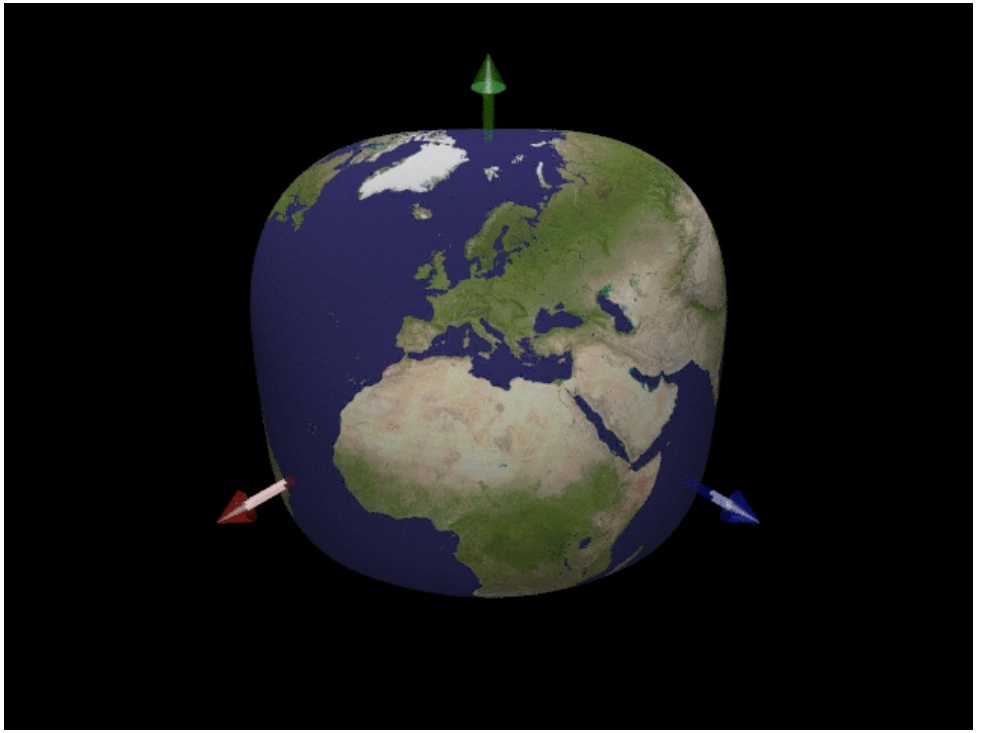
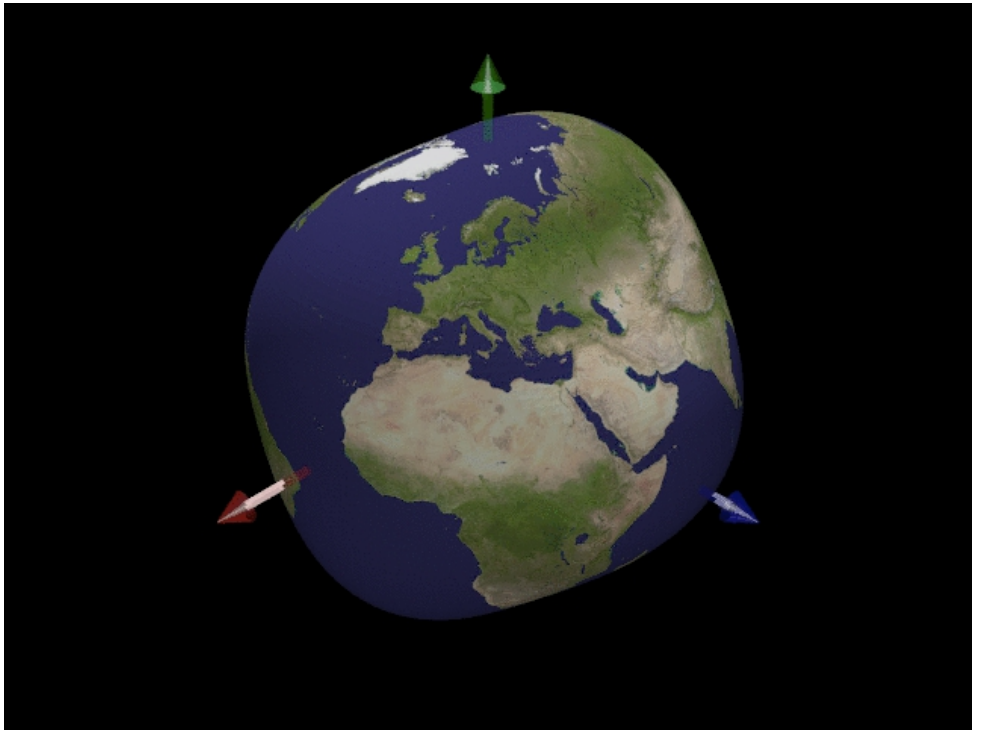
Normal modes





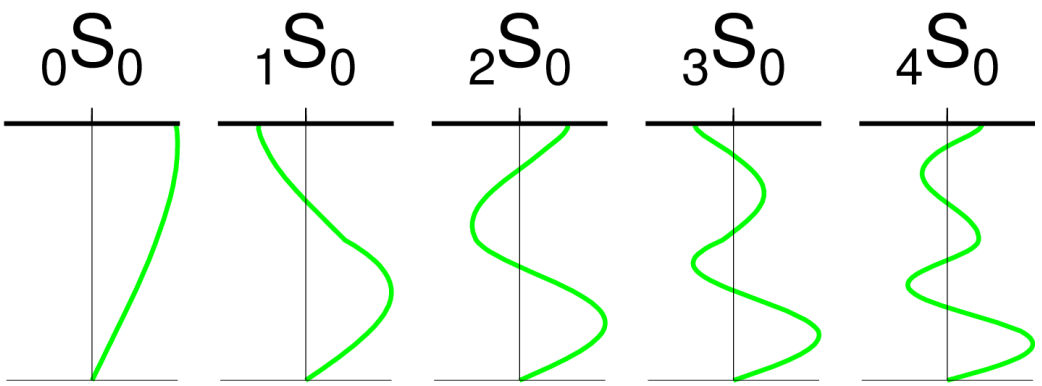
Spheroidal, n=0, ℓ=4

period ≈ 26 minutes

Visualisations from http://icb.u-bourgogne.fr/nano/manapi/saviot/terre/index.en.html

How are normal modes labelled?

