Geodynamics 5: Data-oriented geodynamic modeling

-- Forward & backward in time modeling

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Thanks to many colleagues and students who contributed the development of this research.

Outline

- **Why do and what is data-oriented modeling?**
- **Different approaches of data assimilation and their evolution**
- **Examples of forward and inverse data-assimilation models**

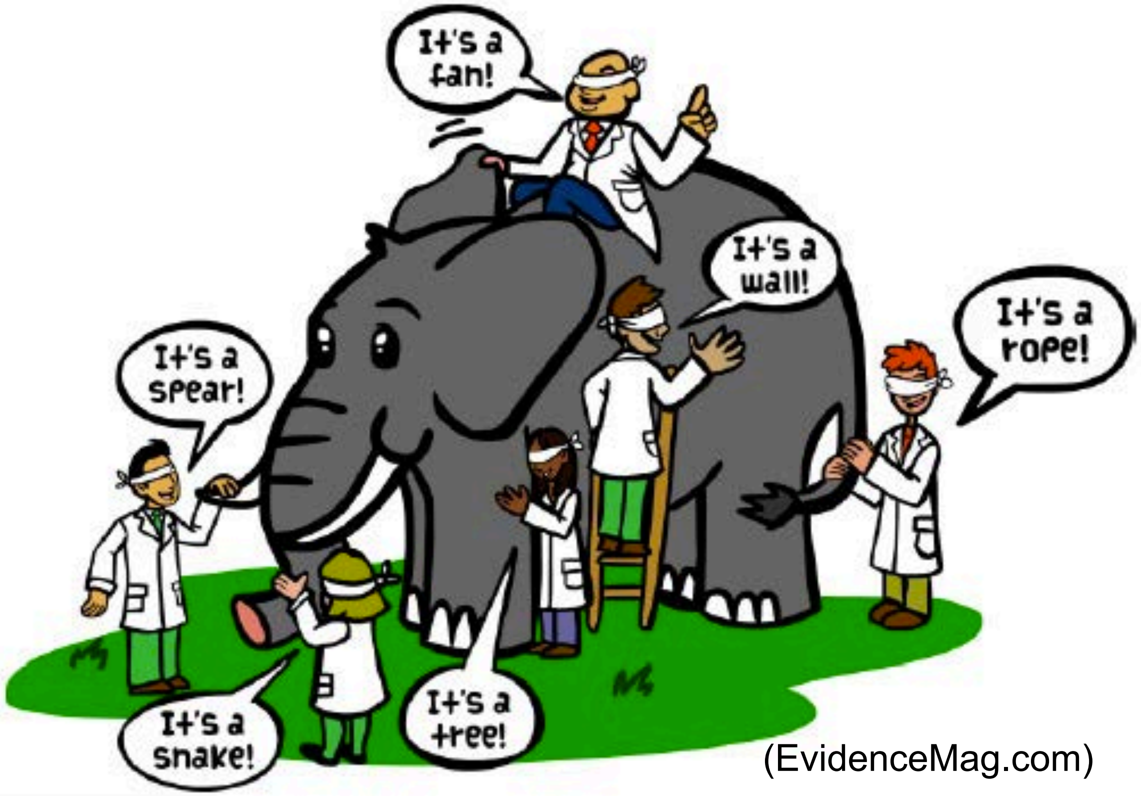
Outline

- Why do modeling and what is data-oriented geodynamic modeling?
- Different approaches of data assimilation and their evolution
- Examples of forward and inverse data-assimilation models

“All models are wrong but some are useful”

Q: why bother doing modeling then?

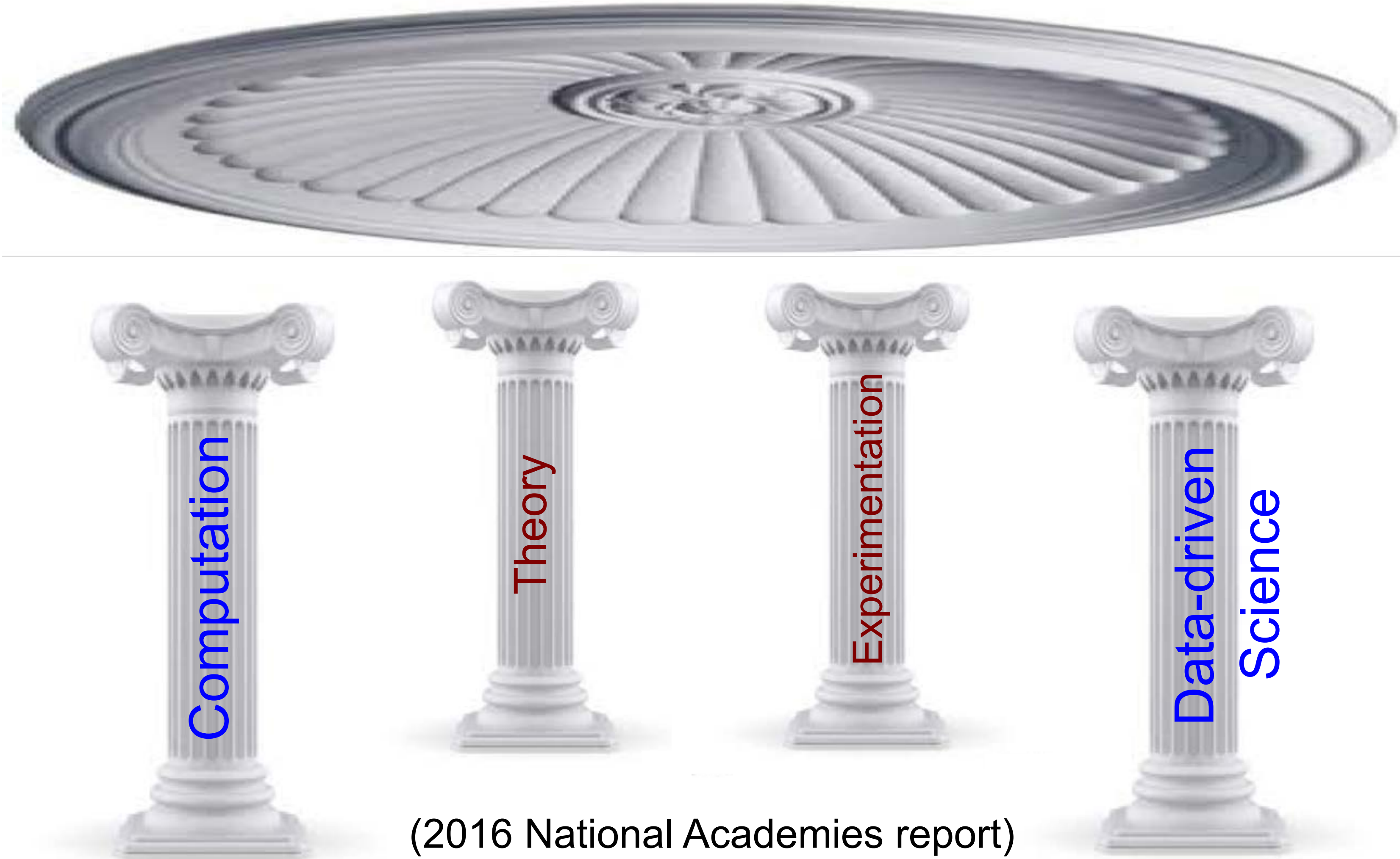
A: Because all models are useful, more or less



(EvidenceMag.com)

Theory & Model vs. Truth

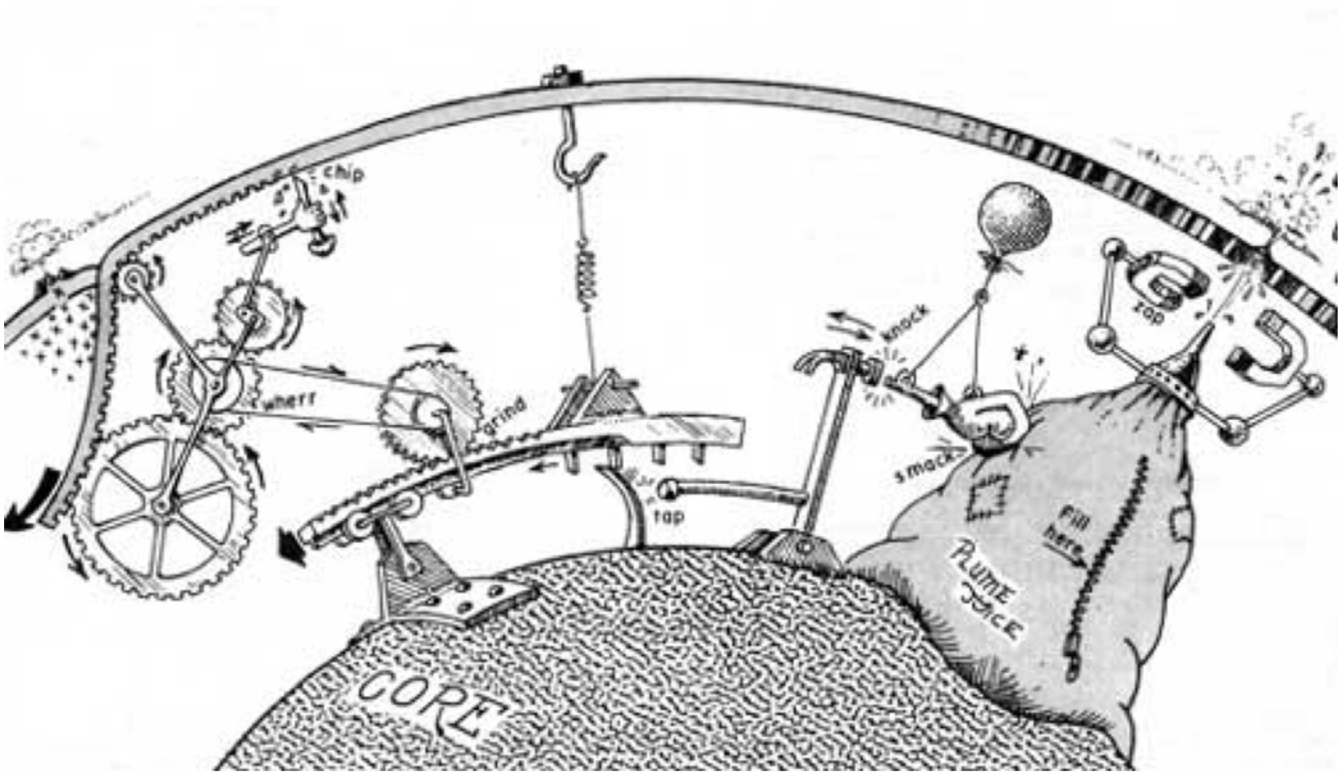
Pillars of scientific discovery



(2016 National Academies report)

What is a data-driven Earth model?

- Broadly speaking, all Earth models are based on data.

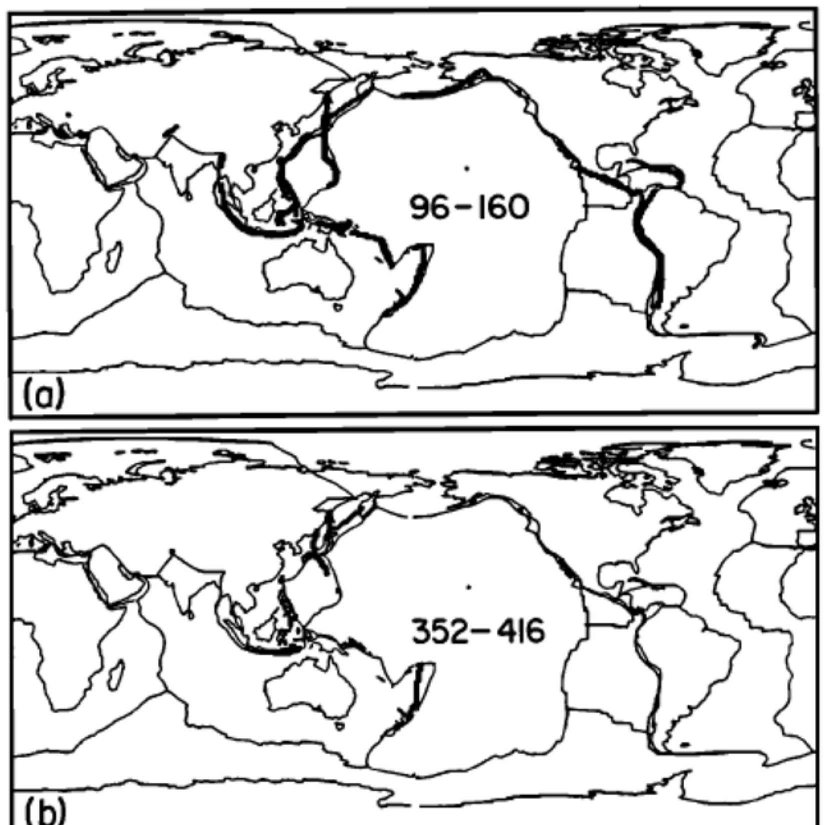


(Holden & Vogt, 1977)

What is a data-driven Earth model?

- Here, we define data-driven models as those assimilating **geologically** and **geographically** inferred observational constraints.

Instantaneous model



(Hager, *JGR,*1984)

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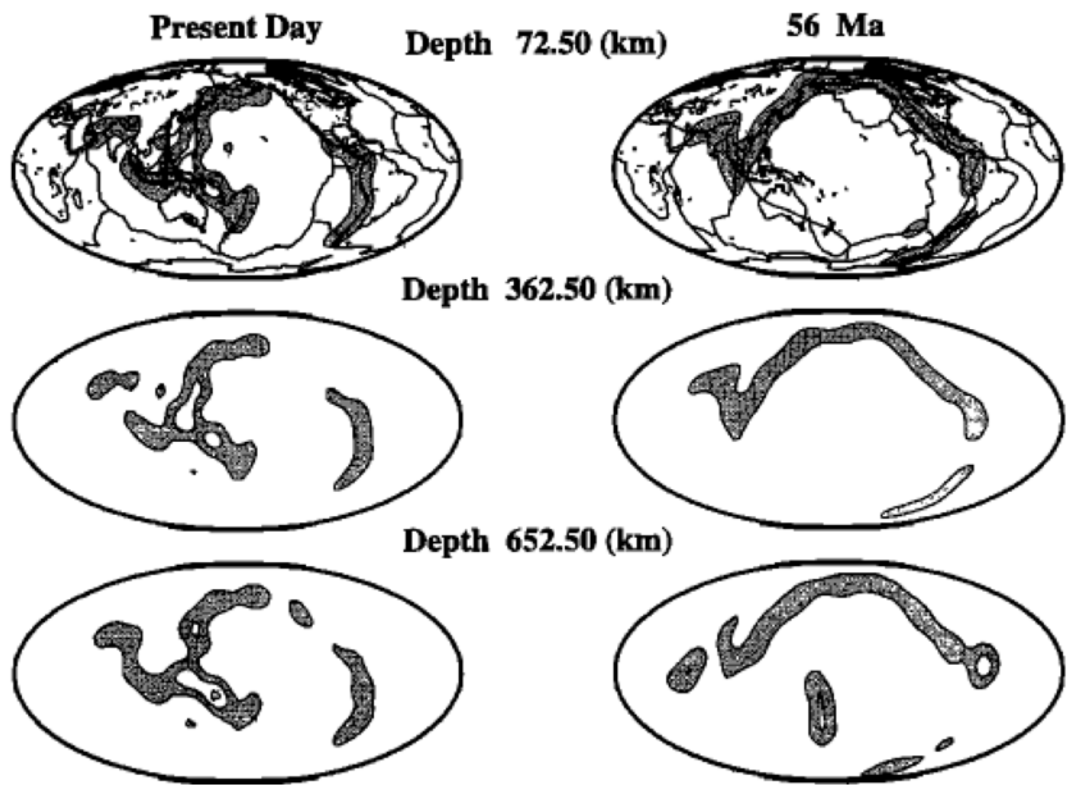
Evolutionary forward model



(Lithgow-Bertelloni & Richards, *Rev. Geophy.*, 1998)

What is a data-driven Earth model?

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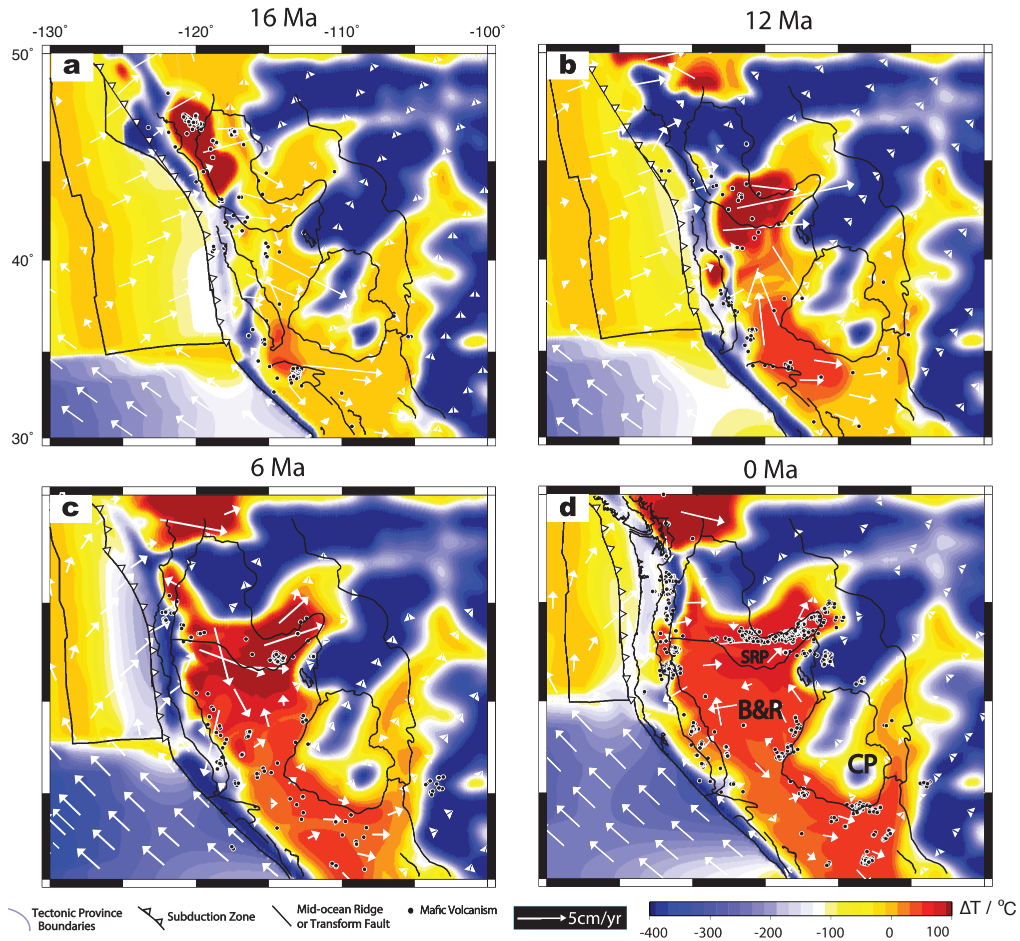
Evolutionary backward model

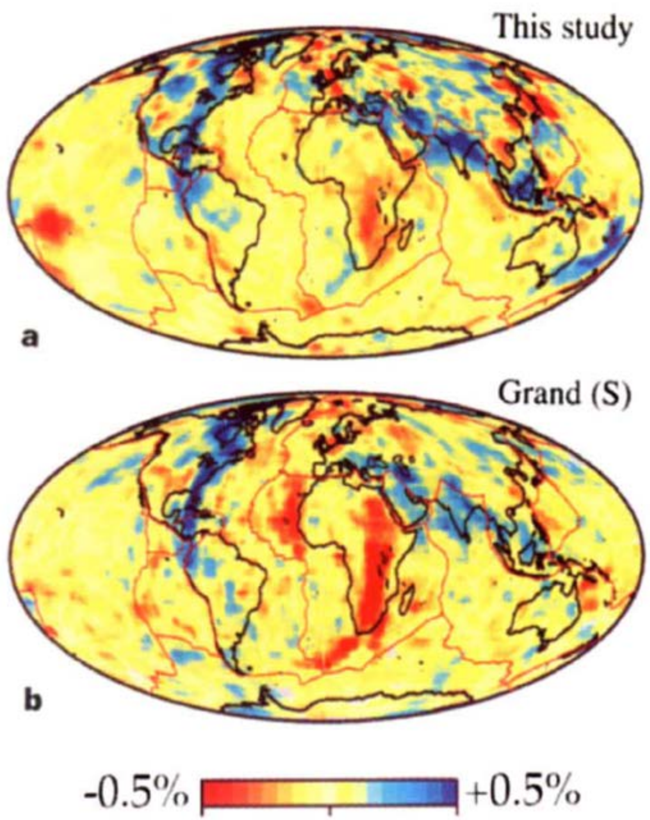


(Zhou et al., *Nature Geosci.*, 2018)

Data constraints for present Earth internal structure

Seismic Tomography:

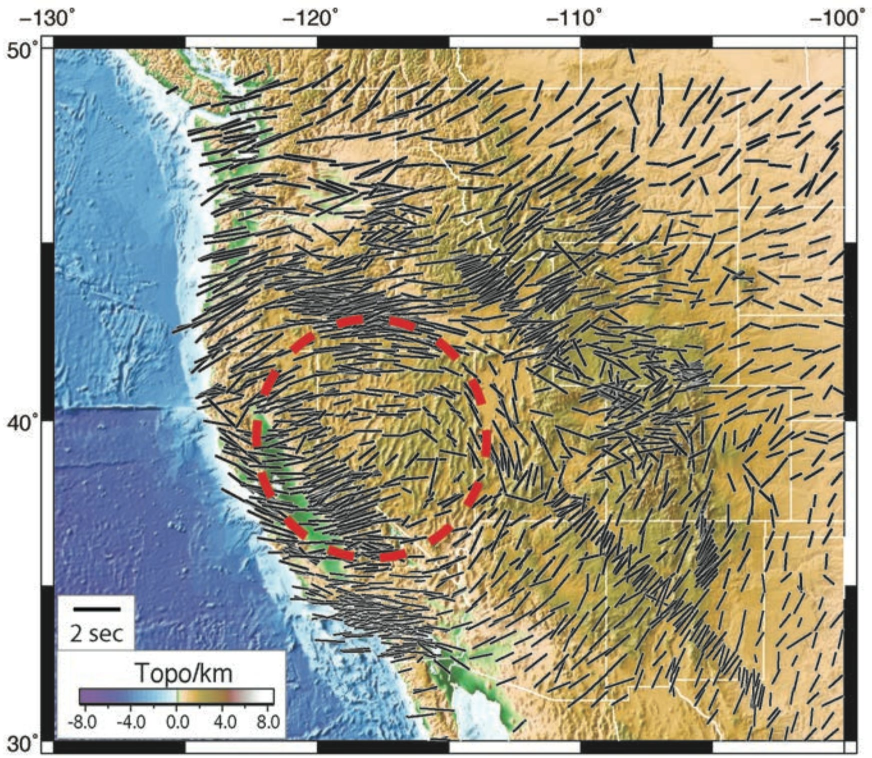
A present-day snapshot of the convecting mantle



(van der Hilst, *Nature*, 1997)

Seismic Anisotropy:

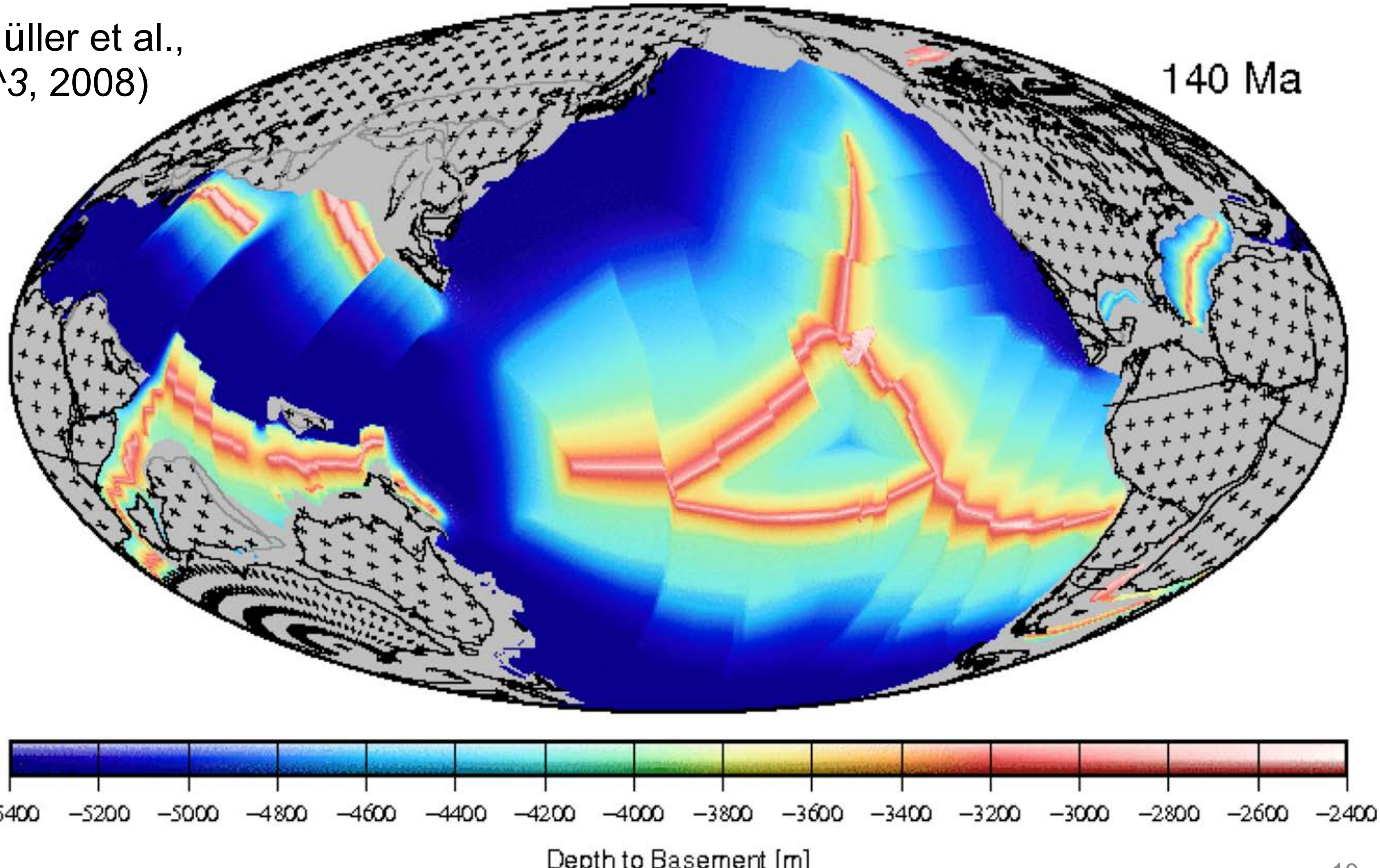
A cumulative effect of recent mantle deformation



(Zhou et al., *EPSL*, 2018)

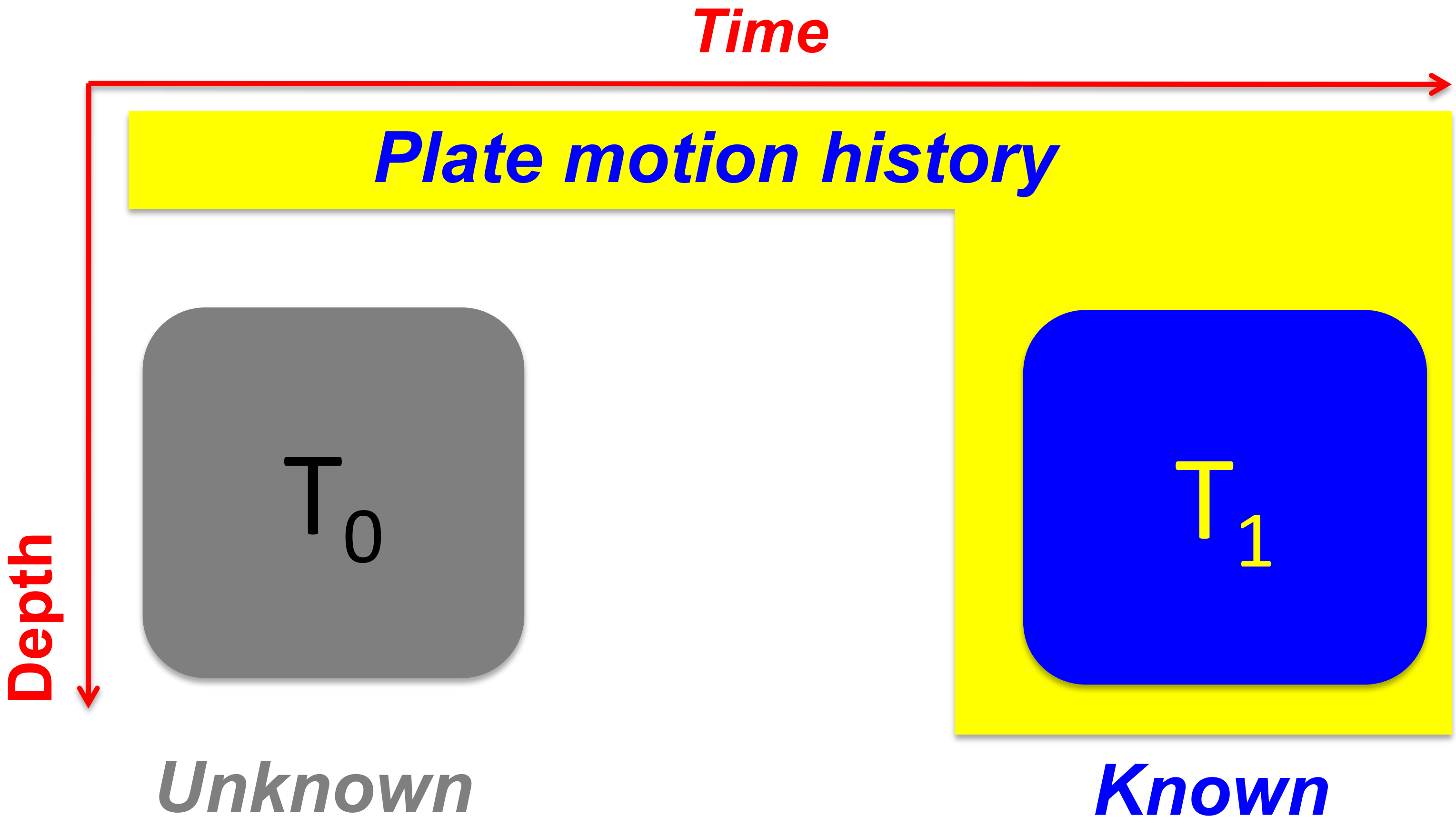
Data constraints for past Earth surface motion

(Müller et al., *G^3*, 2008)



Equation of Earth temporal evolution

--- knowns vs. unknowns



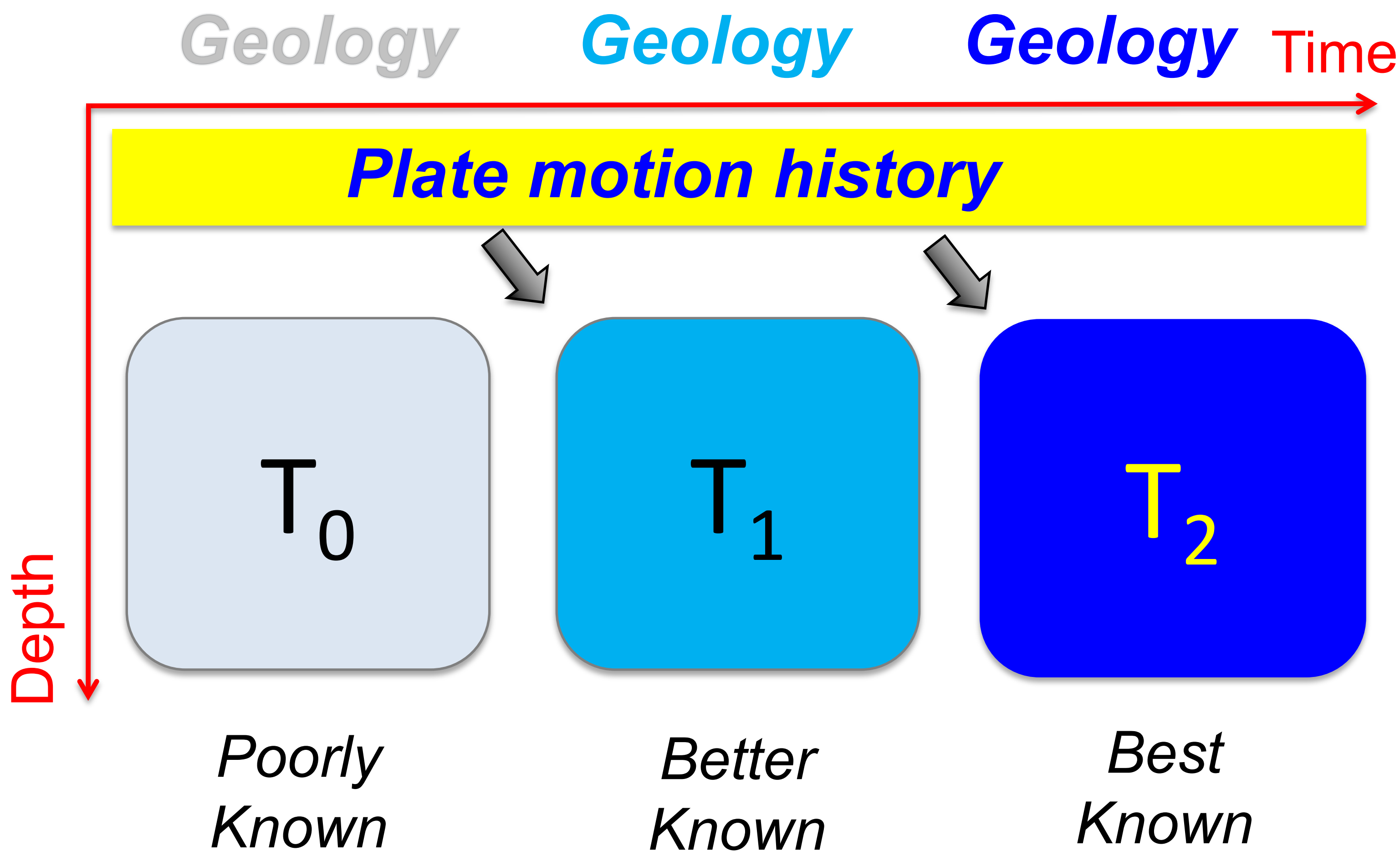
Outline

- Why do and what is data-oriented modeling?
- **Different approaches of data assimilation and their evolution**
- Examples of forward and inverse data-assimilation models

Common approaches of data assimilation

- Forward in time integration
  - **Sequential approach:** Assimilate data and train model behavior toward the present
    - Best take advantage of plate reconstruction
    - Suffer from uncertain initial condition
- Backward in time integration
  - **Variational approach:** Derive past mantle states from the present-day state
    - Best take advantage of present-day mantle information
    - Suffer from limited resolution of seismic tomography

Forward simulation of Earth evolution



Progress in forward model development

- Instantaneous geodynamic models, for studying present-day dynamics:
  - Gravity
  - Plate motion
- Time-dependent subduction models:
  - Subduction & mantle structure
  - Plate motion
  - Dynamic topography

Progress in forward model development

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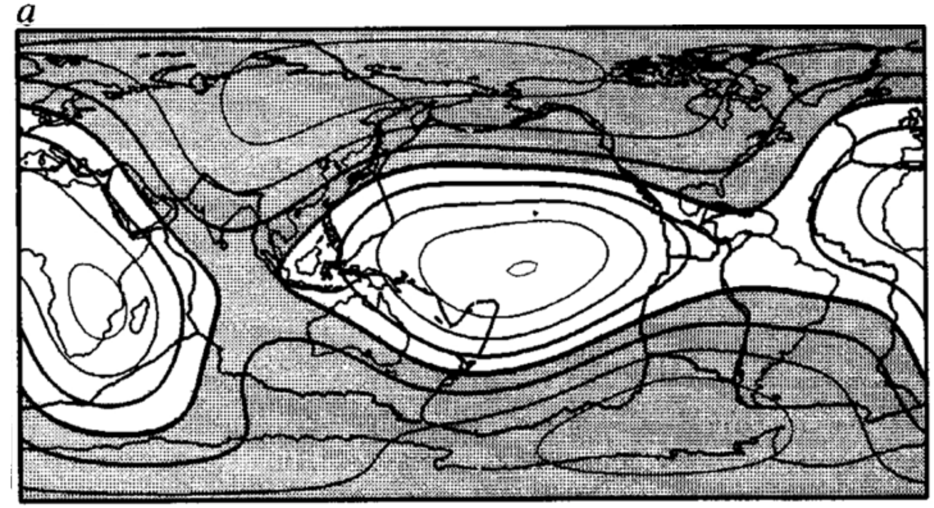
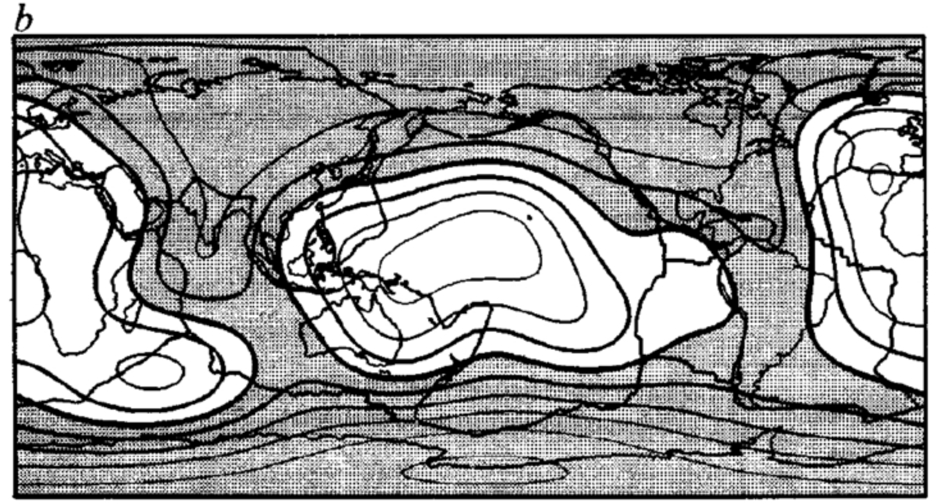
Instantaneous models

