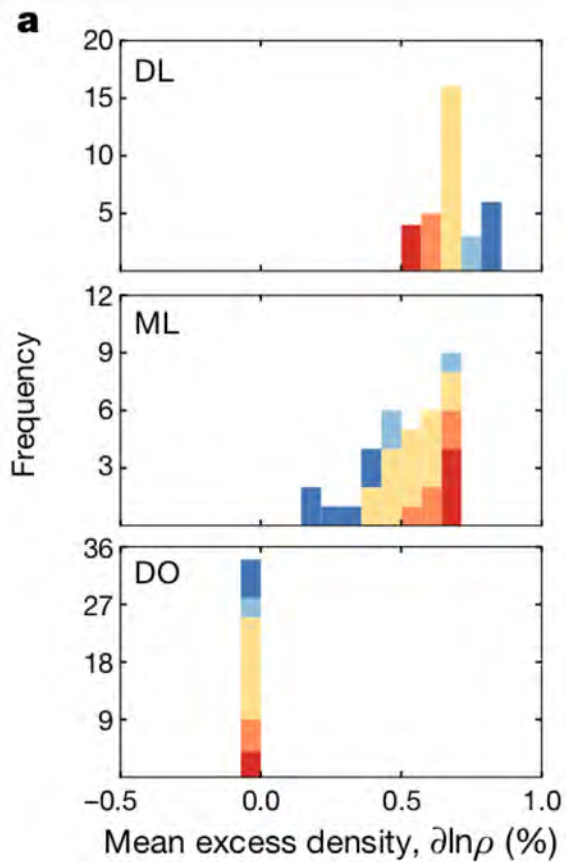
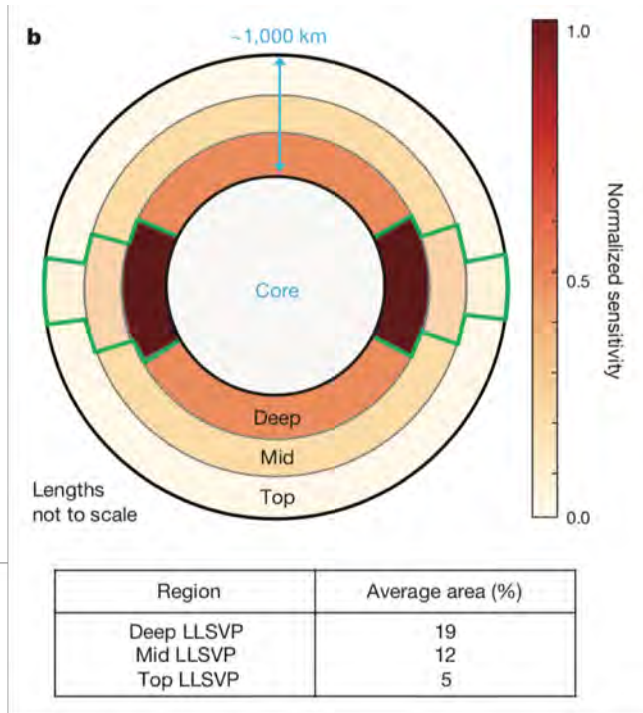


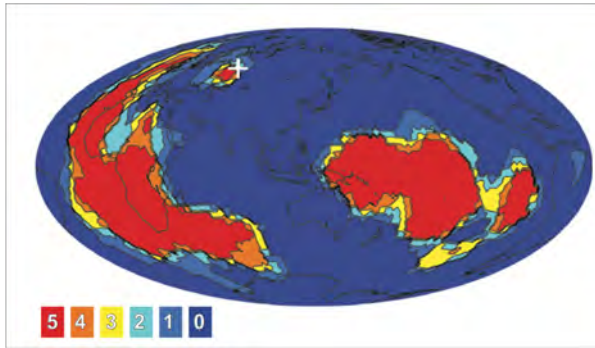
Sensitivity to density structure

Moulik and Ekström 2016

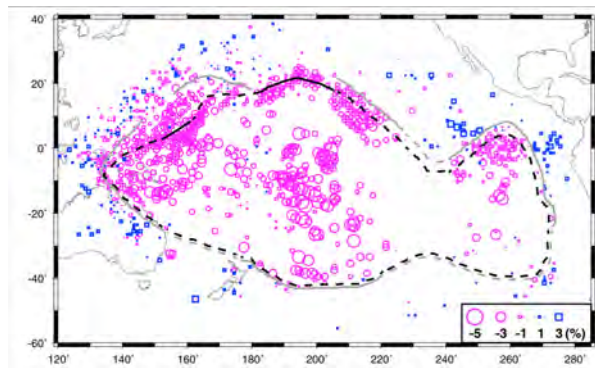


Moulik and Ekström (2016)





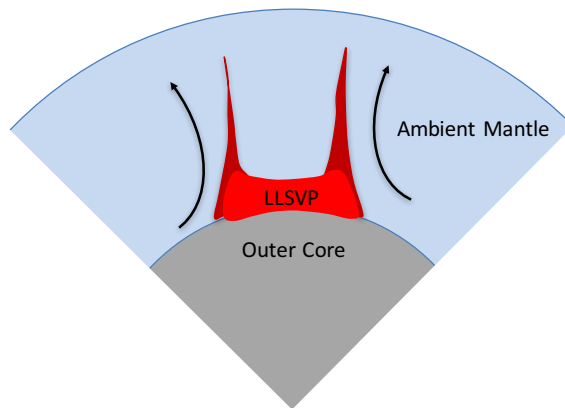
LLSVP boundaries from cluster analysis
Lekic et al. 2012



Shear and bulk sound speed anticorrelated

Waveform modeling reveals sharp boundaries

Tidal tomography

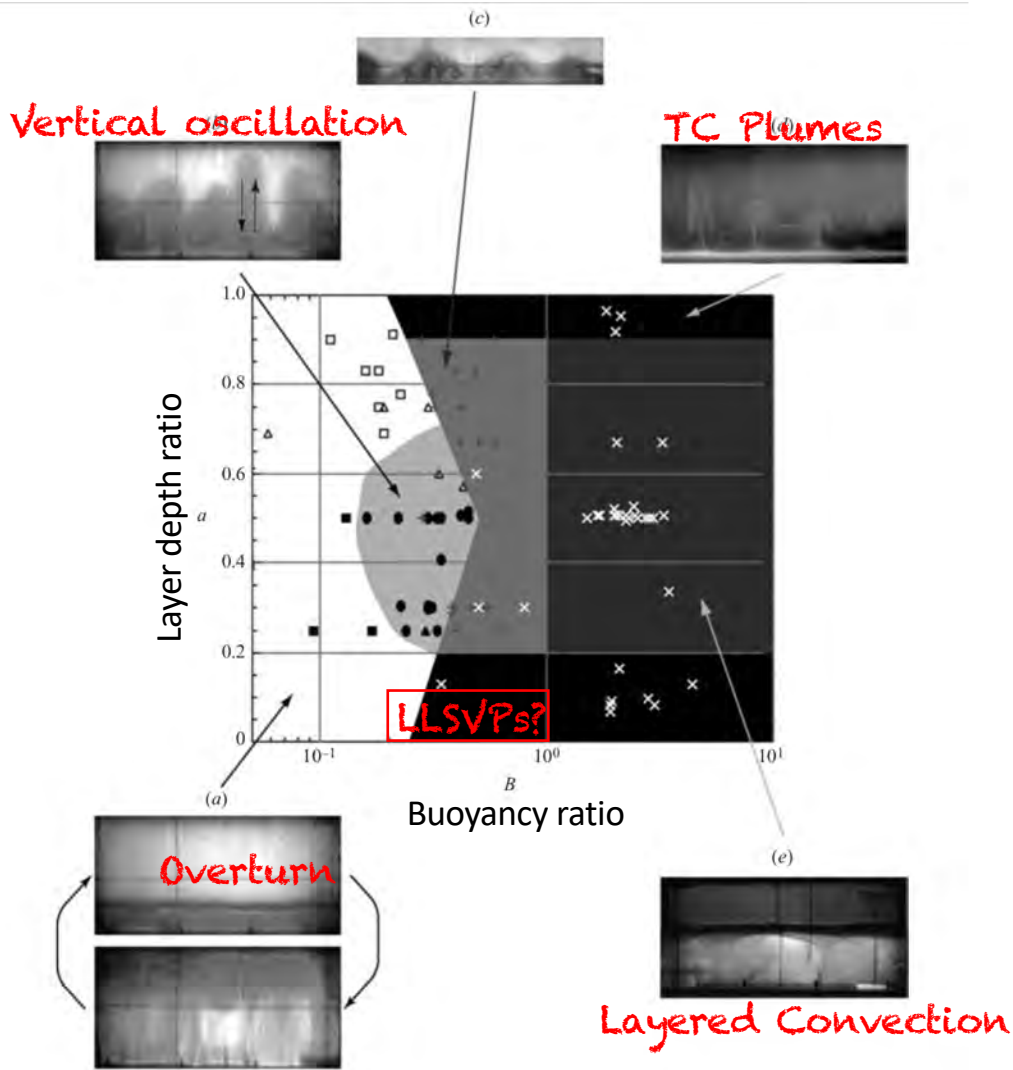


Thermal Buoyancy

Compensated over bottom 1000 km?