Each mode is made up of lateral and radial patterns.



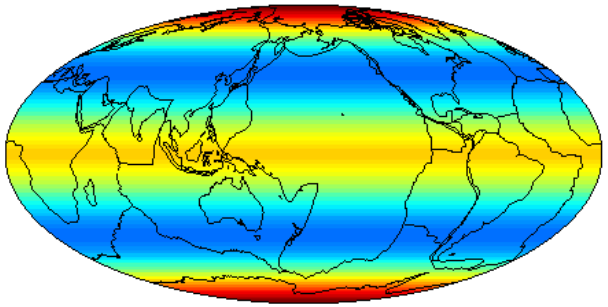
n – overtone number or radial order –
think of this as the number of zeros along the radius of the earth for a mode with l=0

l – angular order –
think of this as the number of zeros across the surface of the sphere the radius of the earth for a mode

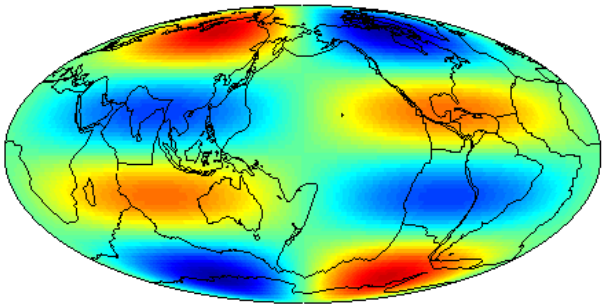
m – azimuthal order –
think of this as telling us about how those lines zeros are ordered on the surface of the sphere

Upper panel shows radial functions for normal modes of a fluid, gravity free sphere.
Lower panels from Stein & Wysession, p 106

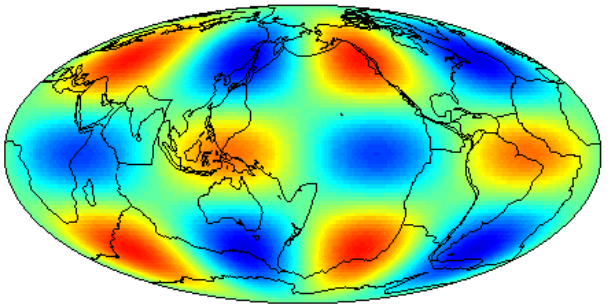
Spherical harmonics



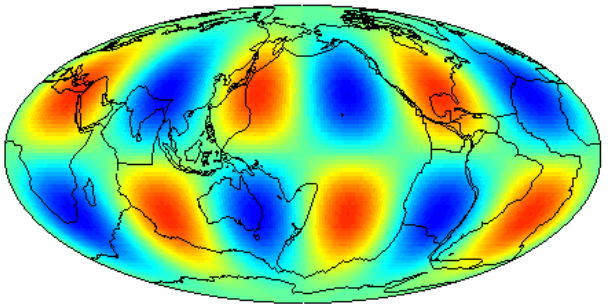
$Y_{4,0}(\theta,\phi)$



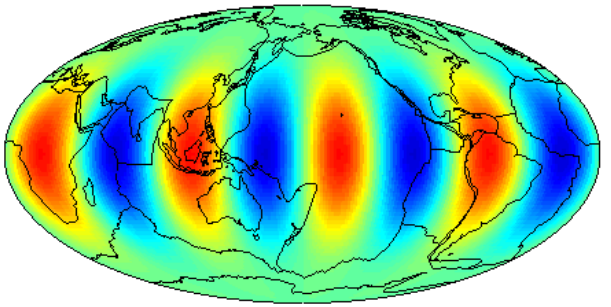
$Y_{4,1}(\theta,\phi)$



$Y_{4,2}(\theta,\phi)$



$Y_{4,3}(\theta,\phi)$



$Y_{4,4}(\theta,\phi)$

Plotted are spherical harmonics for l=4, m=0,1,2,3,4

There is lots of lovely matlab code on Frederik Simons' webpage (frederik.net) for spherical harmonics and other geophysical applications, some of which was used to make these figures.

REALISTIC NORMAL MODES

We will use three different vector spherical harmonics:

$$\boldsymbol{R}_l^m(\theta,\phi)=Y_l^m\,\hat{\boldsymbol{r}} \qquad \boldsymbol{S}_l^m(\theta,\phi)=\frac{1}{\sqrt{l(l+1)}}\left(\frac{\partial Y_l^m}{\partial\theta}\hat{\boldsymbol{\theta}}+\frac{1}{\sin\theta}\frac{\partial Y_l^m}{\partial\phi}\hat{\boldsymbol{\phi}}\right)$$

$$\boldsymbol{T}_l^m(\theta,\phi)=\frac{1}{\sqrt{l(l+1)}}\left(\frac{1}{\sin\theta}\frac{\partial Y_l^m}{\partial\phi}\hat{\boldsymbol{\theta}}-\frac{\partial Y_l^m}{\partial\theta}\hat{\boldsymbol{\phi}}\right)$$

Let us write the displacement of a normal mode with a particular n, l and m as:

$$\boldsymbol{u}(r,\theta,\phi,t)=\left[{}_nU_l(r)\,\boldsymbol{R}_l^m(\theta,\phi)+{}_nV_l(r)\,\boldsymbol{S}_l^m(\theta,\phi)+{}_nW_l(r)\,\boldsymbol{T}_l^m(\theta,\phi)\right]\exp(-i\,{}_n\omega_l t)$$

We could then develop and solve equations for U, V, W and ω so we could say how each normal mode will behave, and how the earth would move under each different oscillation.