- Geoid study

Predicted geoid (degree 2-6)

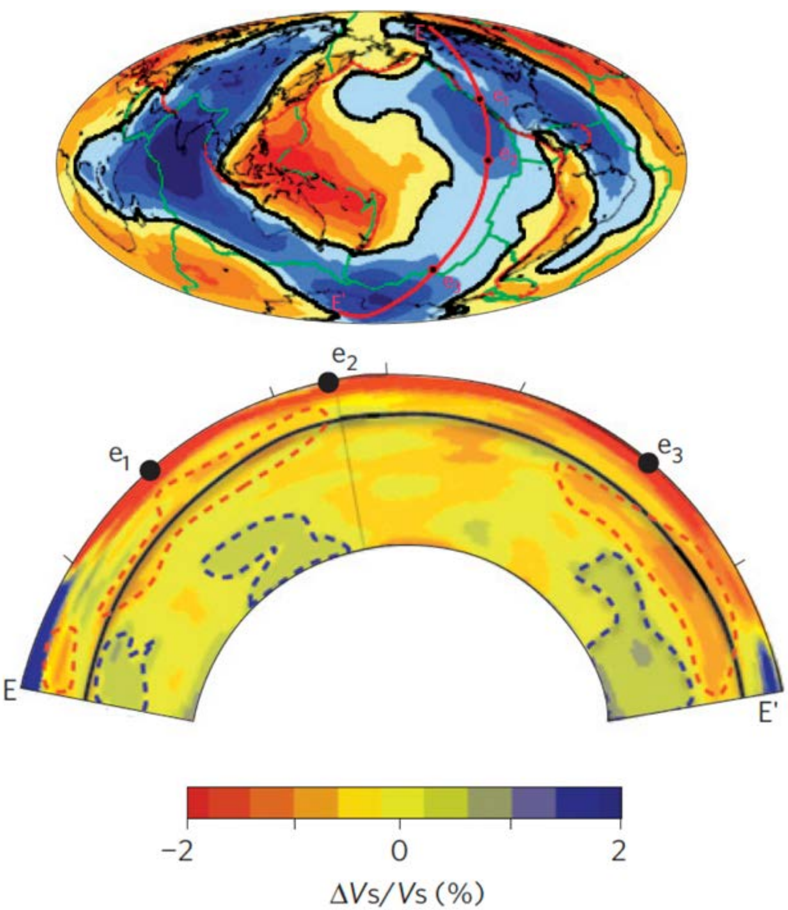


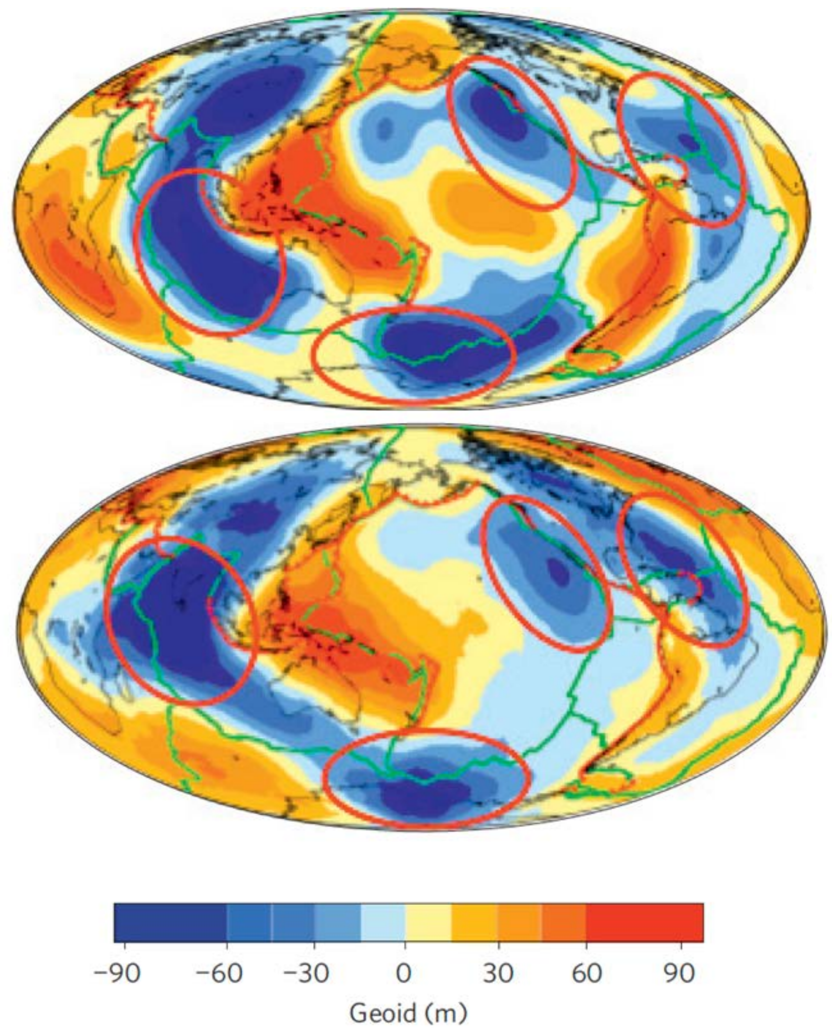
Observed geoid (degree 2-6)



(Hager, *Nature,*1985)

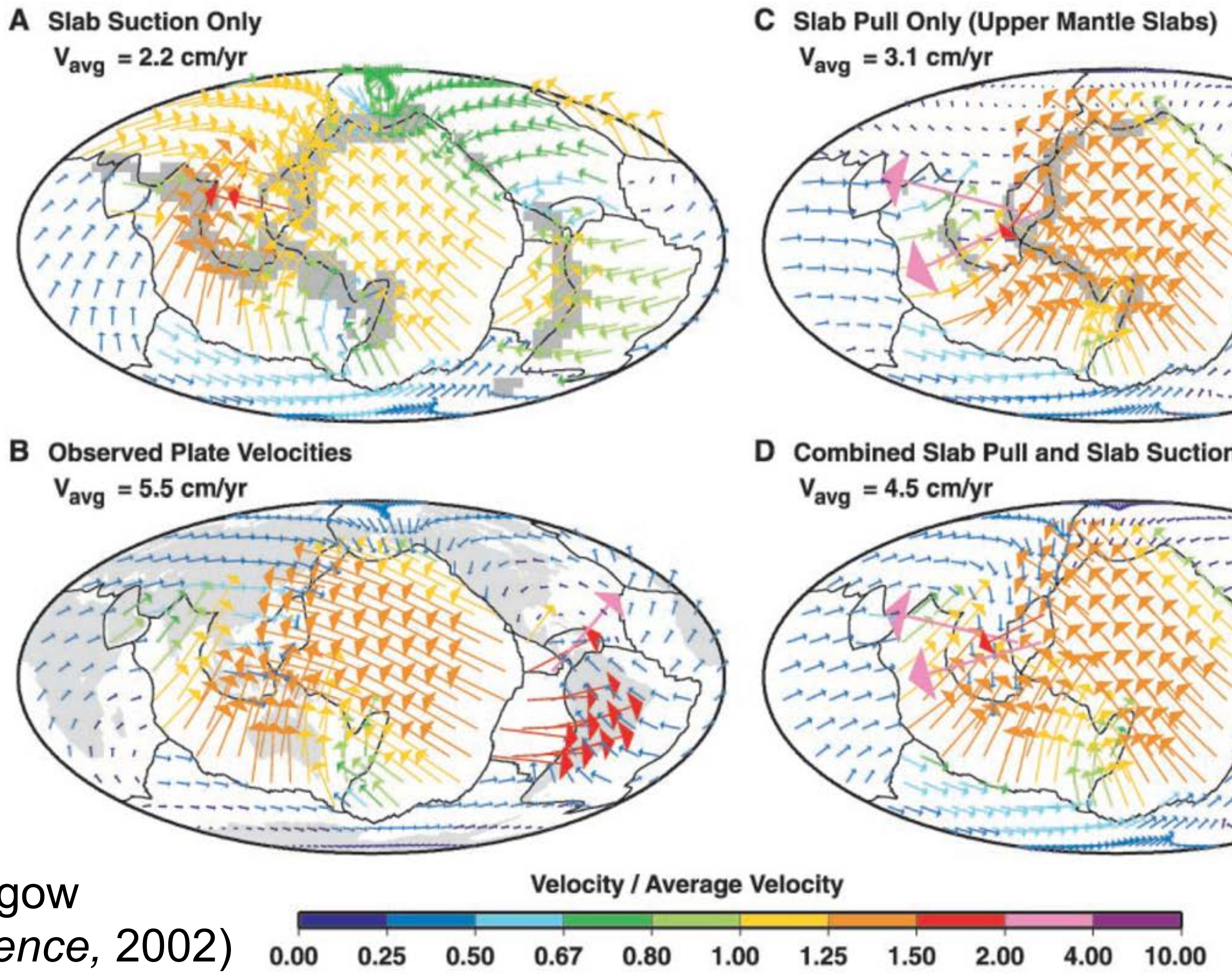
Instantaneous models
- Geoid study





(Spasojevic et al., *Nat. Geosci.,* 2010)

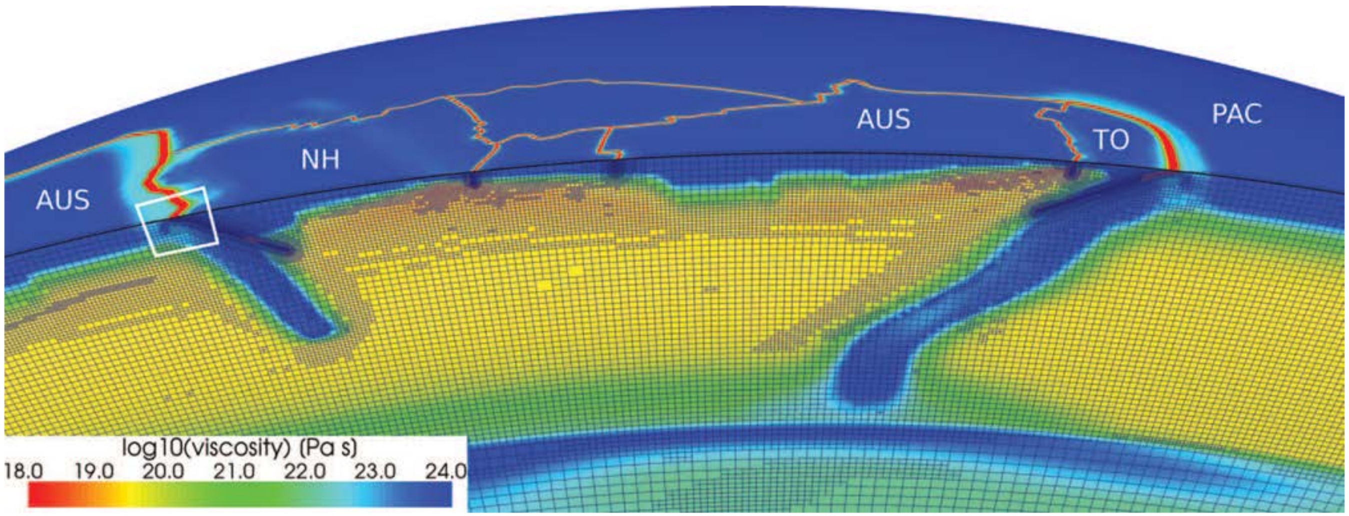
Instantaneous models
- Plate motion

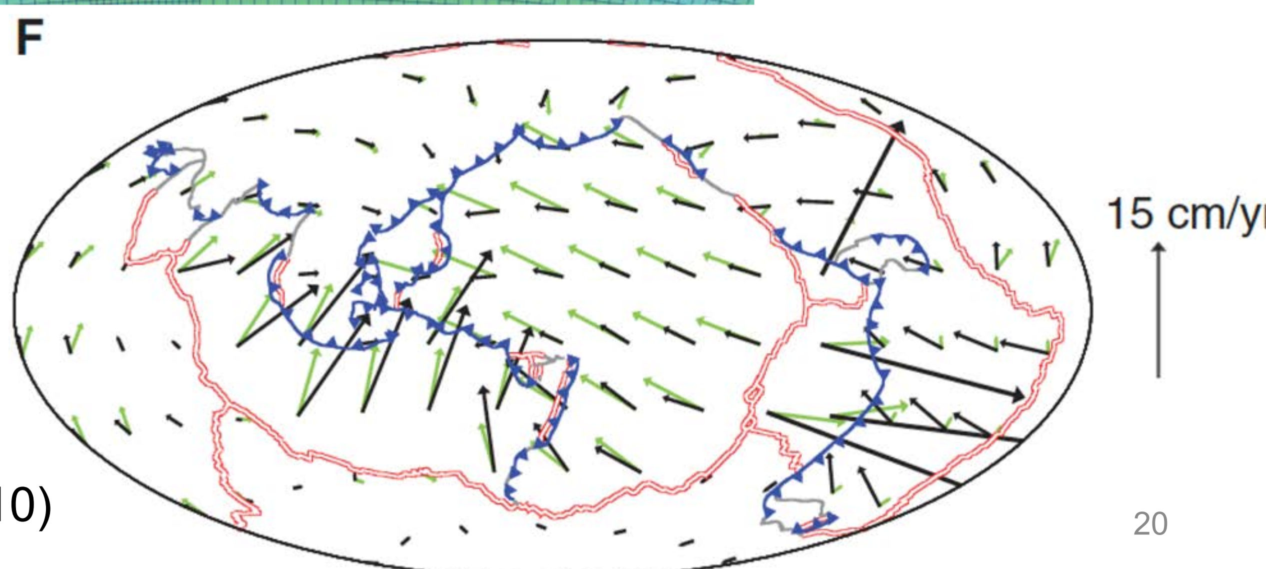


(Conrad & Lithgow -bertelloni, *Science*, 2002)

Instantaneous models

- Plate motion





(Stadler et al., *Science,* 2010)

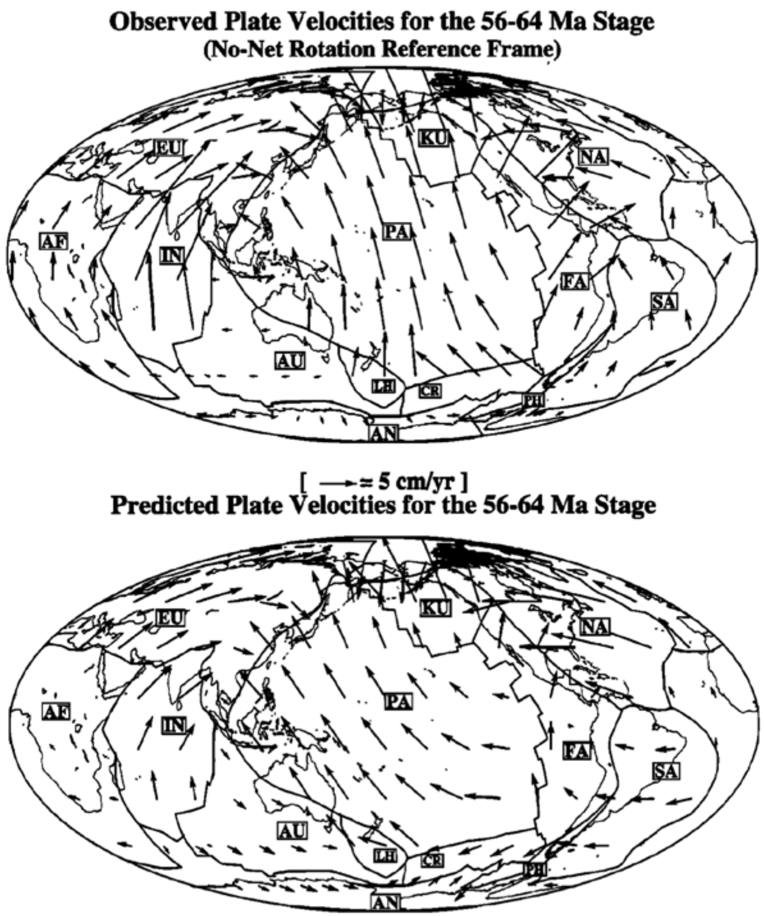
Progress in forward model development

- Instantaneous geodynamic models, for studying present-day dynamics:
  - Gravity
  - Plate motion

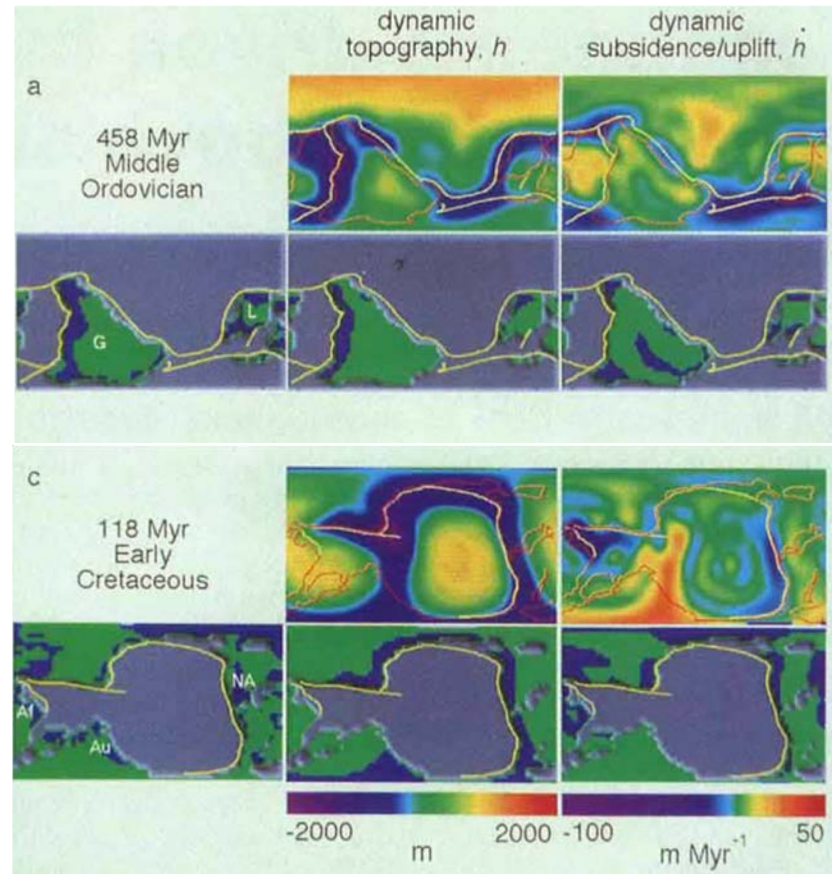
- Time-dependent subduction models:
  - The increasing knowledge of plate tectonic reconstruction allows estimating past subduction and mantle structures and associated surface responses.
  - Subduction & mantle structure
  - Plate motion
  - Dynamic topography

Time-dependent models

- Subduction & mantle structure



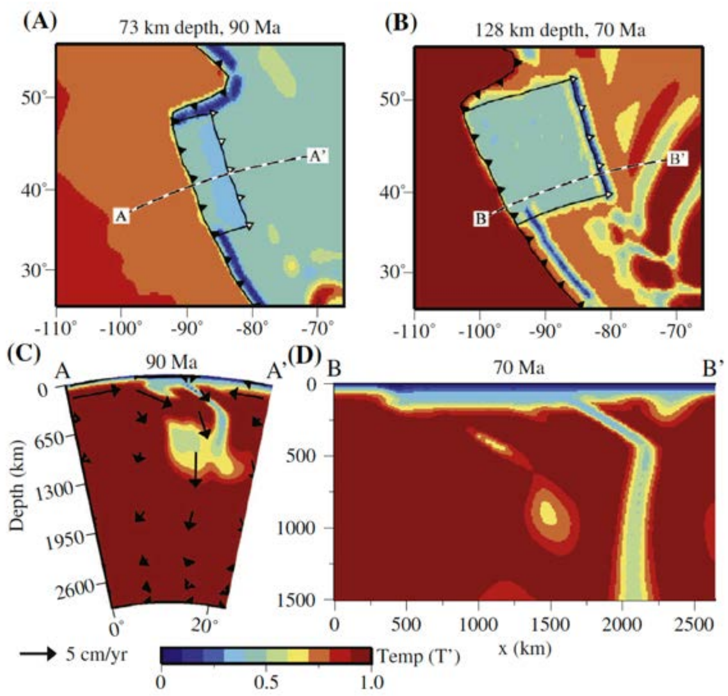
(Lithgow-Bertelloni & Richards, *Rev. Geophy.,* 1998)

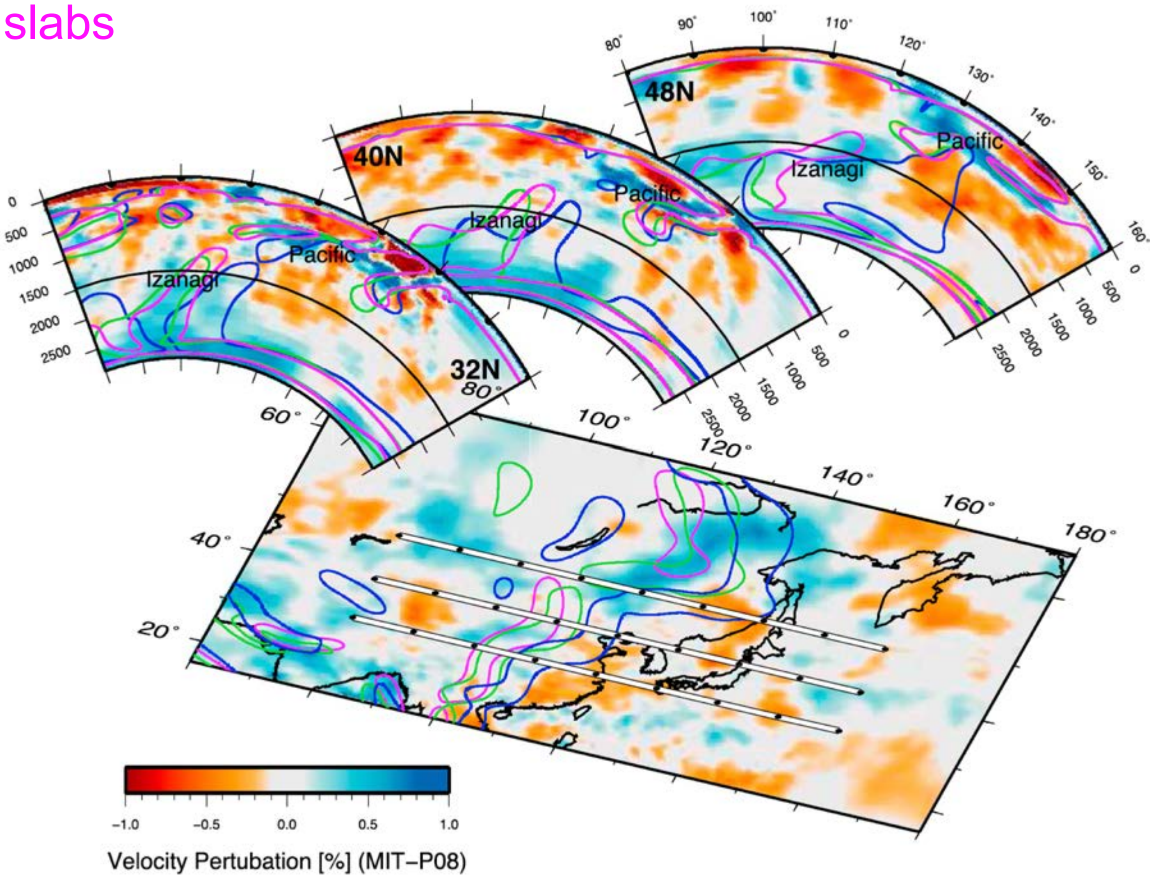


(Gurnis, *Nature,* 2000)

Time-dependent models
- Subduction & mantle structure

- With imposed upper-mantle slabs





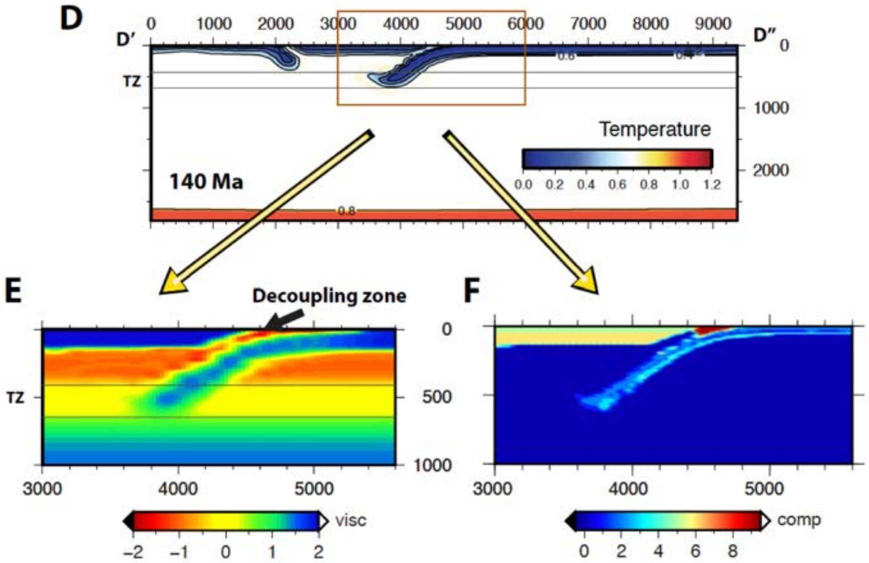
(Bower et al., *PEPI,* 2015)

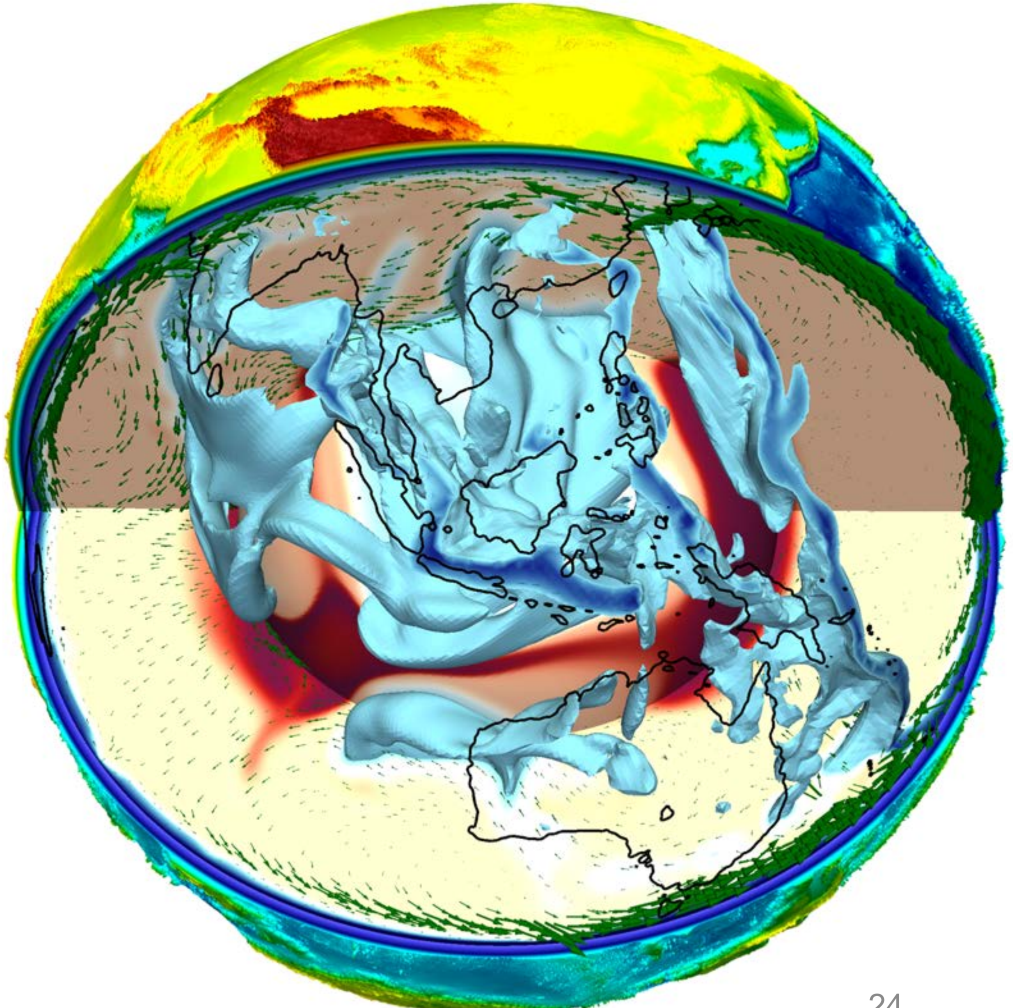
(Seton et al., *GRL,* 2015)

Time-dependent models

- Subduction & mantle structure

- More natural subduction





(Hu et al., *EPP,* 2018)

Outline

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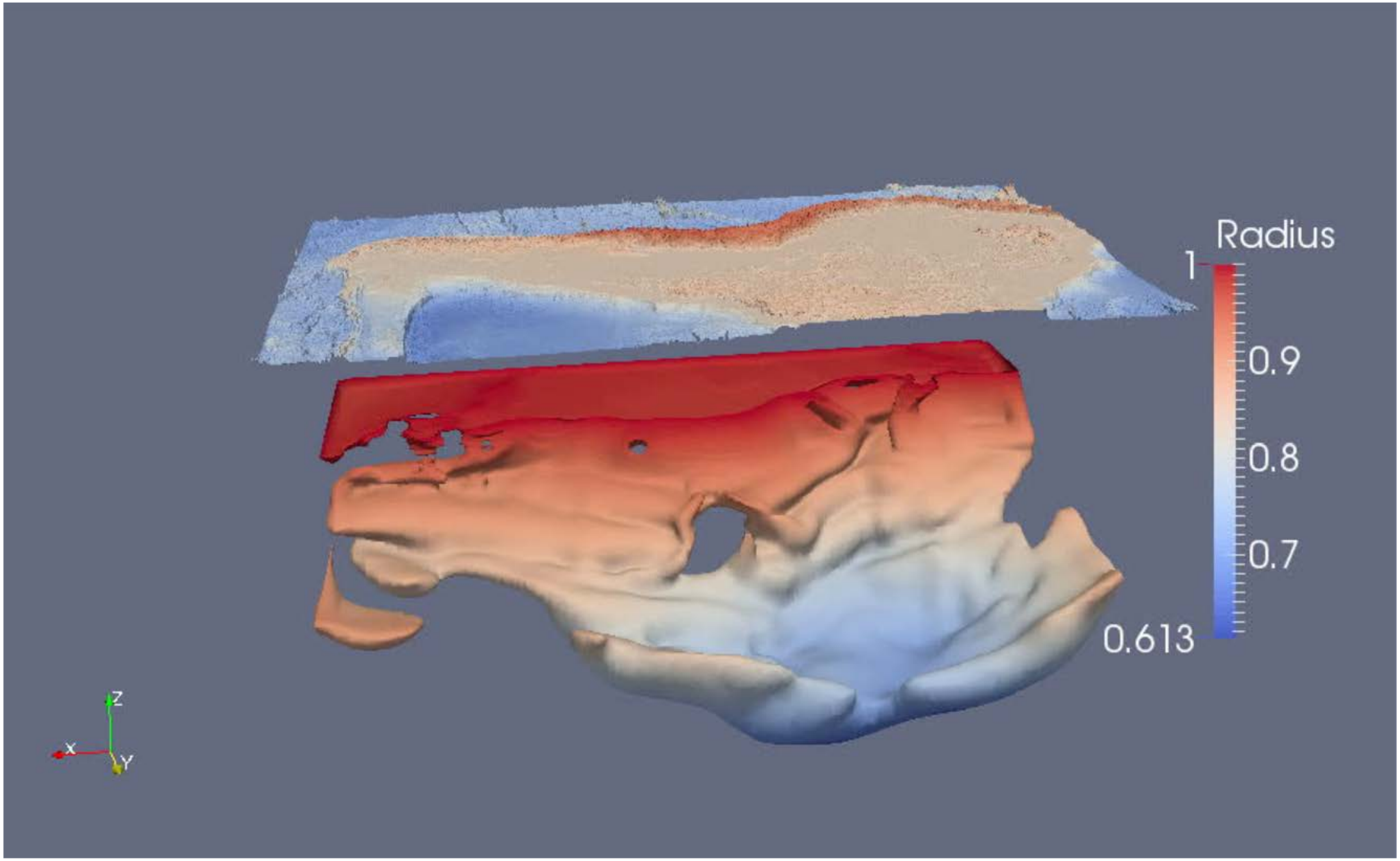
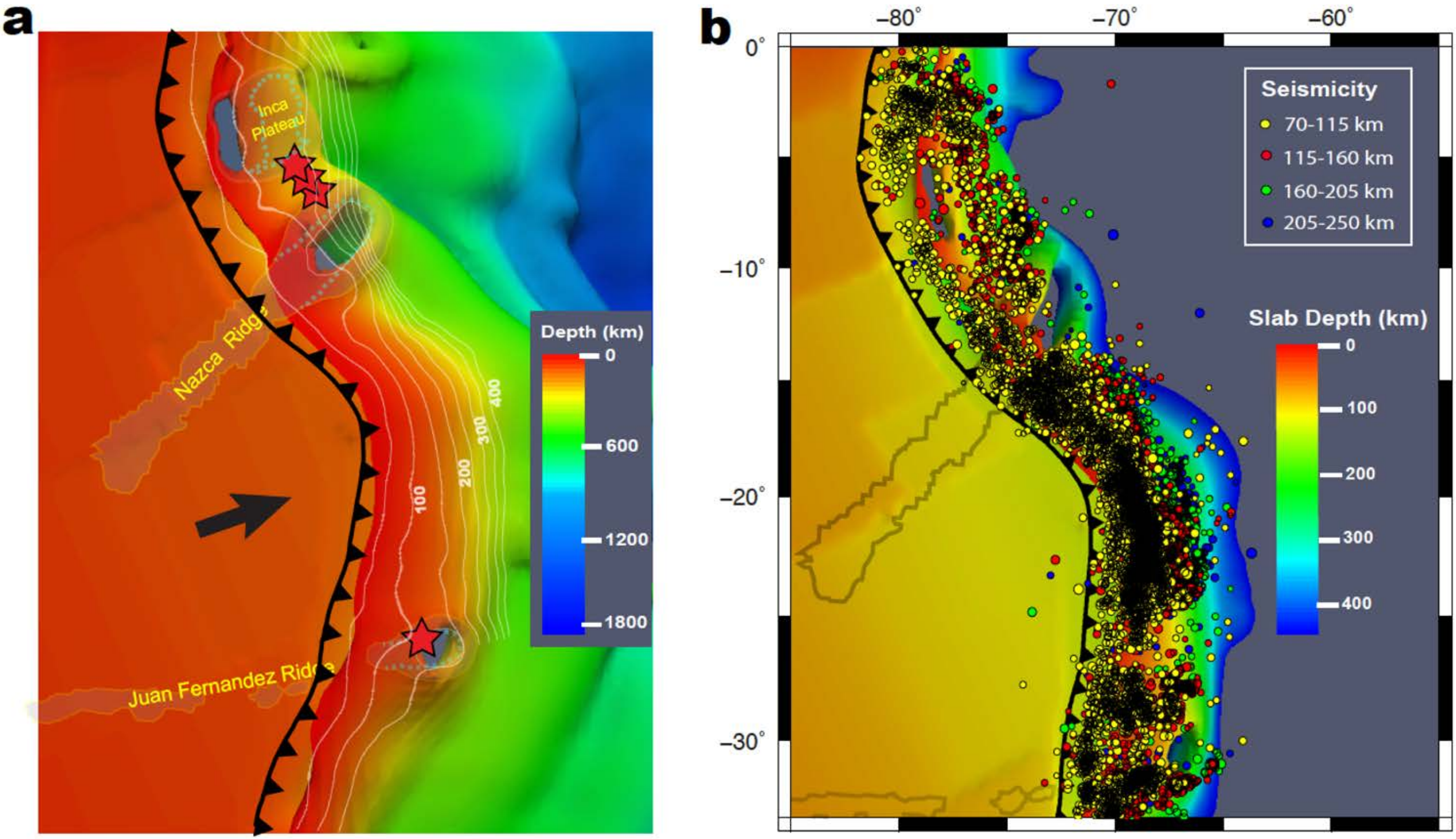
Example 1: Modeling Nazca subduction



(Hu et al., *EPSL*, 2016)

Torn flat slabs vs. intra-slab seismicity

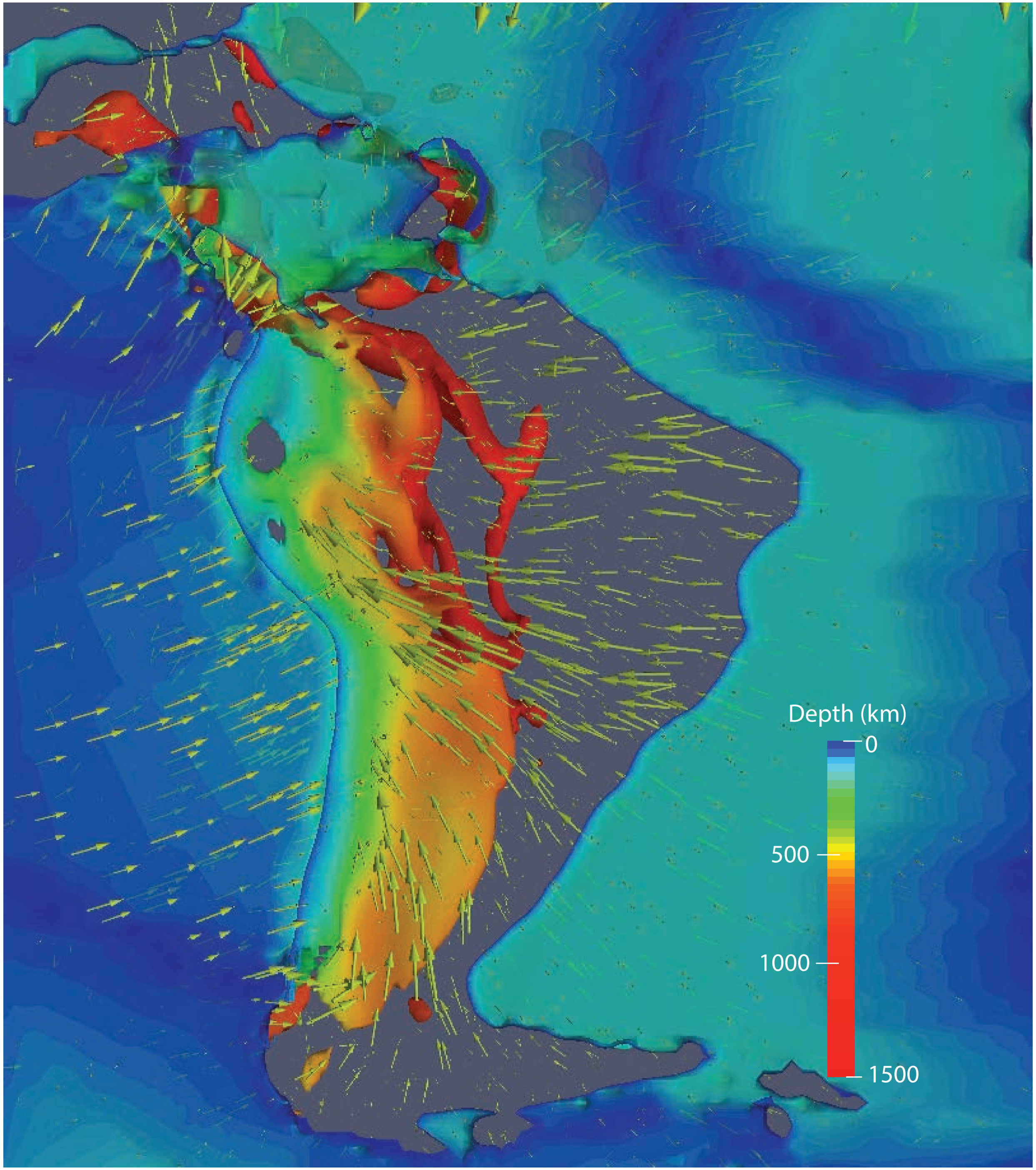


(Hu & Liu, *EPSL*, 2016)

Can be easily expanded to assimilating more constraints.