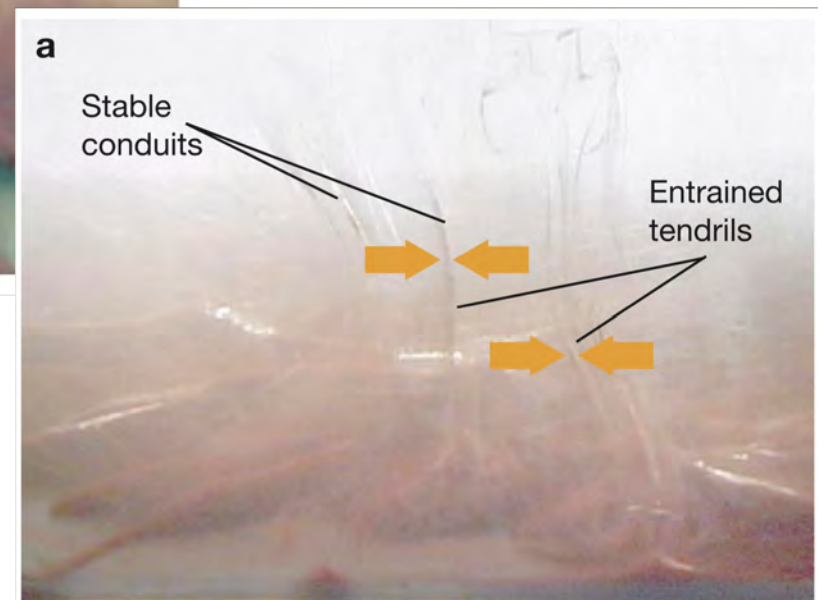
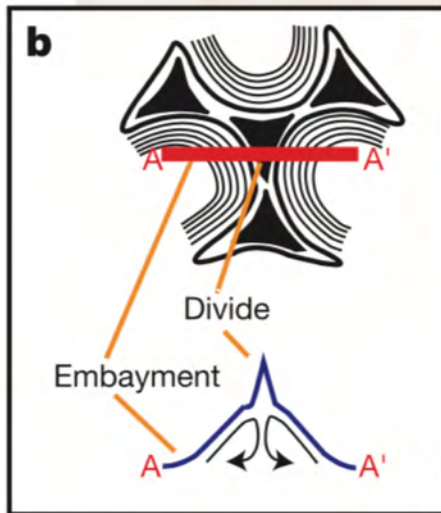
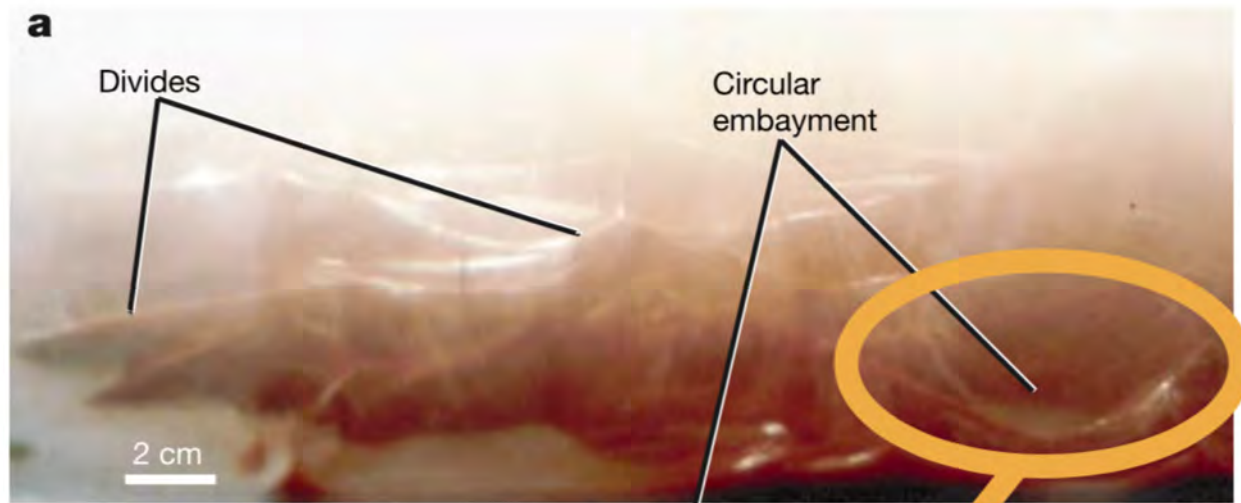
Compositional Buoyancy

Stratified



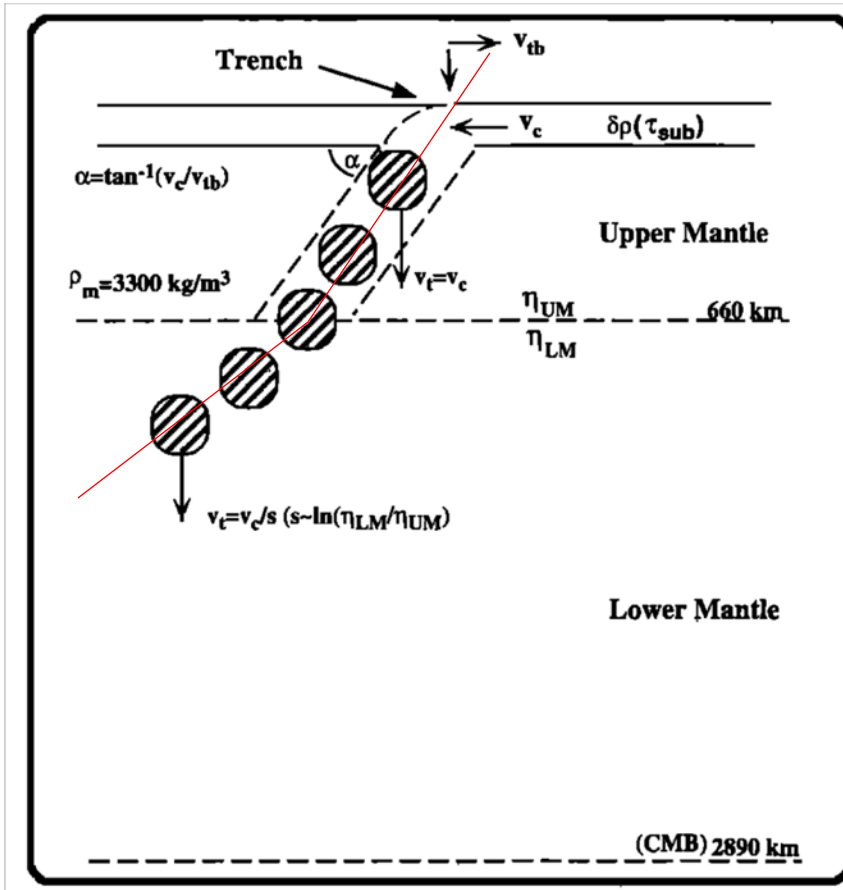
Regime diagram from Laboratory analogue experiments

$$B = \frac{\Delta\rho_\chi}{\alpha\rho_0\Delta T},$$



Jellinek and Manga 2002

Chemical piles – what controls shape?