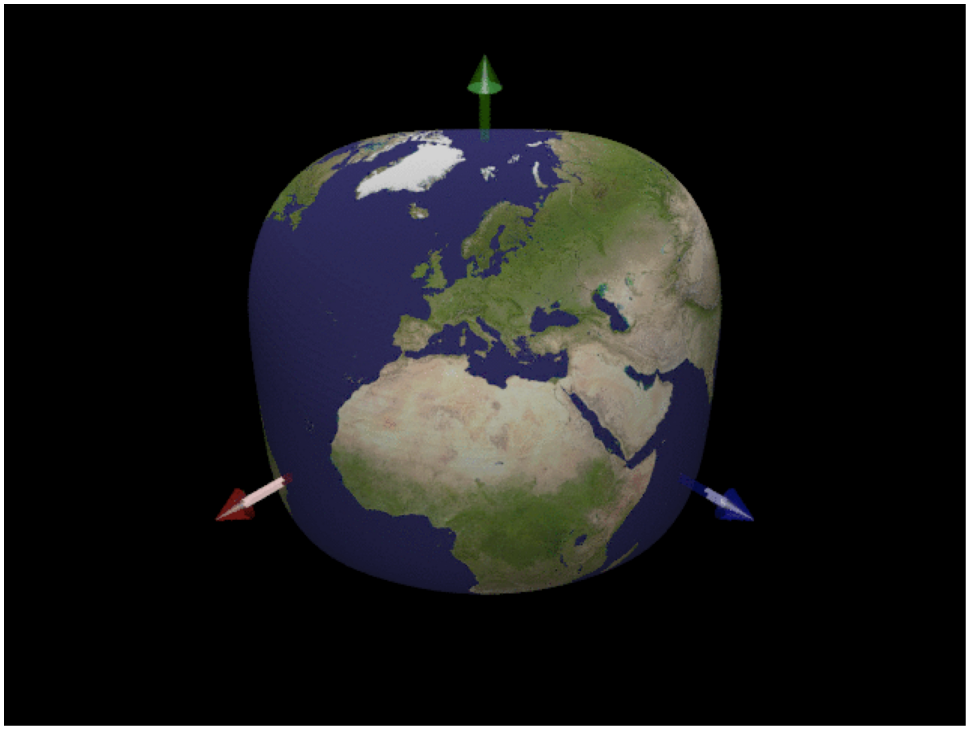
Steve Grand showed us how we could consider displacement in the Earth as either P wave motion or S-wave motion. We also heard that P and SV energy can be converted at interfaces, but that SH energy is not. The same type of behaviour is true of normal modes. Just as we did with body waves, and surface waves, we can break the motions of the earth into two types – toroidal and spheroidal. The P-SV motions are the counterpart of the spheroidal (sometimes poloidal) normal modes, and the SH motions are the toroidal modes (which were forbidden in the liquid planet we just considered).

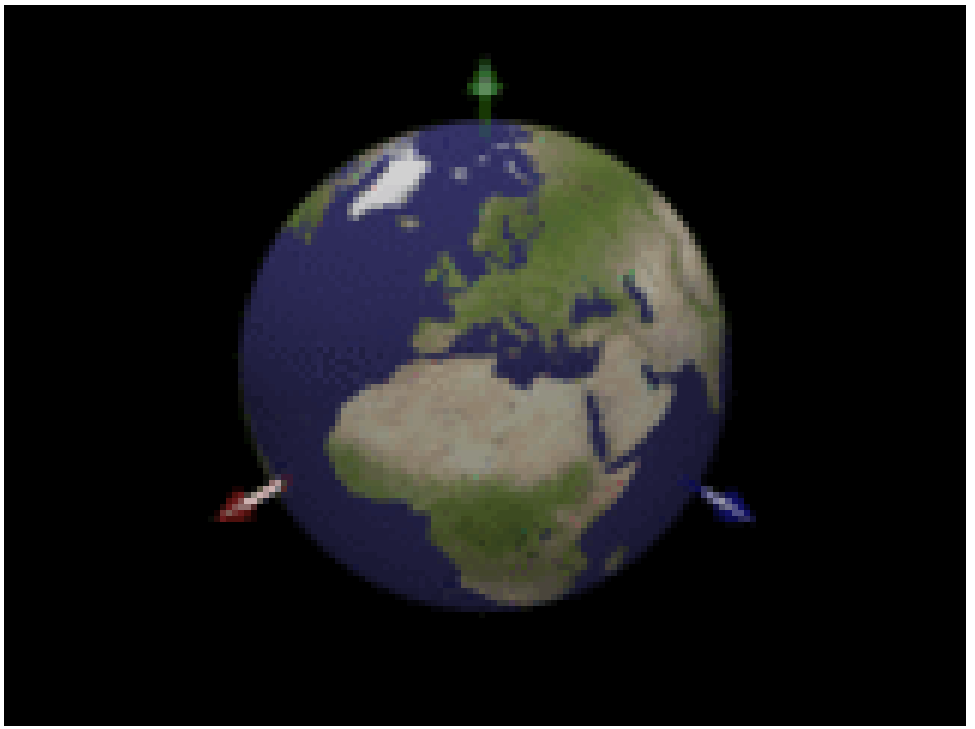
Two types of normal modes

Doing some rearranging, we would find we have split the modes into two parts – the one which contains motion described by the $\boldsymbol{R}_l^m$ and $\boldsymbol{S}_l^m$ vector fields, and one whose motion is described by the $\boldsymbol{T}_l^m$ vector field.