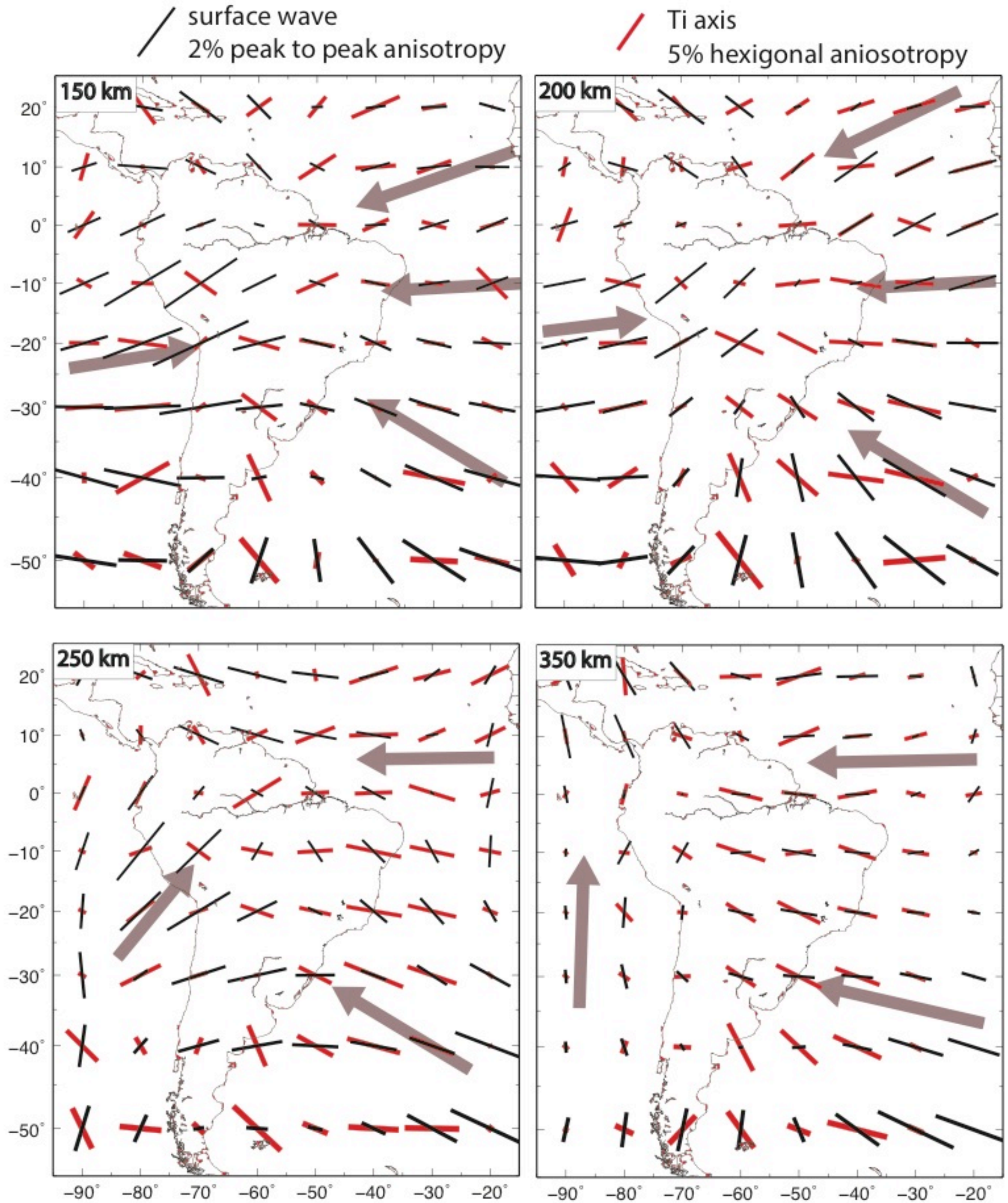
Mantle flow history used for predicting ***LPO*** and ***SKS***

(Using the 4-D mantle flow history to train an initially isotropic pyrolitic mantle)



(Hu et al., *EPSL*, 2017)

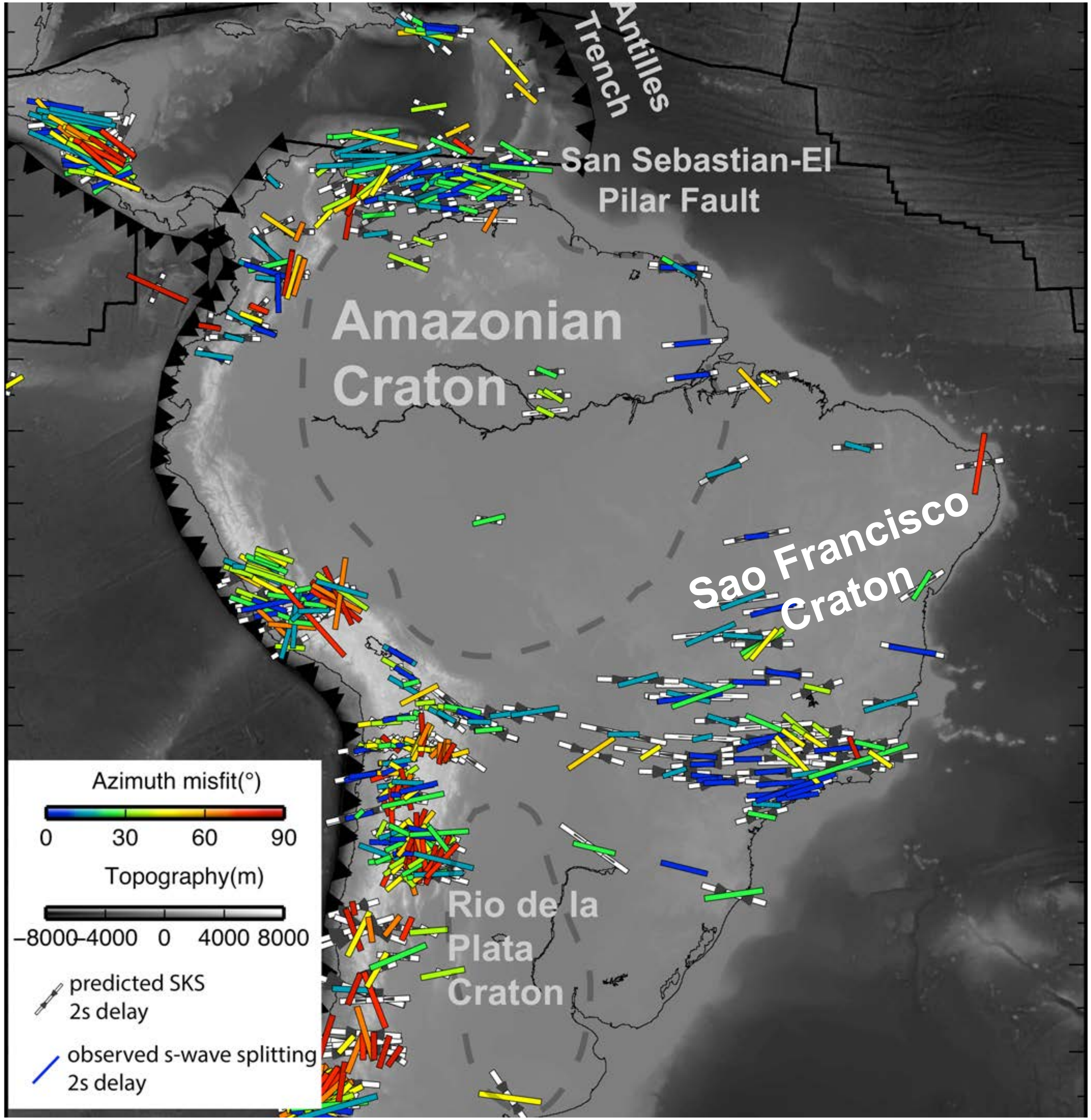
Predicted LPO (red) vs. surface wave anisotropy (black)



A radiation pattern outlines the large-scale mantle flow induced by the down-going Nazca slab.

(Hu et al., *EPSL*, 2017)

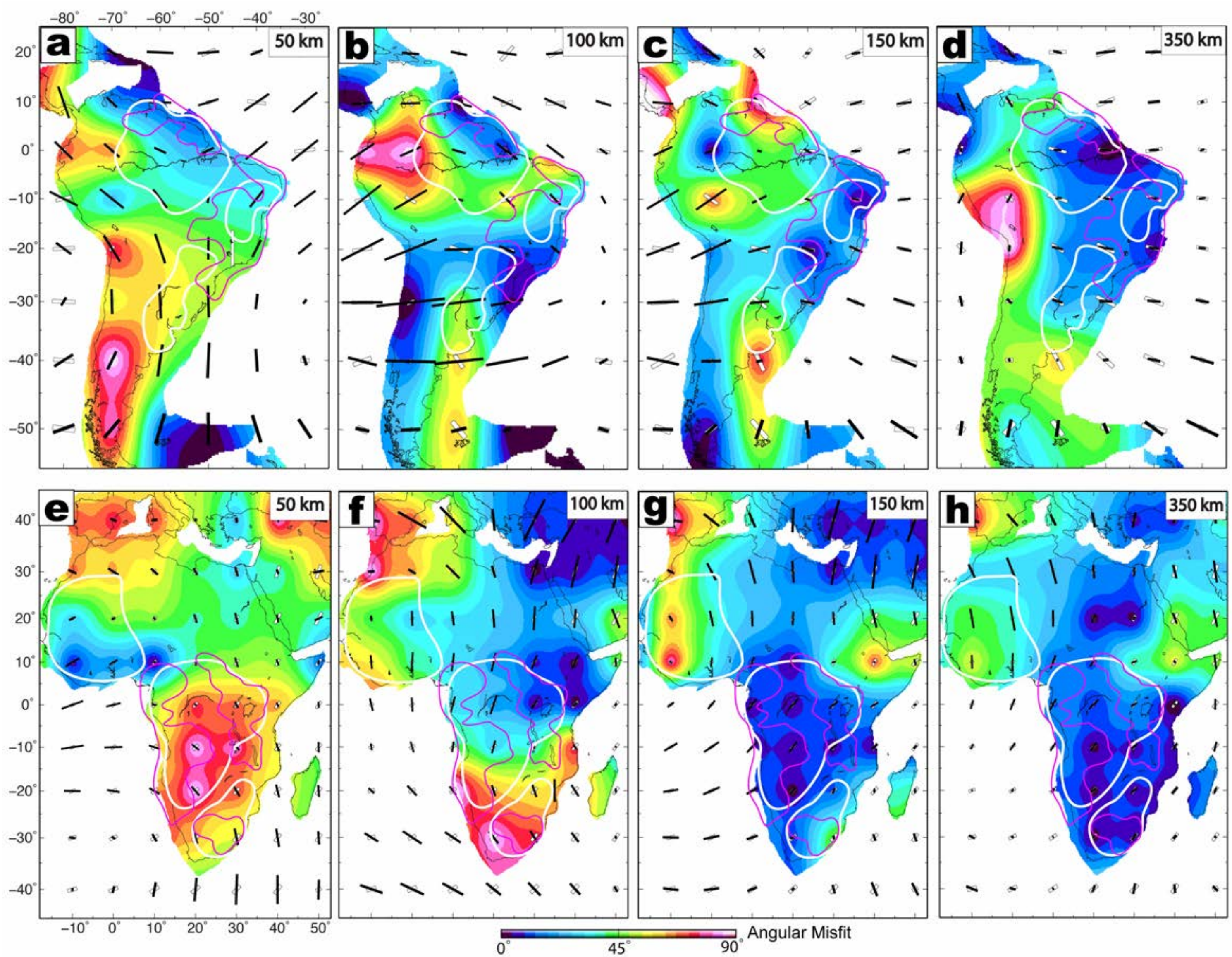
Predicted (white) & observed (color) SKS splitting



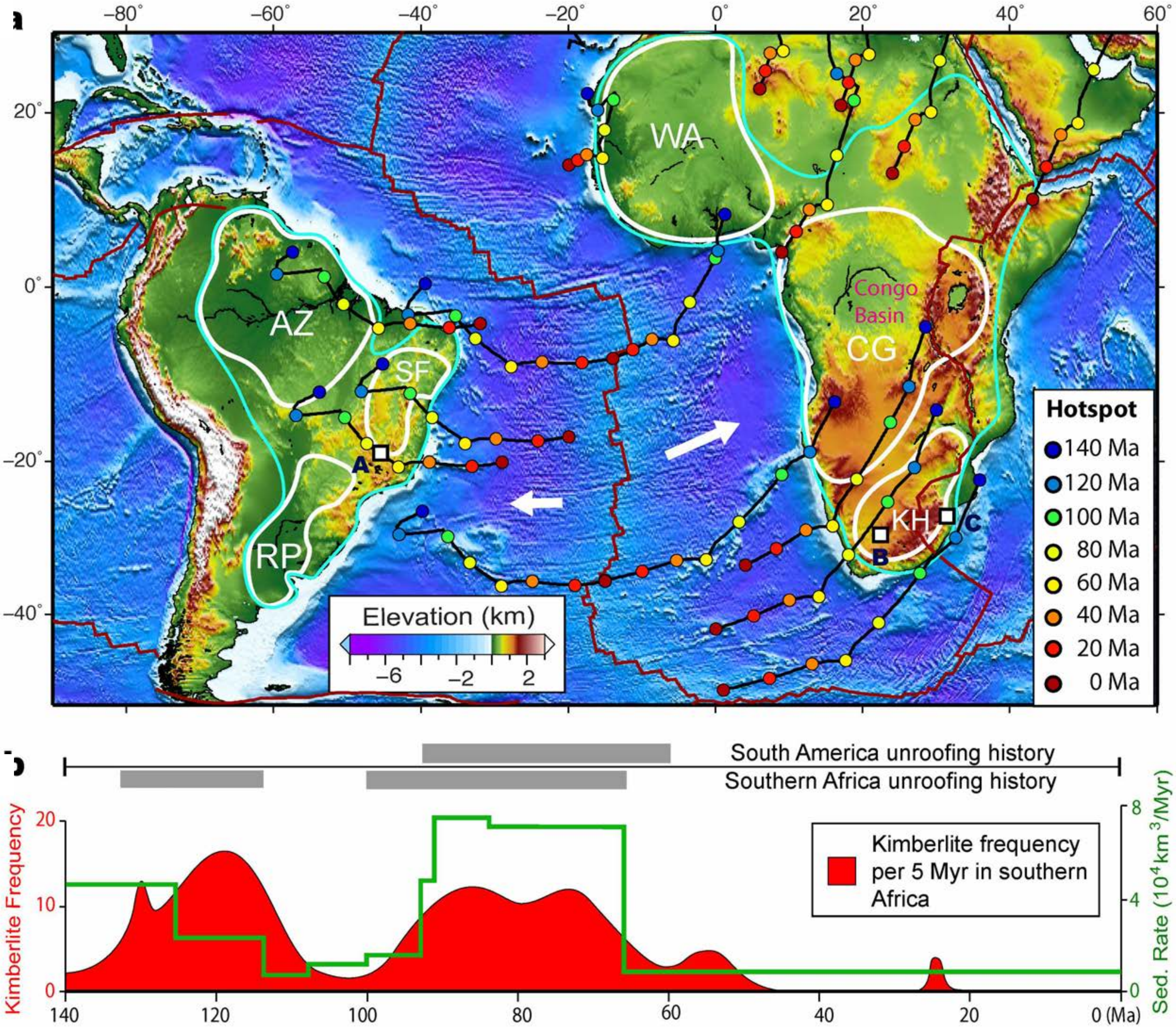
A radiation pattern outlines the large-scale mantle flow induced by the down-going Nazca slab.

(Hu et al., *EPSL*, 2017)

Anisotropy alignment with Cenozoic mantle flow

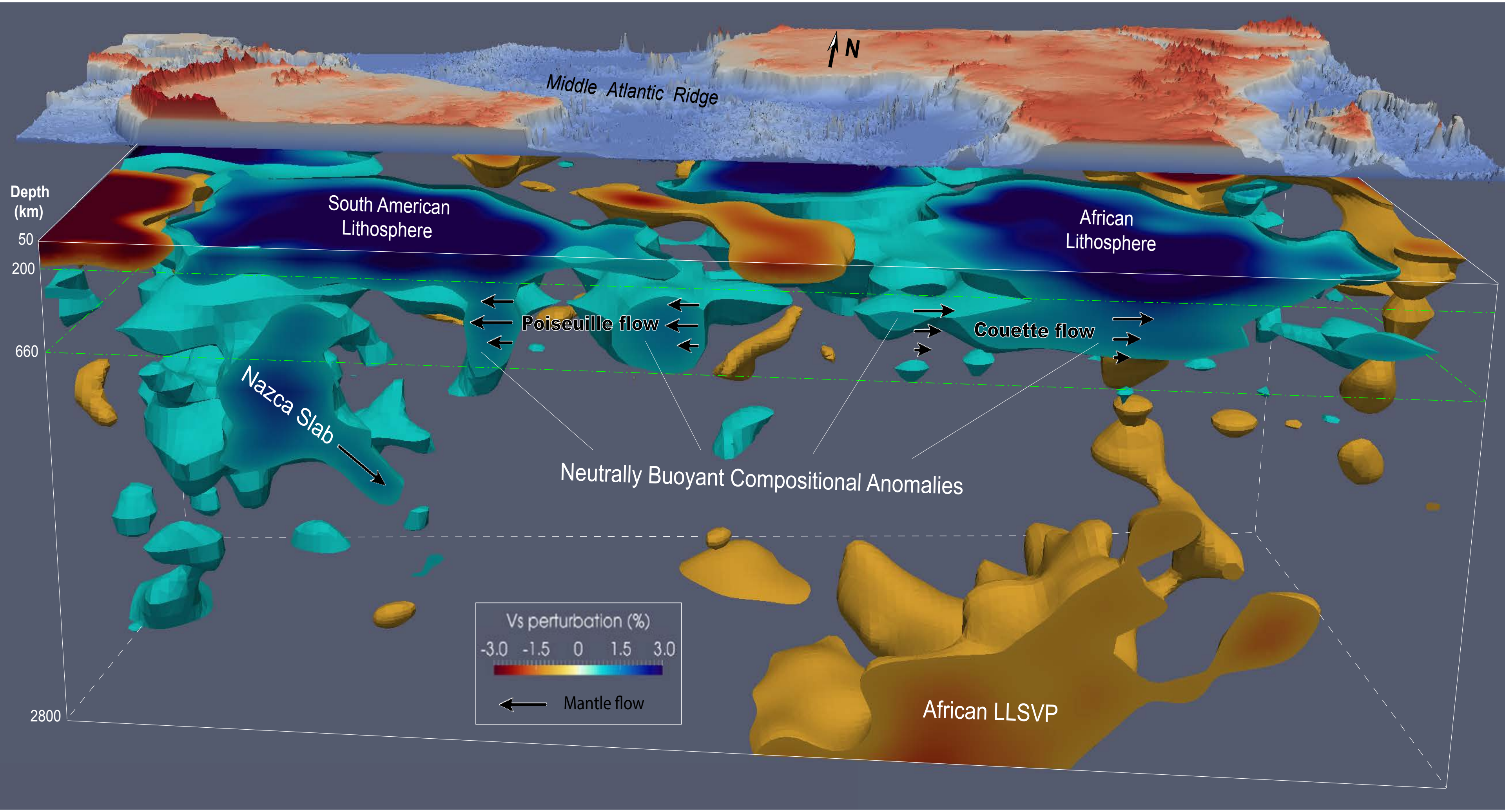


(Hu et al., *Nature Geosci.*, *2018*)



−80°
−60°
−40°
−20°
0°
20°
40°
60°
20°
0°
−20°
−40°
WA
AZ
SF
RP
A
Congo Basin
CG
KH
B
C
Elevation (km)
−6
−2
2
Hotspot
140 Ma
120 Ma
100 Ma
80 Ma
60 Ma
40 Ma
20 Ma
0 Ma
South America unroofing history
Southern Africa unroofing history
Kimberlite Frequency
20
10
0
Sed. Rate (10⁴km³/Myr)
8
4
0
Kimberlite frequency per 5 Myr in southern Africa
140
120
100
80
60
40
20
0 (Ma)

Lithosphere delamination below south Atlantic

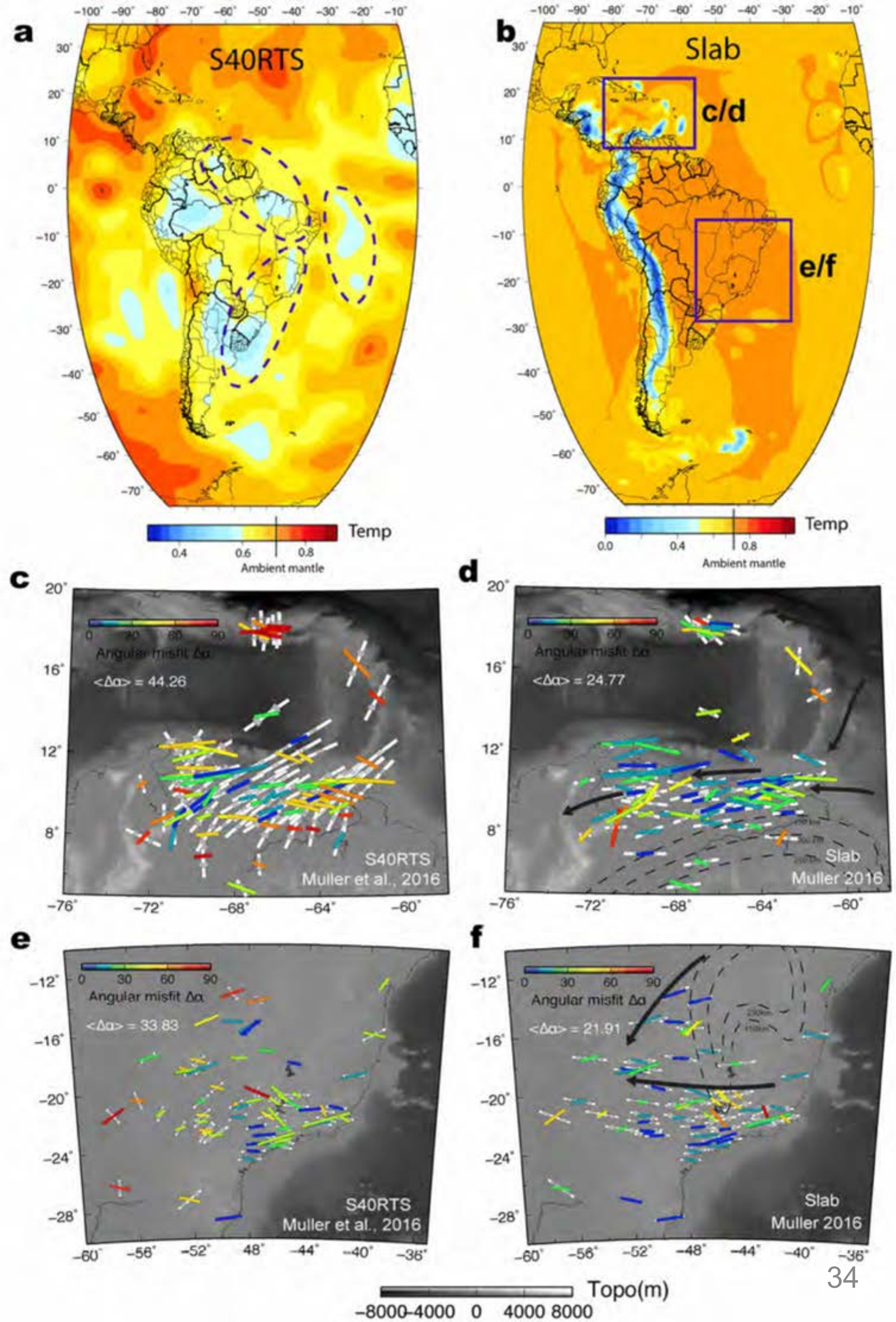


(Hu et al., *Nature Geosci.*, 2018)

Assuming **upper-mantle fast anomalies** as high density structures greatly degrades the fit to observed seismic anisotropy.

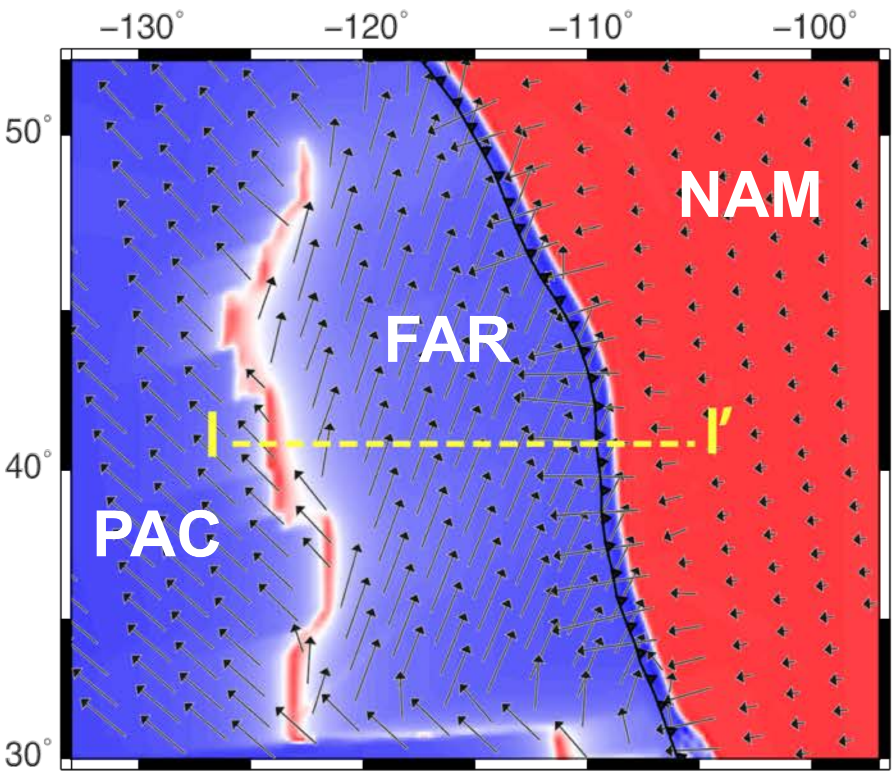
Neutrally buoyant

Delaminated Lithosphere

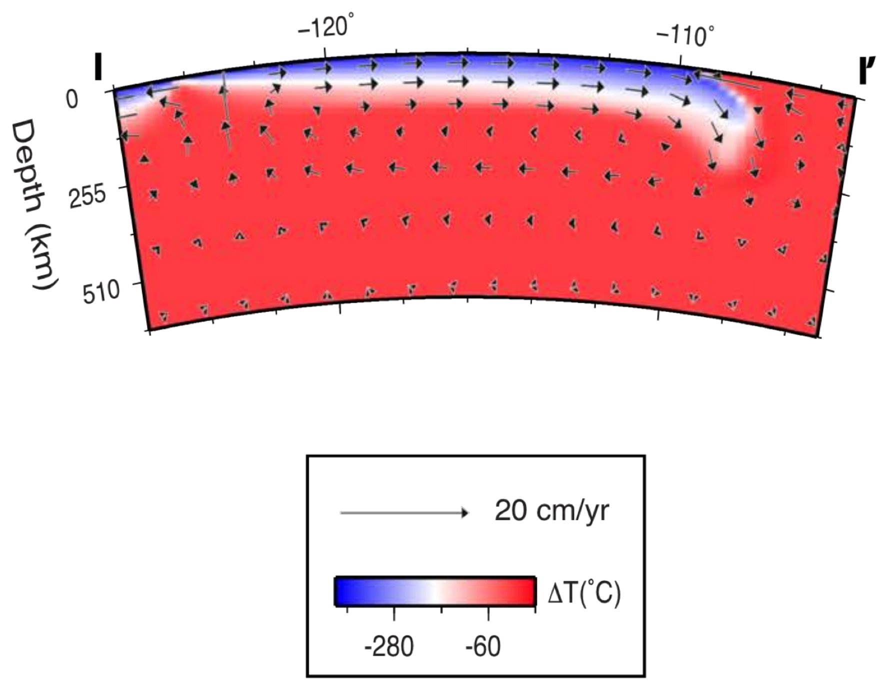


Example 2: Simulating Juan de Fuca subduction

(Age=35Ma; depth=23 km)

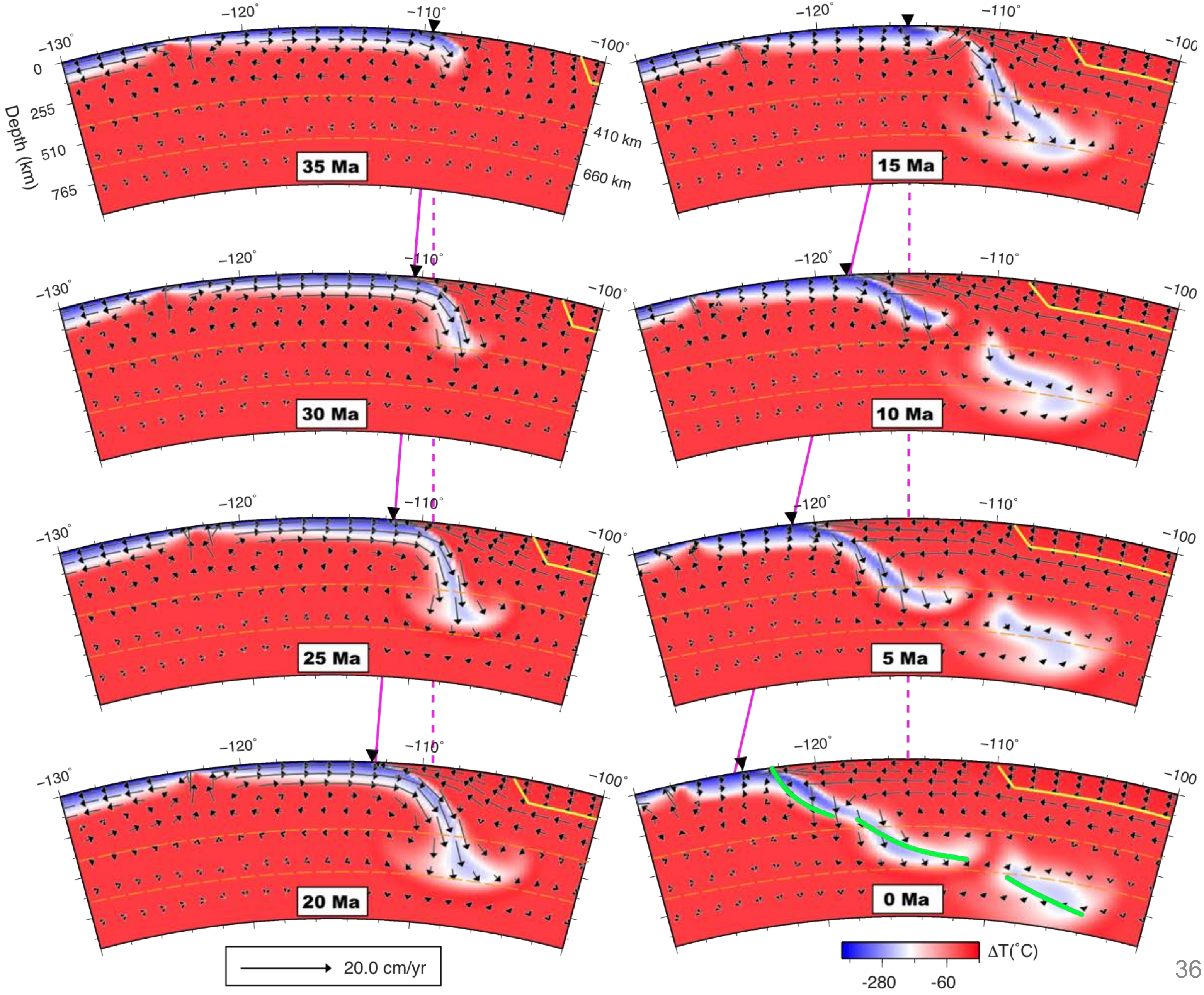


Self-emerging slab

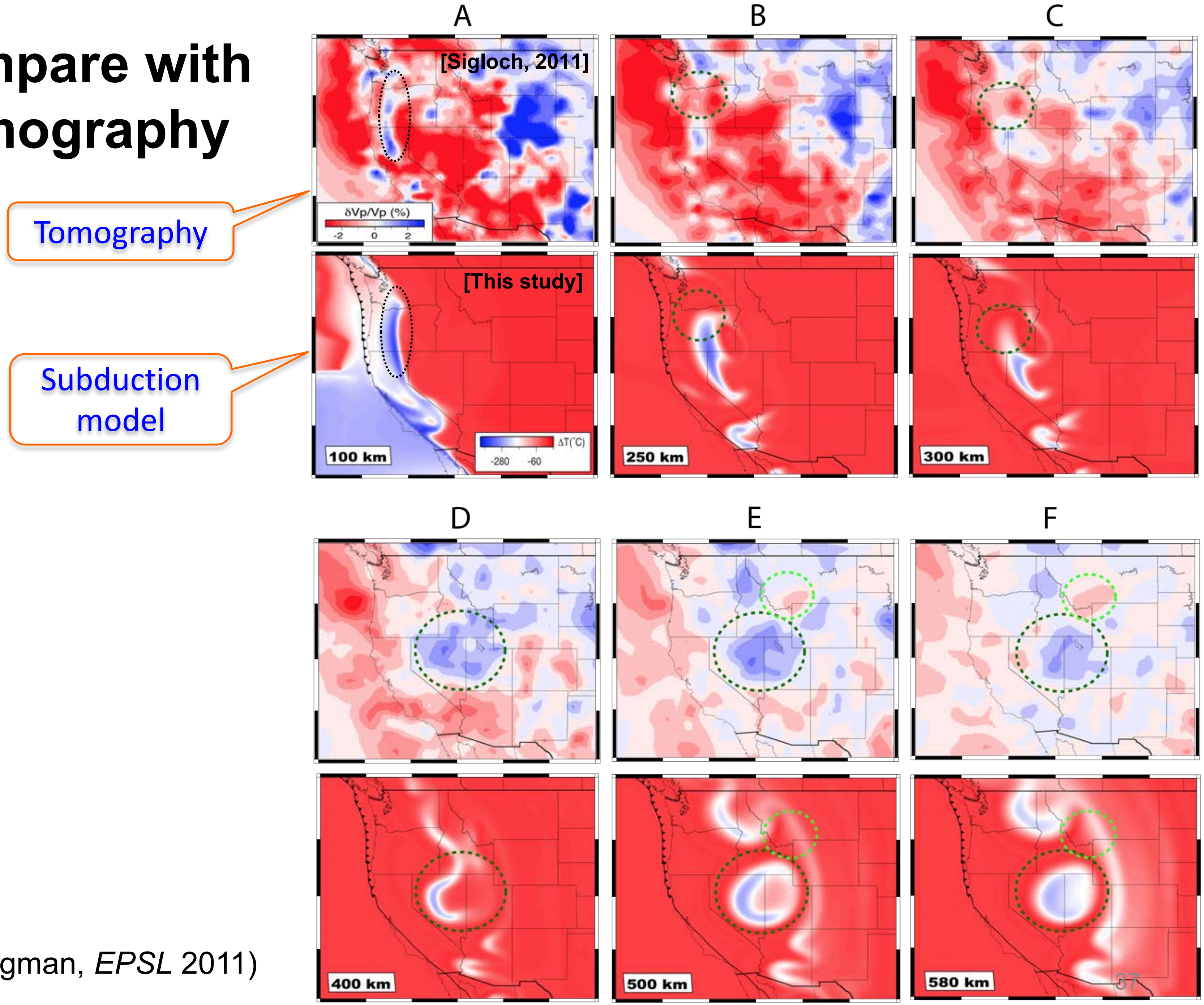


(Liu & Stegman, *EPSL* 2011)

Farallon segmentation: ***best-fit in 2D*** (41° N)