Simplest models of slab sinking

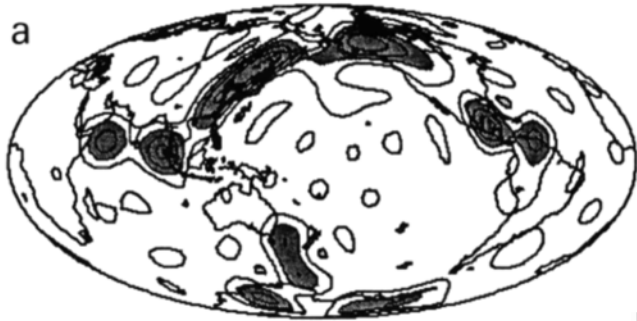
'Slablets' introduced, sink vertically.

Ricard et al. 1993

Lithgow-Bertelloni and Richards 1998 *Rev. Geophys.*

SLABS (DEPTH 2000 KM, DEGREES 1-15)

a



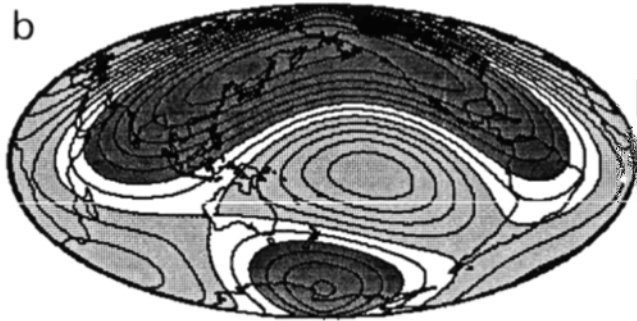
Put slabs into the mantle based on 180 Myr subduction history.

Use resulting 'slablet' density model to drive
Mantle flow

Lower mantle density structure not so different from tomography.

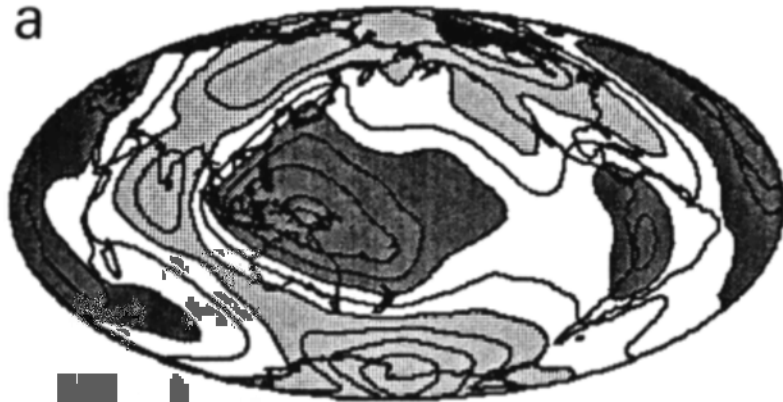
SLABS (DEPTH 2000 KM, DEGREES 1-3)