The first of these, the spheroidal modes, have the radial component of $\nabla \times \boldsymbol{u}$ equal to zero. (Radial modes are a special case of spheroidal modes, they have $u_\theta, u_\phi = 0$).

The second of these, the toroidal modes have both $\nabla \cdot \boldsymbol{u} = 0$ and $u_r = 0$

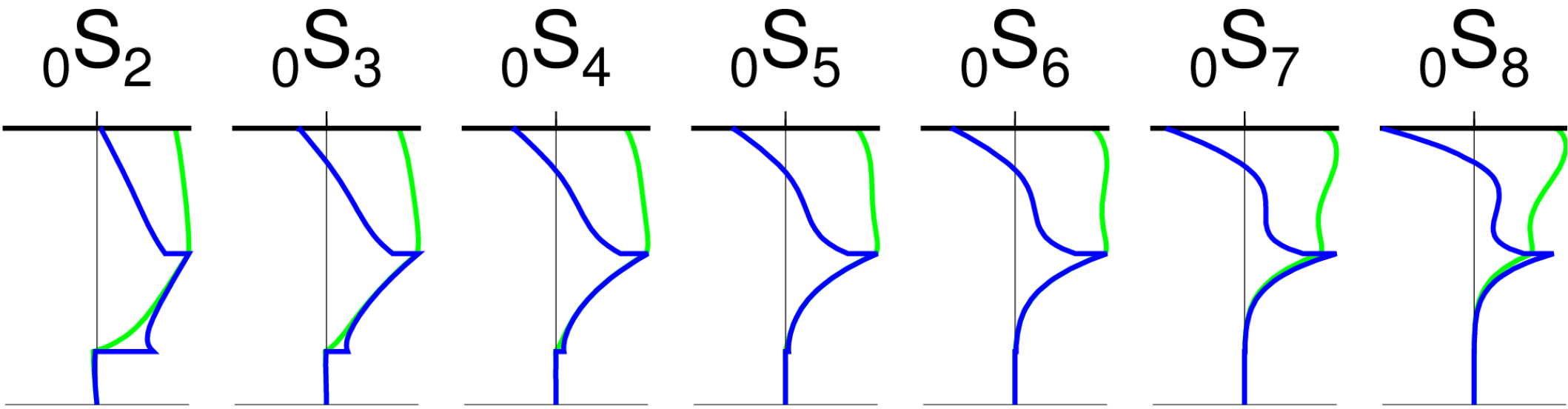




Eigenfunctions of normal modes

These equations can also be written in matrix form and solved for the ${}_n\omega_l$ and the displacements which are given by U(r) and V(r).

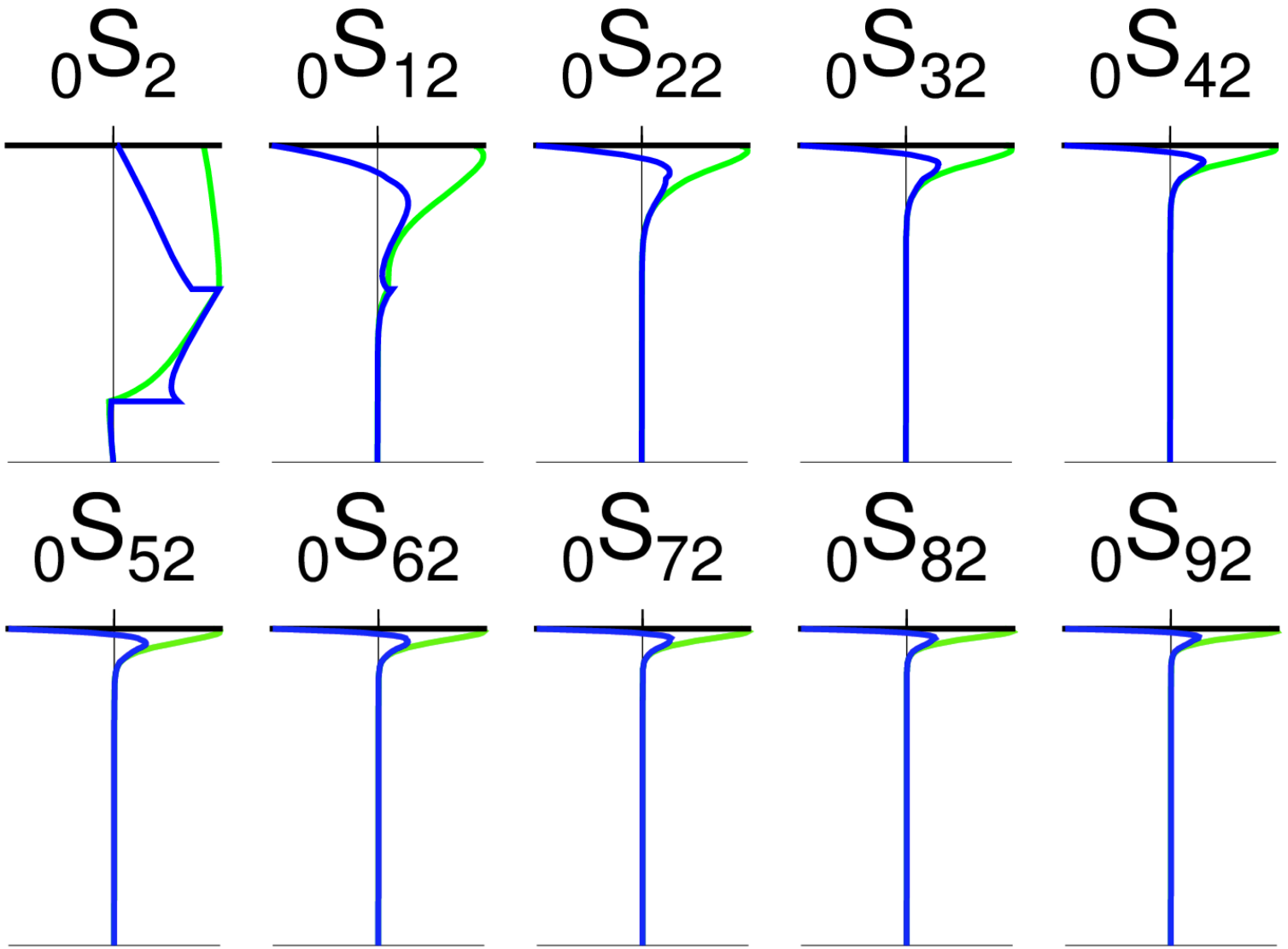
Gravity does not affect the toroidal modes, so the matrix equation for W(r) and T(r) is simpler.