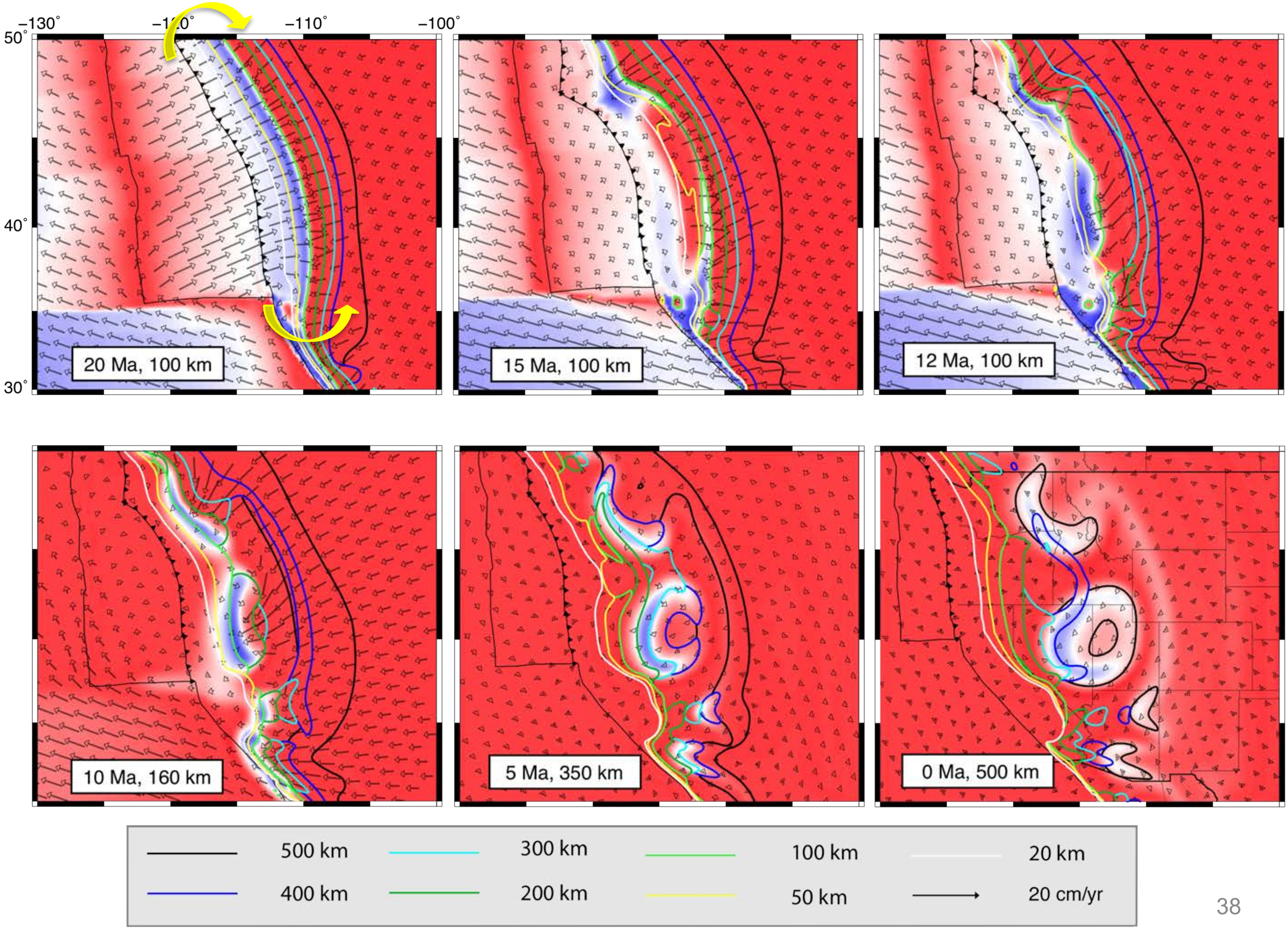


Compare with tomography

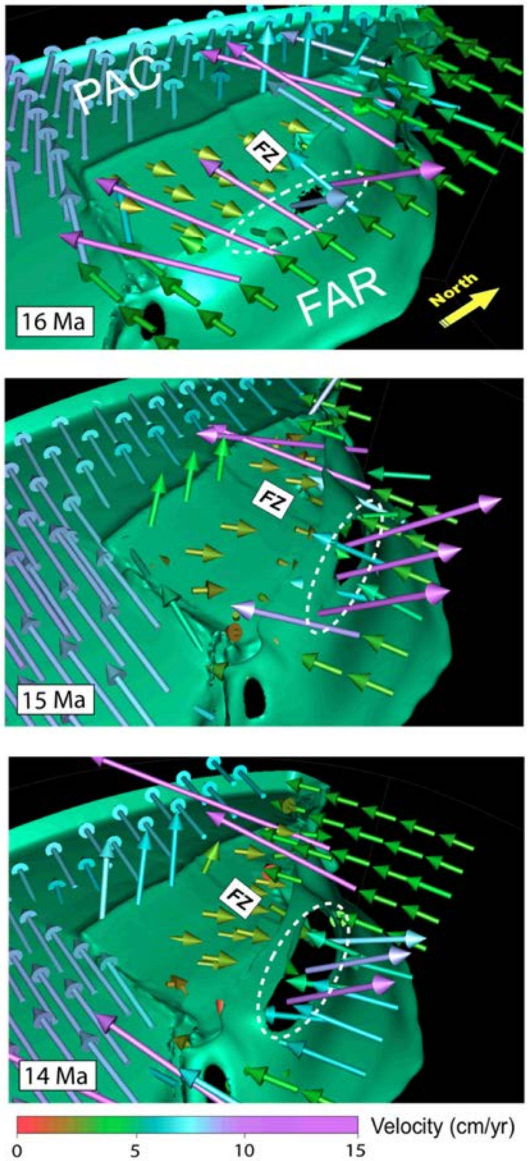


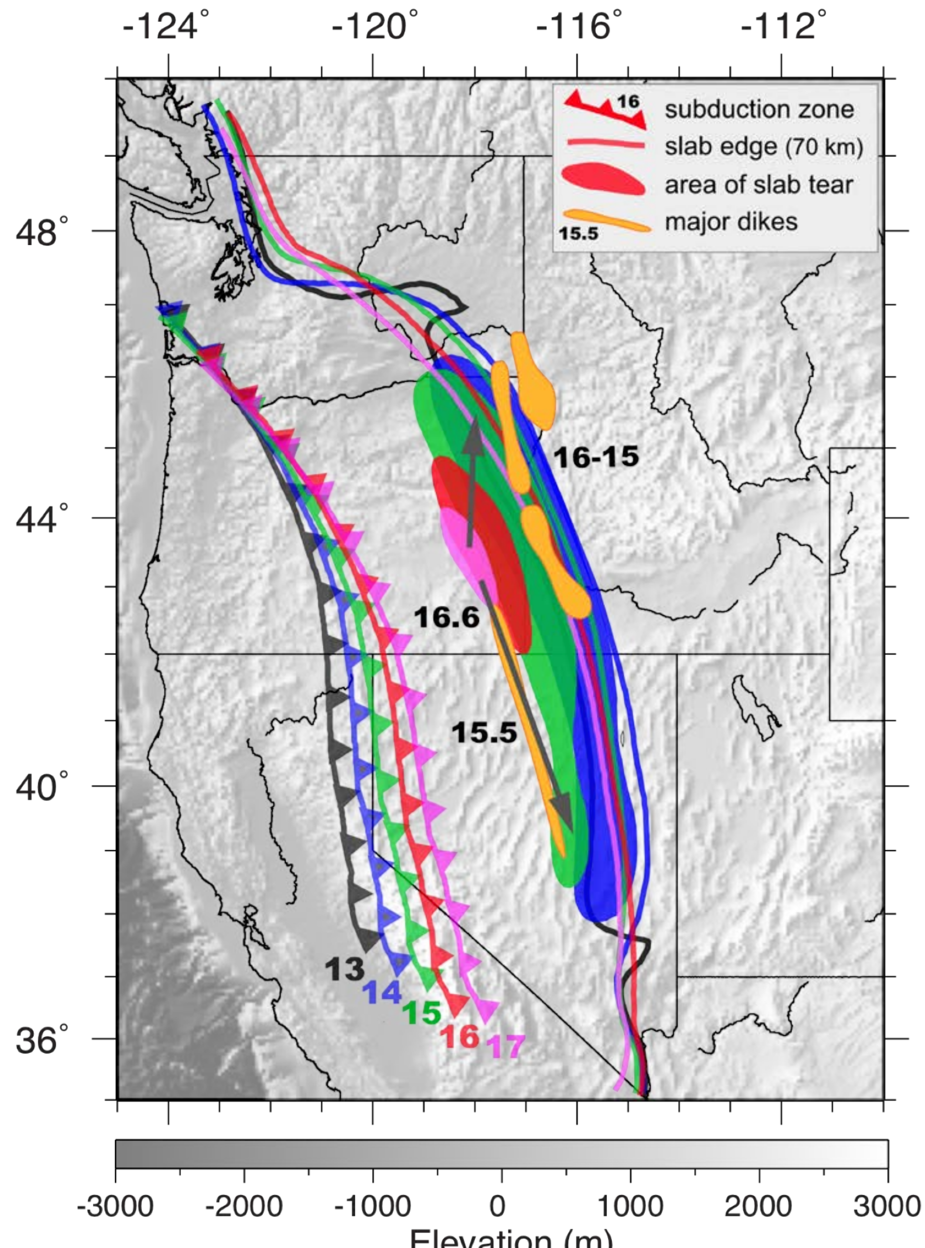
(Liu & Stegman, *EPSL* 2011)

Farallon subduction and segmentation



Formation of LIP due to slab tearing





(Liu & Stegman, *Nature,* 2012)

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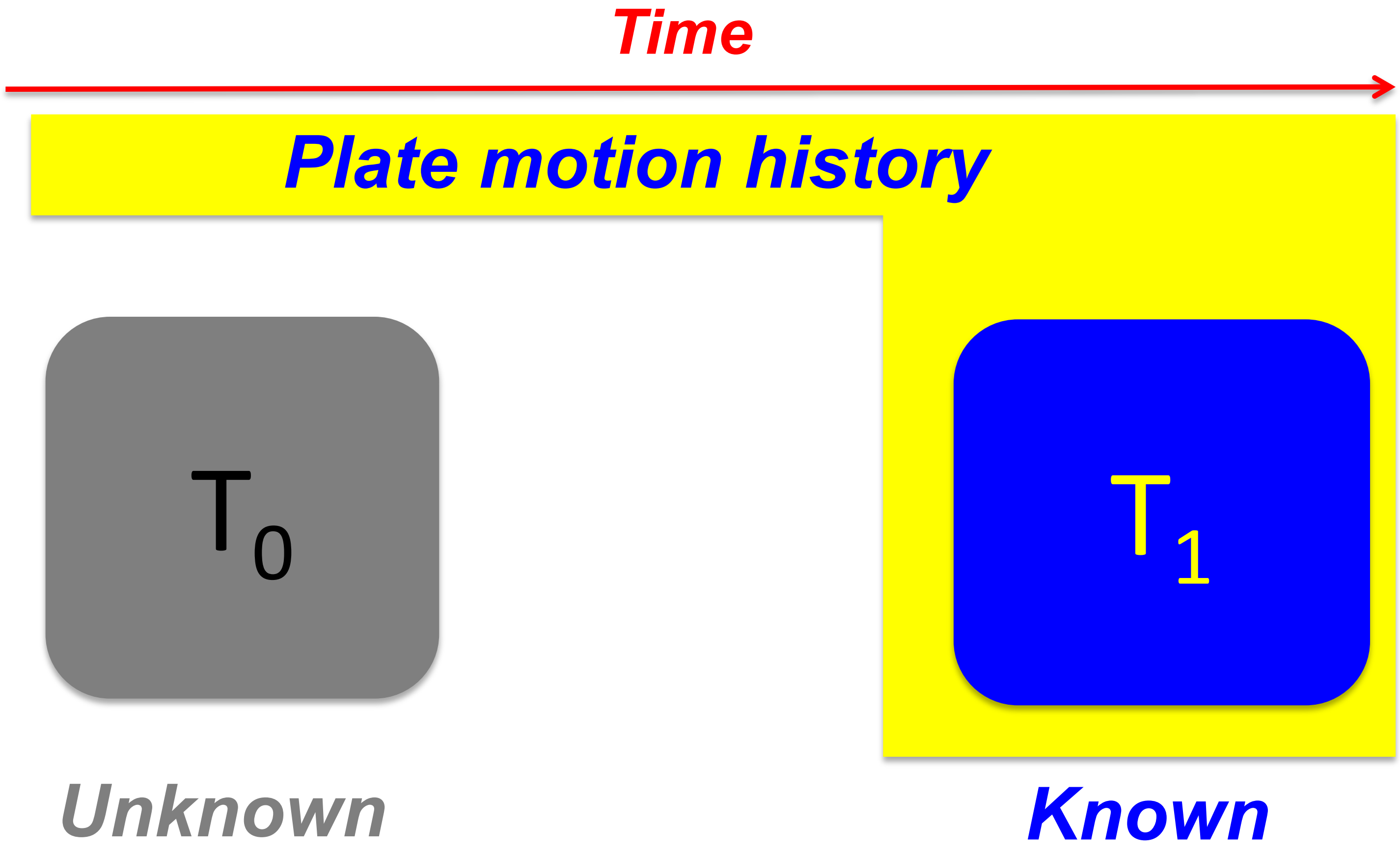
Forward models with data assimilation

- Although suffering from the uncertain initial condition, provides useful insight into the dynamics of Earth, especially when equipped geological/geophysical constraints.
- Serves as the basis of all inverse models, and thus determines the “usefulness” of them.
- Complements the inverse models by taking care of the fine-scale, complex behavior of mantle processes like subduction.

Inverse simulation of Earth evolution



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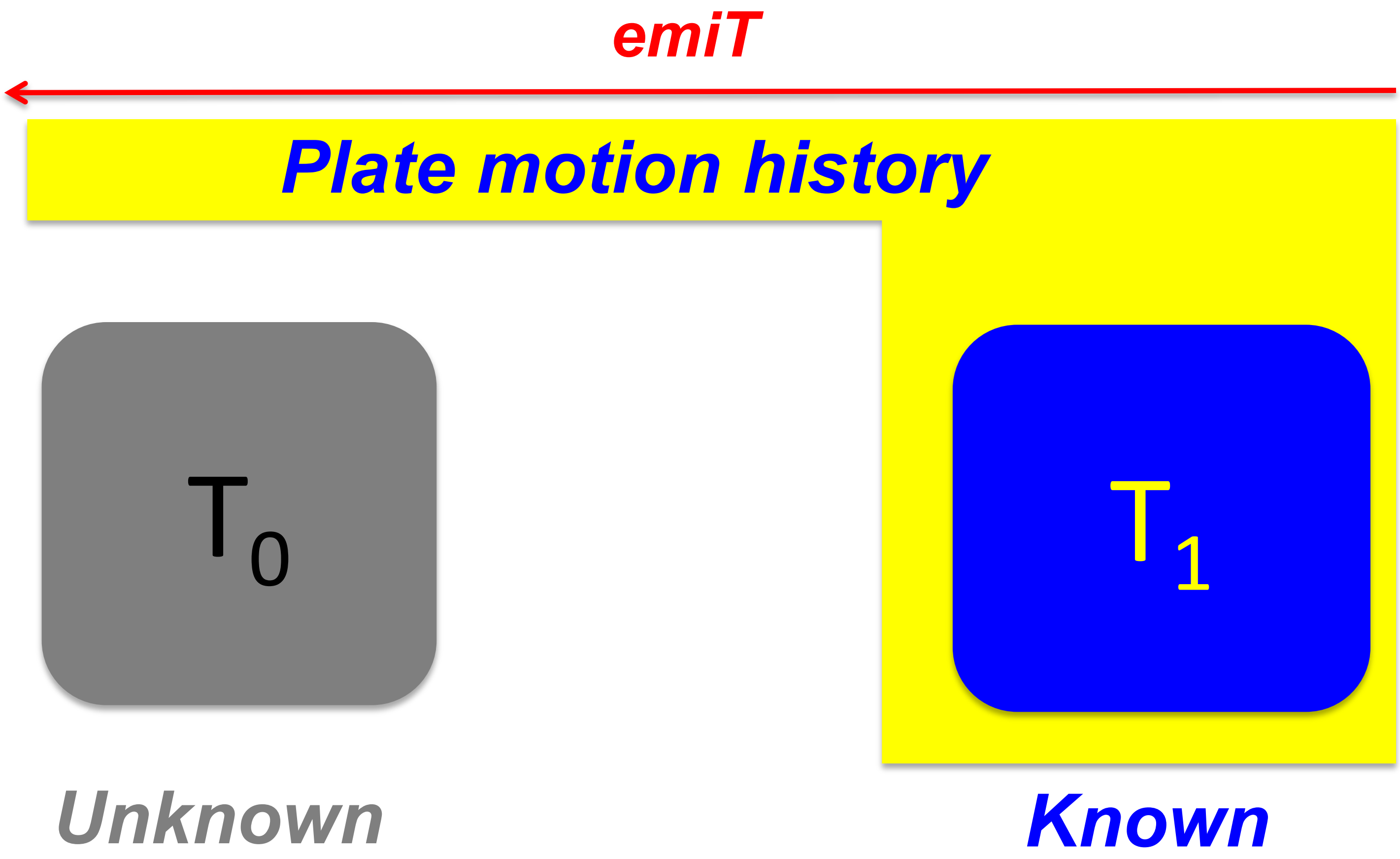


Reversibility of mantle evolution

$$\nabla \cdot \vec{u} = 0$$ **(continuity)**

$$-\nabla P + \nabla \cdot (\eta \nabla \vec{u}) = \Delta \rho \vec{g}$$ **(momentum)**

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \kappa \nabla^2 T + H$$ **(energy)**

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T - \vec{u} \cdot \nabla T = \textbf{Diffusion rate} + \textbf{Advection rate}$$

$$Pe = \frac{Advection\,rate}{Diffusion\,rate} = \frac{Lu}{\kappa}$$

$L = 10^6\,m$
$u = 3\,cm/yr$
$\kappa = 10^{-6}\,\mathrm{m^2/s}$

$$\boldsymbol{Pe = 1000}$$

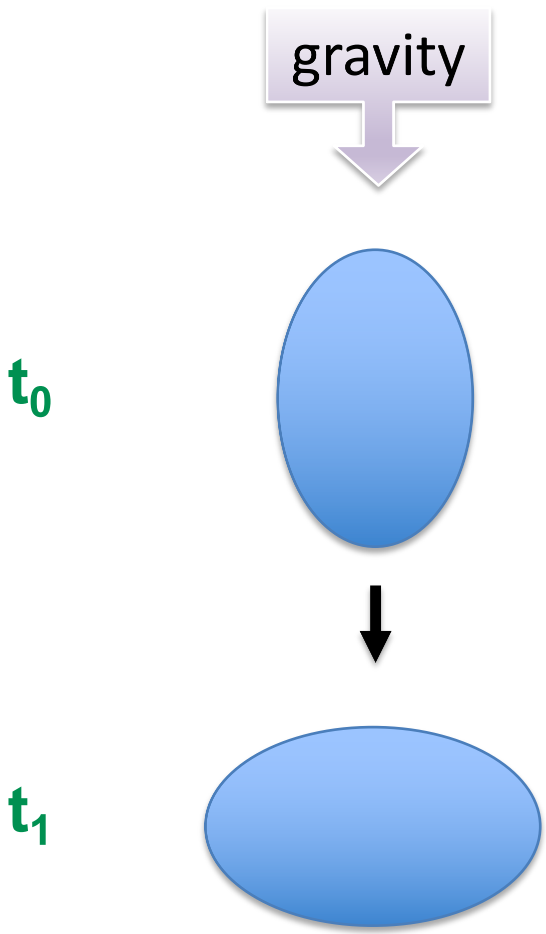
Mantle interior is dominated by advection

Kinematic reversibility

- **Low Reynolds number (Stokes) flows are quite reversible (a property of its laminar nature), assuming the kinematic history (flow velocity) is known.**

- **Let's watch a movie…**

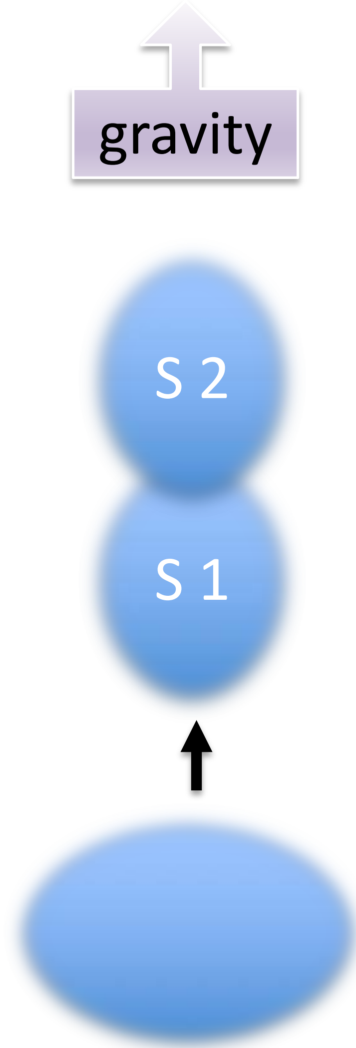
- **This led to the idea of 'simple backward flow' model, because:**
  - **Flow within the mantle interior is dominantly advection (Pe=1000; diffusion is negligible).**
  - **Gravity that controls mantle flow is easily reversible (merely a sign issue) for a given density structure.**



Uncertainties

- **Density**
- **Viscosity**

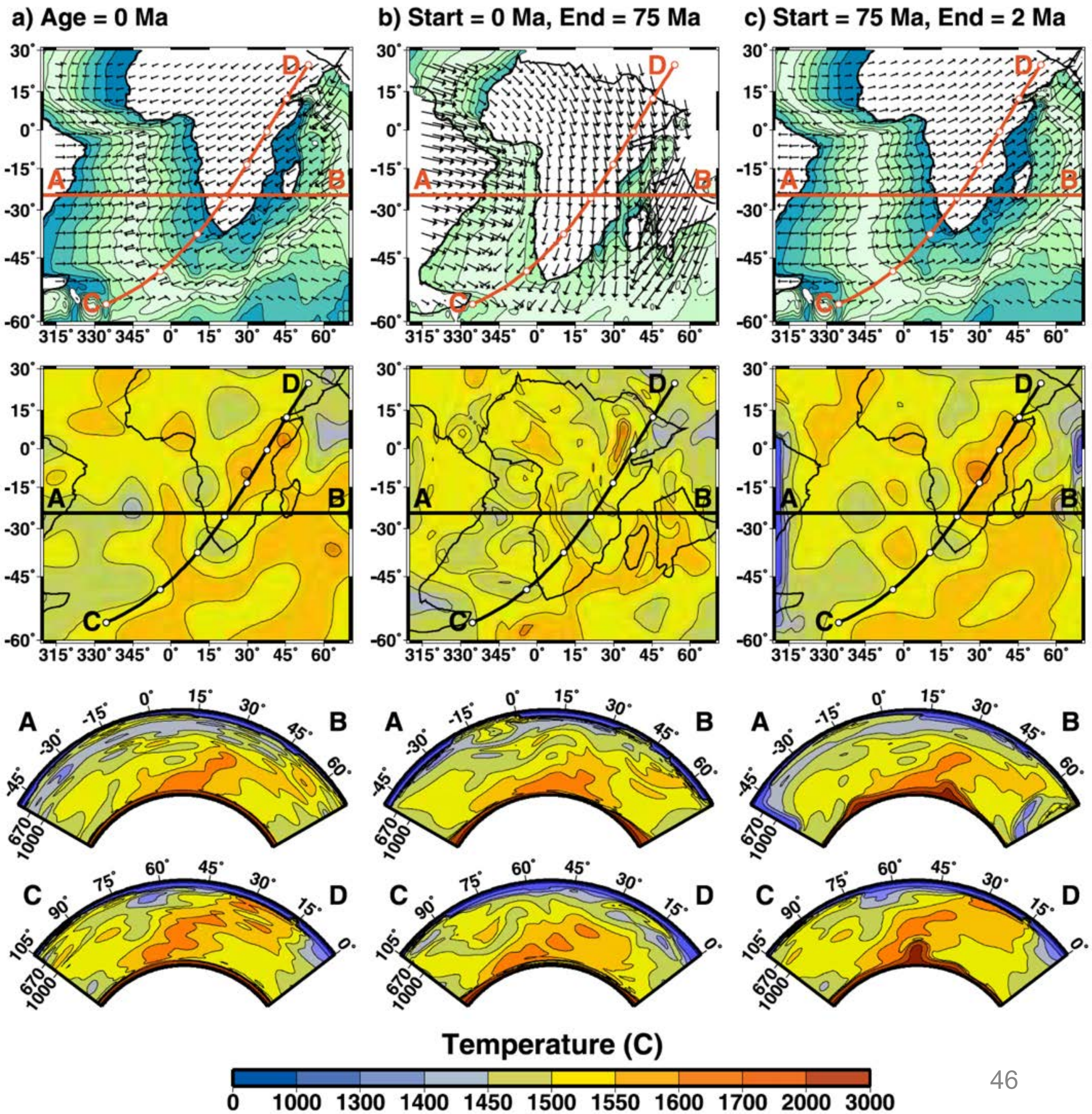
=



Mantle density converted from tomography + two thermal boundary layers

Restores the large-scale flow away from BLs

Note errors growing off the lower thermal boundary

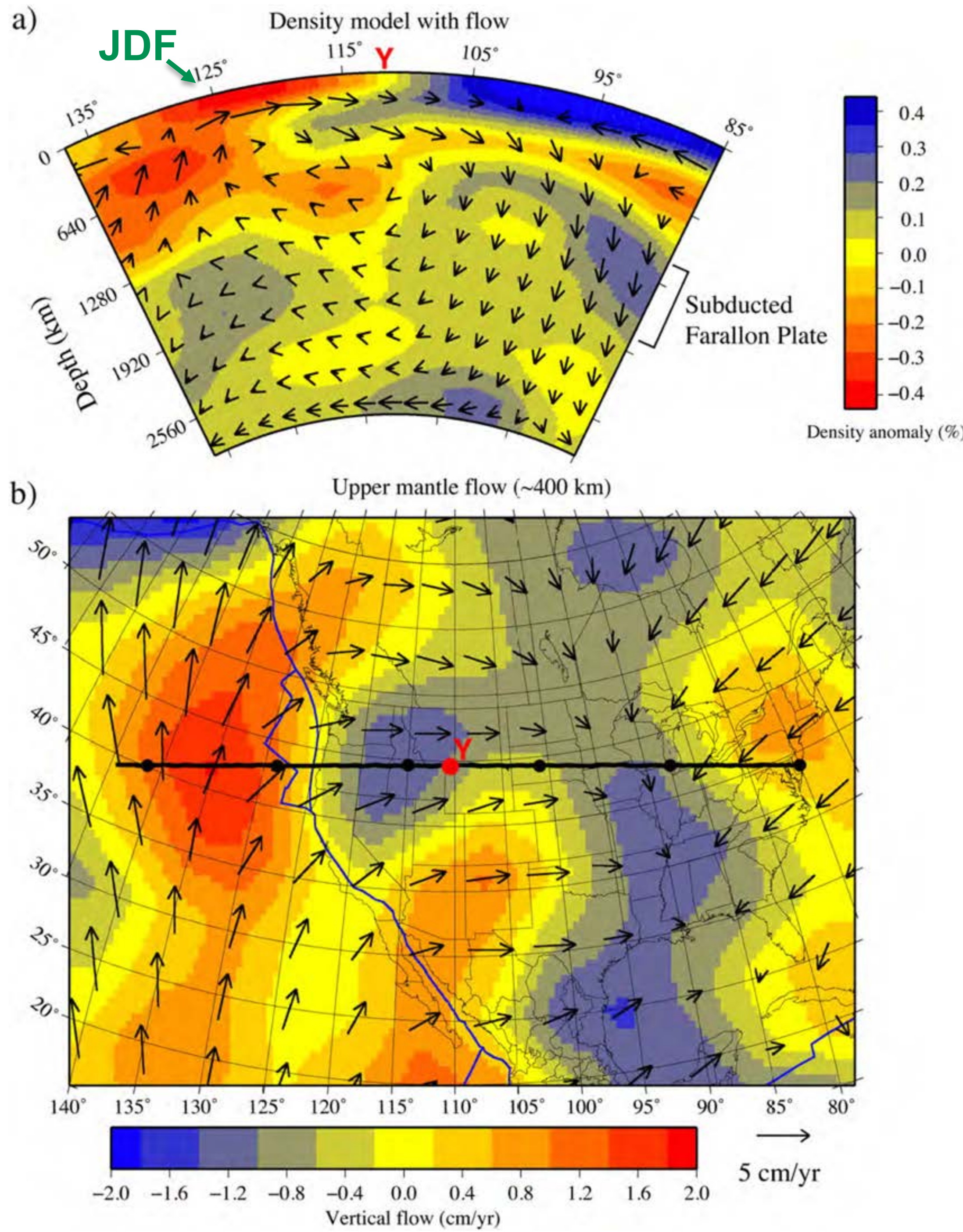


(Conrad & Gurnis, G-cubed, 2003)

Mantle density converted from tomography only (no thermal BLs)

Restores the large-scale flow away from BLs

Misses BL dynamics, such as that associated with subduction (Juan de Fuca).

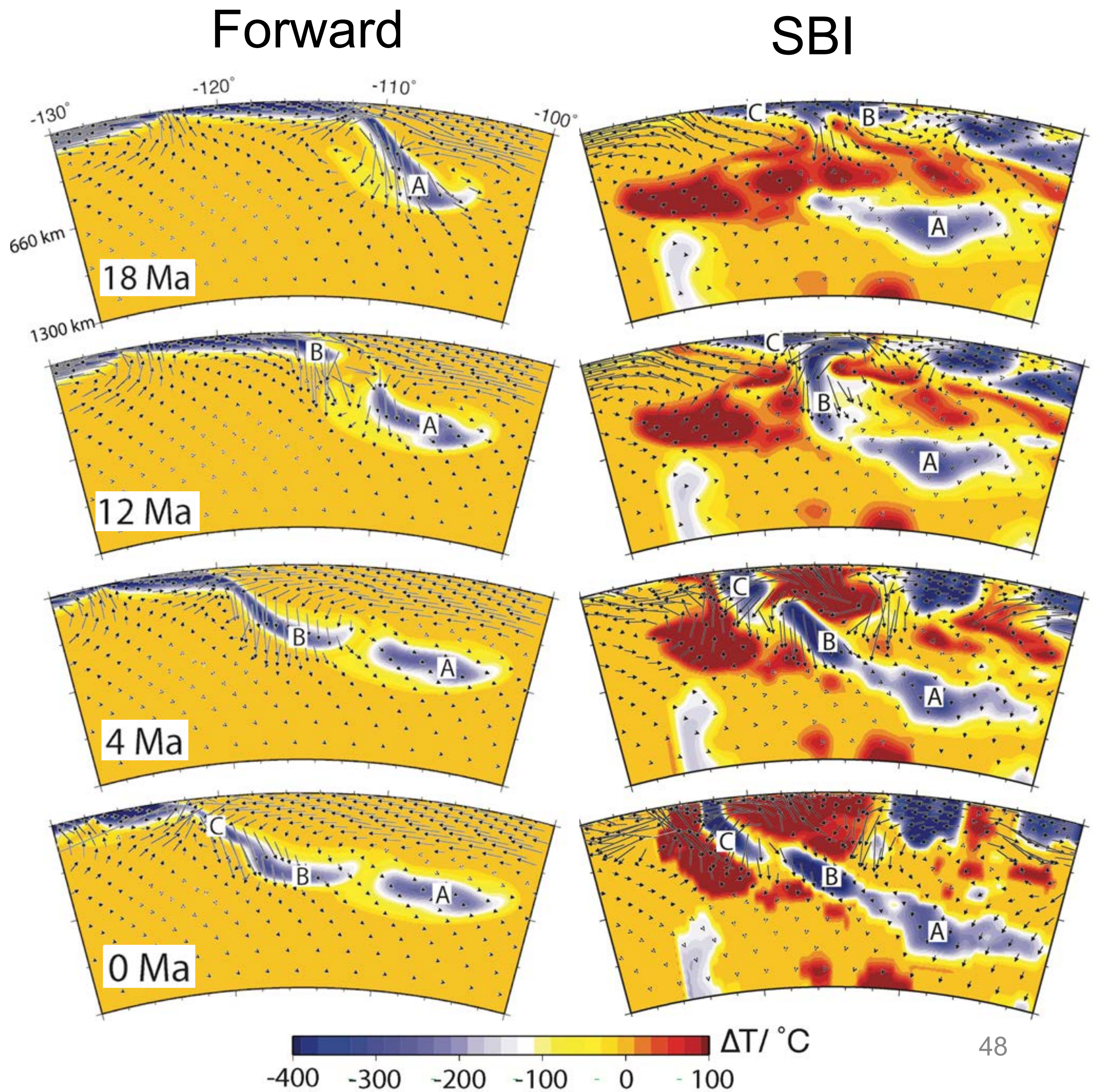


(Smith et al., JVGR, 2009)

Mantle density converted from tomography only (no thermal BLs)

With a more realistic viscosity structure, the slab dynamics is poorly recovered:

- Subduction from upper plate side.
- Progressively separated slab pieces backward.
- Hot mantle distributed everywhere.
- …



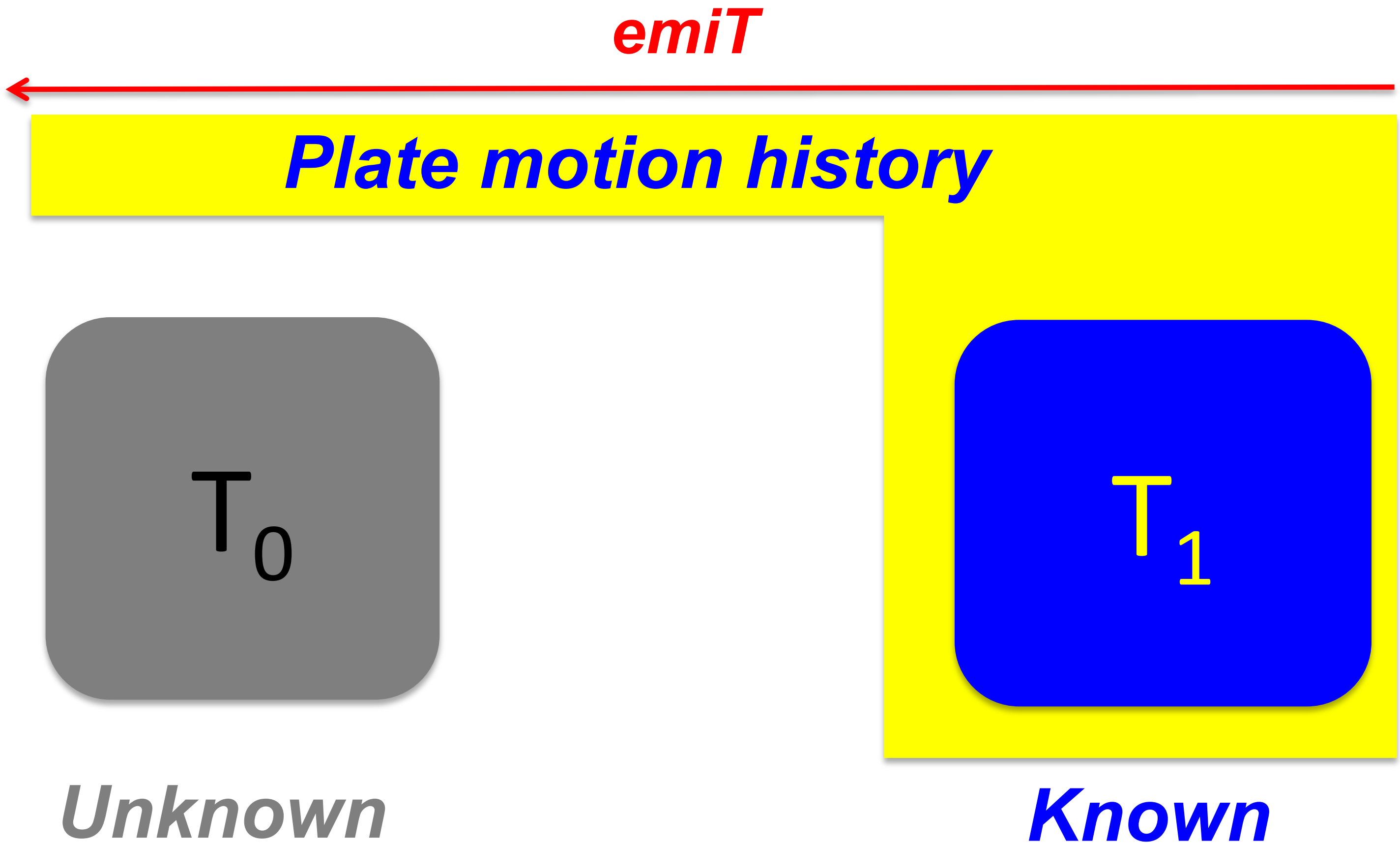
(Zhou & Liu, *G-cubed*, 2009)

Simple backward flow model

- Captures the large-scale mantle flow and surface expression, such as dynamic topography and gravity/geoid.
- Recovery of fine-scale mantle dynamics remains challenging due to the neglect of thermal diffusion and, therefore, processes associated with thermal boundary layers.
- Errors quickly accumulate backward in time.