b



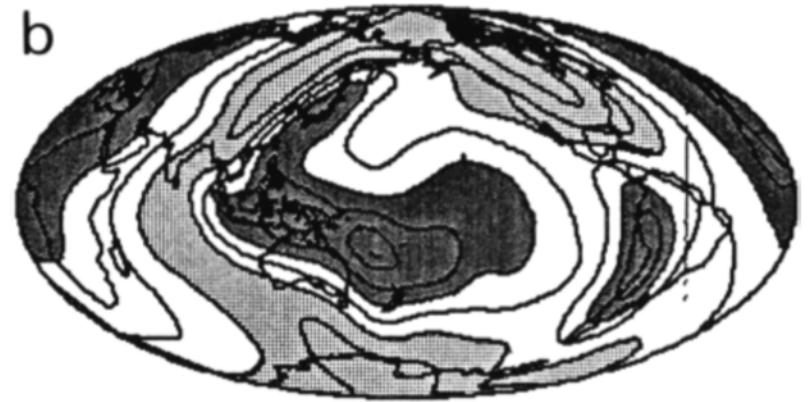
OBSERVED GEOID

a



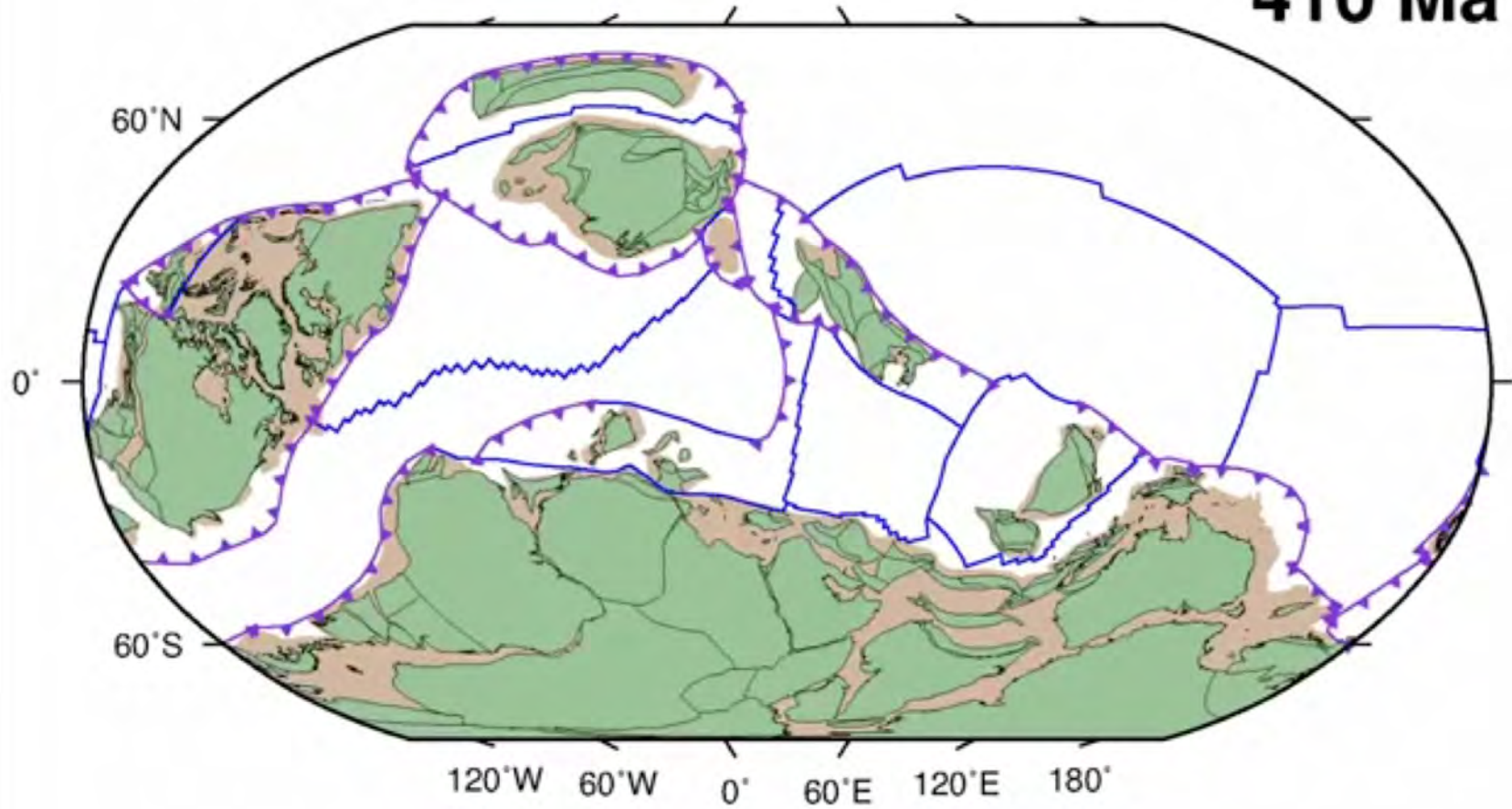
COMPUTED GEOID WITH C21 S21

b

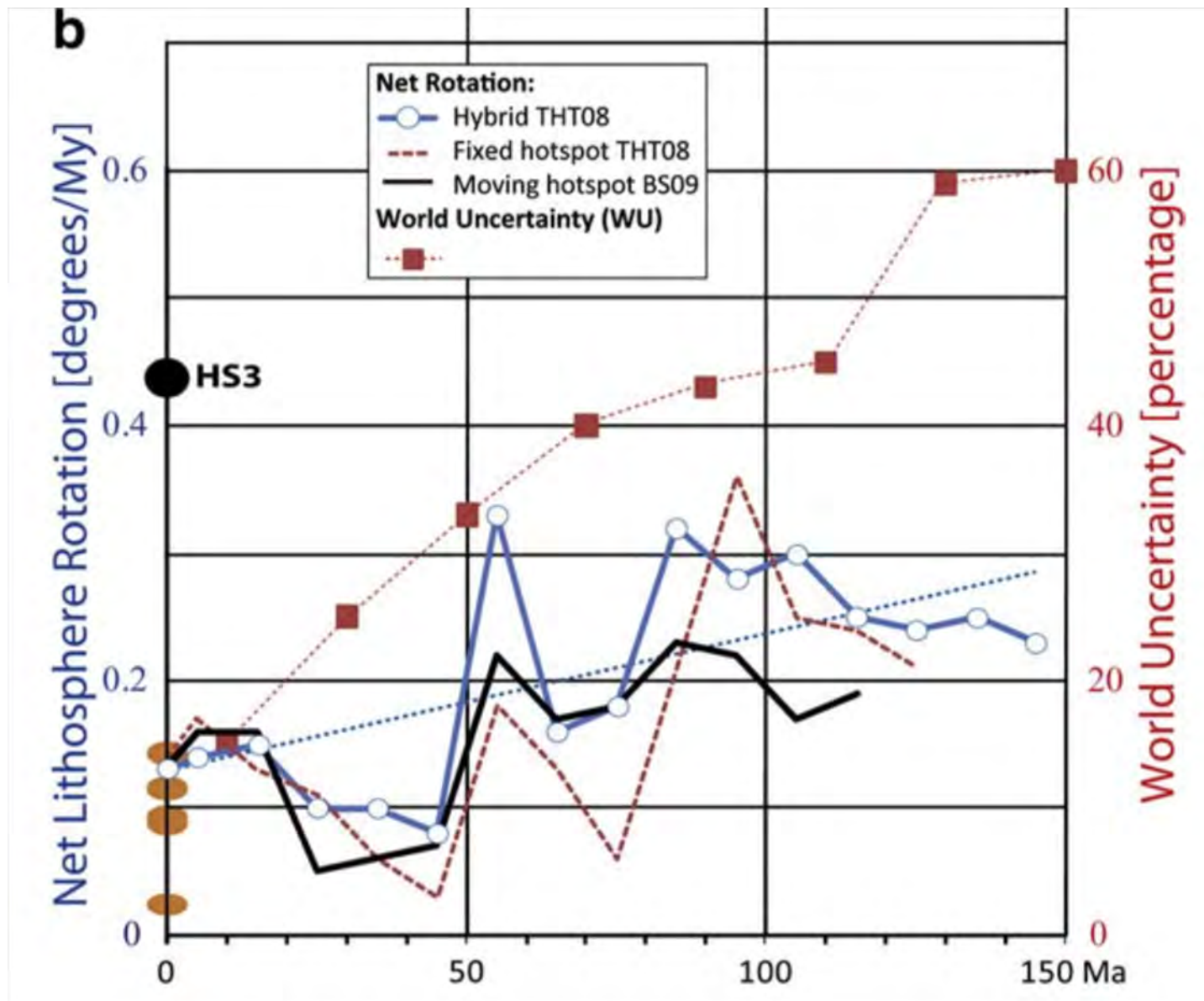


Ricard et al. 1993

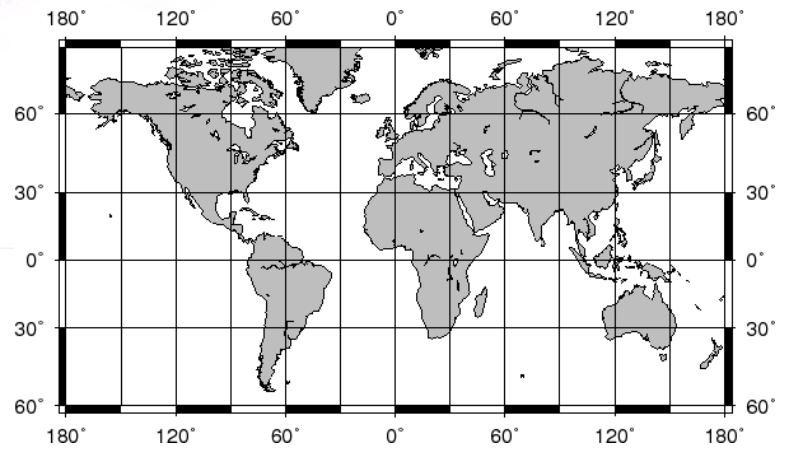
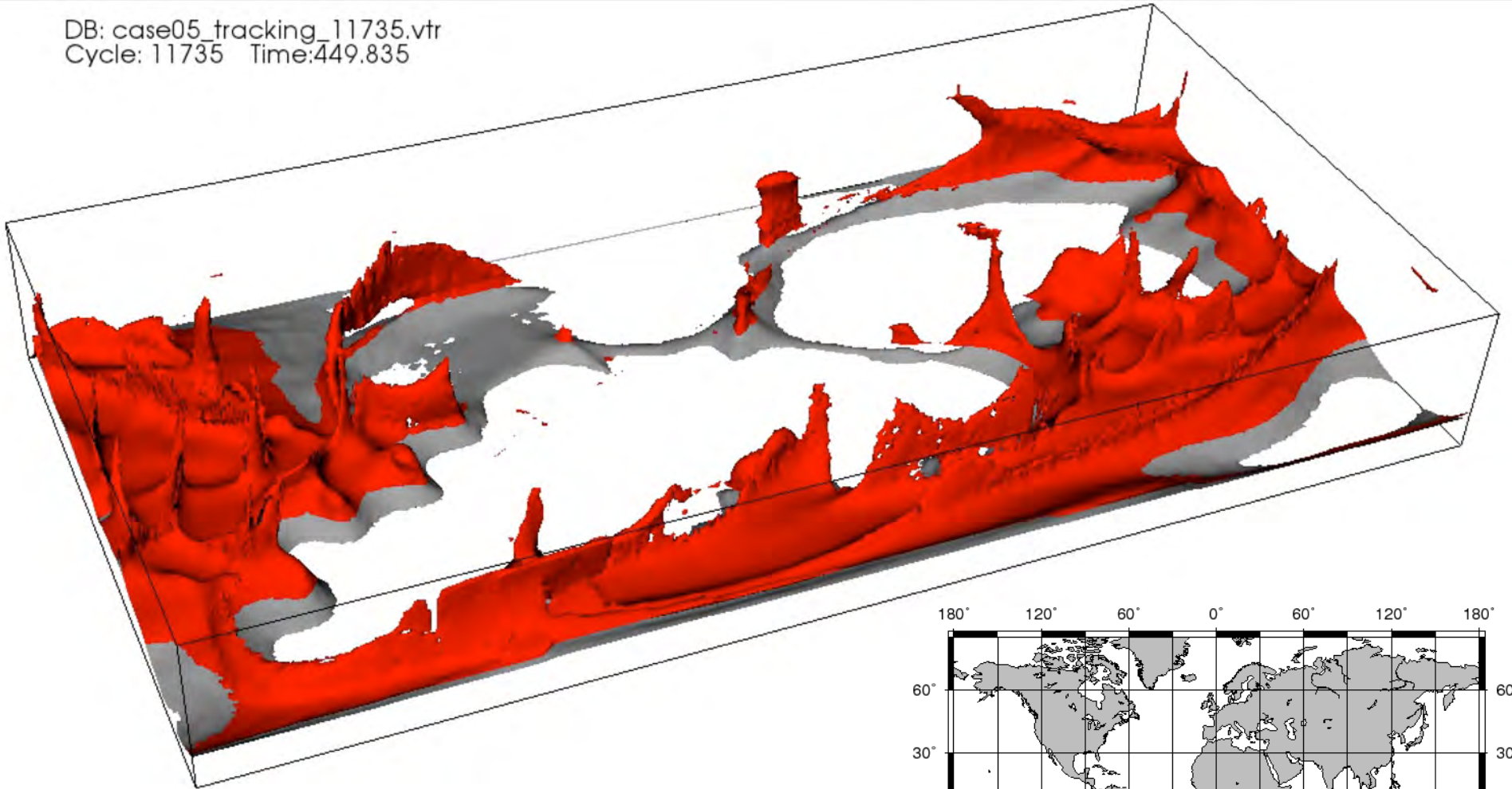
410 Ma