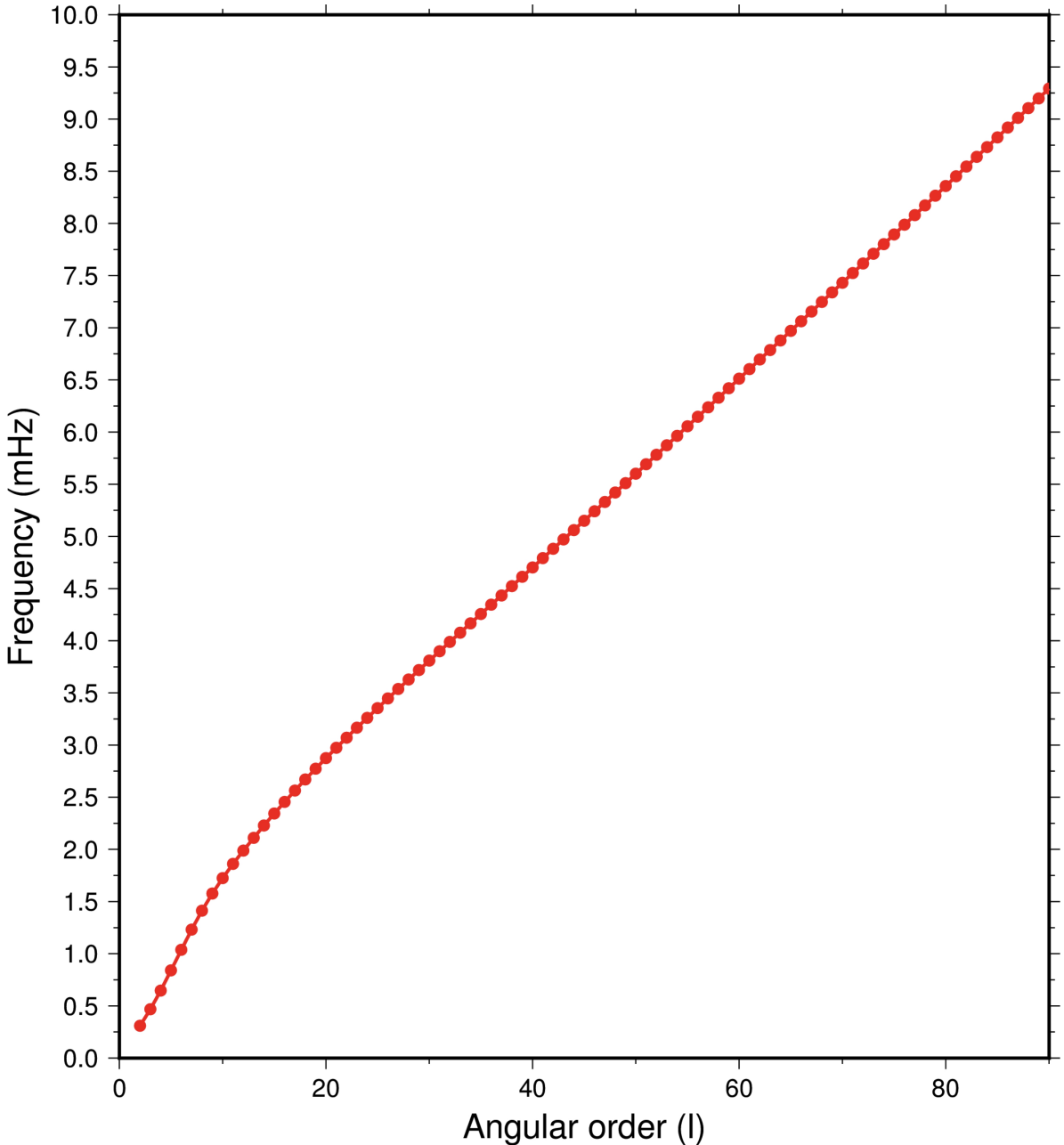
remember displacement looks like $\left[{}_nU_l(r)\,\boldsymbol{R}_l^m(\theta,\phi)+{}_nV_l(r)\,\boldsymbol{S}_l^m(\theta,\phi)+{}_nW_l(r)\,\boldsymbol{T}_l^m(\theta,\phi)\right]\exp(-i\,{}_n\omega_l t)$

Normal mode branches

These equations can also be written in matrix form and solved for the ${}_n\omega_l$ and the displacements which are given by U(r) and V(r).