Inverse simulation of Earth evolution

--- with the adjoint method



Inverse simulation of Earth evolution

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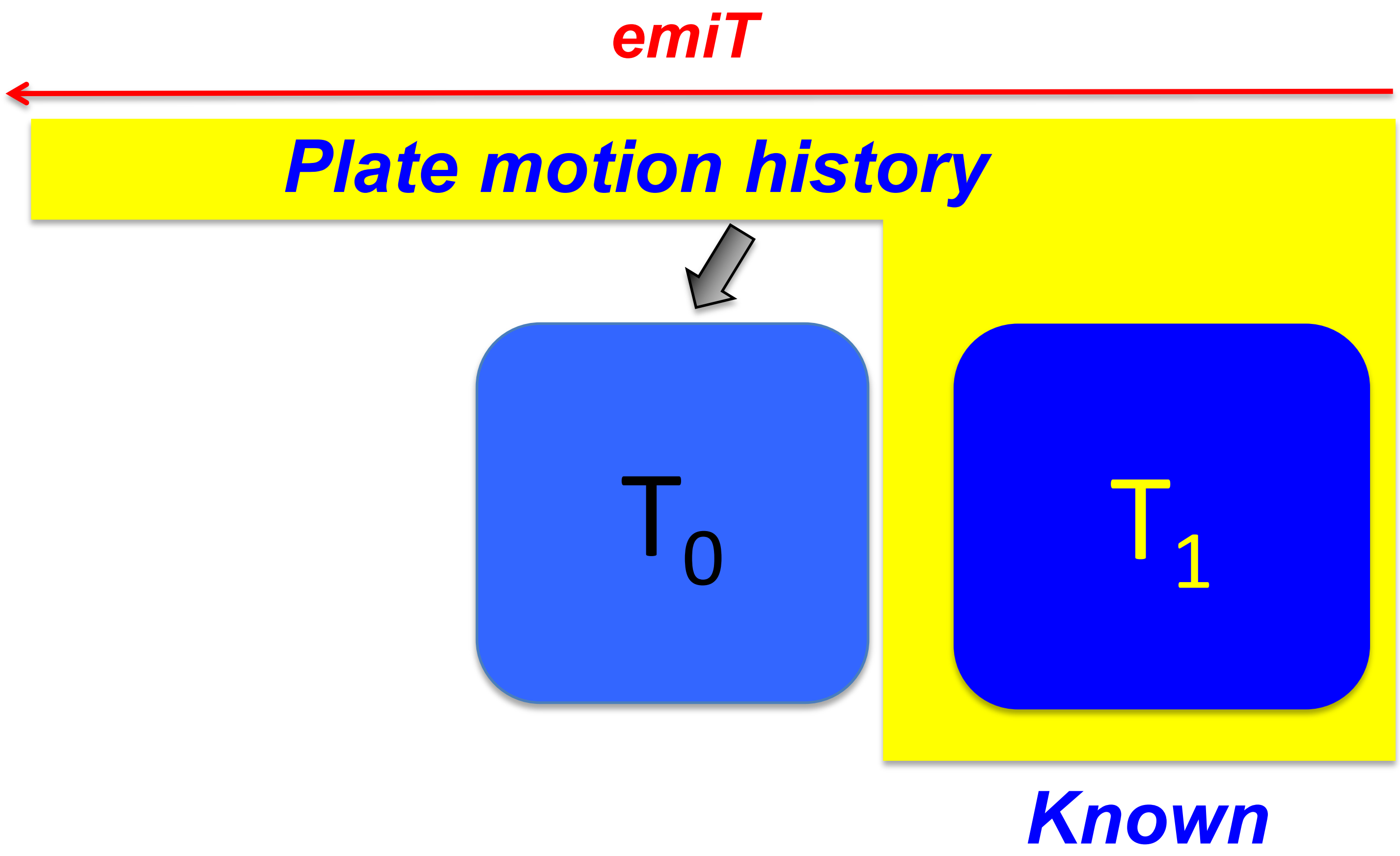

Inverse simulation of Earth evolution

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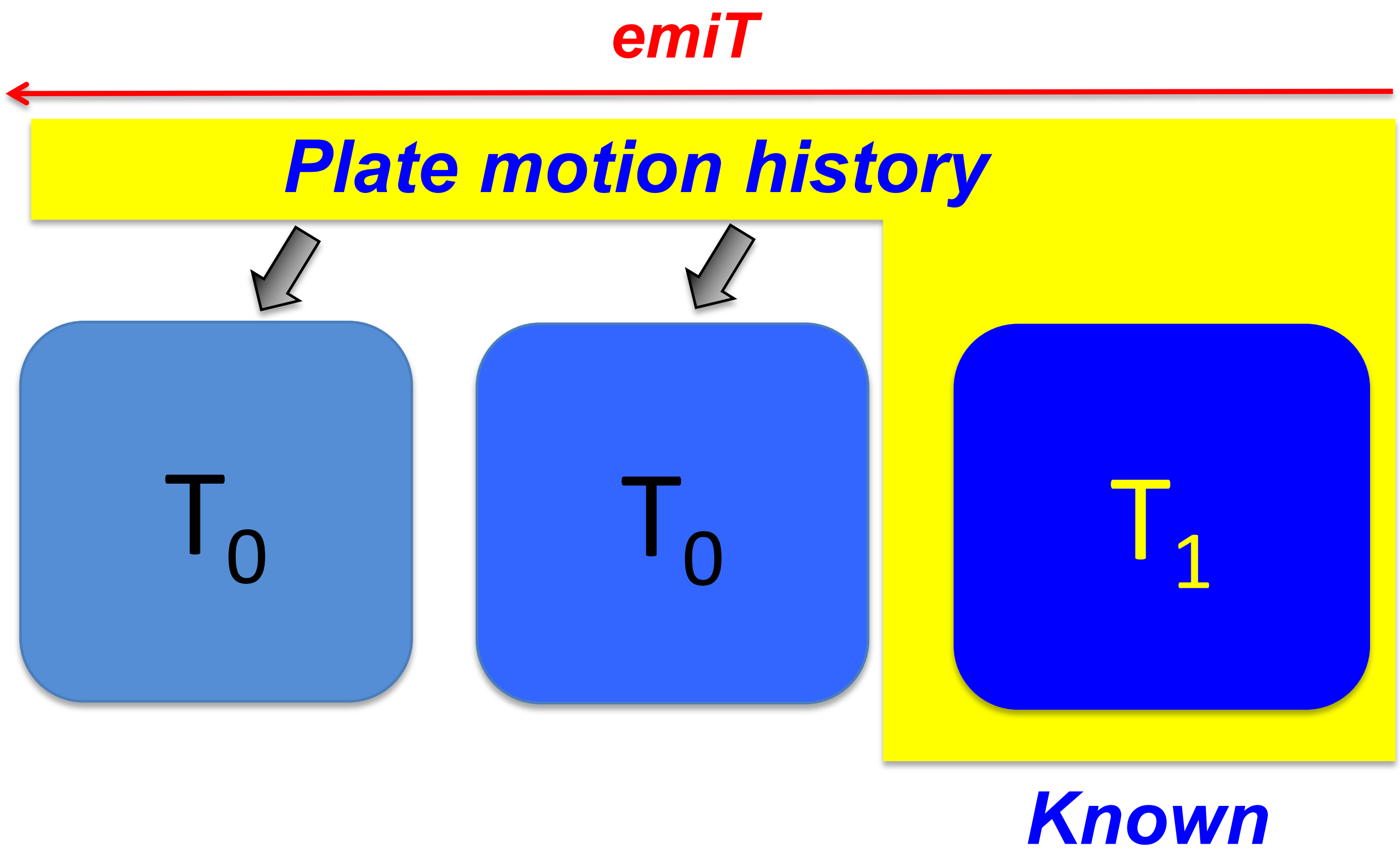


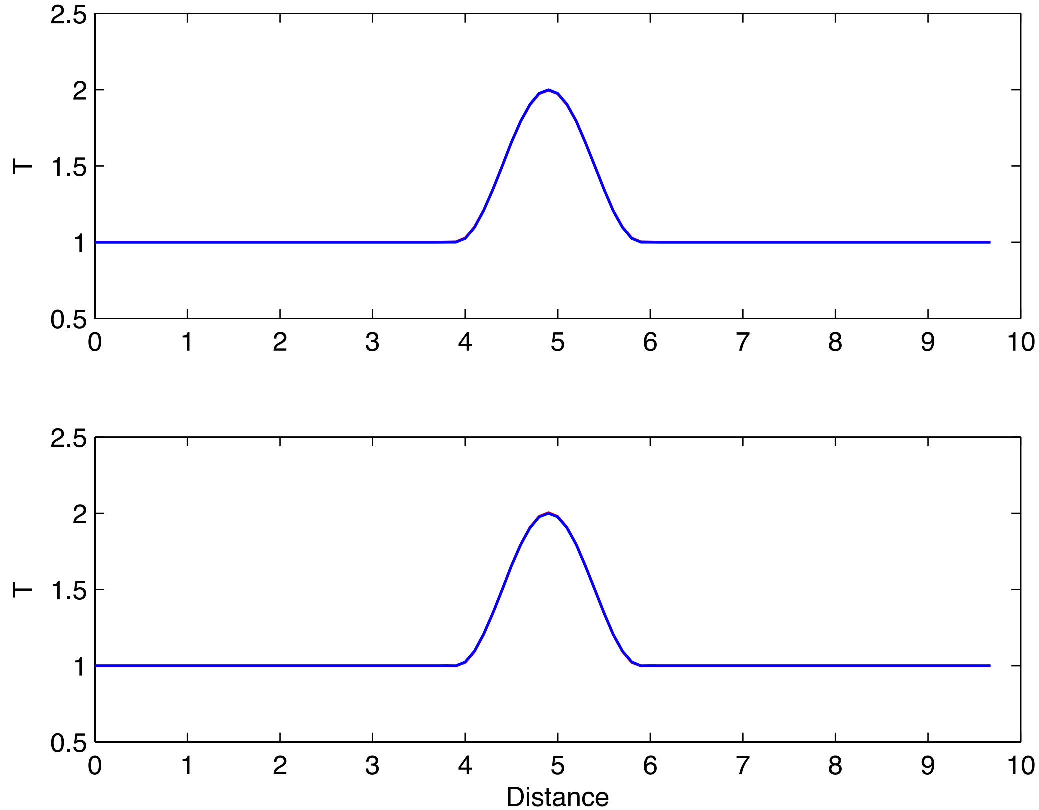
Can we easily reverse mantle evolution?

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \kappa \nabla^2 T$$

Thermal diffusion

Positive time

Negative time



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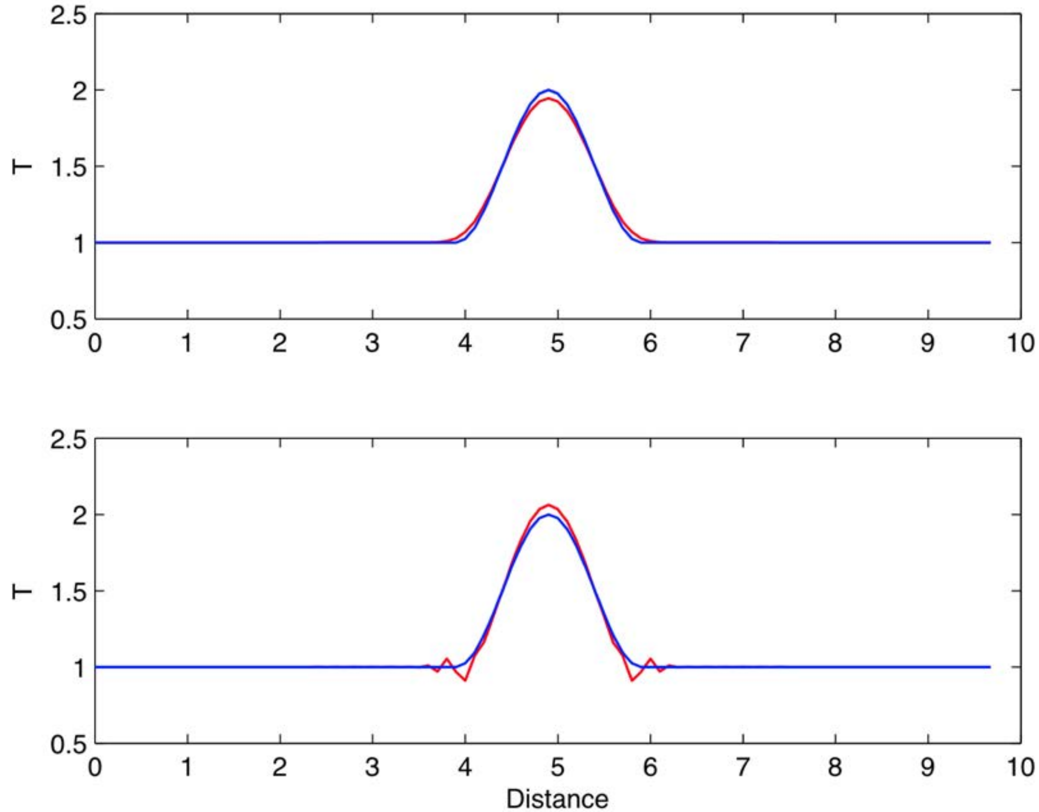
Positive time

Negative time



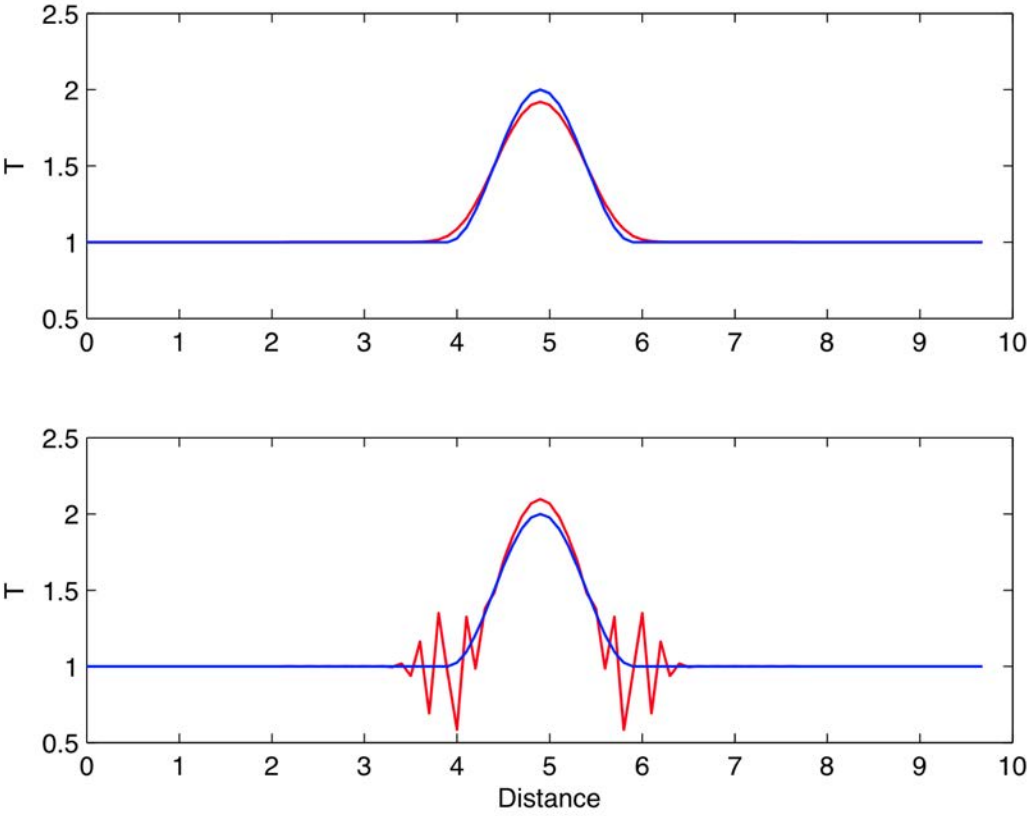
Can we easily reverse mantle evolution?

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \kappa \nabla^2 T$$

Thermal diffusion

Positive time

Negative time



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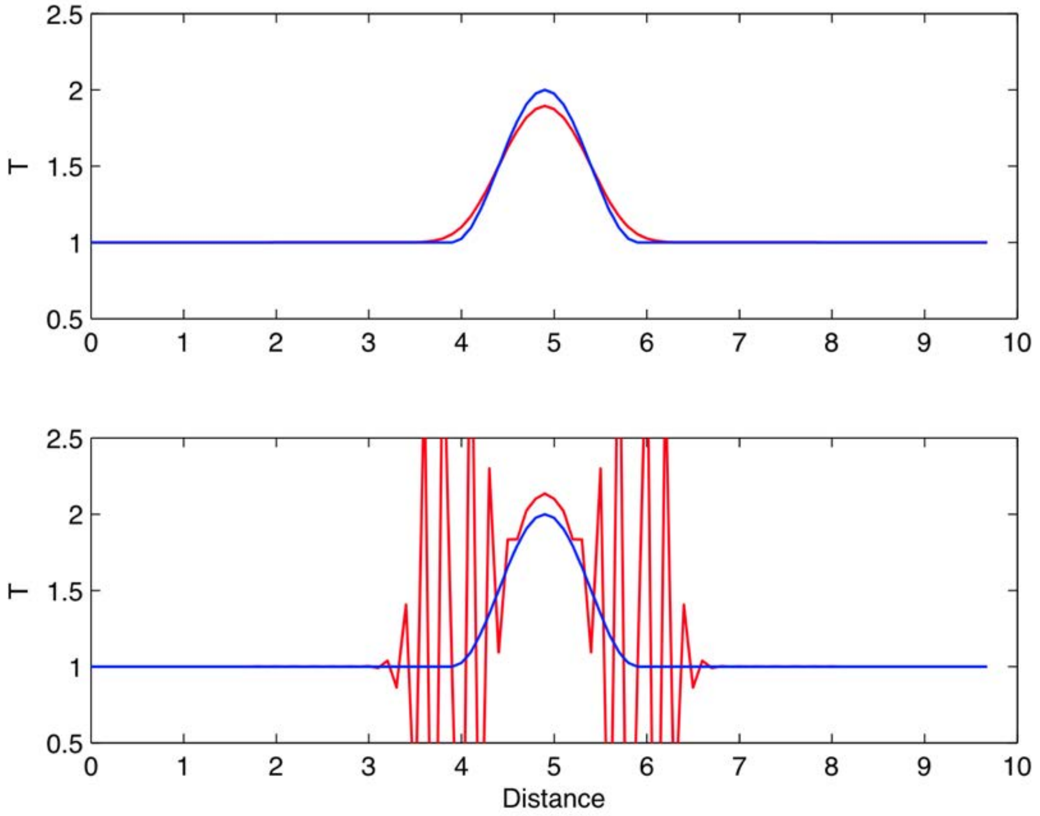
Negative time

Thermal diffusion



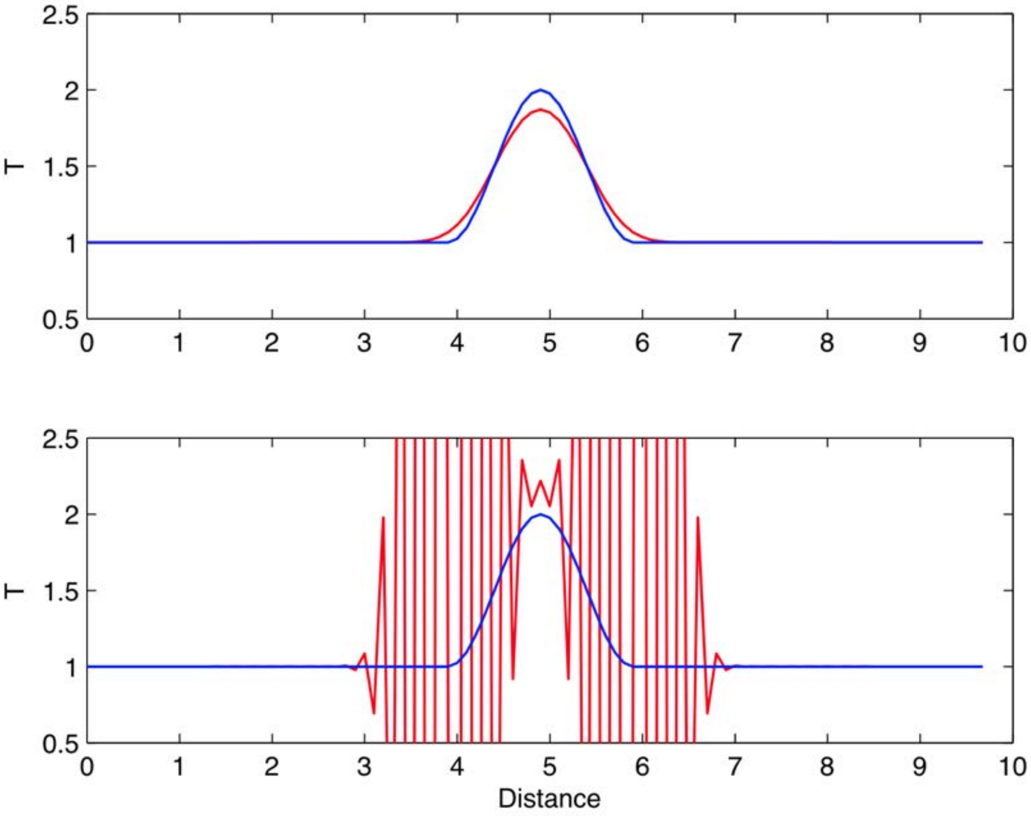
Can we easily reverse mantle evolution?

$$\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T = \kappa \nabla^2 T$$

Positive time

Negative time

Thermal diffusion



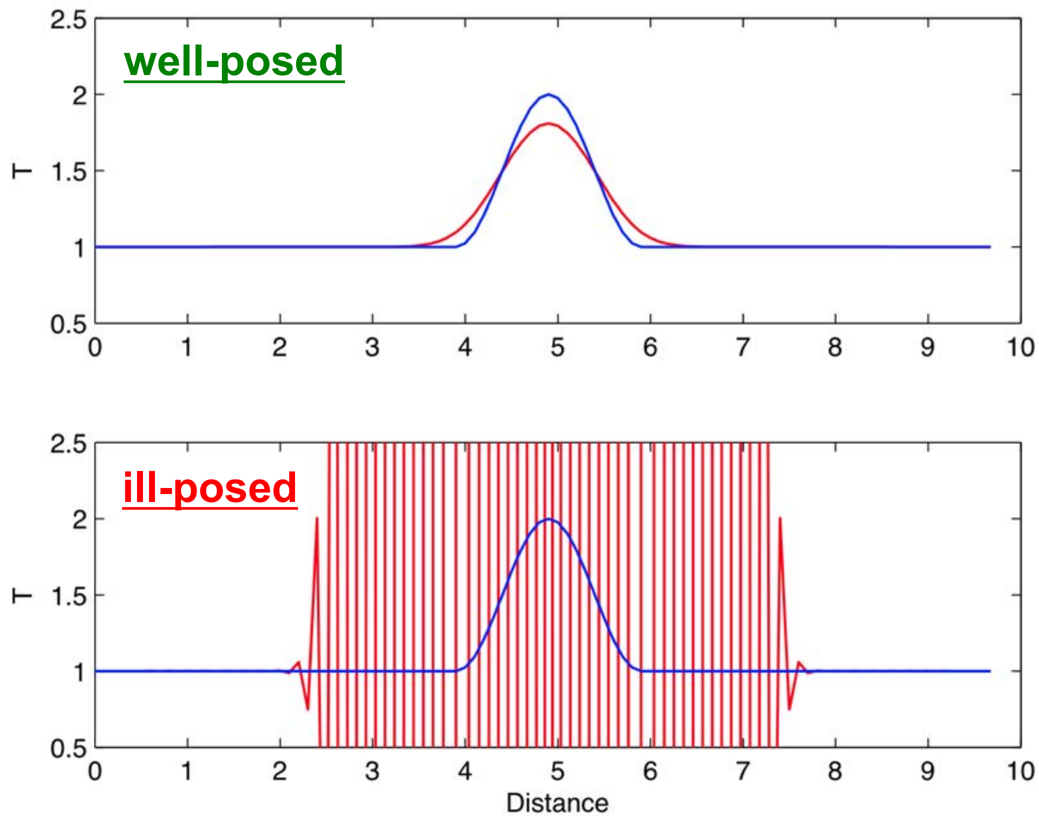
Can we easily reverse mantle evolution?

Mathematically & physically impossible!

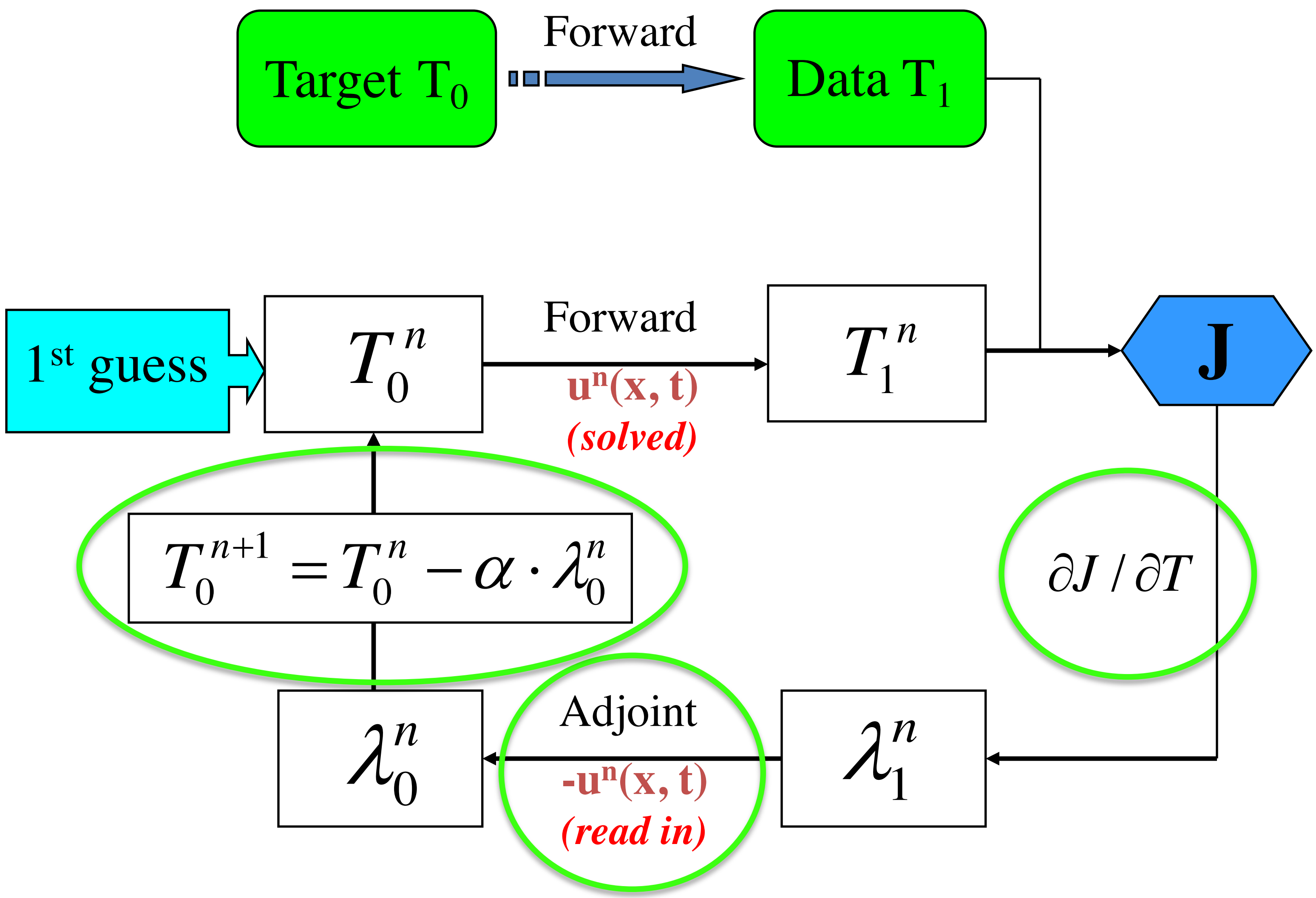
Thermal diffusion

Positive time

Negative time



Adjoint algorithm



The idea of adjoint: $dJ = (\partial J / \partial \vec{a}) d\vec{a}$ (e.g. R. Errico, 1997)

$$J = \int_t \int_{\vec{x}} (T_p - T_d)^2 d\vec{x} dt \cdot \delta(t - t_1)$$ ***(cost function)***

$d\vec{a}$: error in the initial; T_p: prediction; T_d: data

Lagrange function:

$$L = J + \iint_{t,\vec{x}} \lambda\left(\partial T / \partial t + \vec{u} \cdot \nabla T - \kappa \nabla^2 T - H\right) d\vec{x} dt$$

$$\delta L = \delta J + \iint_{t,\vec{x}} \lambda\left(\partial \delta T / \partial t + \vec{u} \cdot \nabla \delta T - k \nabla^2 \delta T\right) d\vec{x} dt$$

where λ is the adjoint quantity

Adjoint Equation: integration by part and let $\delta L = 0$

$$2(T_p - T_d) \cdot \delta(t - t_1) = \frac{\partial \lambda}{\partial t} + \vec{u} \cdot \nabla \lambda + \kappa \nabla^2 \lambda$$

$$\frac{\delta J (t = t_0)}{\delta T} = -\lambda(t = t_0)$$

Iterative adjoint solver

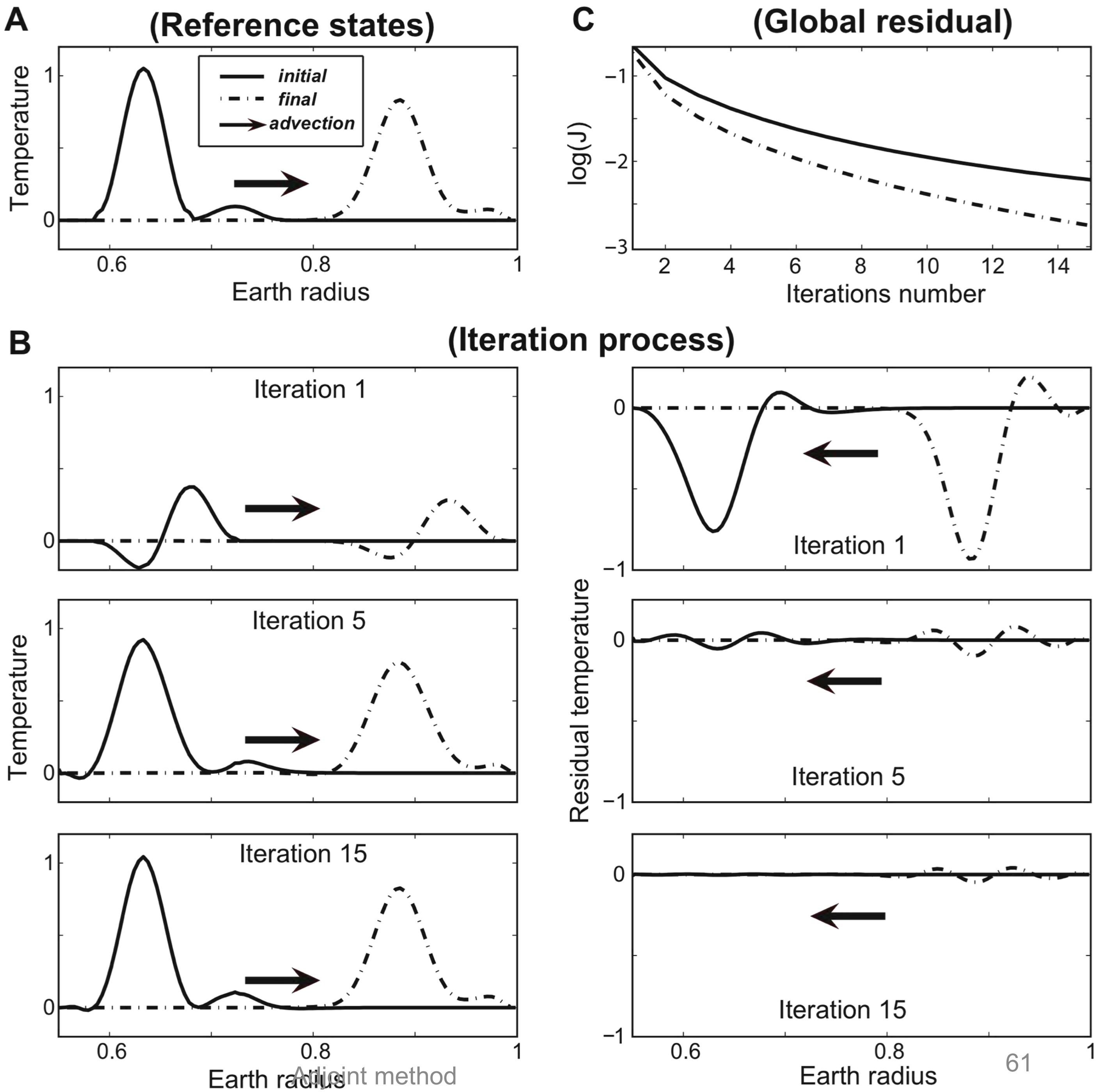
Unconditionally converging for a kinematic (linear) problem.

$$\frac{\partial T}{\partial t} + \boxed{\vec{u} \cdot \nabla T} = \kappa \nabla^2 T$$

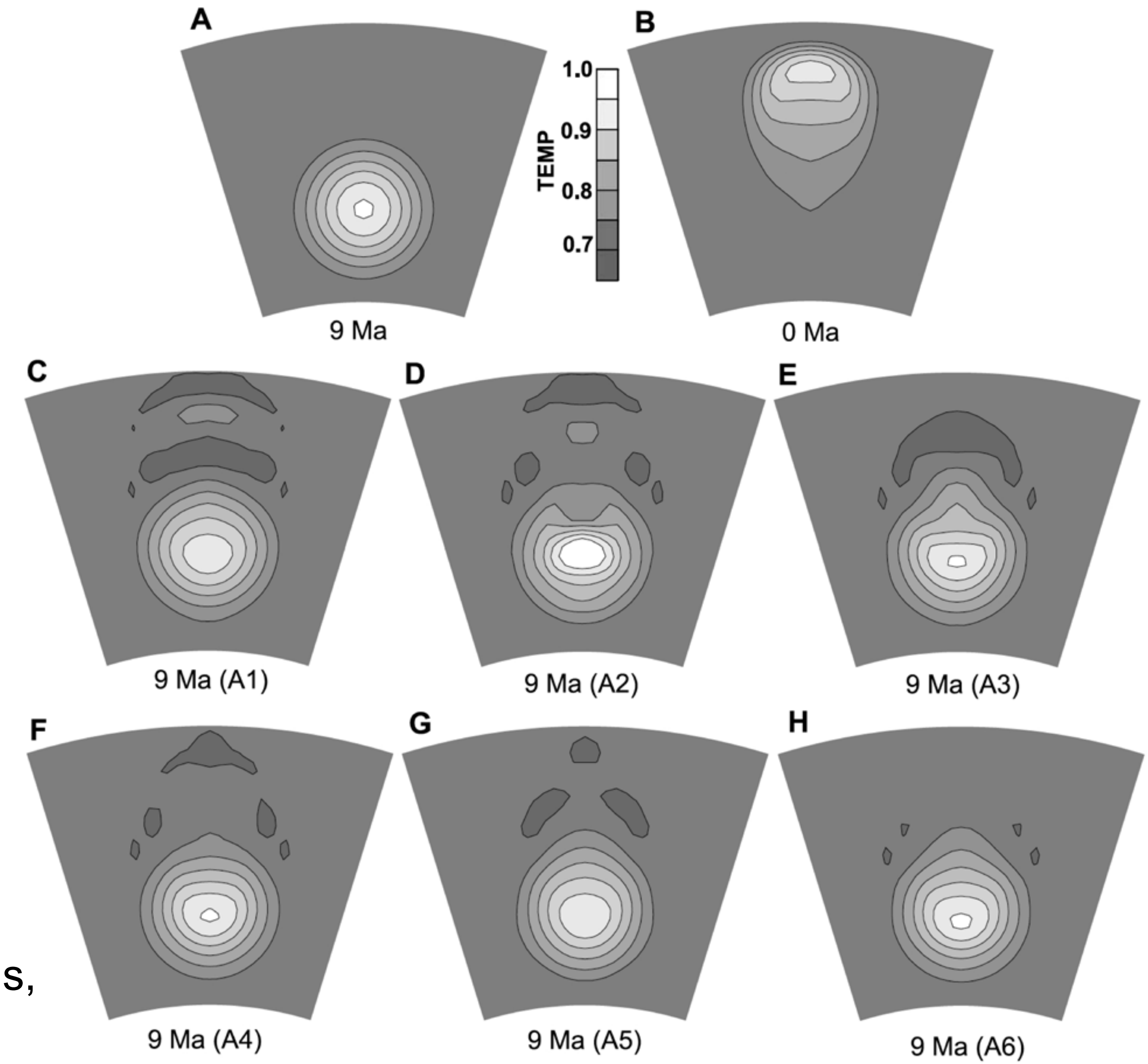
non-linear

First guess matters!