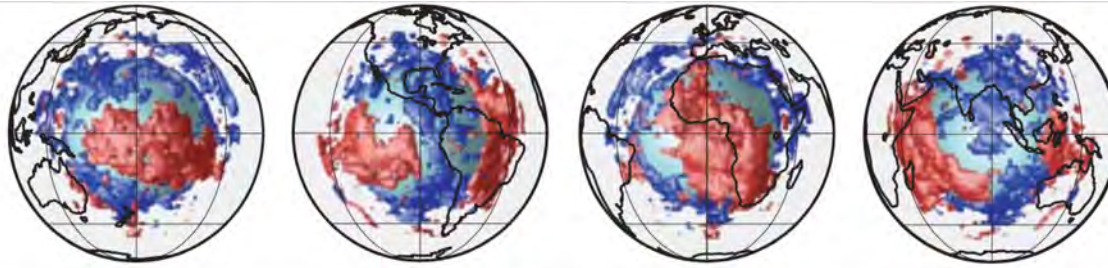
Time-dependent plate boundary configuration from Matthews et al. 2016



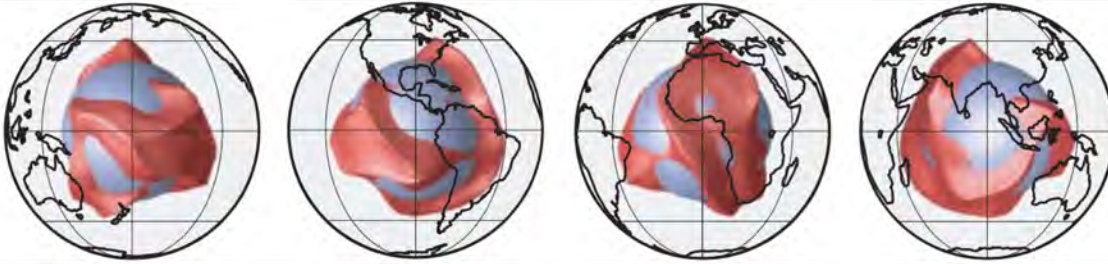
DB: case05_tracking_11735.vtr
Cycle: 11735 Time:449.835



$|\delta v_S| > 0.7\%$
(1,100–2,891 km)
Ref. 7

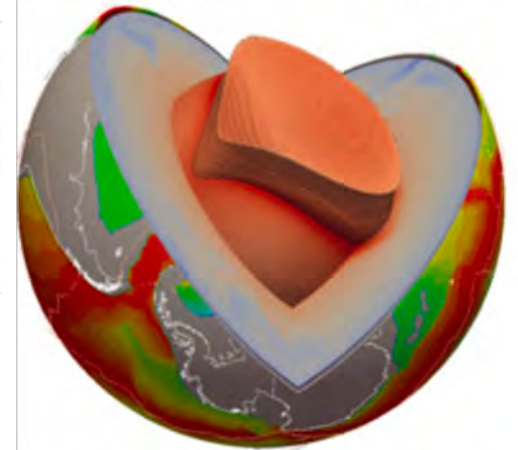


$D_1 = 255$ km

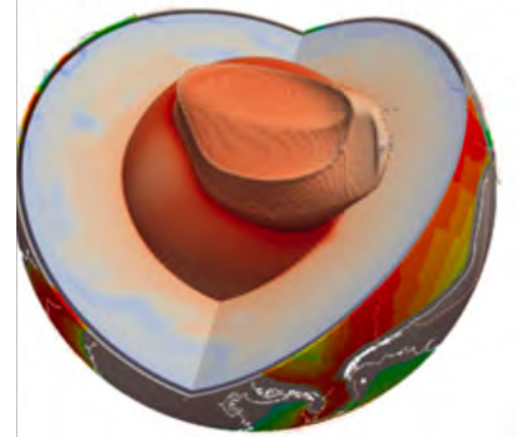


McNamara and Zhong 2004

b) African Structure Present

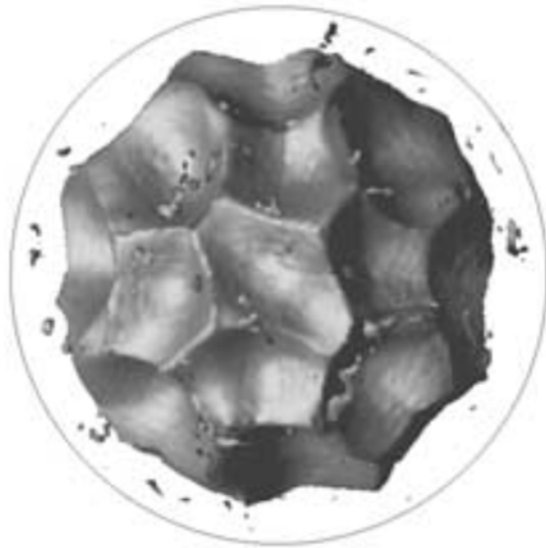


d) Pacific Structure Present



300 Temperature (K) 4000

Bower et al. 2013

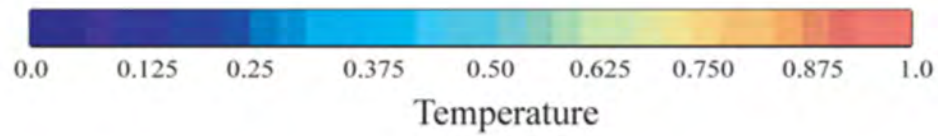
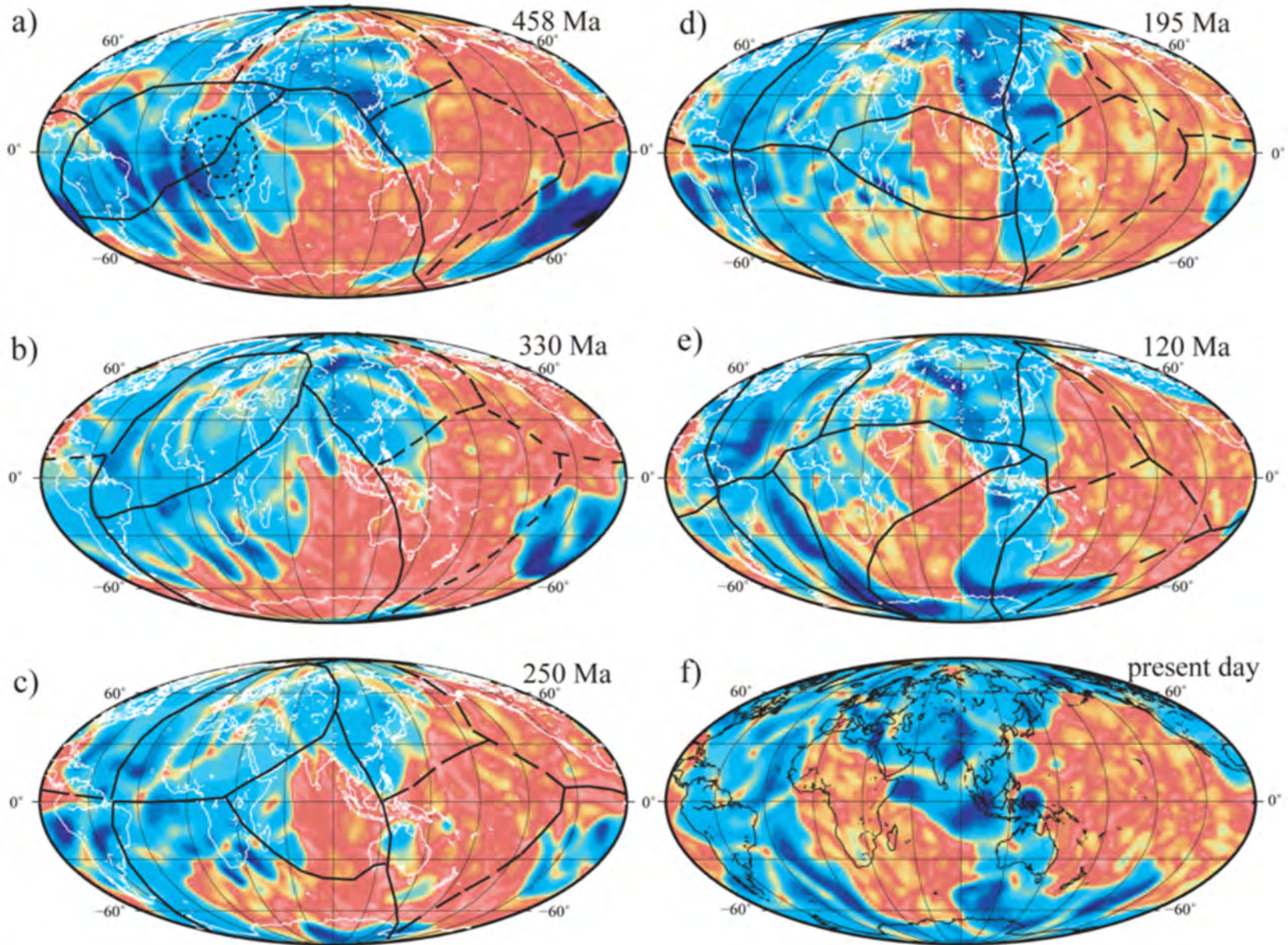