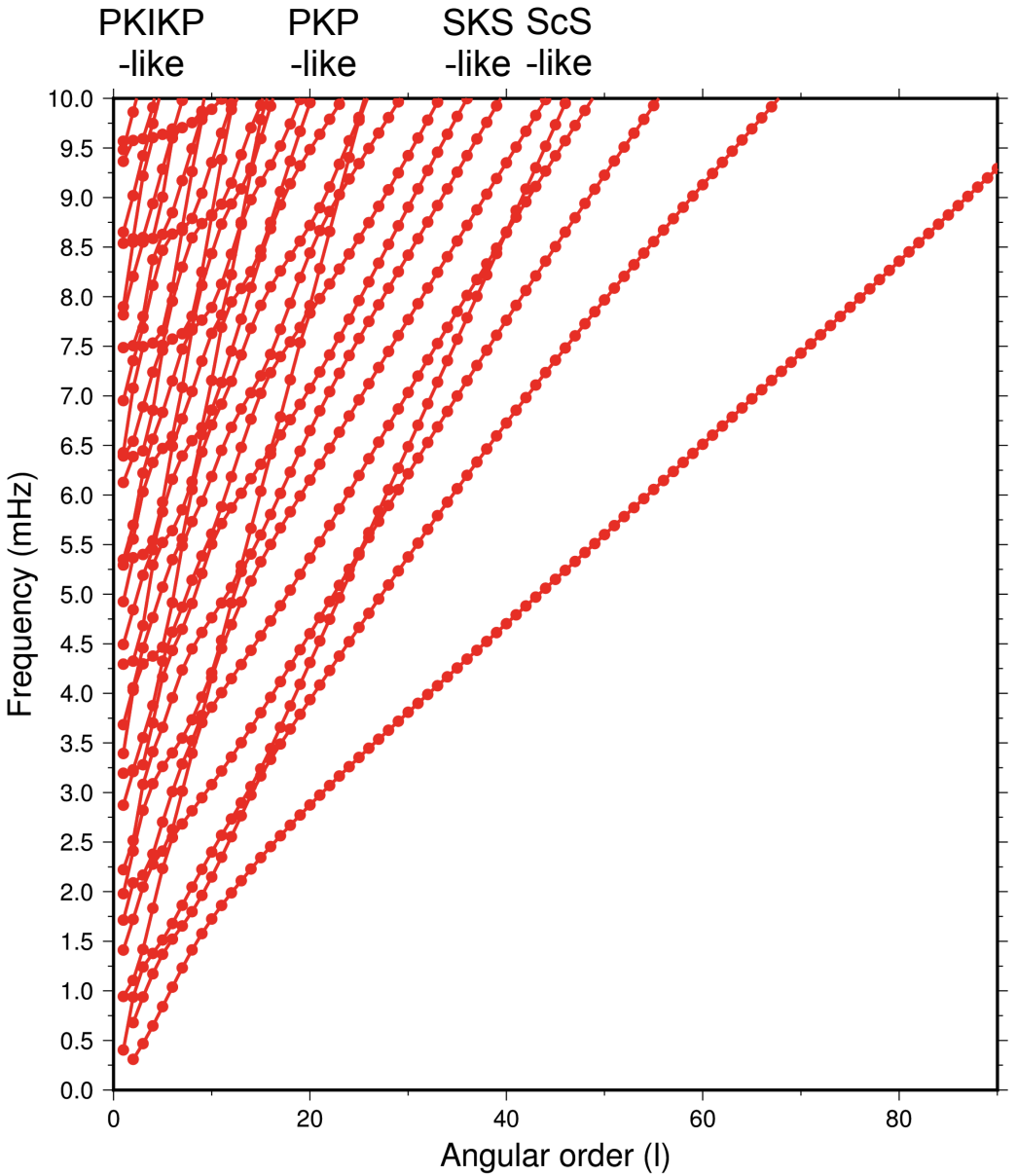


Normal mode branches



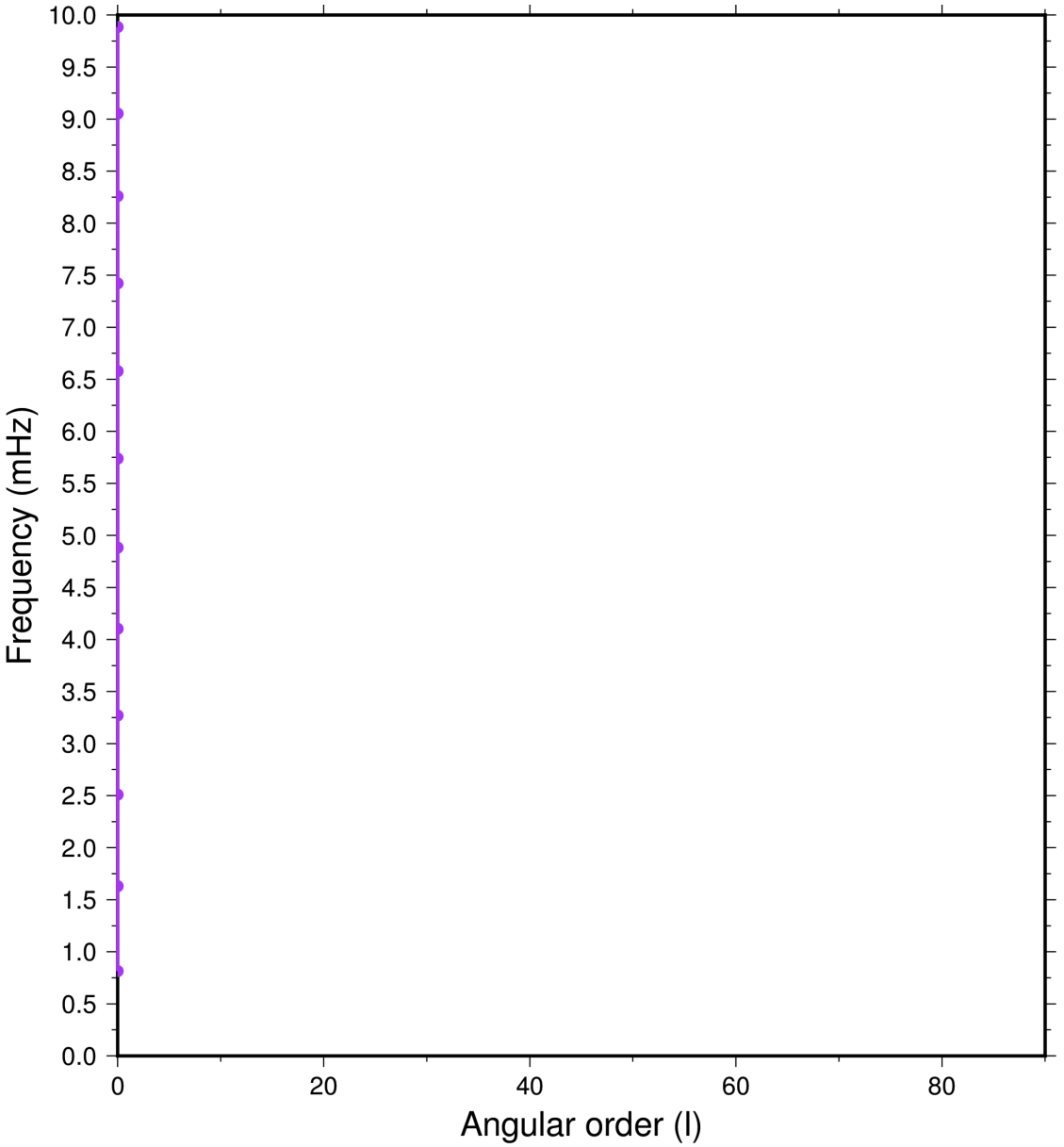
n=0;