[Liu & Gurnis, *Ear. Sci. Front.* 2010]

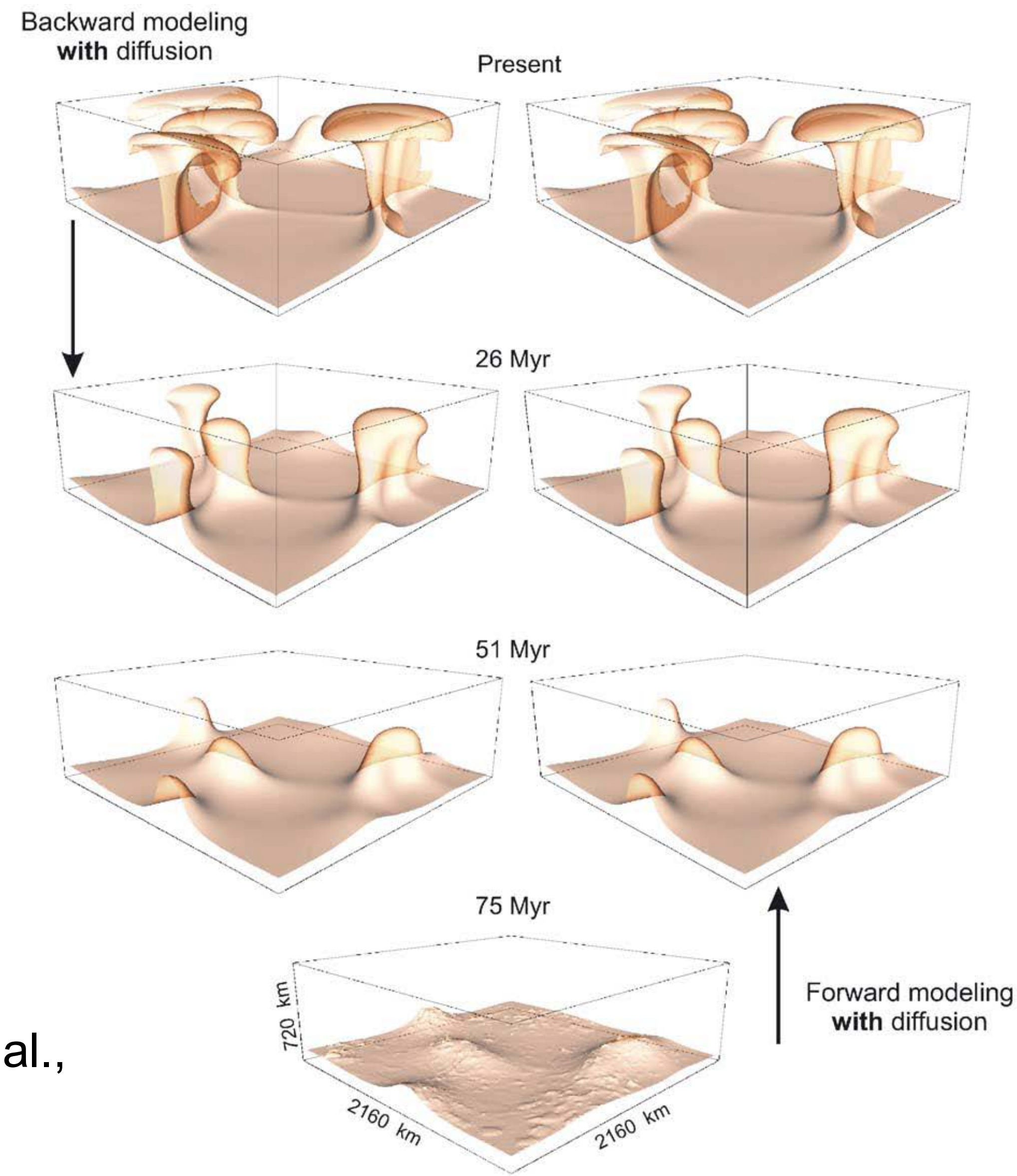


An optimal starting initial condition is necessary



(Liu & Gurnis, *JGR*, 2008)

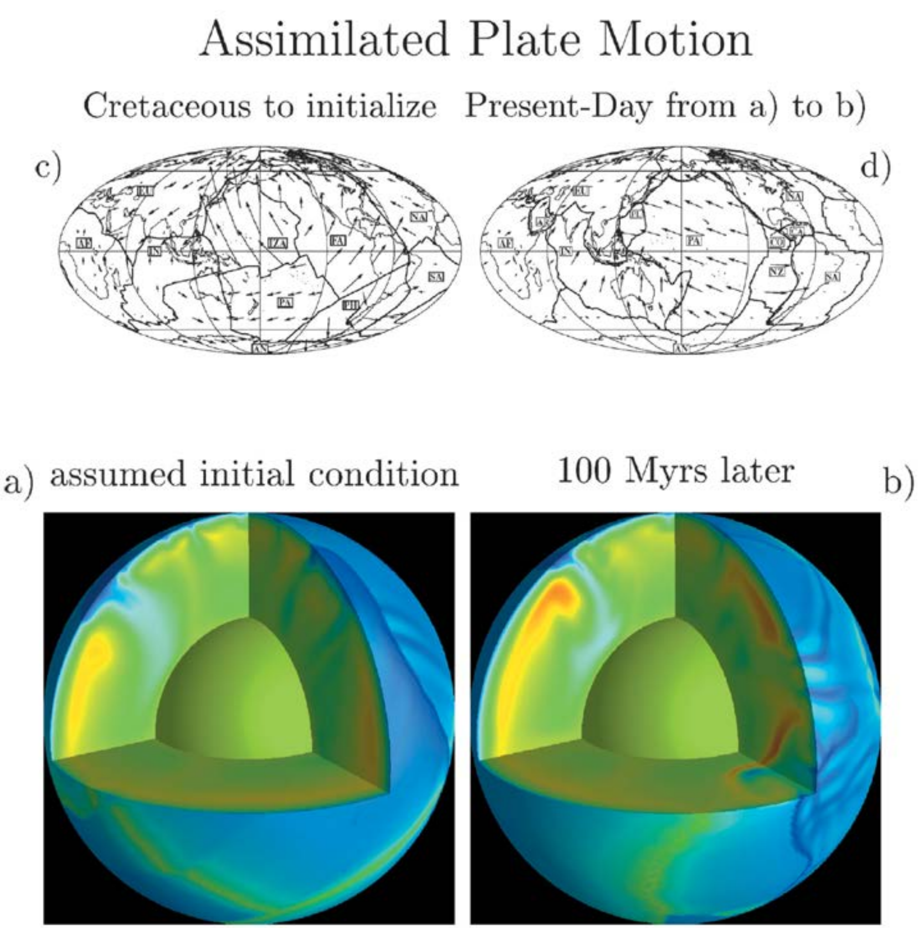
An optimal starting initial condition is necessary



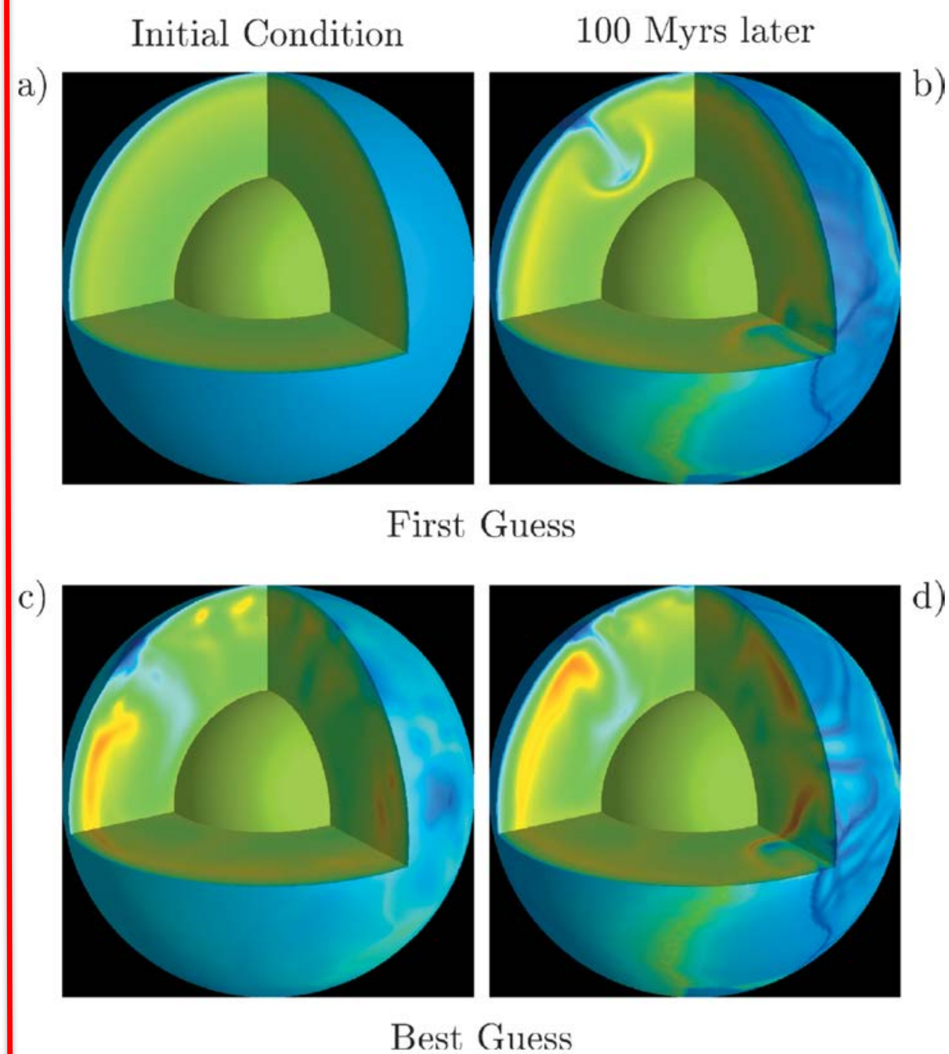
(Ismail-Zadeh et al., *PEPI*, 2004)

Known surface kinematics help the convergence

(Reference states)

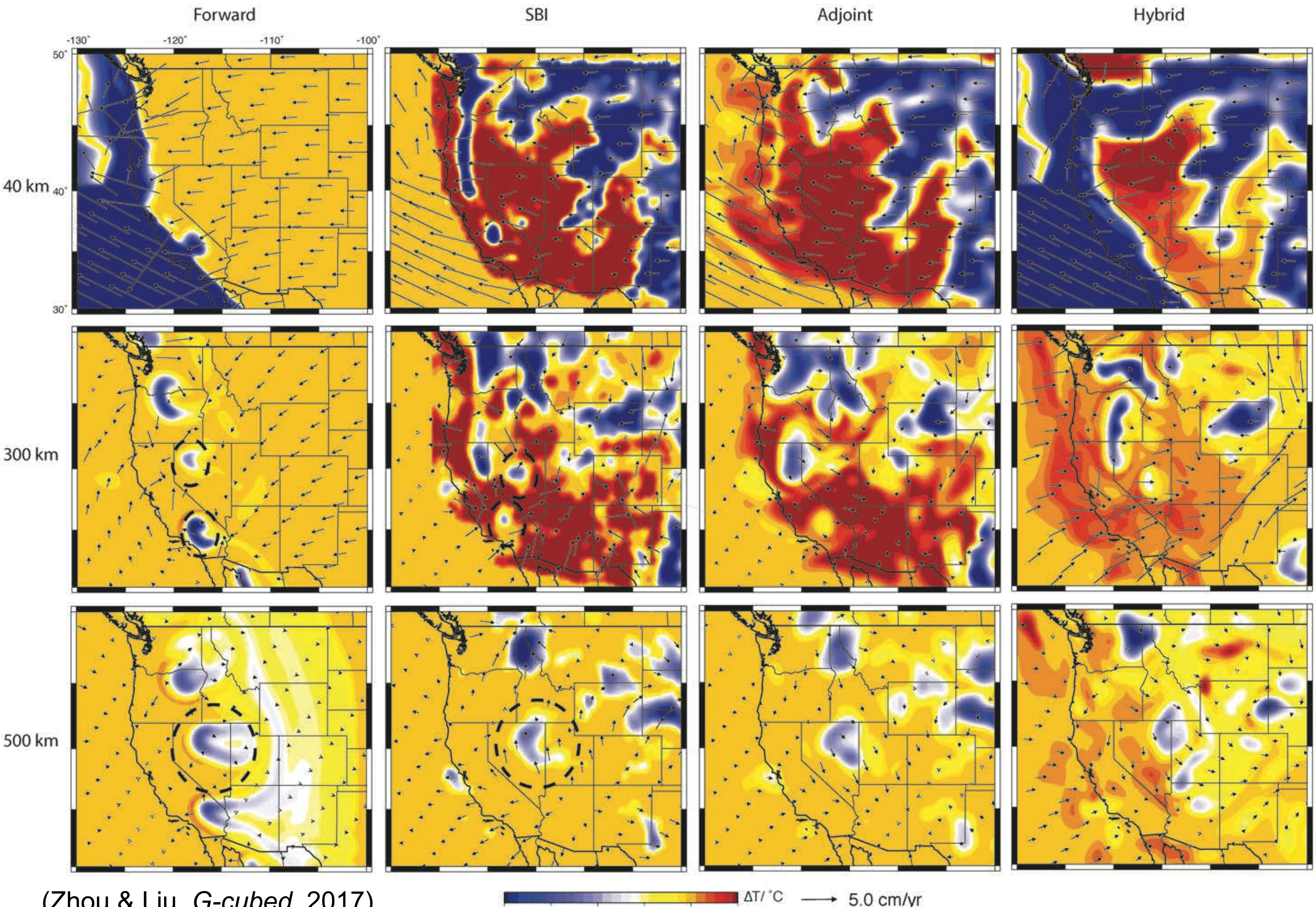


(Bunge et al., *GJI*, 2003)



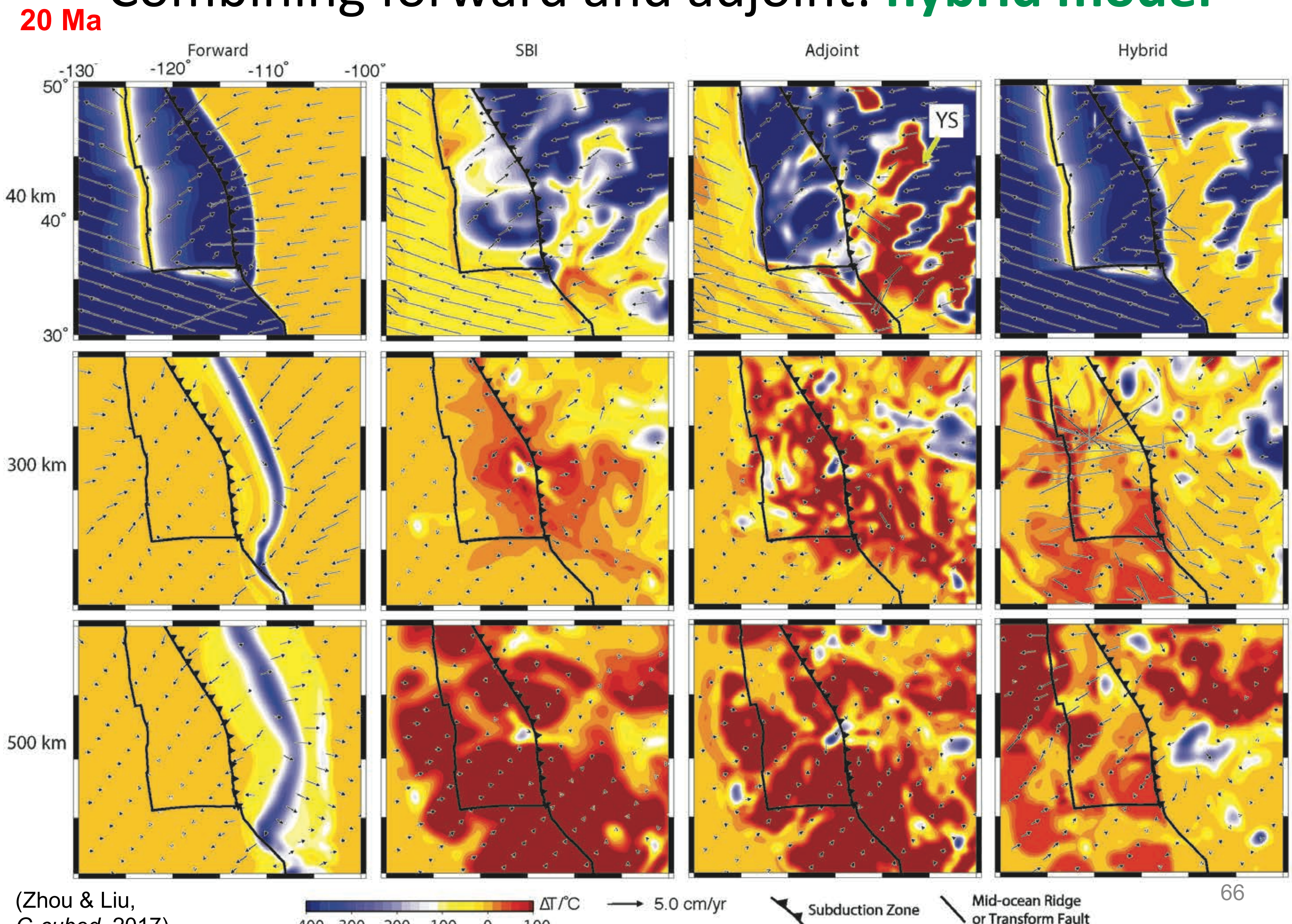
Combining forward and adjoint: **hybrid model**

0 Ma



(Zhou & Liu, *G-cubed*, 2017)

Combining forward and adjoint: hybrid model



(Zhou & Liu, *G-cubed*, 2017)

Comparison of various data assimilation models

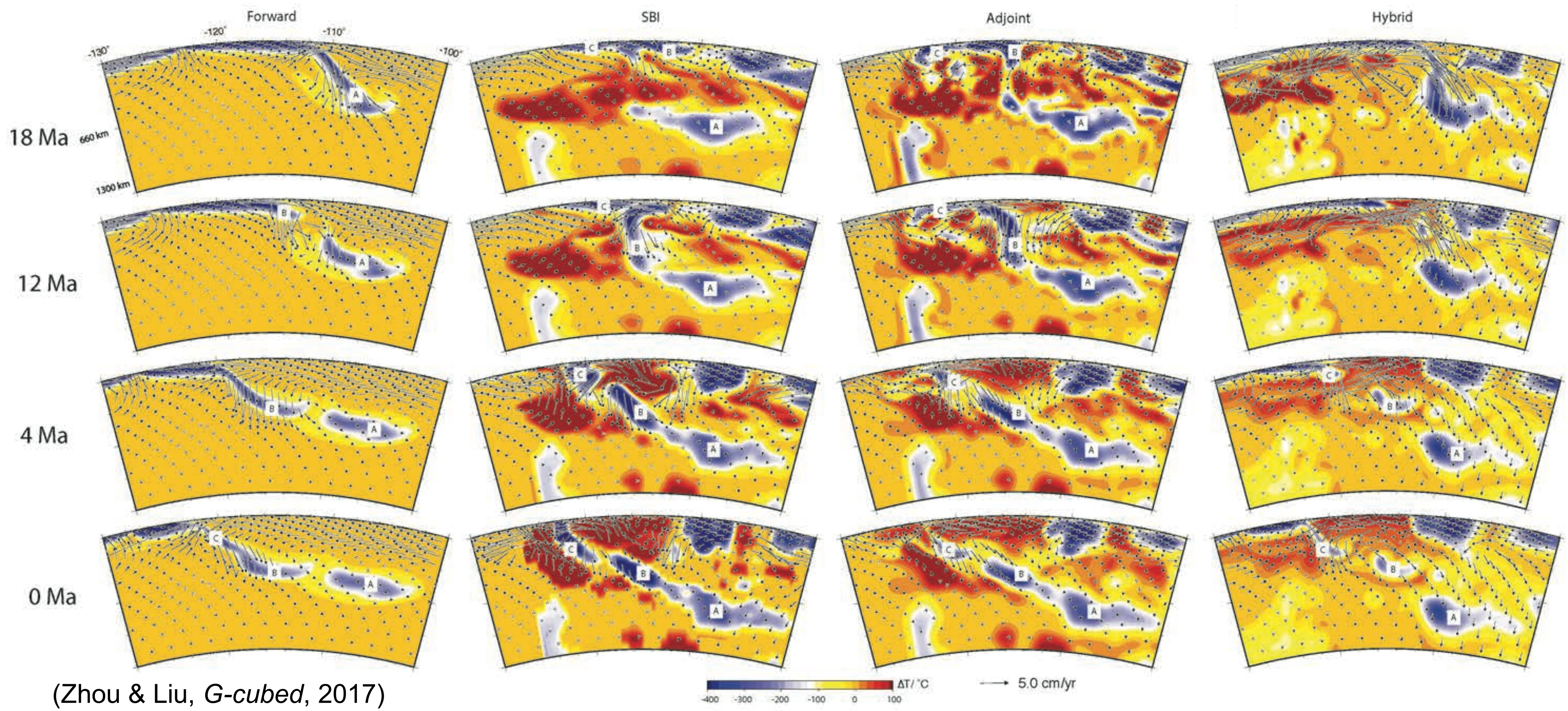
Resulting in different temporal histories

Realistic slab behavior, but many structures are missing

Anchored slab not recovered, strange evolution of slab & hot mantle

Unrealistic slab below the MOR; hot mantle persists below continent

Realistic recovery of slab and hot mantle dynamics



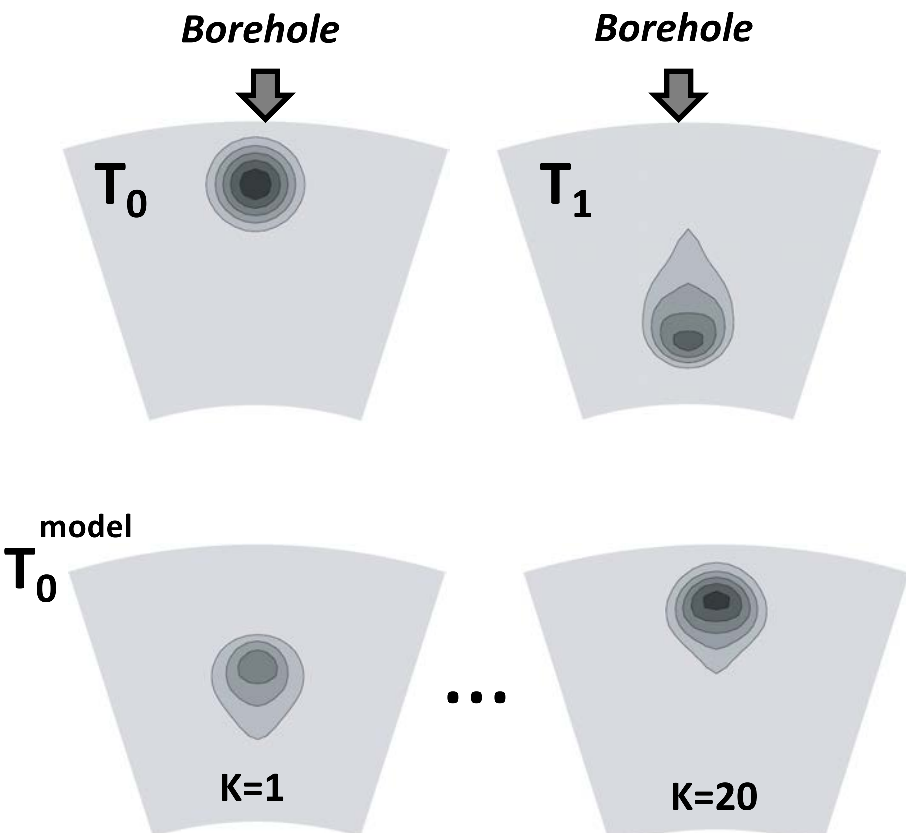
(Zhou & Liu, *G-cubed*, 2017)

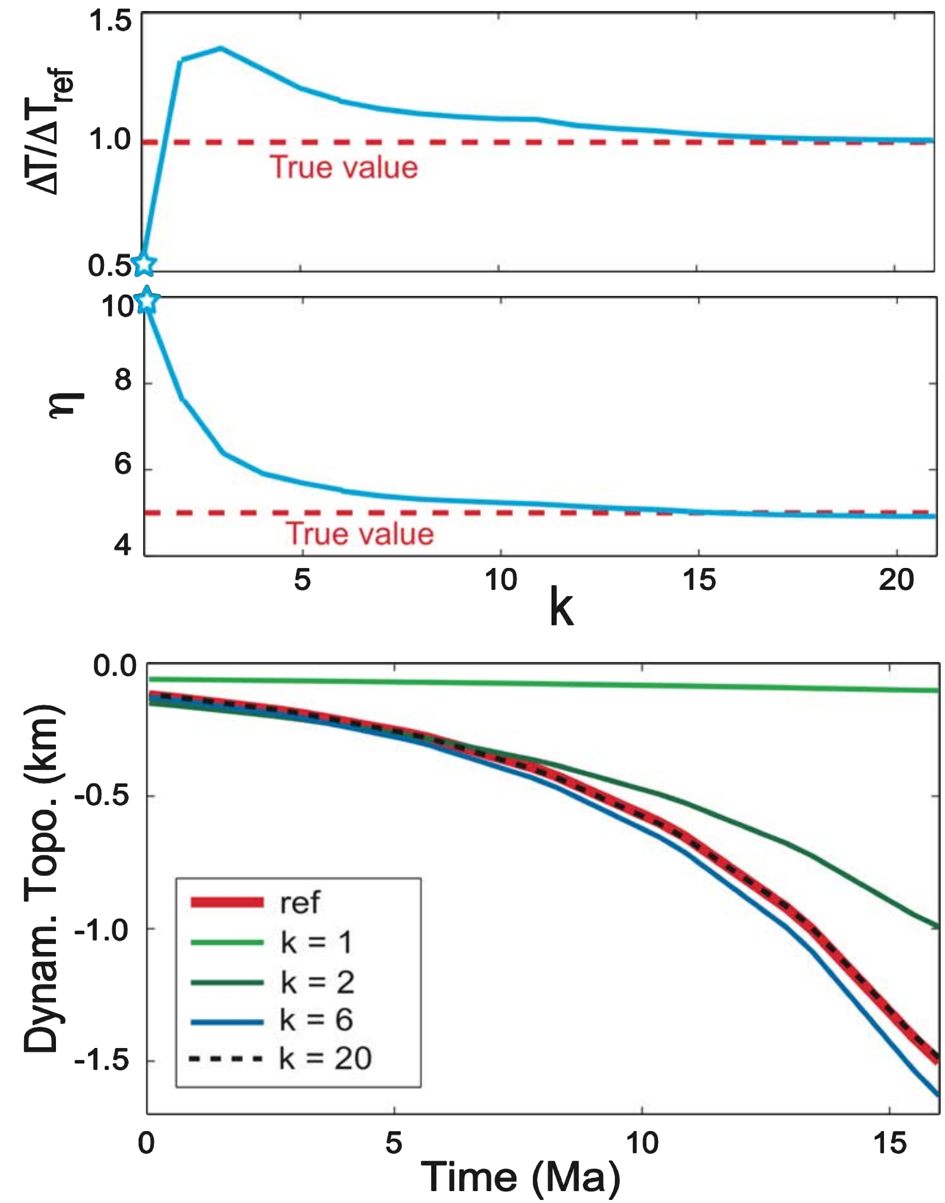
However, all above inverse algorithms

- Work only for idealized mantle scenarios.
- Their application to real Earth suffers from uncertain model parameters:
  - Density amplitude inferred from tomography
  - Mantle viscosity structure
- These uncertainties could result in drastically different initial conditions and associated surface responses!

Uncertain parameters need to be constrained

- *Using dynamic topography*



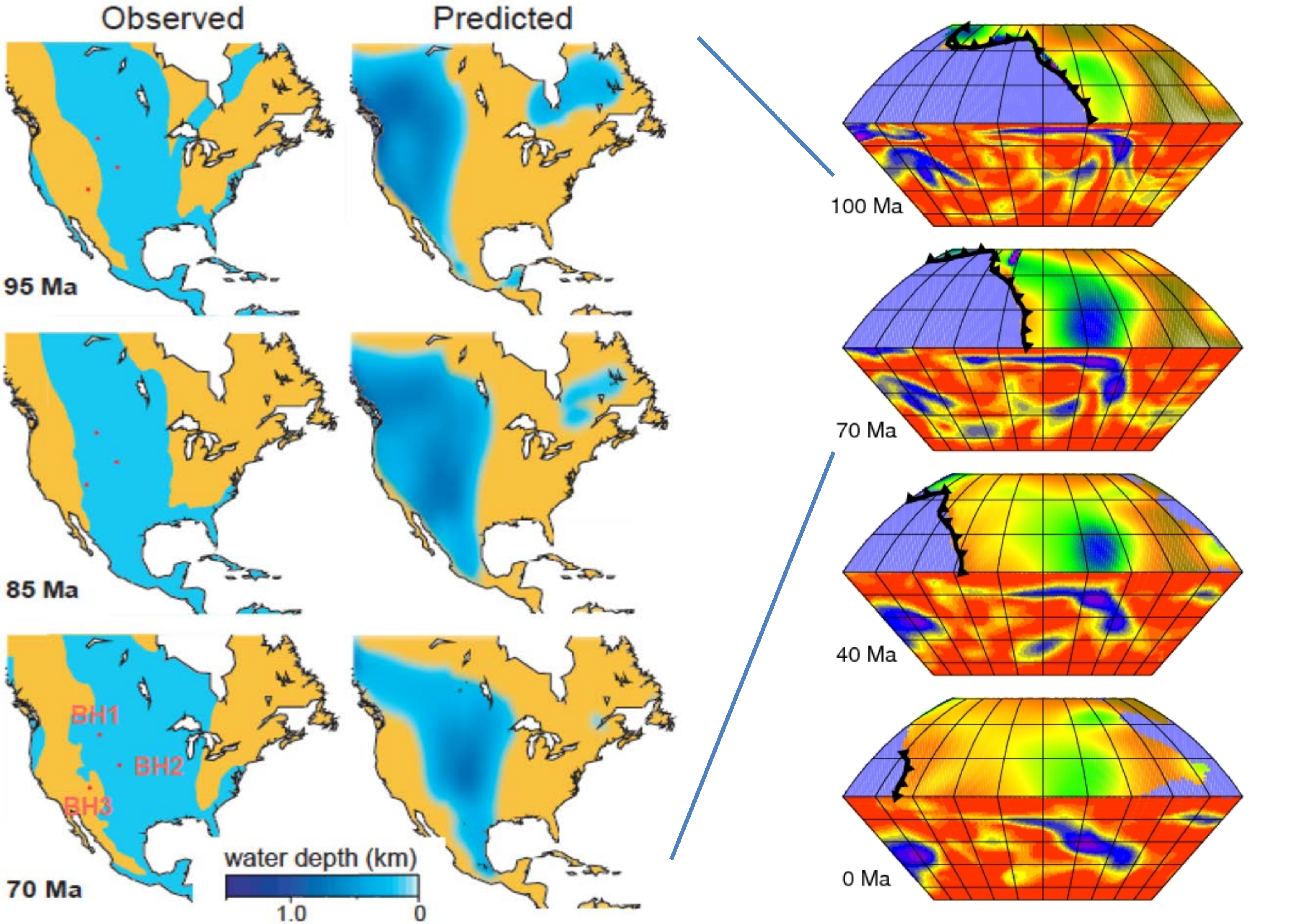


[Liu& Gurnis, *JGR*, 2008]

Outline

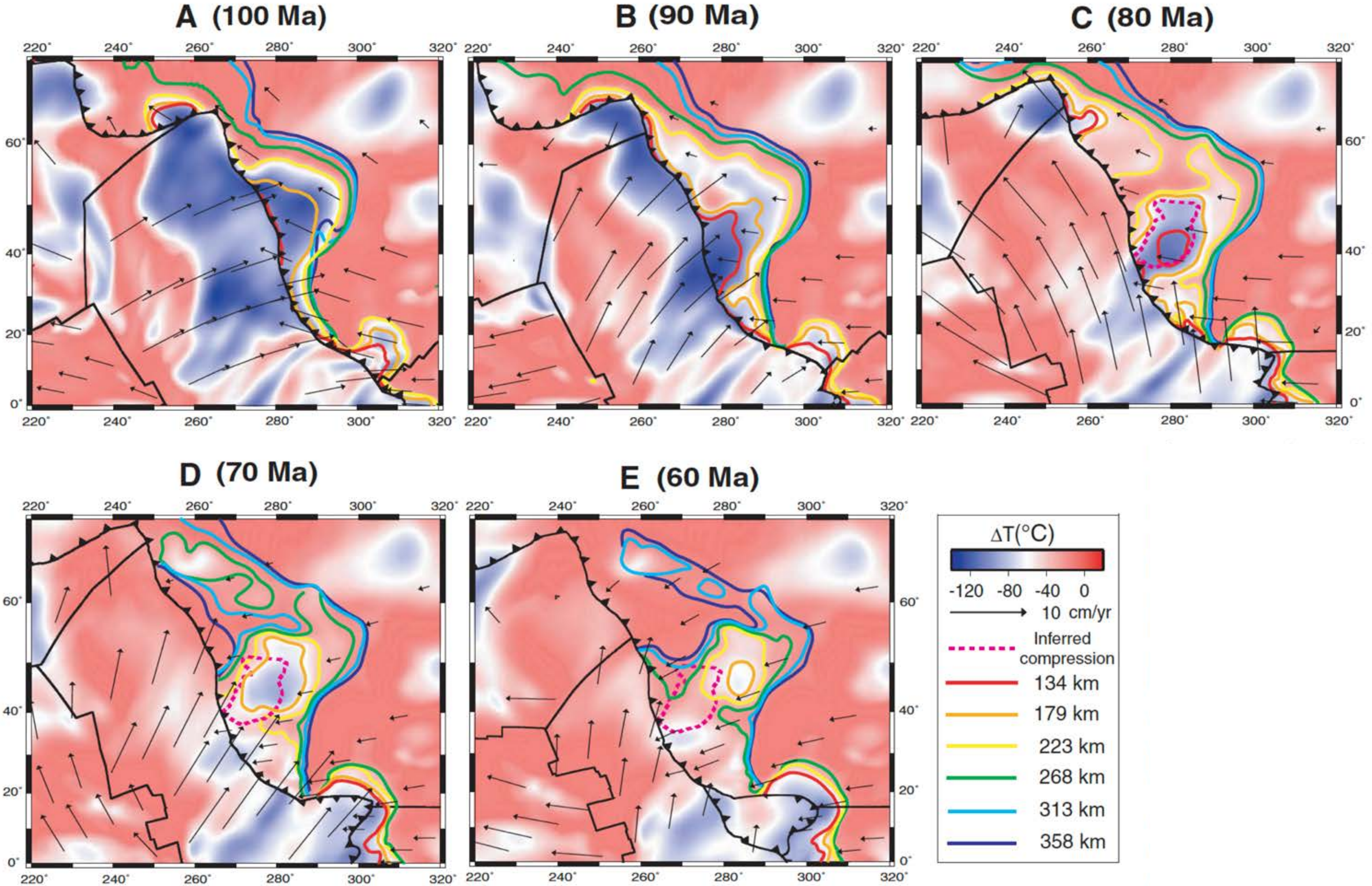
- Why do and what is data-oriented modeling?
- Different approaches of data assimilation and their evolution
- **Examples of inverse data-assimilation models**

Application of the **adjoint** method



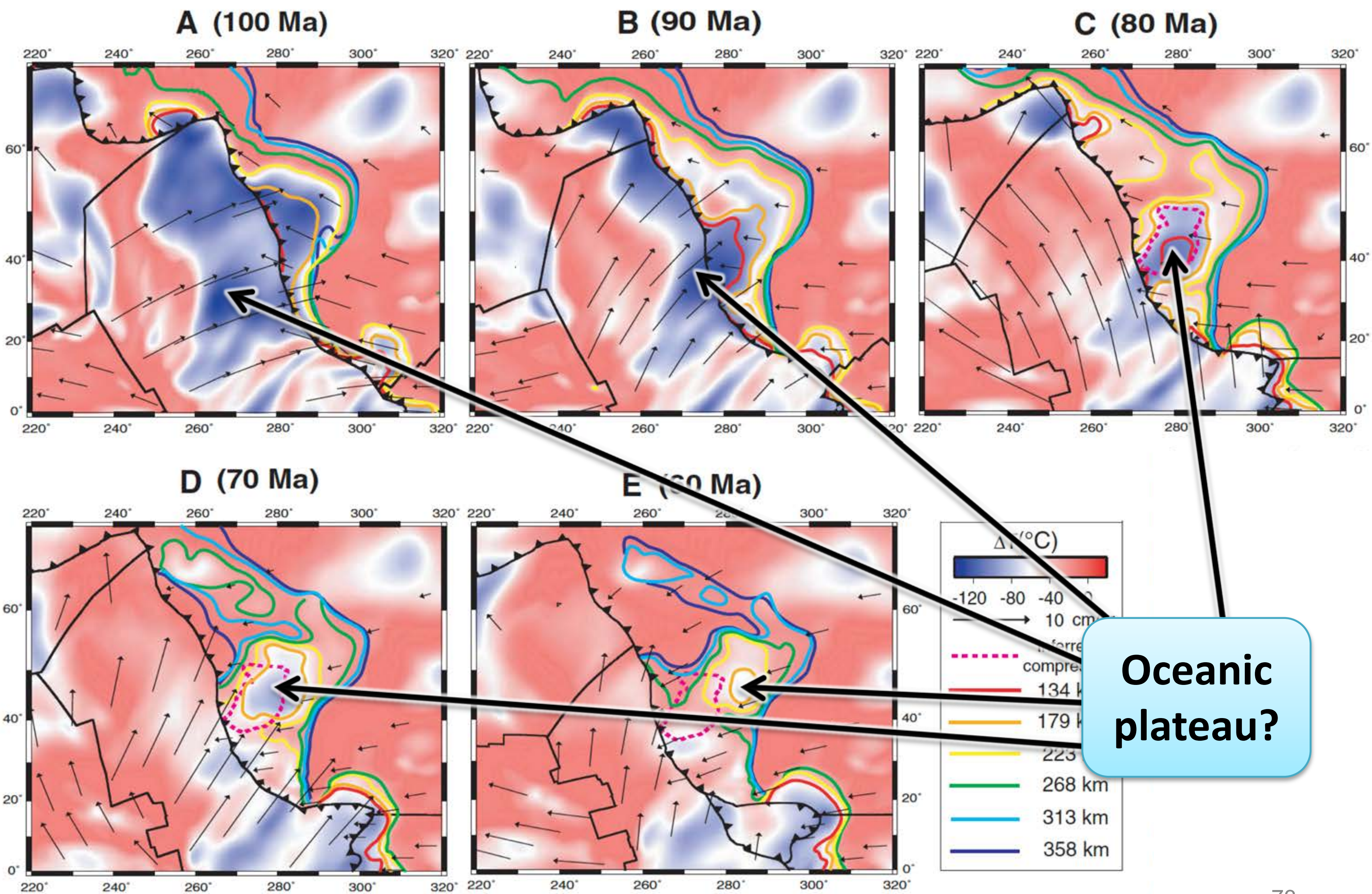
(Liu et al., *Science,* 2008)

Reconstructed a Cretaceous flat slab



(Liu et al., *Science,* 2008)

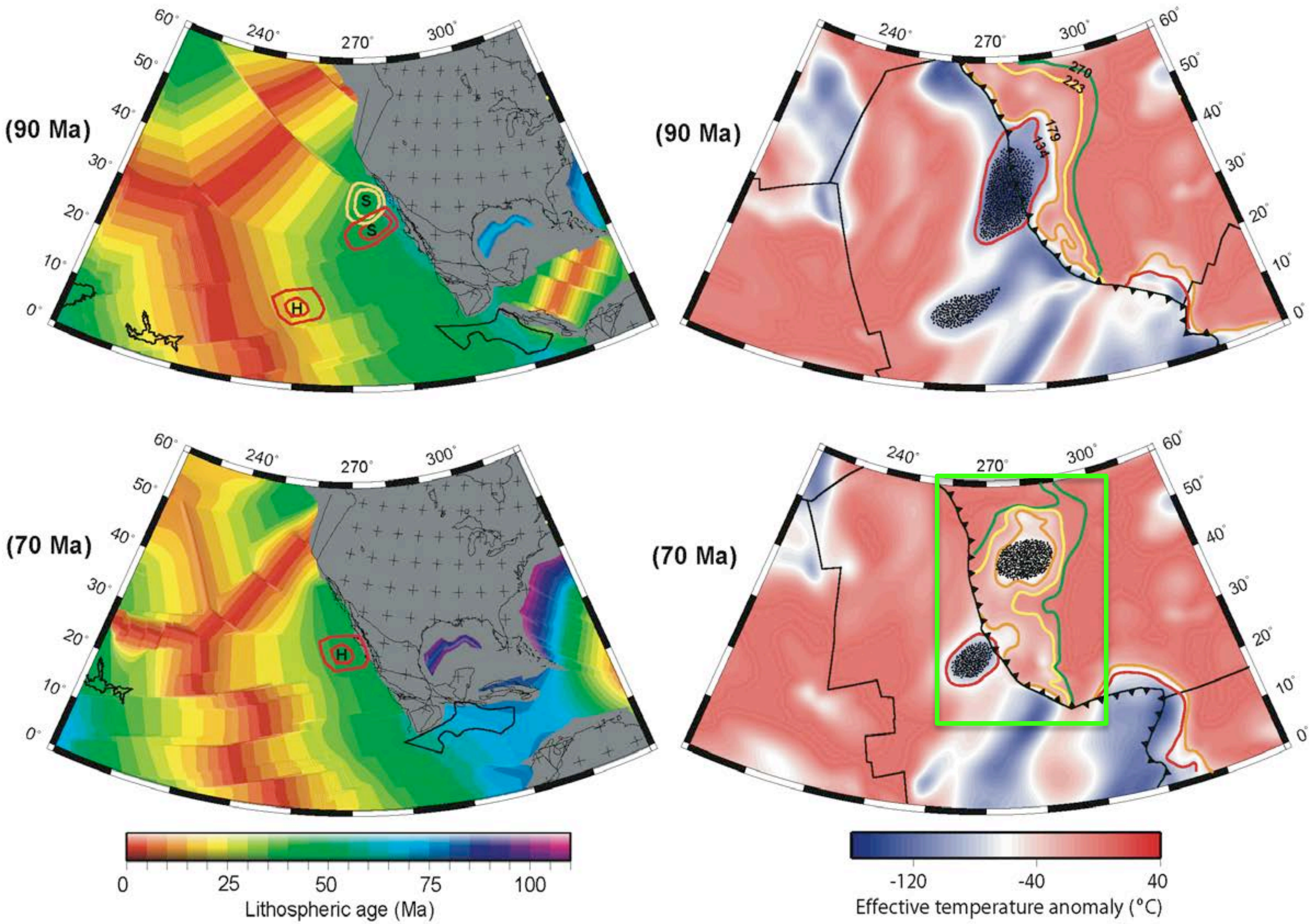
What caused the flat subduction?