50X lower viscosity



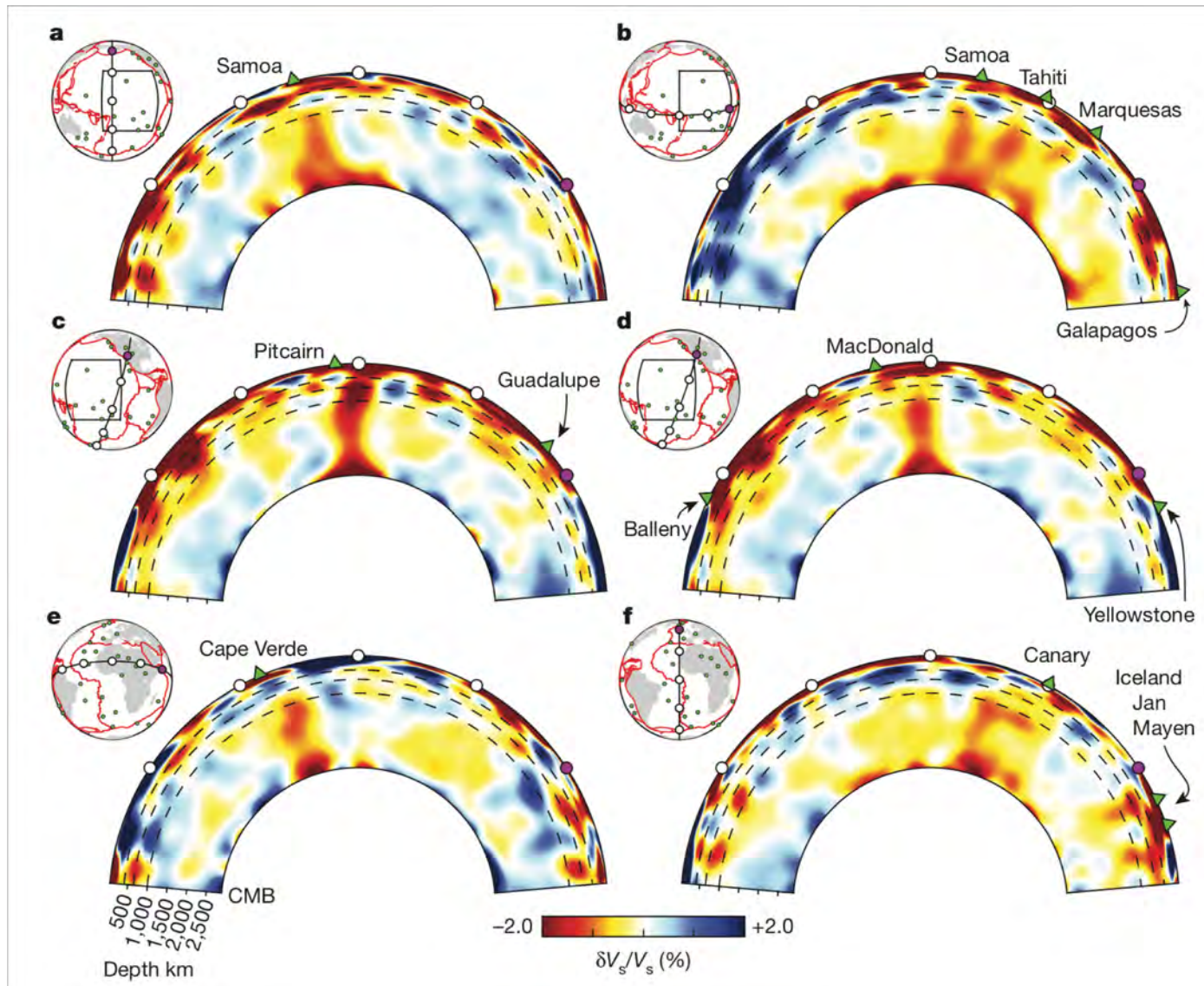
b)

50X higher viscosity



Zhang et al. 2010

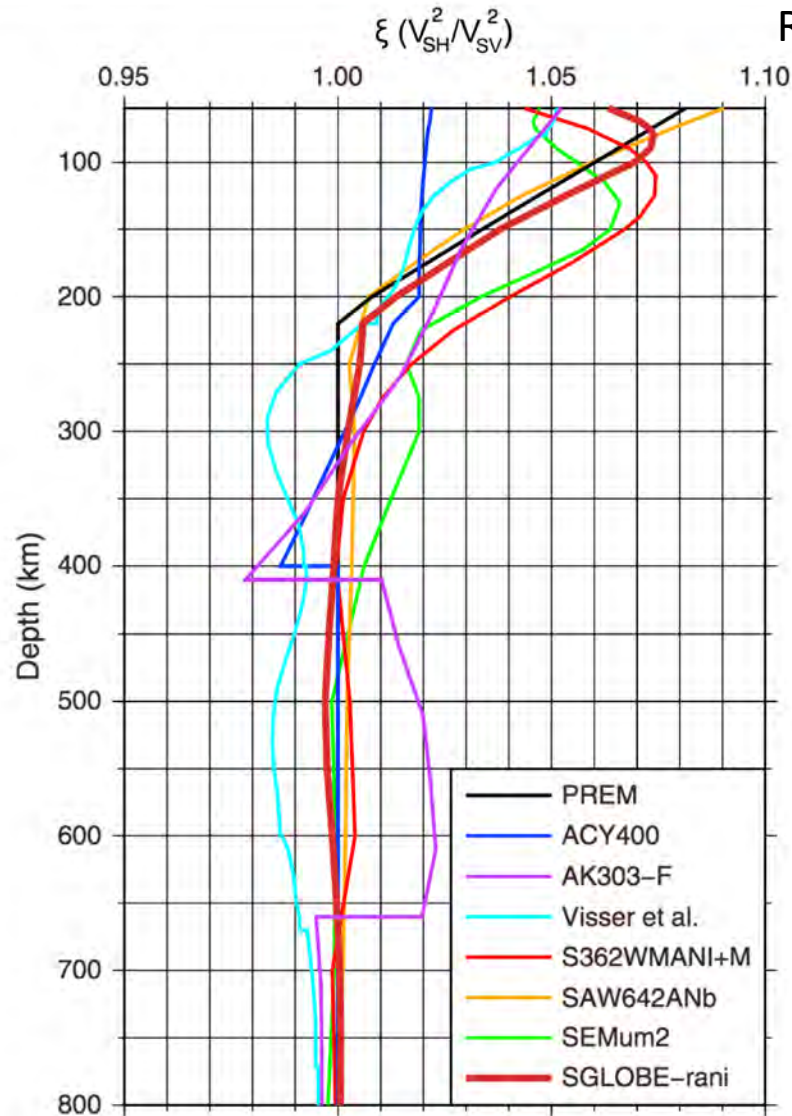
Lower-mantle mysteries



French and Romanowicz 2015

Rheological explanation for broad plumes, ponding below 1000?