l increasing

Normal mode branches



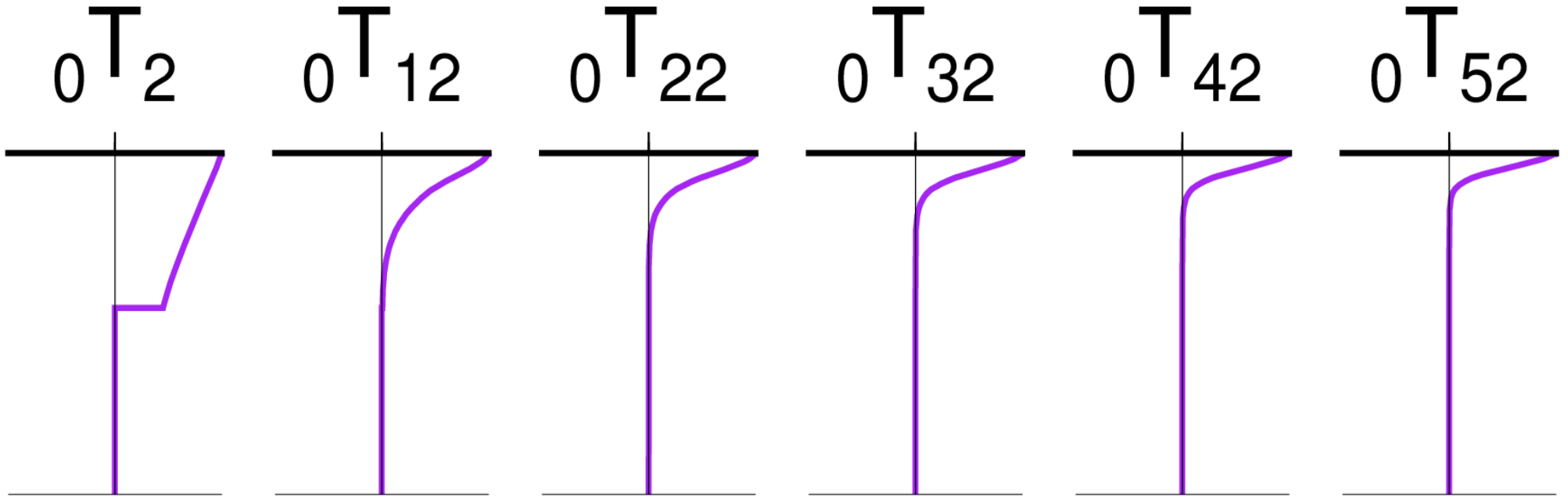
Labels after Lay & Wallace (1995)

Radial normal modes



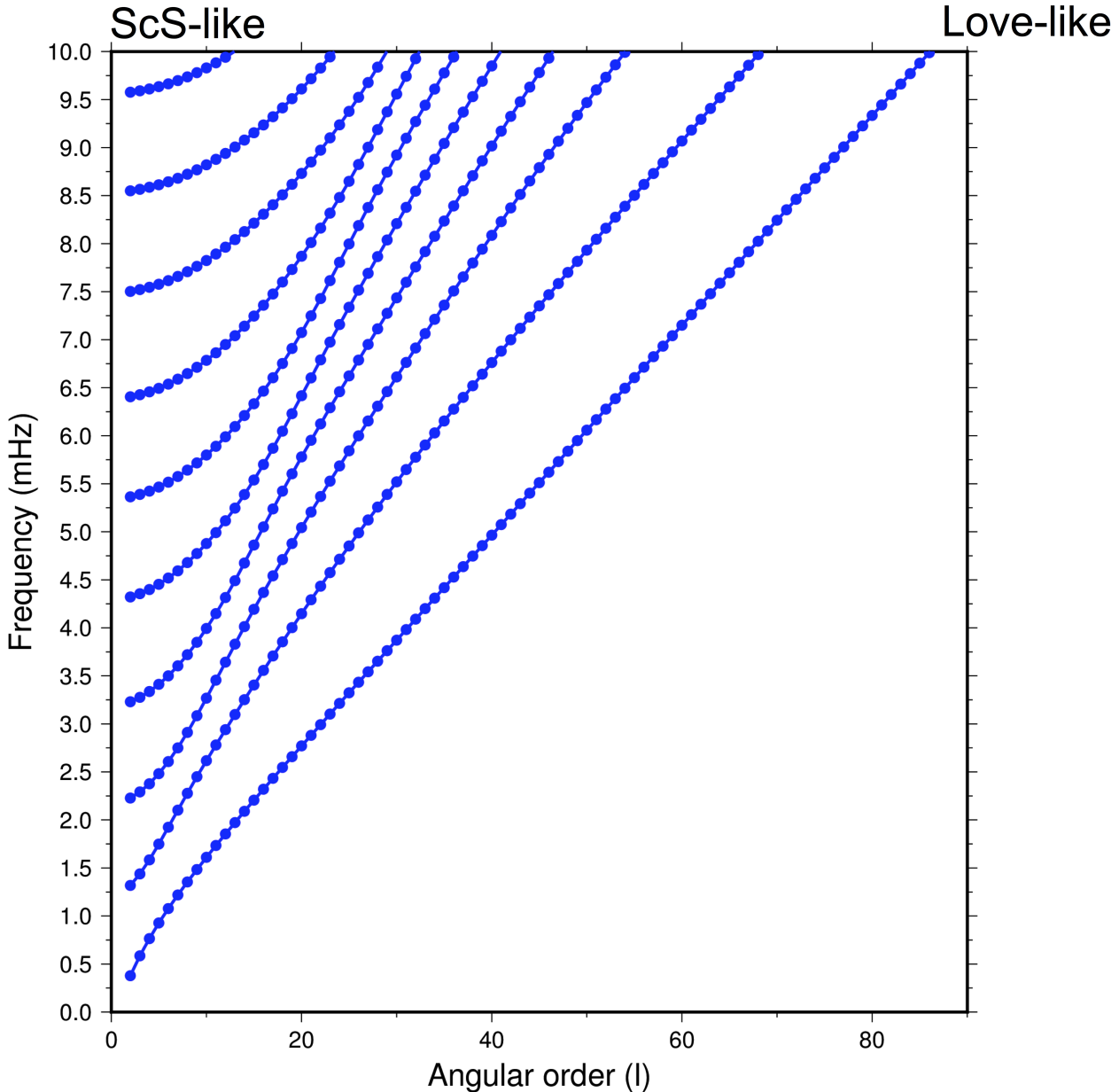
l=0;
n increasing

Normal mode branches



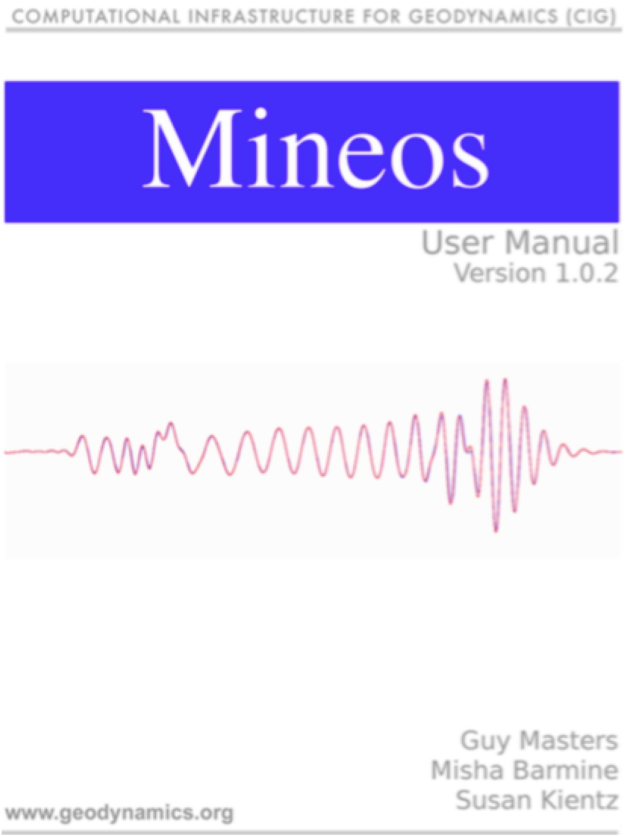
Normal mode branches

For toroidal modes:



Normal mode Calculations

We will be using Mineos.



But keep an eye out for FrOsT:

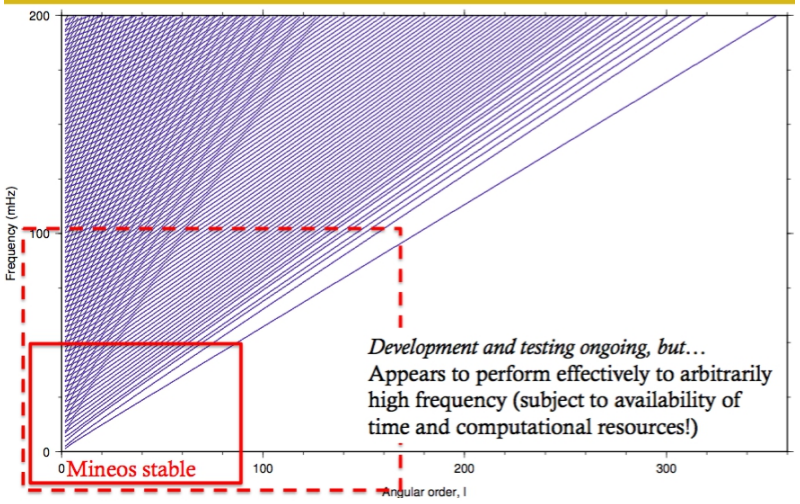
FrOsT: Enabling the next generation of normal-mode seismology

Andrew Valentine, David Al-Attar, Jeannot Trampert & John Woodhouse

a.p.valentine@uu.nl ◆ www.geo.uu.nl/~andrew

Universiteit Utrecht · UNIVERSITY OF CAMBRIDGE · UNIVERSITY OF OXFORD

Example: Inner core spheroidal modes, PREM



Normal modes!

A quick reminder of the terminology we've got so far:

n = radial order

l = angular order

m = azimuthal order

Labeling modes:

$_nS_l$

Spheroidal mode

$_nT_l$

Toroidal mode