(Liu et al., *Science*, 2008)

Data assimilation models make testable predictions

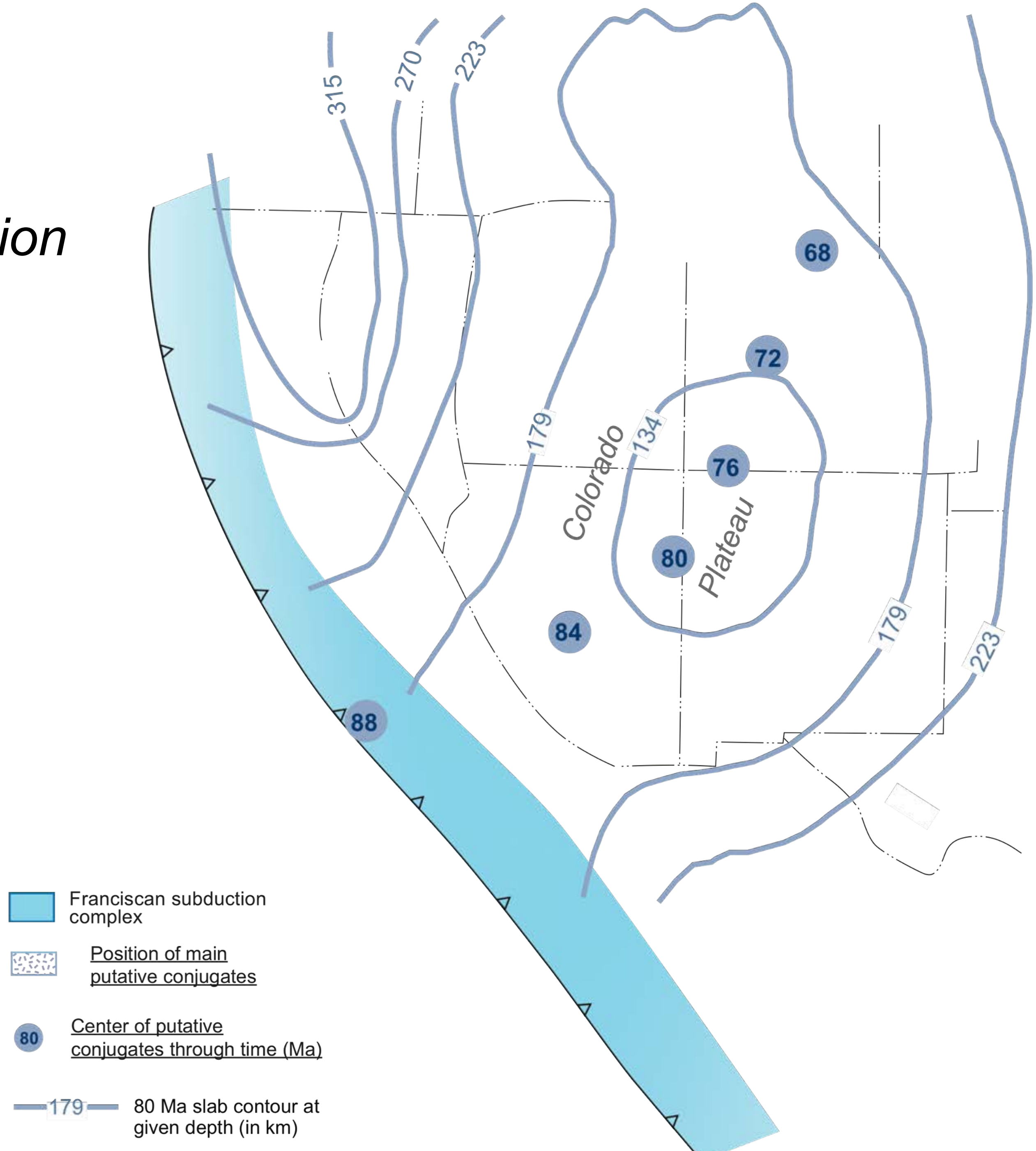
Verification of plateau subduction



(Liu *et al.*, *Nature Geosci.,* 2010)

Laramide Orogeny

--- A prediction



(Liu *et al.*, *Nature Geosci.* 2010)

Laramide Orogeny

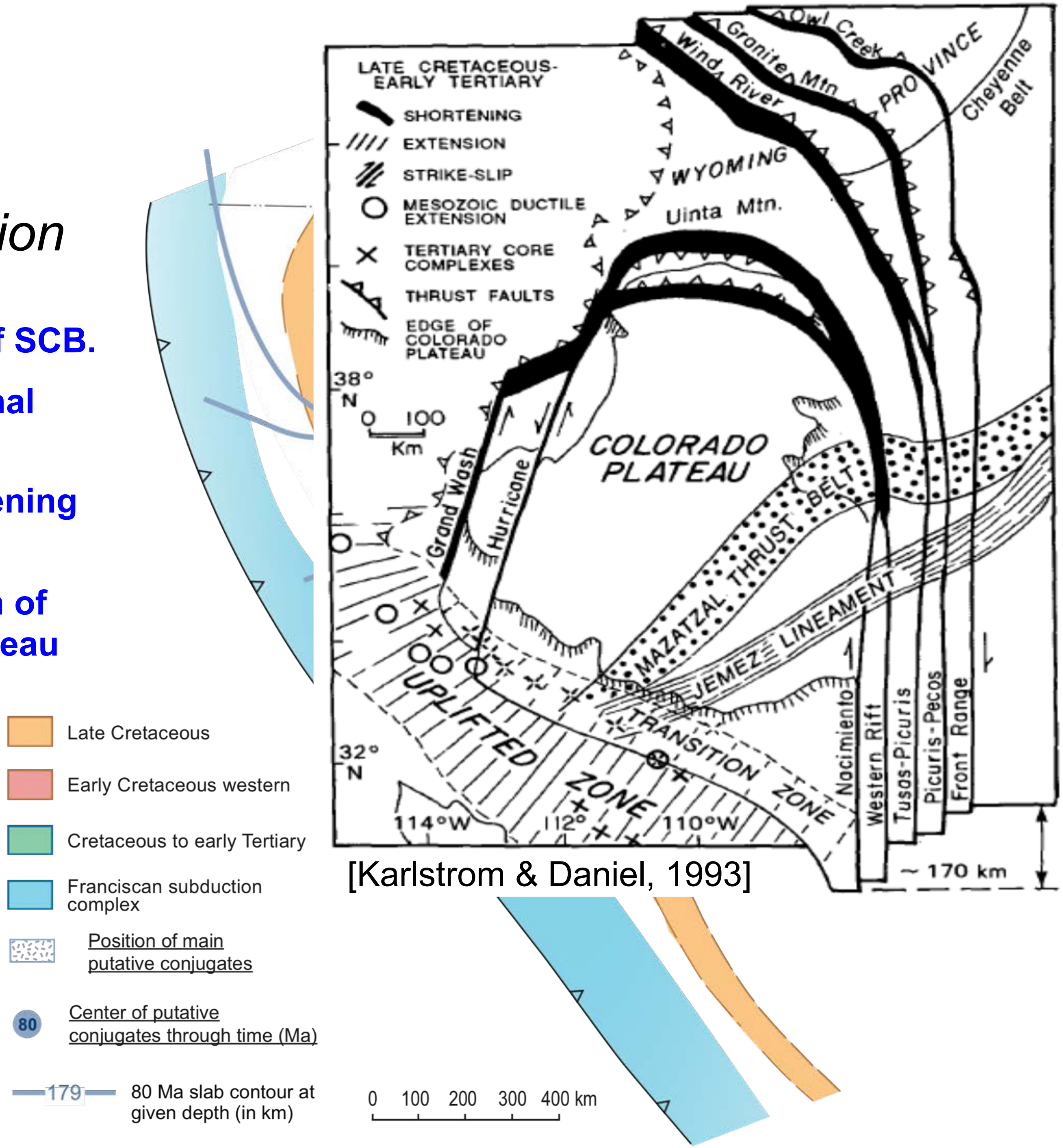
--- *A prediction*

⇒ **Destruction of SCB.**

⇒ **Transpressional Front Range**

⇒ **Normal shortening in Wyoming**

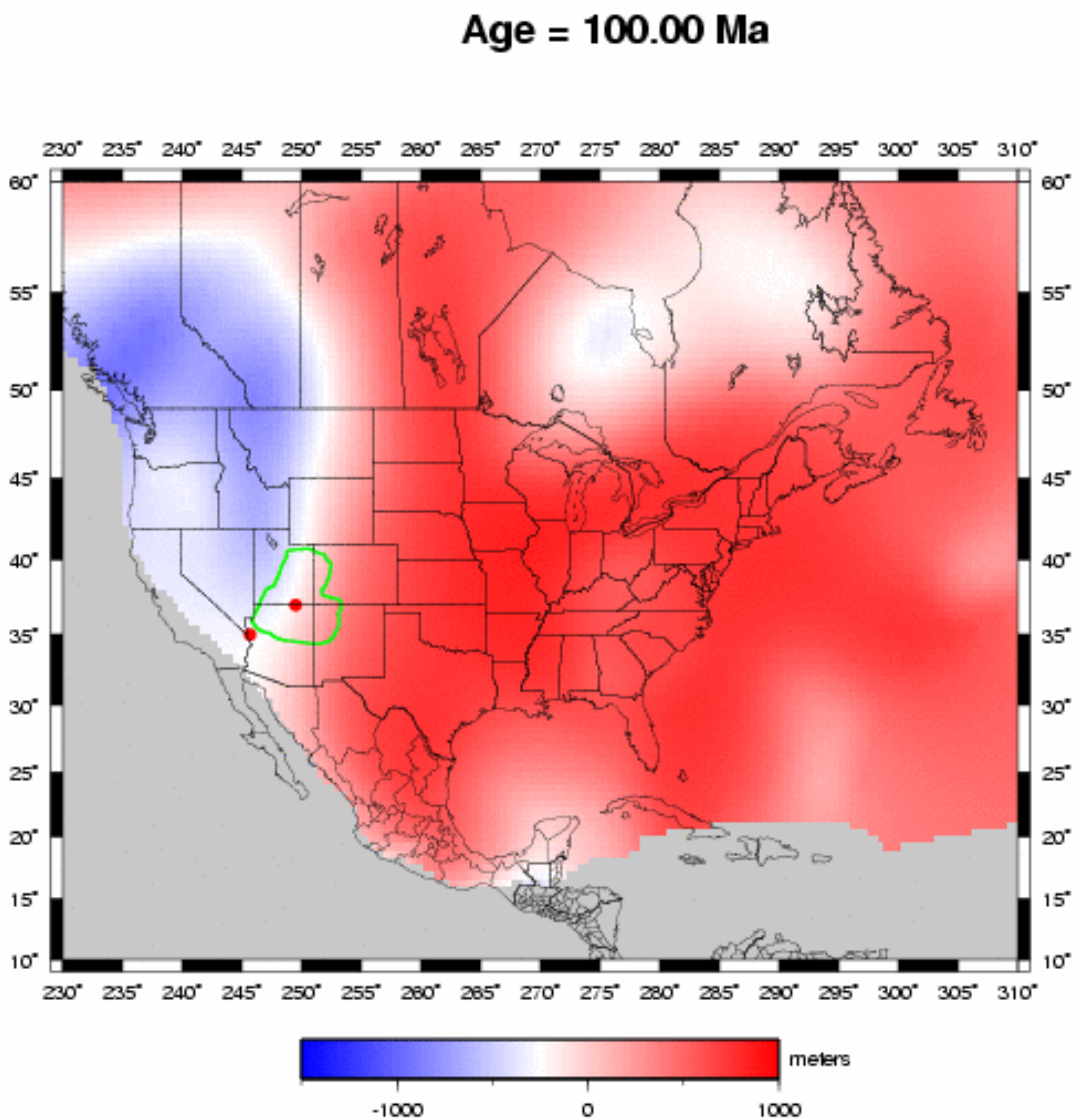
⇒ **NE translation of Colorado Plateau**

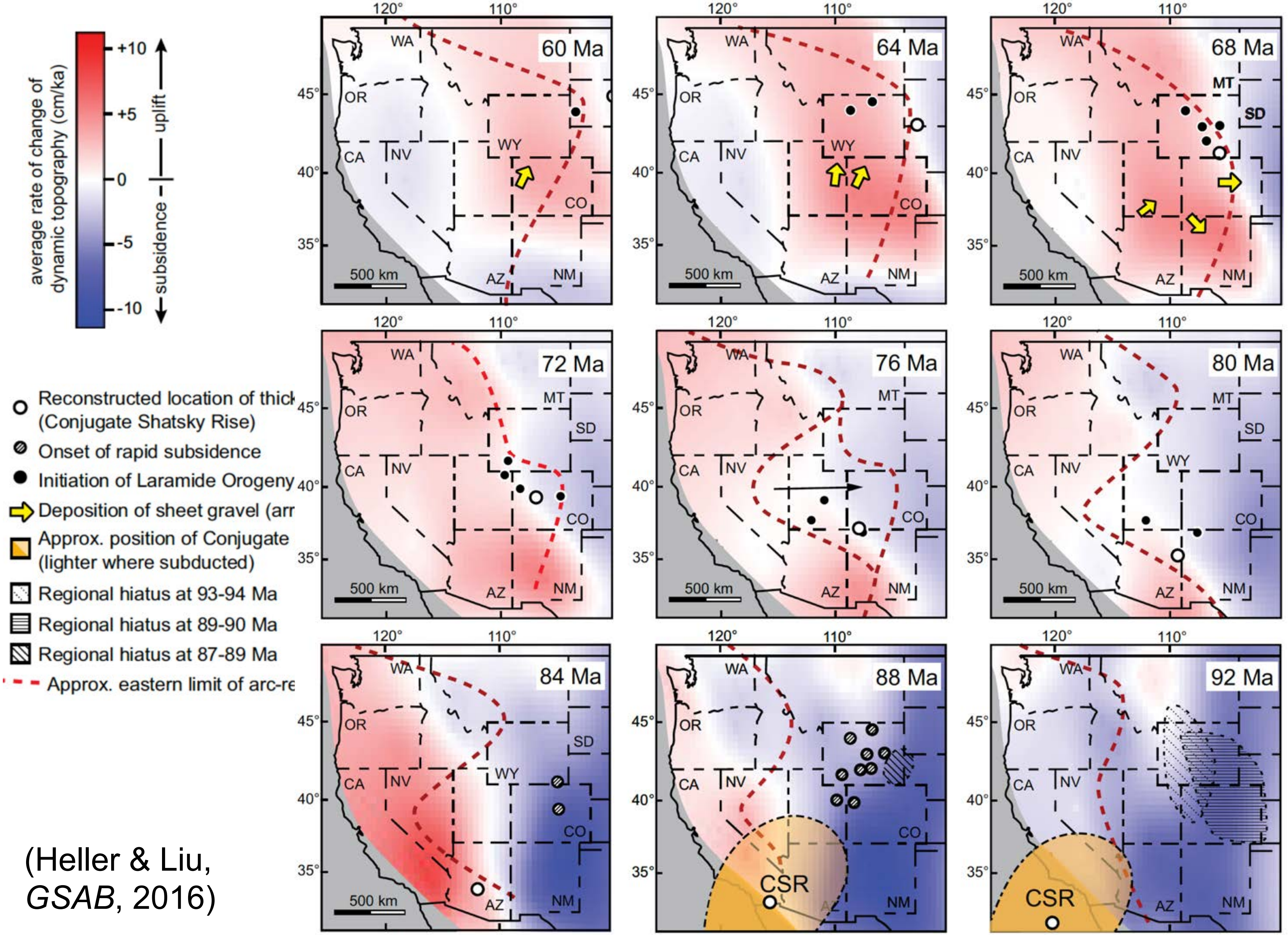


[Karlstrom & Daniel, 1993]

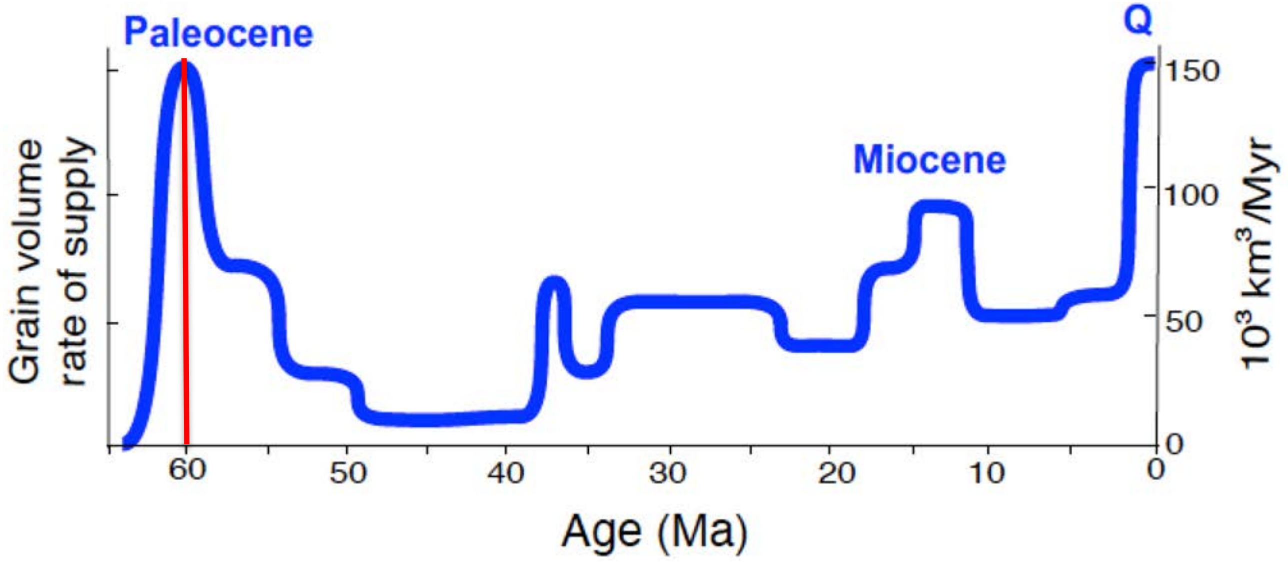
(Liu *et al.*, *Nature Geosci.* 2010)

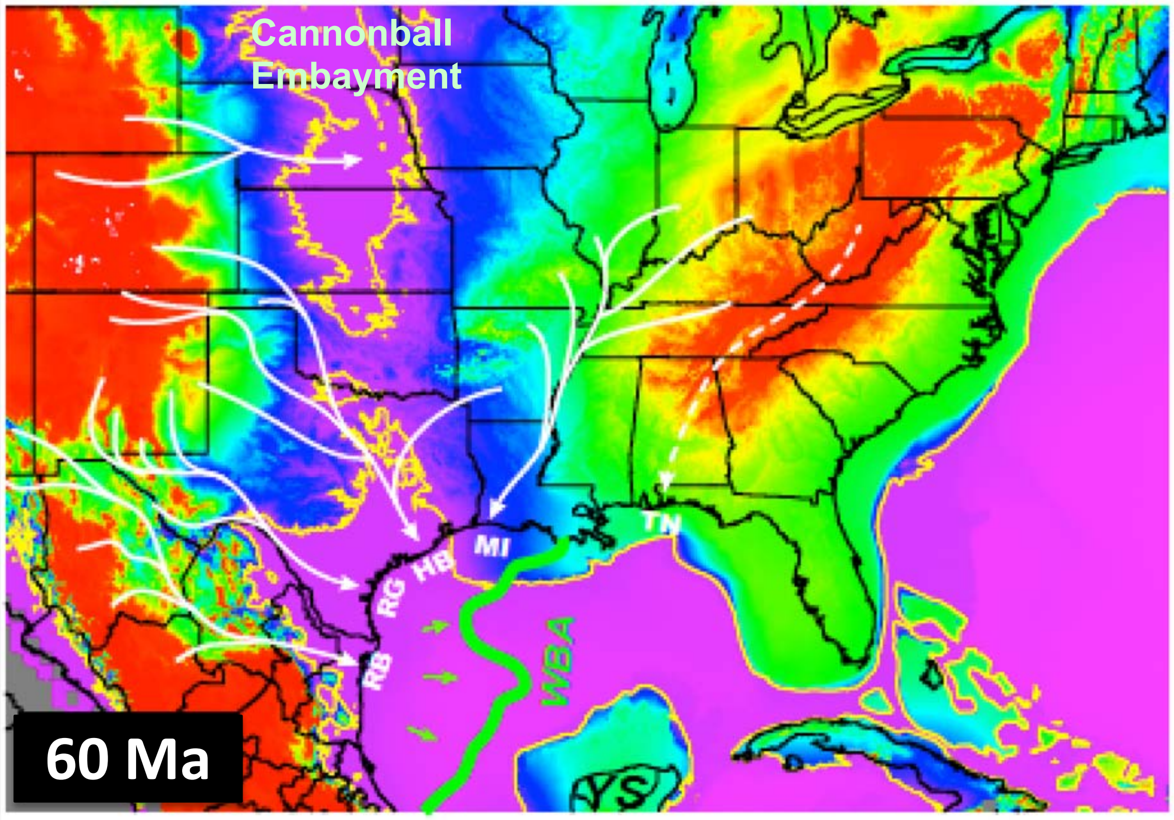
Another prediction: eastward propagating subsidence

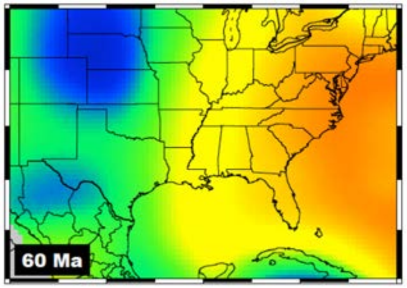




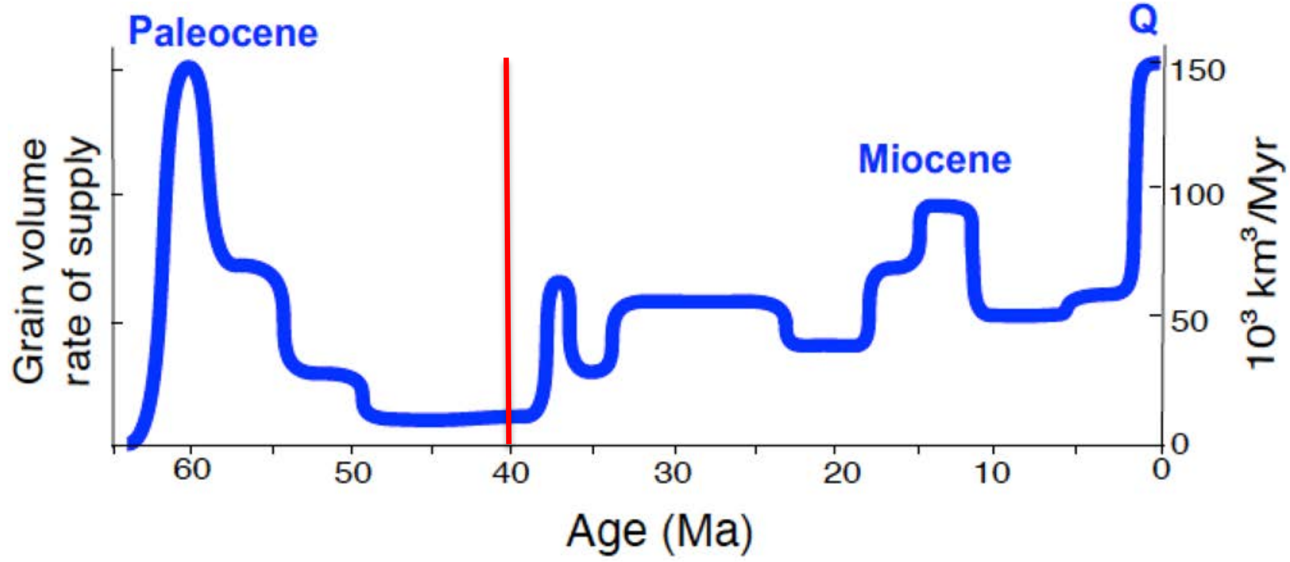
(Heller & Liu, *GSAB*, 2016)

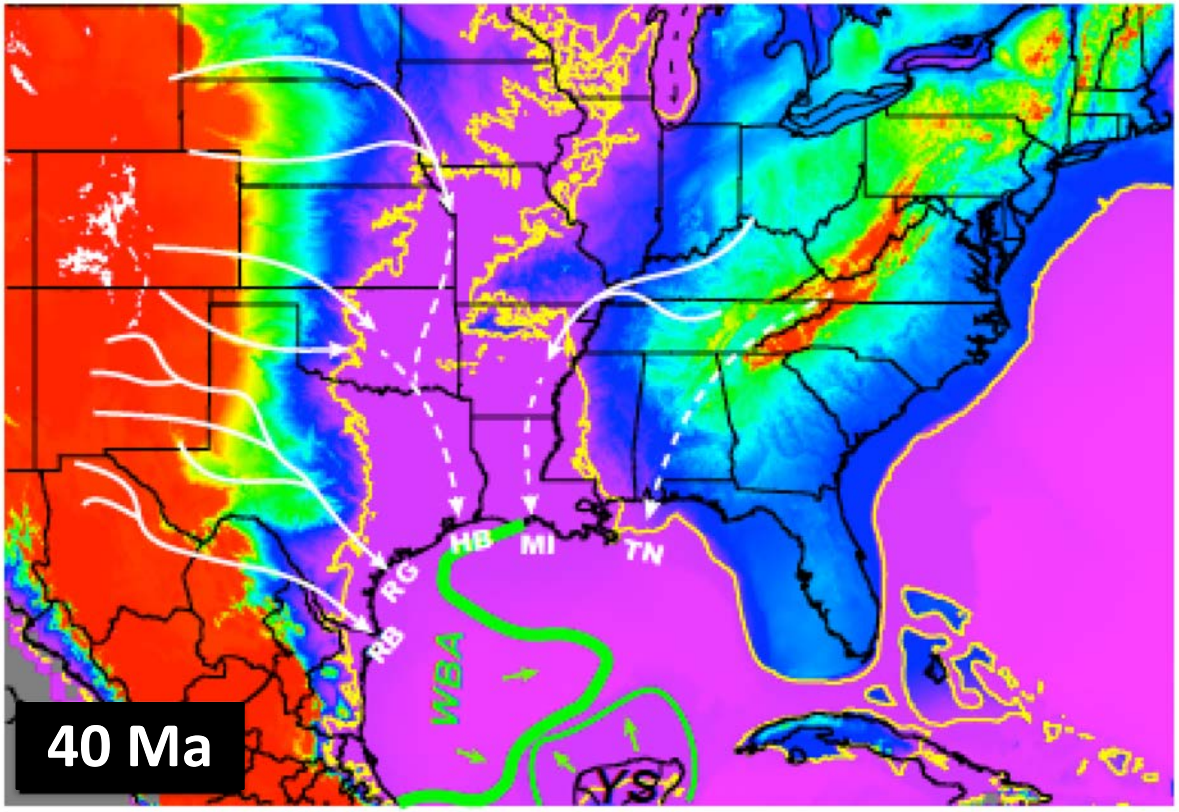


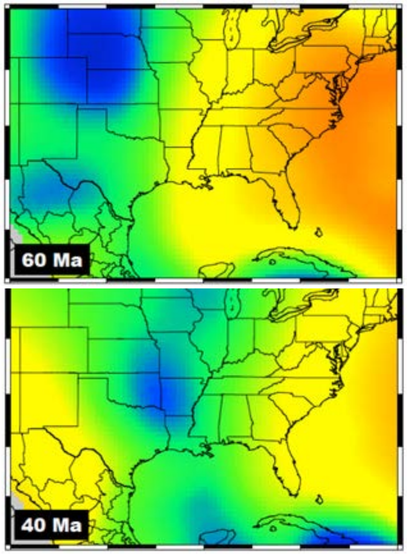




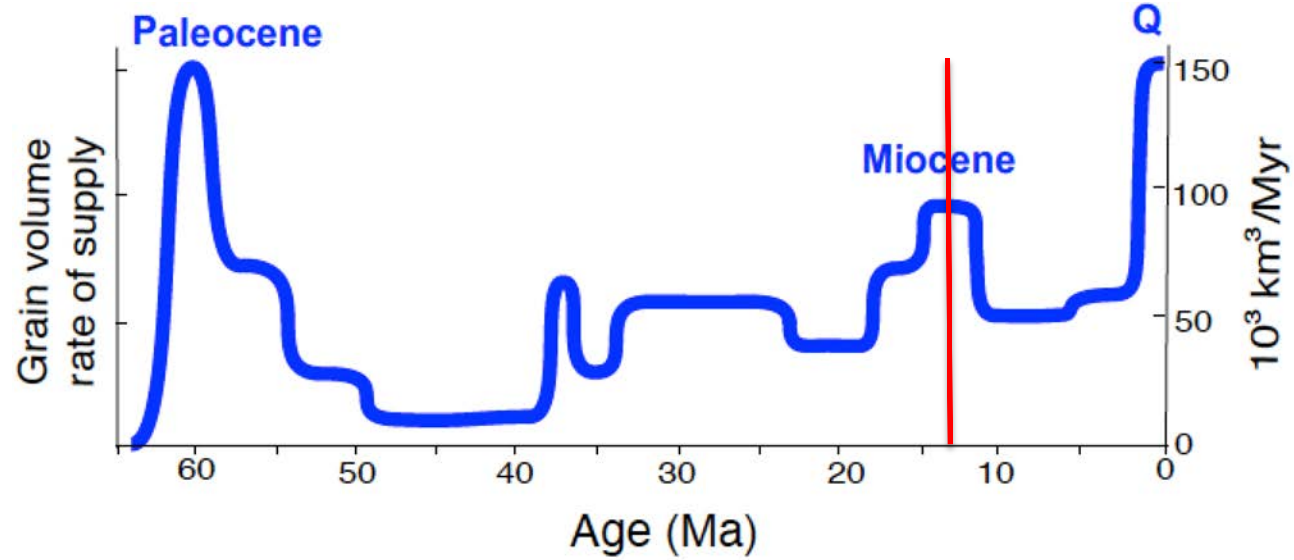
(Liu, *Nature Geosci.*, 2014)

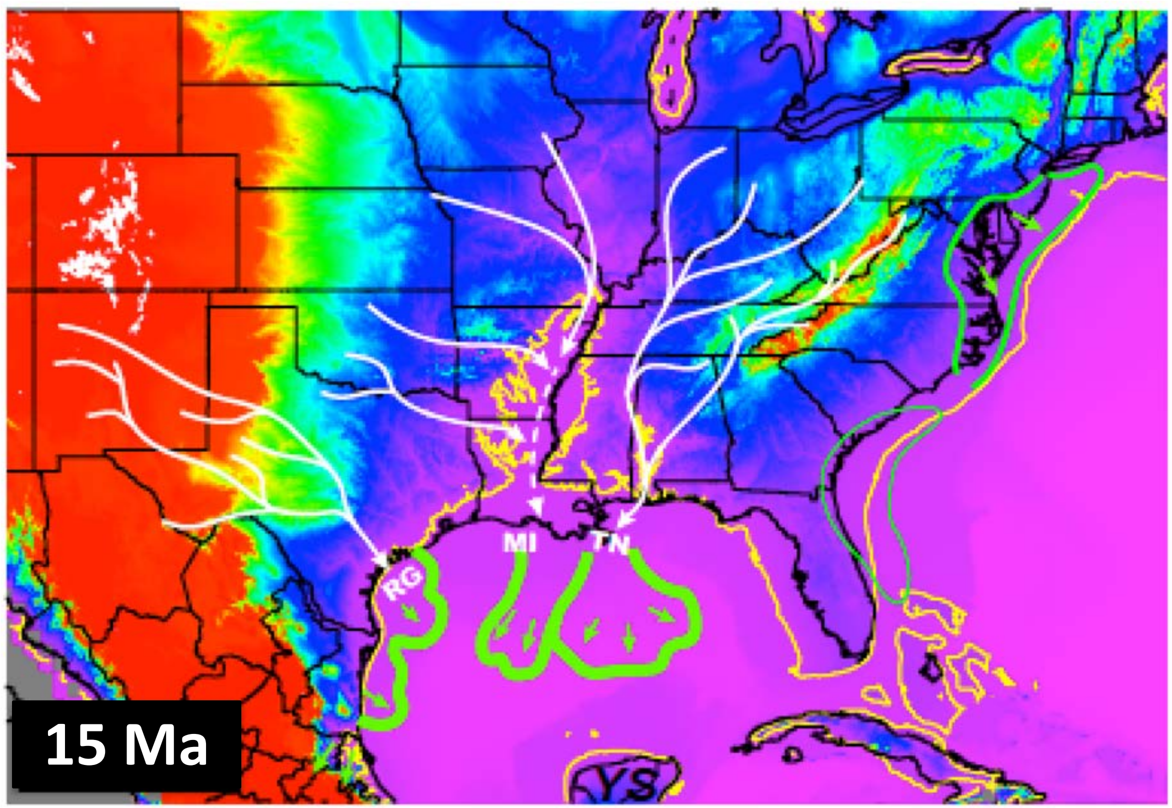


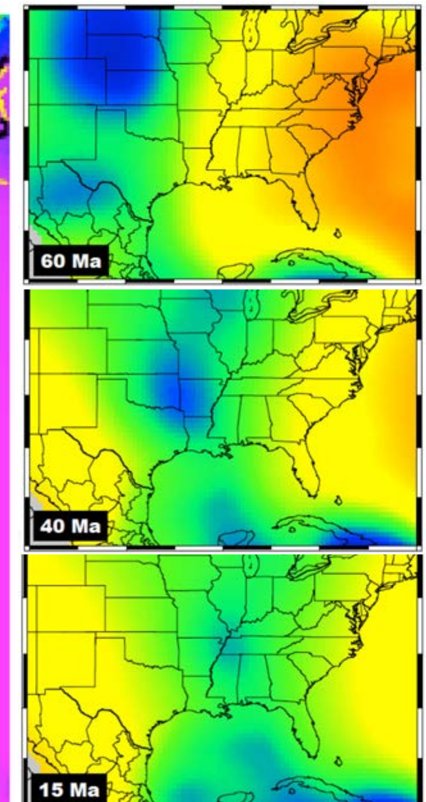




(Liu, *Nature Geosci*., 2014)

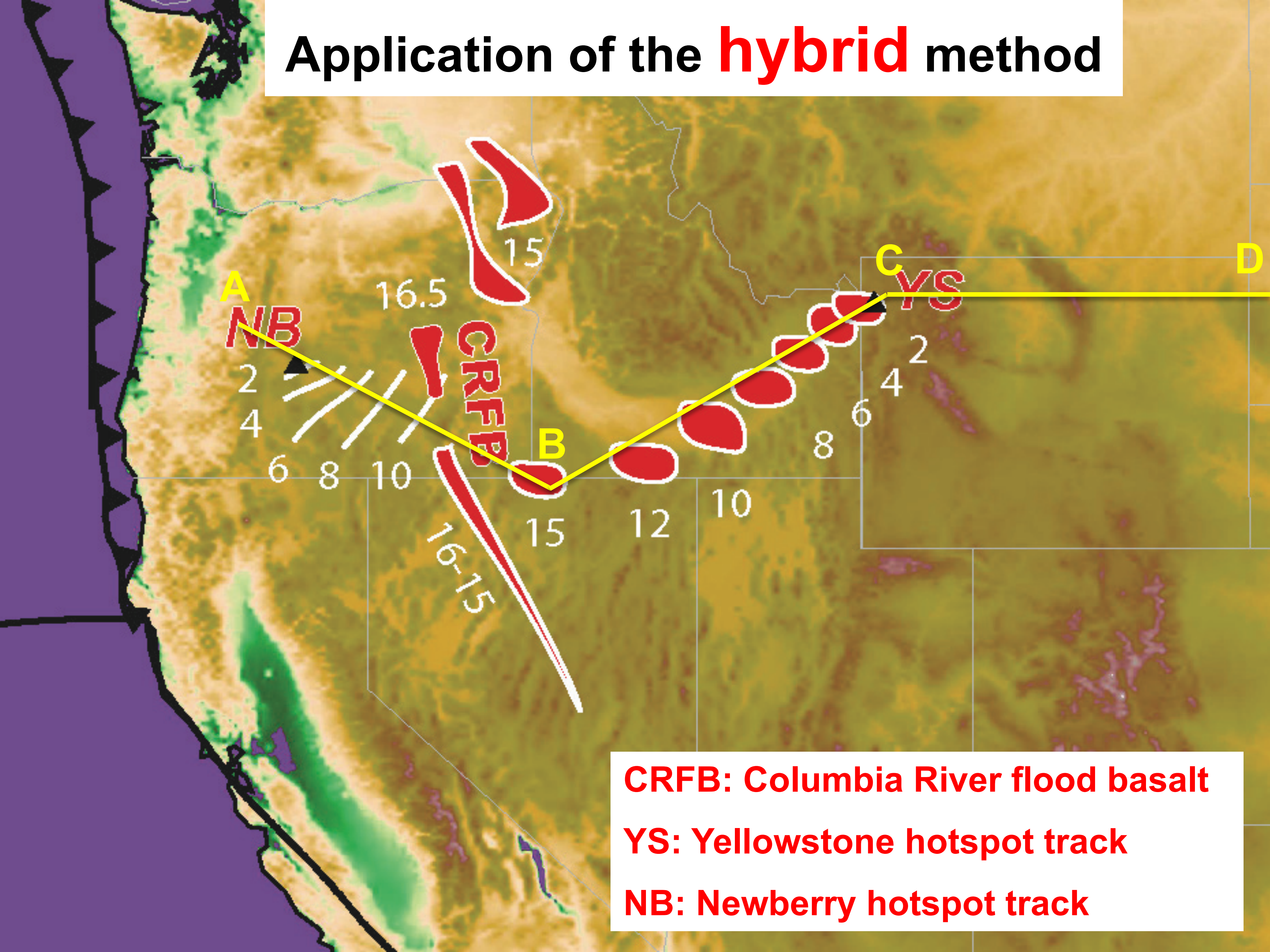






(Liu, *Nature Geosci*., 2014)

Application of the **hybrid** method

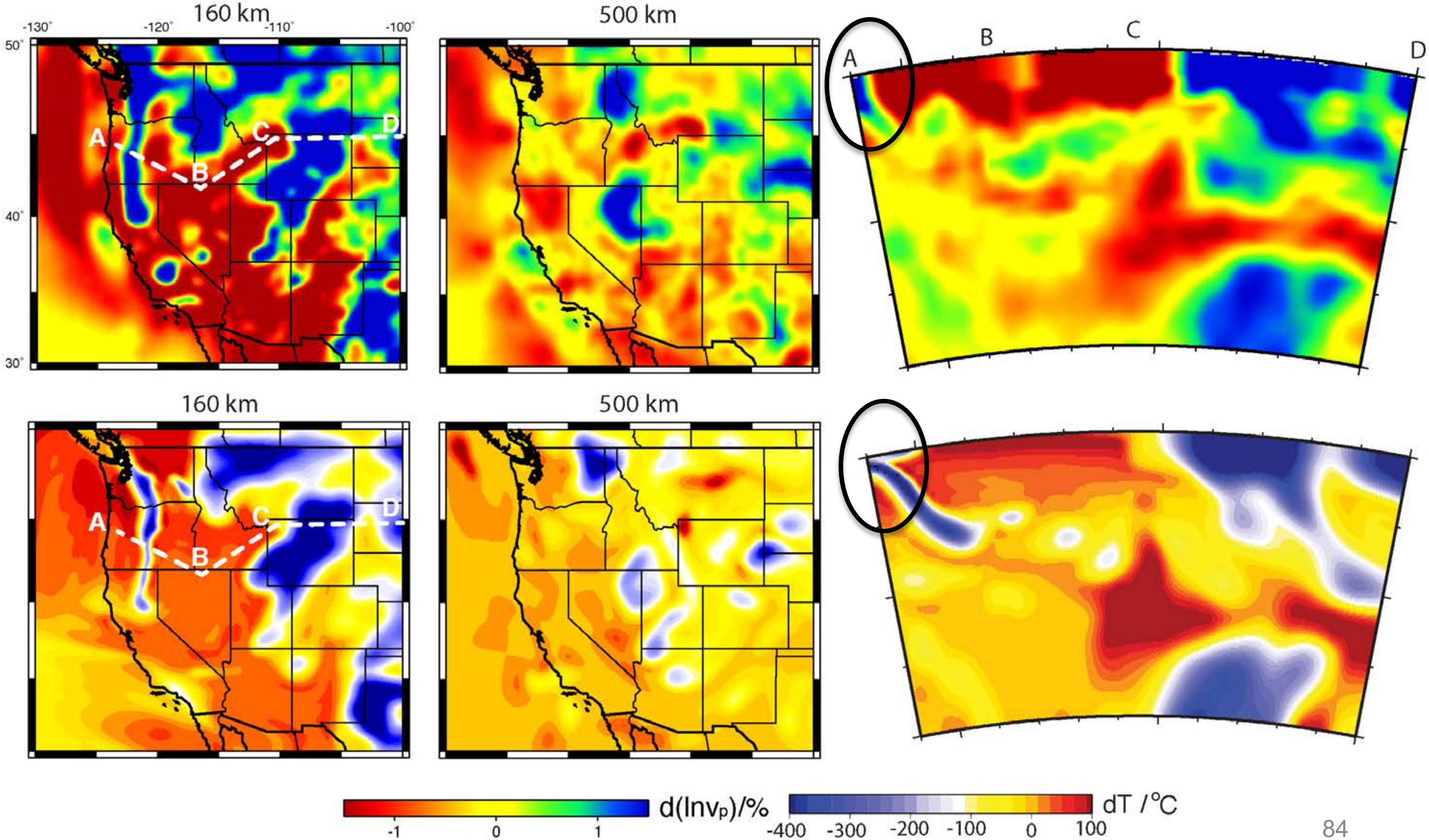


CRFB: Columbia River flood basalt

YS: Yellowstone hotspot track

NB: Newberry hotspot track

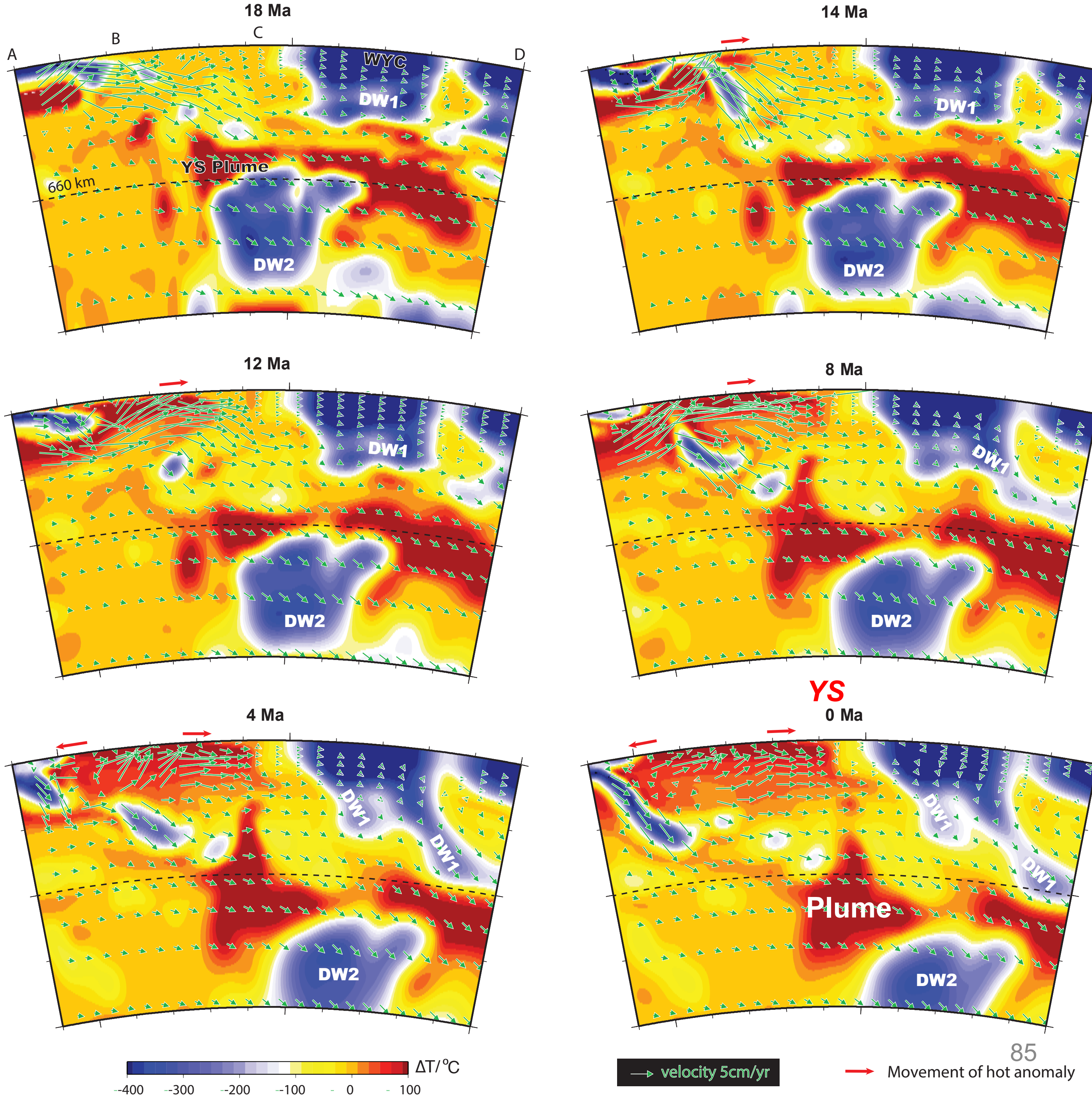
Inversion with the **hybrid** approach



(Zhou et al., *Nature Geosci.*, 2018)

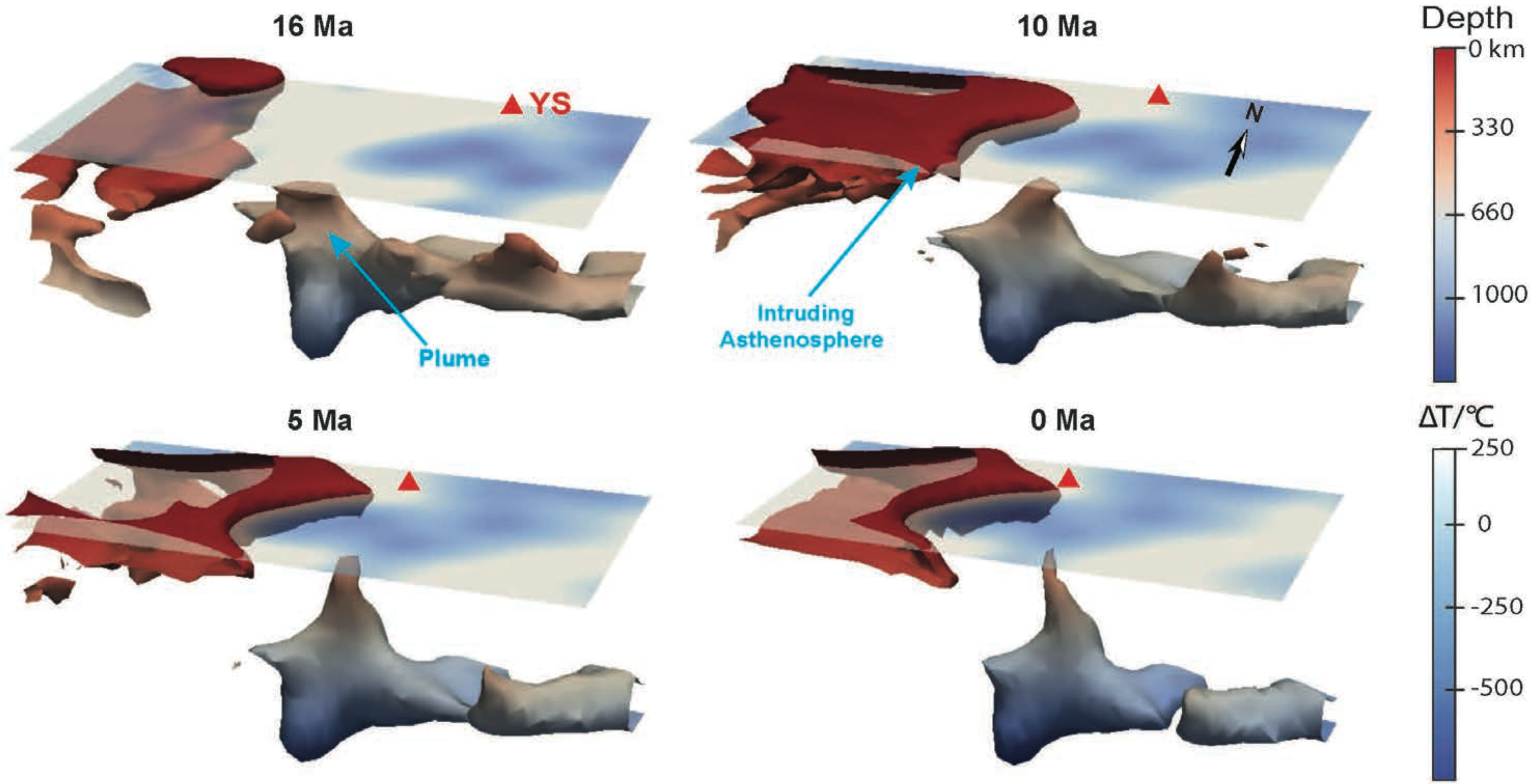
Heat below YS predominantly came from the Pacific mantle.

The mantle plume plays a minor role in generating volcanism.



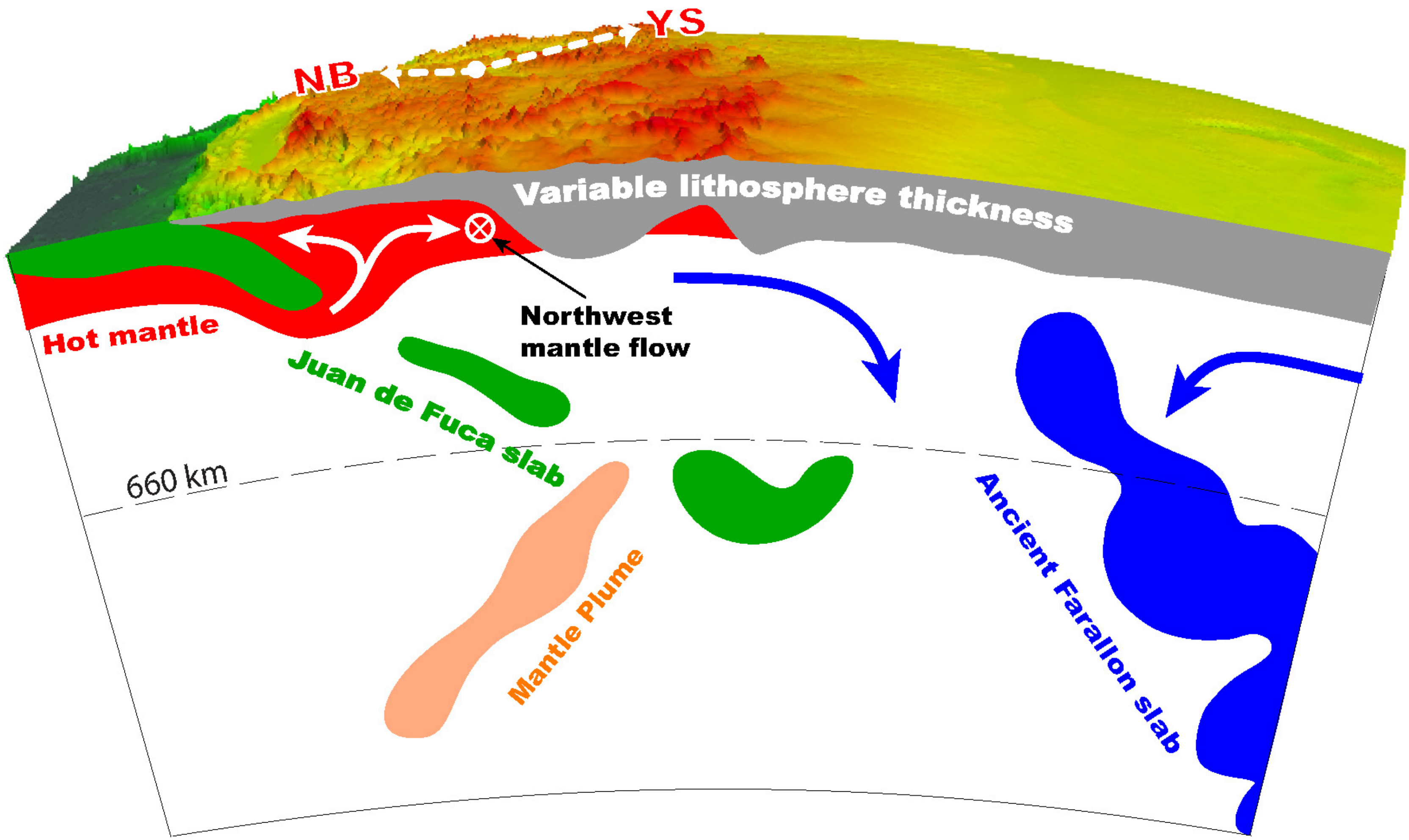
(Zhou et al., *Nature Geosci*., 2018)

Reconstructed hot mantle migration below the western U.S.



(Zhou et al., *Nature Geosci*. 2018)

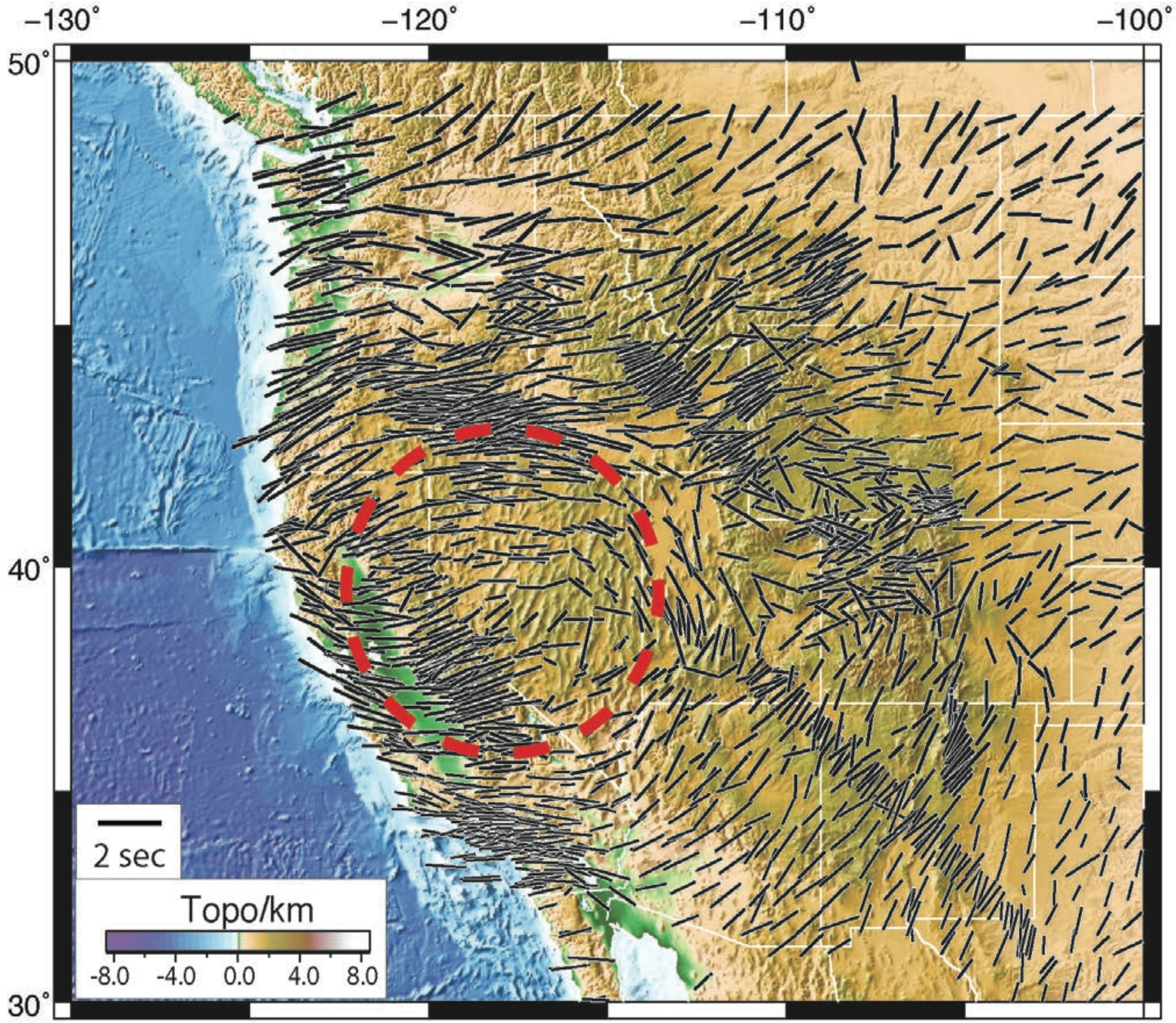
Complex mantle flow below western U.S.



(Zhou et al., *EPSL*, 2018)

Model validation

Formation of the peculiar pattern of western U.S. seismic anisotropy (e.g., SKS)



(Zhou et al., *EPSL*, 2018)

Variable lithosphere thickness

+

Juan de Fuca slab

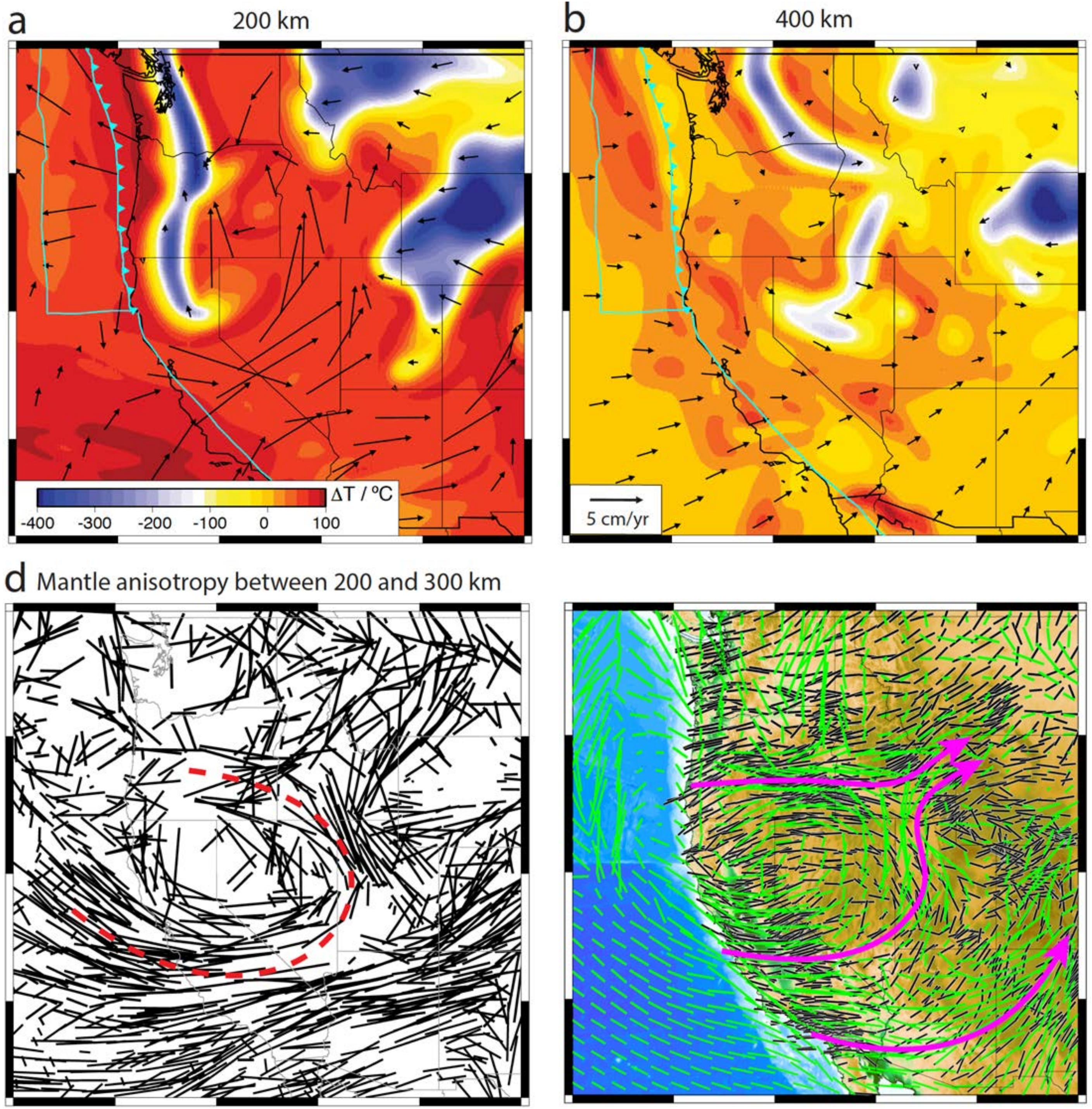
+

Farallon slab

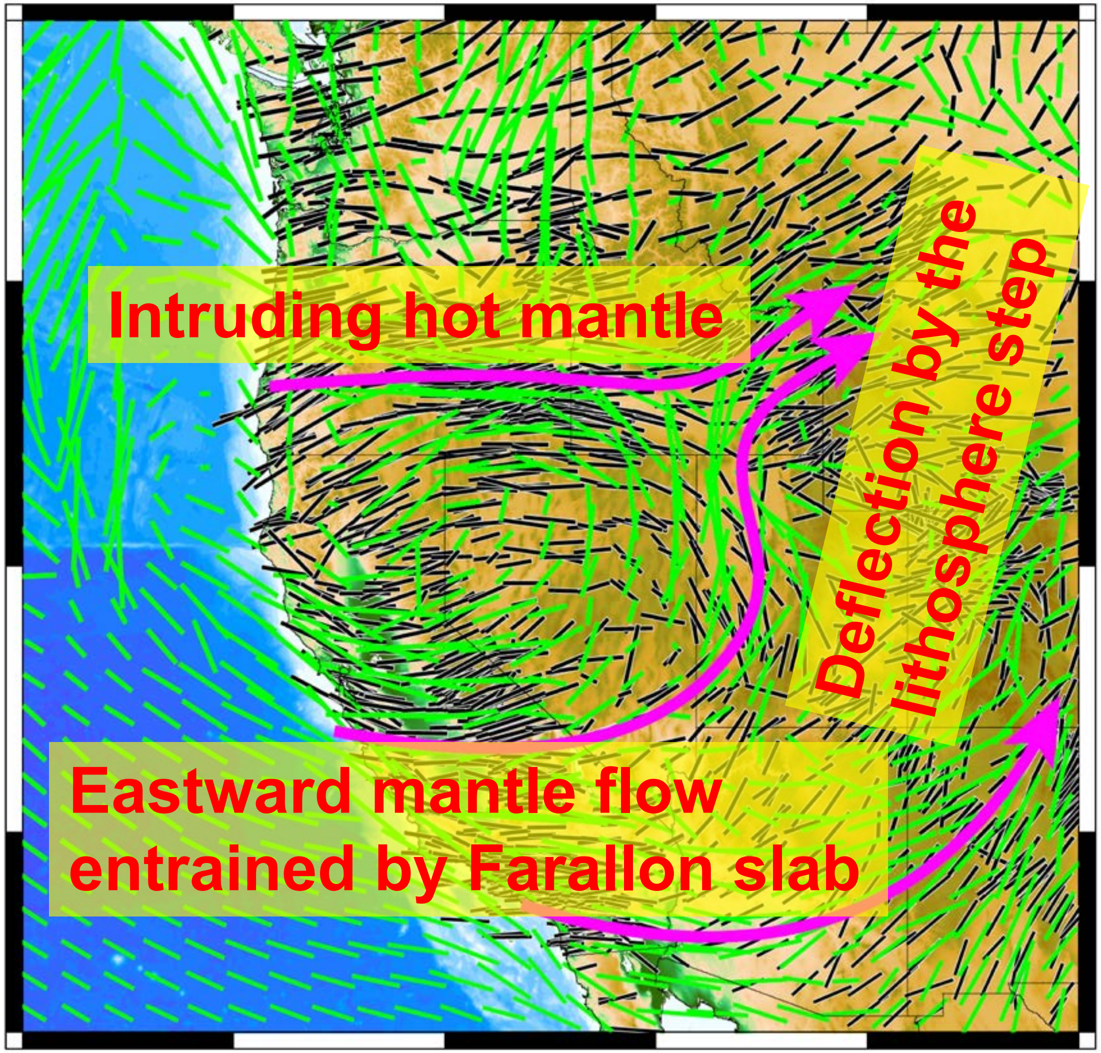
+

Hot Pacific mantle

(Zhou et al., *EPSL*, 2018)

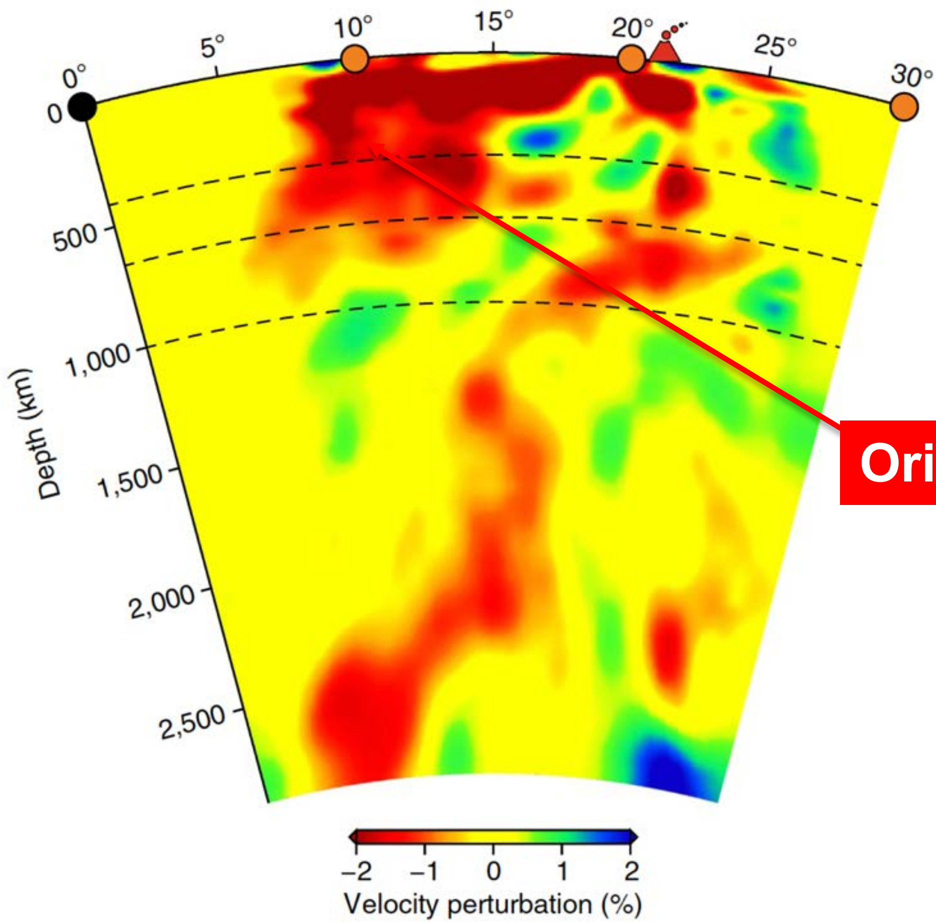


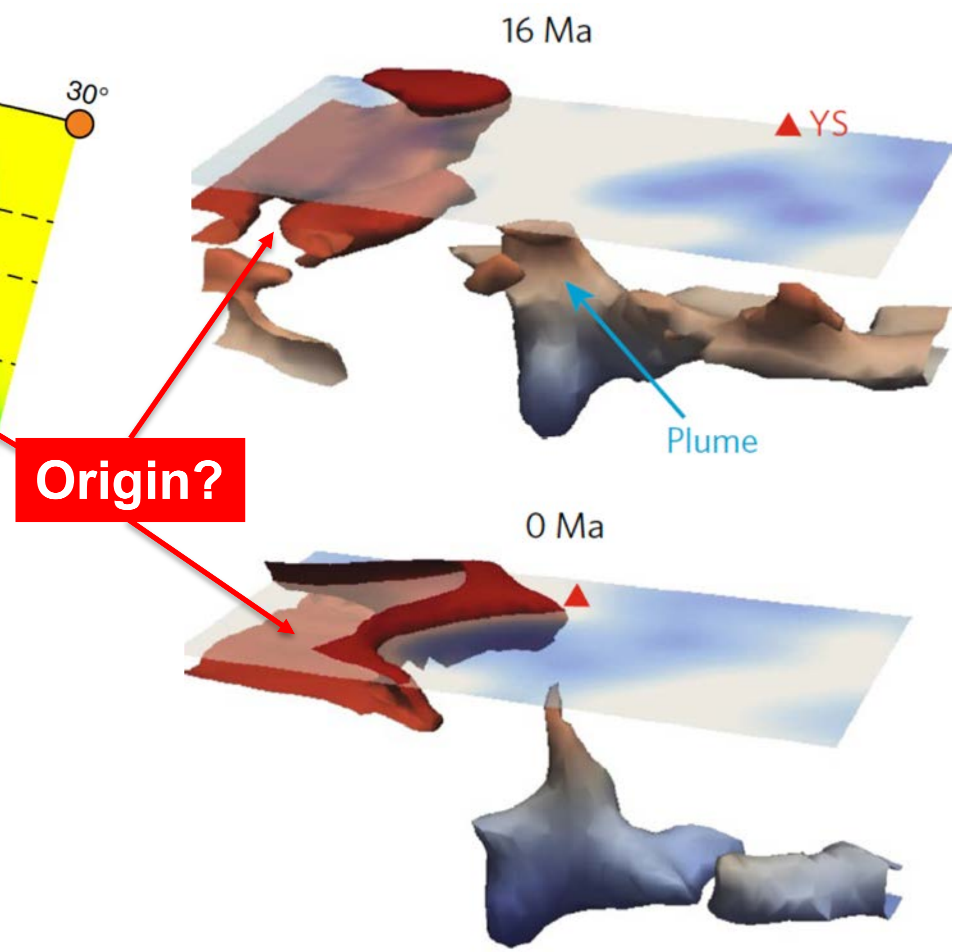
Main features of mantle dynamics



(Zhou et al., *EPSL*, 2018)

New question on slab-plume interaction





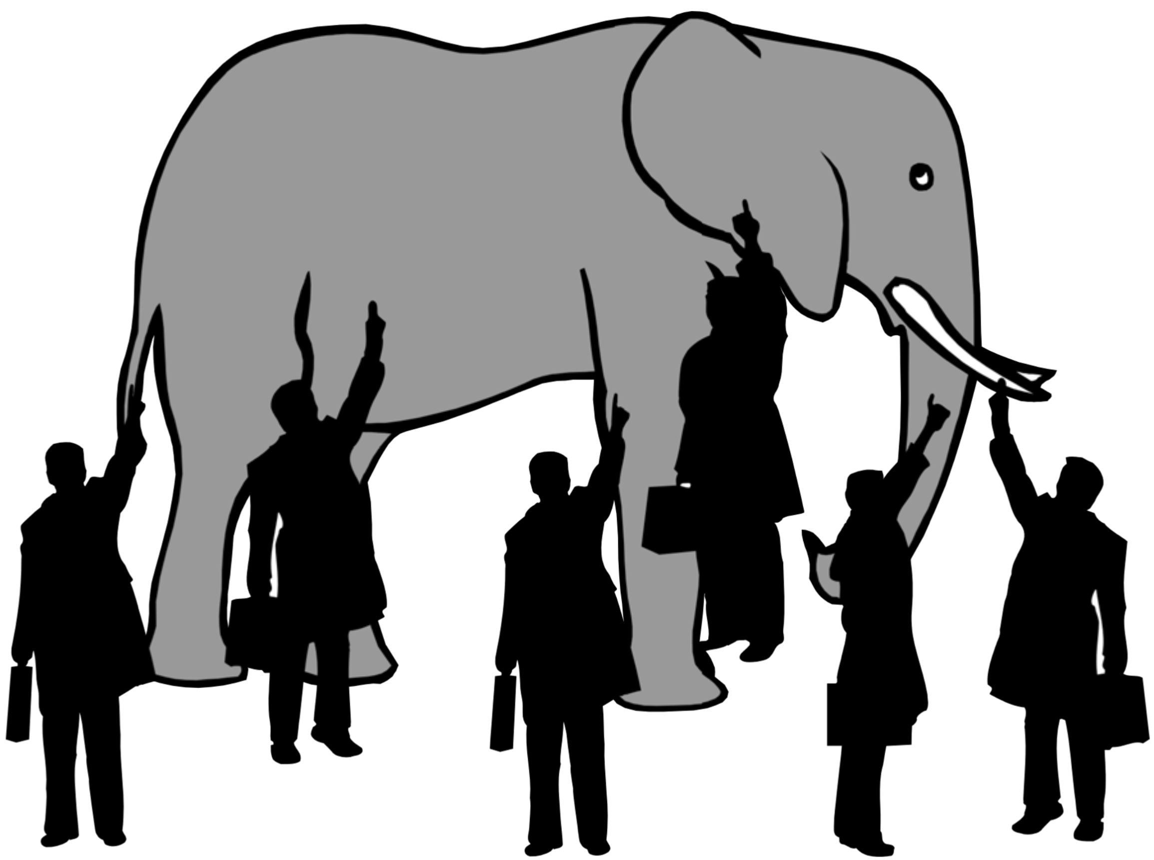
(Nelson & Grand, *Nat. Geosci.*, 2018)

(Zhou et al., *Nat. Geosci.*, 2018)

Concluding remarks

- Geodynamic models with data assimilation (both forward and inverse) represent a promising approach for quantitatively understanding deep Earth processes.
- New research frontiers include further developing assimilation schemes (both forward and inverse), codes coupling (with mineral physics, seismology, surface processes, petrology/geochemistry, etc.).

Data assimilation represents a **system** modeling approach



SAME man examining the elephant