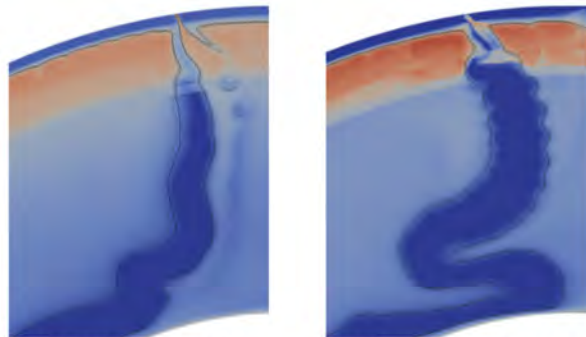
Radial anisotropy from SGLOBE-RANI and other models



Diffusion creep – grain size sensitive but no CPO.

Dislocation creep – not grain size dependent, produces CPO

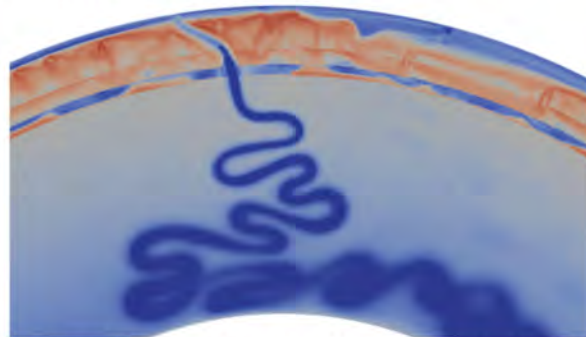
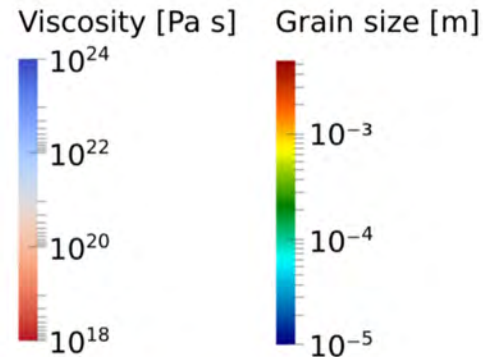
Grain size evolution?



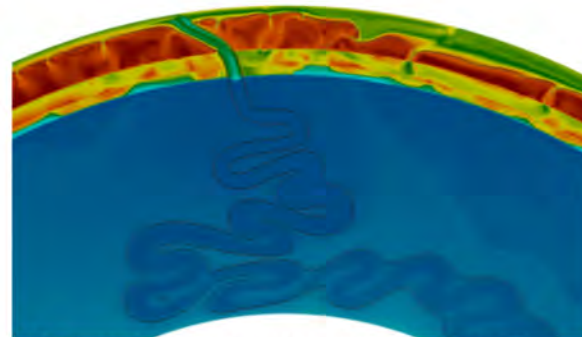
(a) Steinberger & Calderwood, 2006



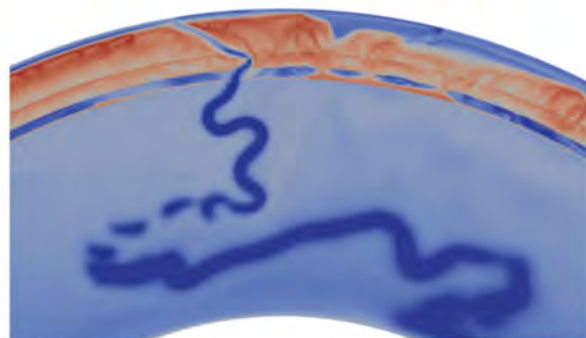
(b) Constant grain size



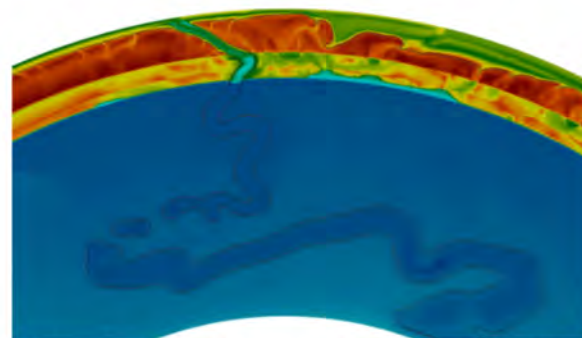
(c) Evolving grain size, LM $V_{\text{diff}} = 2\text{e-}6$



(d) Evolving grain size, LM $V_{\text{diff}} = 2\text{e-}6$



(e) Evolving grain size, LM $V_{\text{diff}} = 1.5\text{e-}6$



(f) Evolving grain size, LM $V_{\text{diff}} = 1.5\text{e-}6$

$$\eta_{\text{diff}} = \frac{1}{2} A_{\text{diff}}^{-1} d^m \exp\left(\frac{E_{\text{diff}}^* + PV_{\text{diff}}^*}{RT}\right),$$

$$\eta_{\text{dis}} = \frac{1}{2} A_{\text{dis}}^{-\frac{1}{n}} \dot{\epsilon}_{\text{dis,II}}^{\frac{1-n}{n}} \exp\left(\frac{E_{\text{dis}}^* + PV_{\text{dis}}^*}{nRT}\right),$$

$$\dot{d}_{\text{growth}} = p_g^{-1} d^{1-p_g} k_g \exp\left(-\frac{E_g + PV_g}{RT}\right),$$

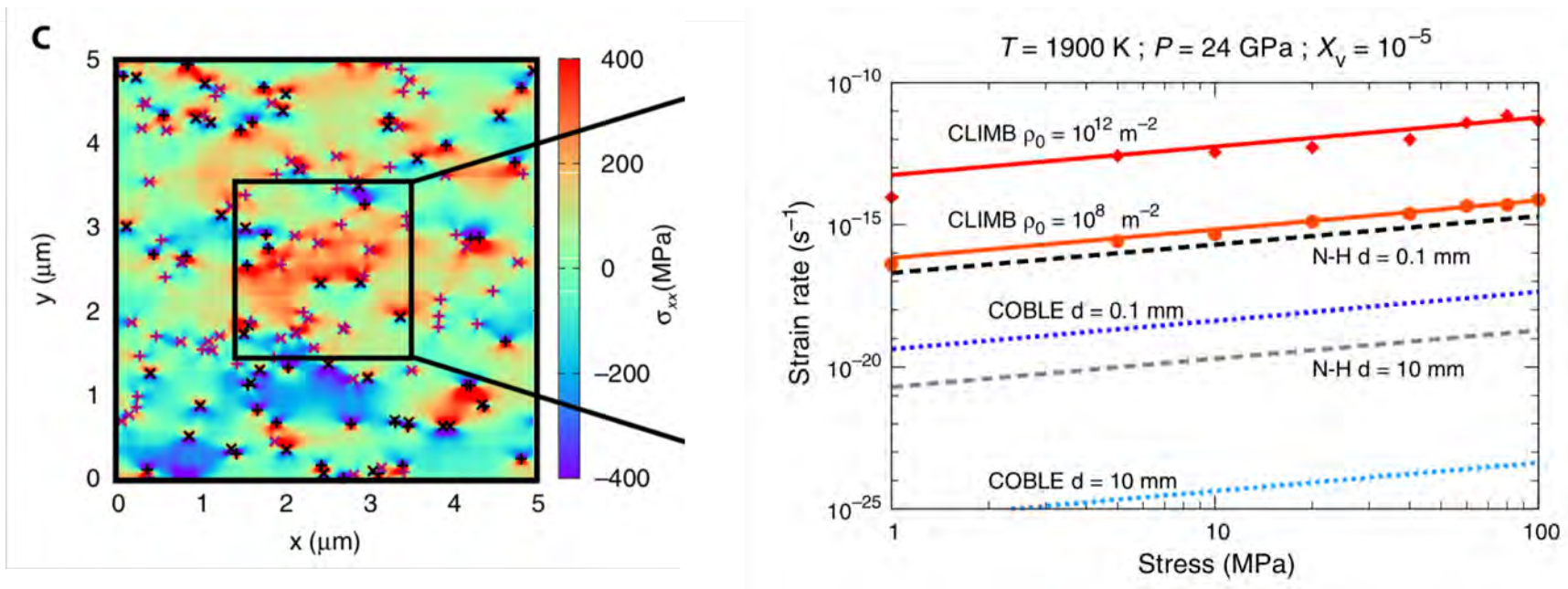
$$\dot{d}_{\text{reduce}} = 4 \dot{\epsilon}_{\text{II}} \dot{\epsilon}_{\text{dis,II}} \eta_{\text{eff}} \frac{\lambda d^2}{c\gamma},$$

Dannberg et al. 2017
CIDER project!

Is diffusion creep the dominant LM mechanism?

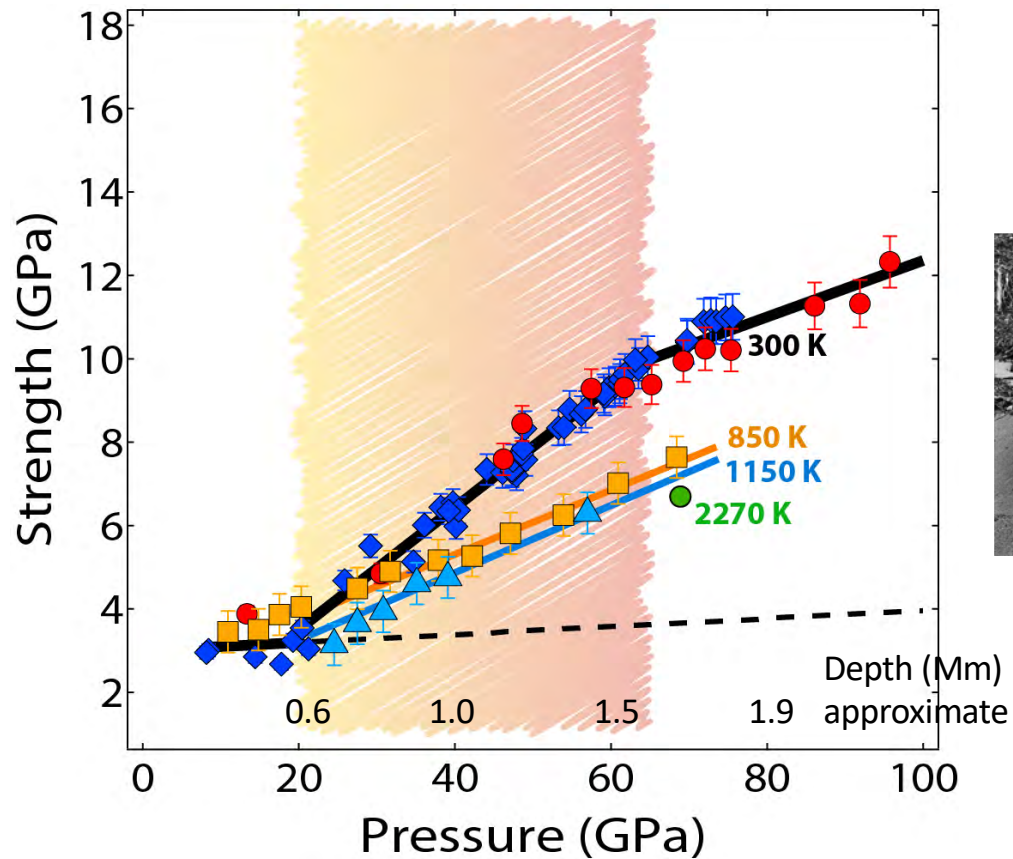
Pure climb creep mechanism drives flow in Earth's lower mantle

Francesca Boioli,^{1*} Philippe Carrez,¹ Patrick Cordier,^{1†} Benoit Devincere,² Karine Gouriet,¹ Pierre Hirel,¹ Antoine Kraych,¹ Sebastian Ritterbex^{1‡}



Newtonian? Probably, but grain size **insensitive** and no **CPO**

Ferropericlasite strength increase?



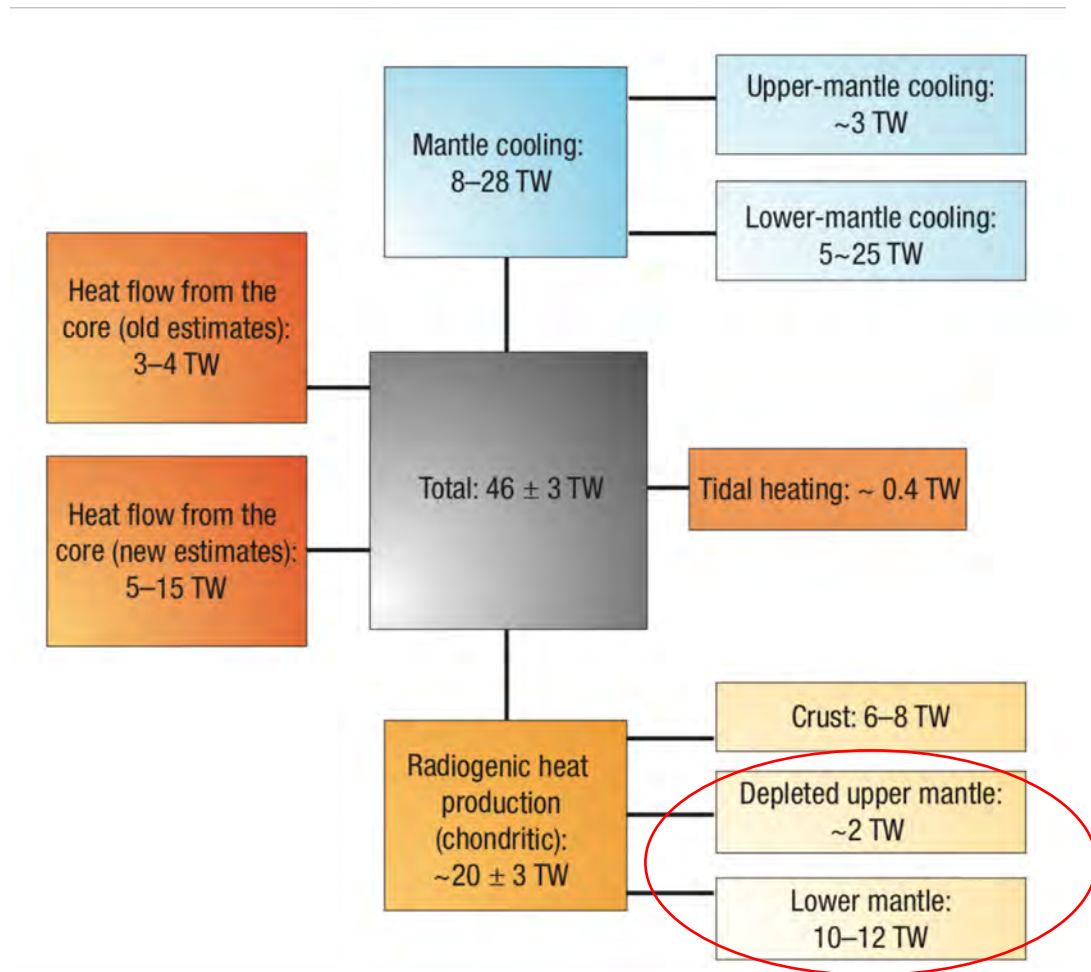
Increase in strength of Ferropericlasite. Minor phase but could control rheology if arranged in sheets.



Effective viscosity could be nonlinear

Marquardt and Miyagi 2015 and unpublished data

Energy budget



Viability of chemical blobs etc. sensitive to this and how it is distributed

Thursday Tutorial Preview



With thermochemical piles:

