Geodynamics IV: Mixing

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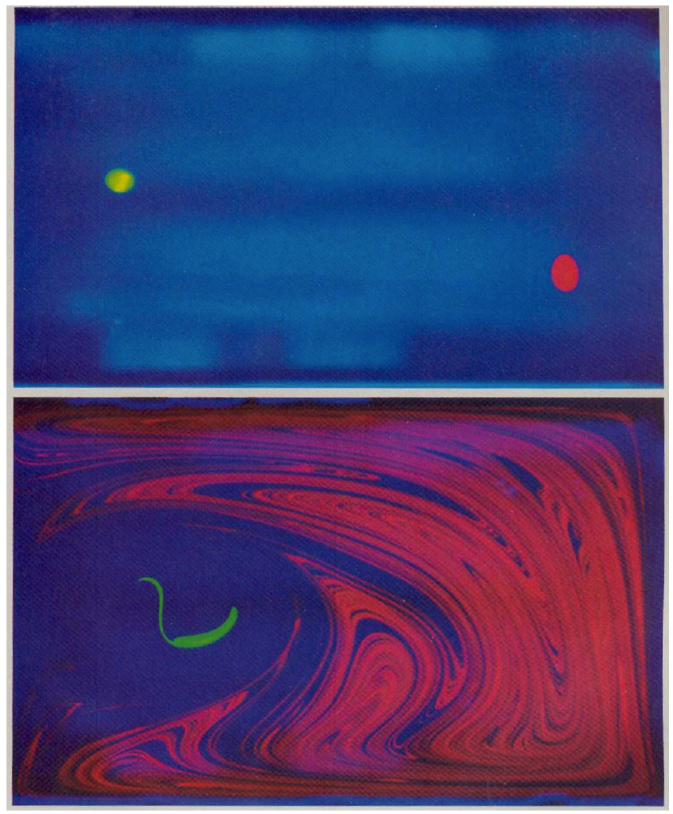
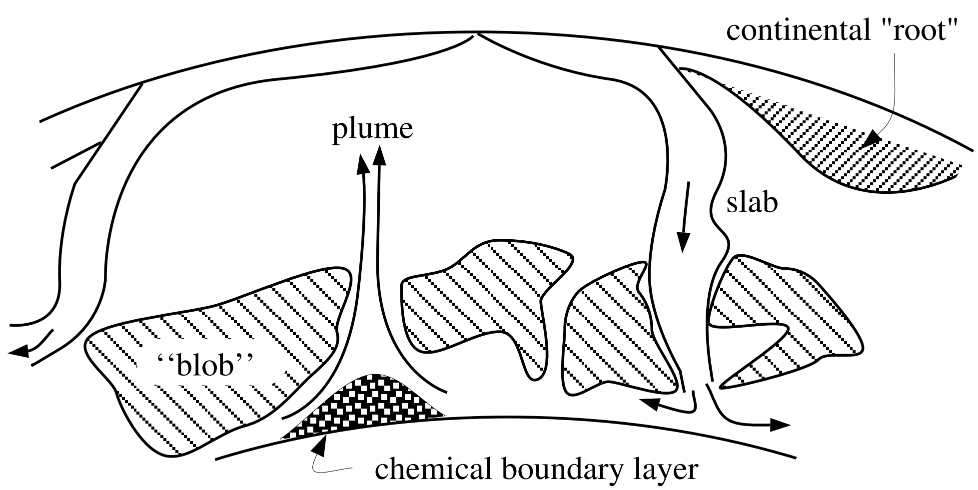


“Virtually everyone agrees that mixing in complicated”
Ottino, Ann Rev Fluid Mech (1990)

Why study mixing?

To provide a quantitative framework to interpret geochemical and isotopic variations in magmas; present structure and evolution of structures, rates of mass and energy exchange, evolution of mantle composition; magmatic processes within the crust



Outline

- A bit of terminology
- Physics of mixing
- Characterization of mixing
- Introduction to mixing in the mantle

Not covered

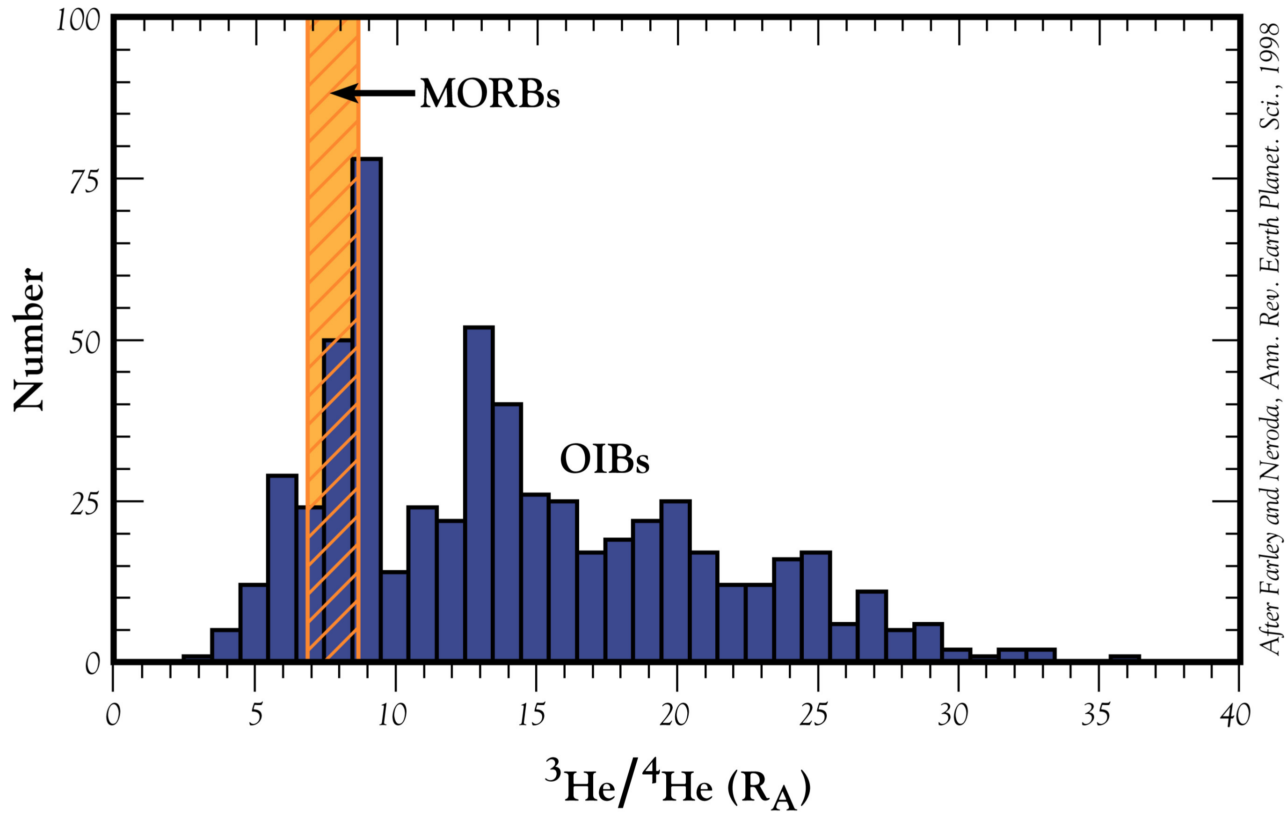
- How convection works (see other lectures)
- The geochemistry we want to interpret (see other lectures)
- Numerical and computational challenges (see van Keken et al *JGR* 1997 for a discussion)
- Turbulent mixing (only low Reynolds number, laminar mixing)
- Mixing of mechanically disrupted material (e.g., by impacts)

Main points

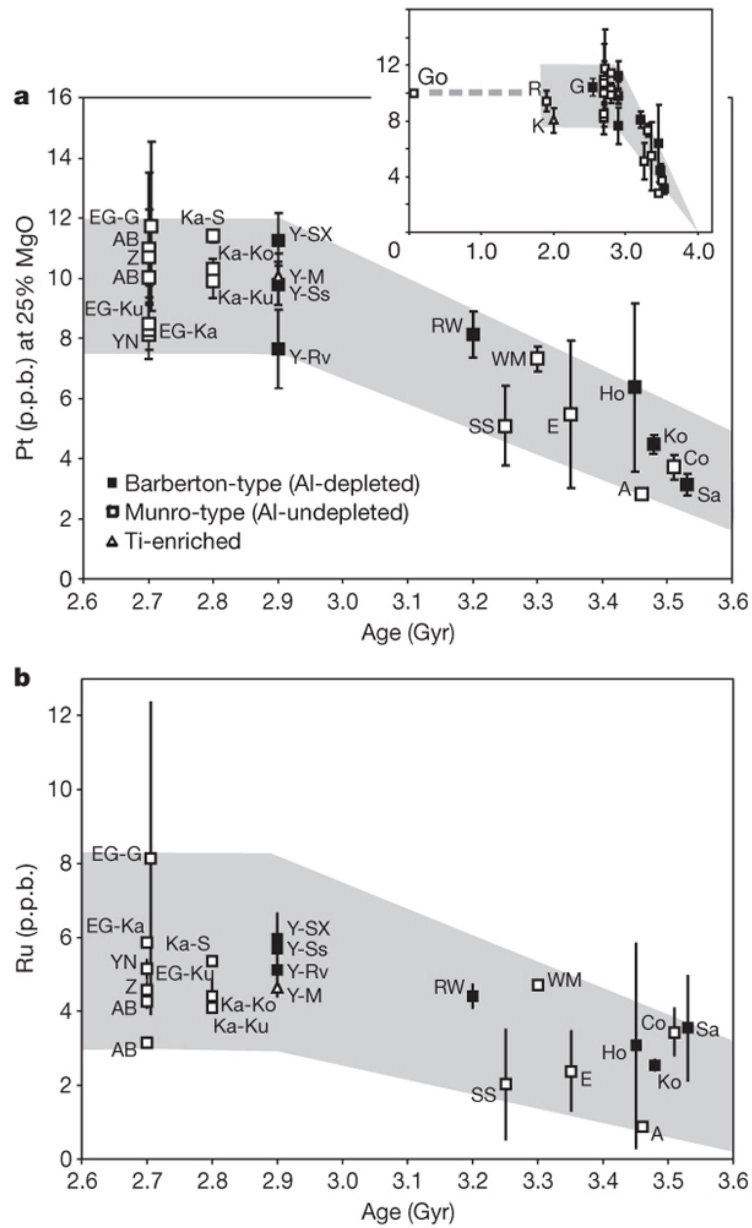
- Flow type matters
- Time dependence matters
- Properties of heterogeneity matter
- Convection both creates and destroys heterogeneity

Some observations that we can interpret in the context of mixing

1. Global scale: mantle contains well-mixed regions and heterogeneity



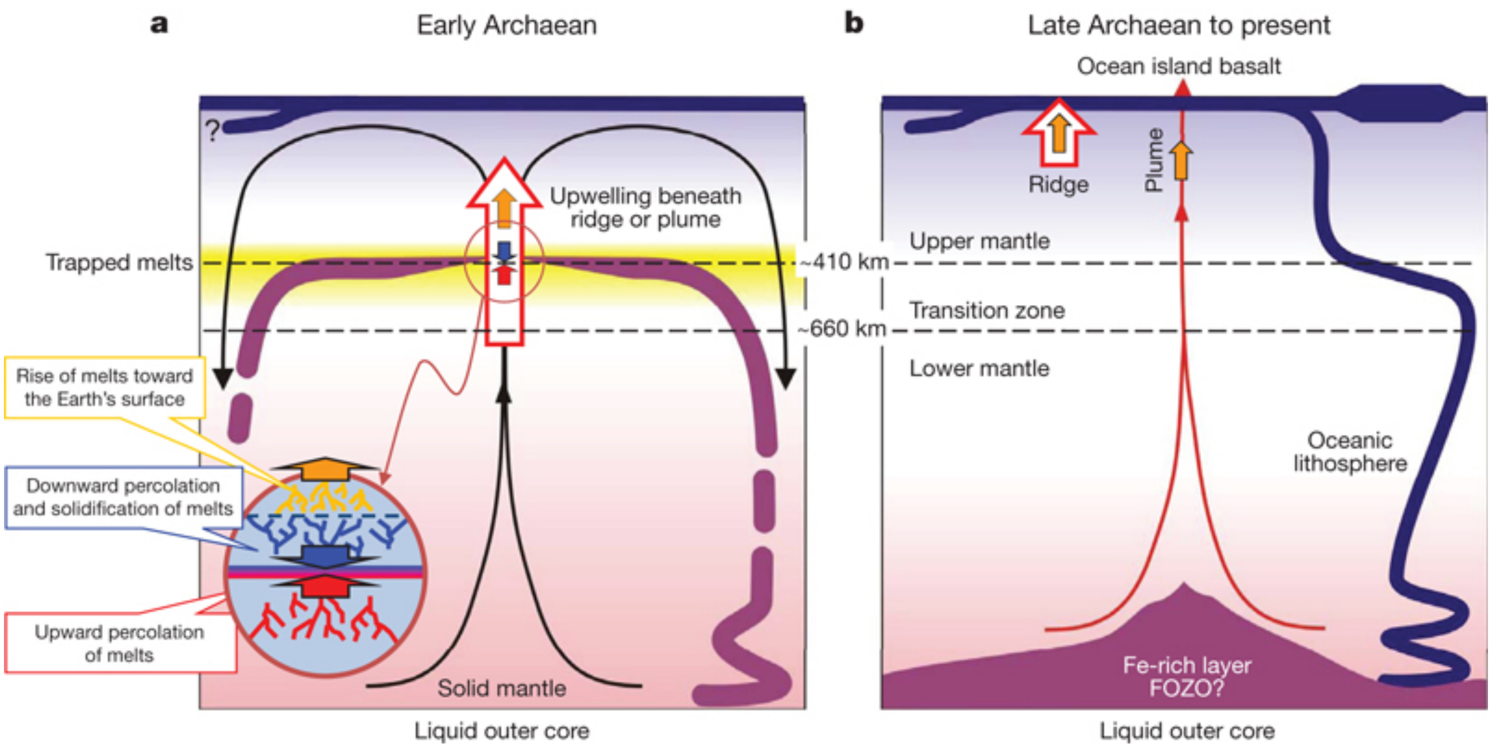
2. Mixing time for early Earth?



mixing of a PGE-rich late veneer?

Maier et al., *Nature* 2009

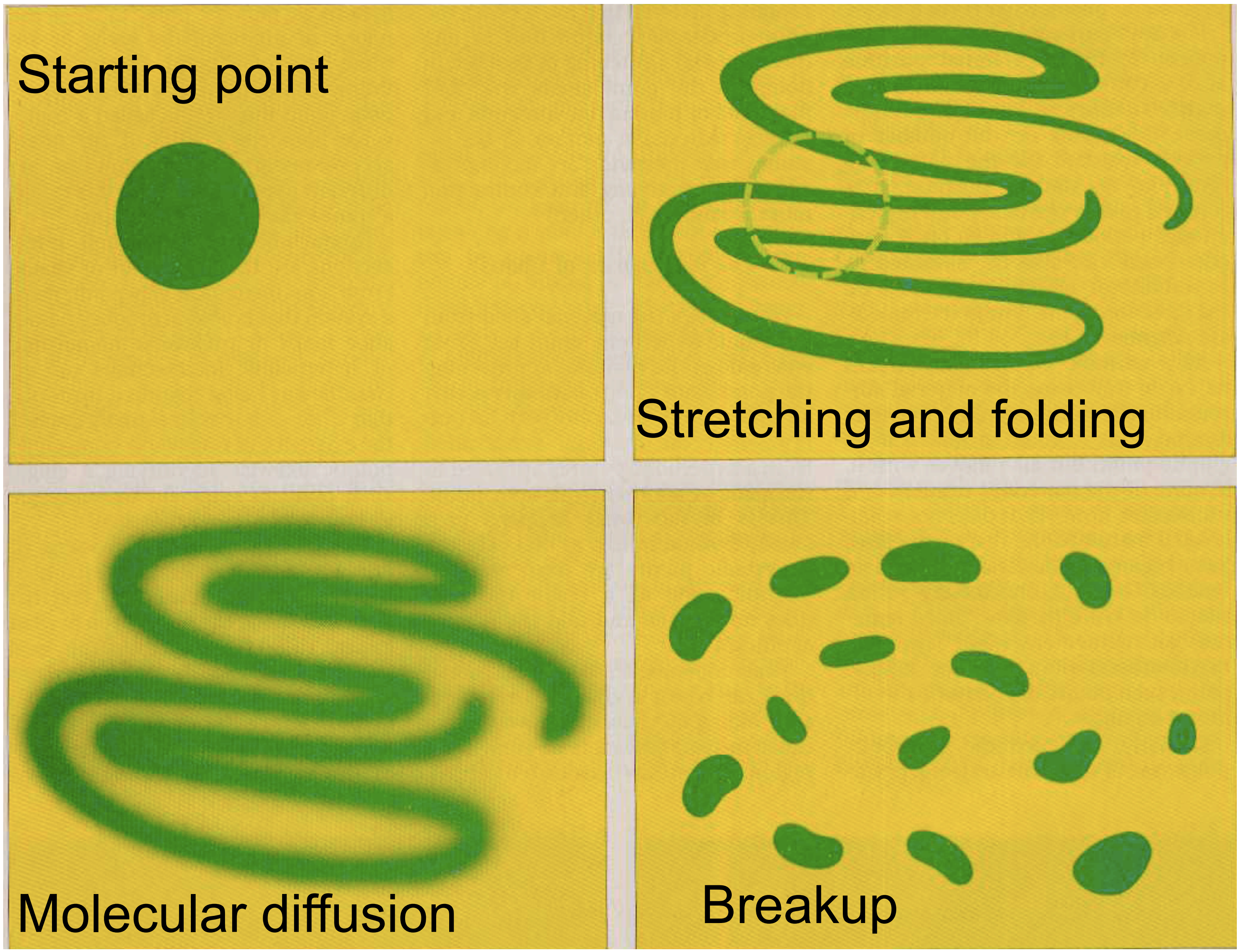
3. More early Earth mixing



Lee et al., *Nature* 2010 (a CIDER product)

When? How much? Entrainment rate?

How does mixing occur?



Ottino, *Scientific American* 1989

Definitions

Stirring: *stretching* and *folding* of material surfaces to reduce length scales

Mixing: homogenization by stirring and diffusion

Passive tracer: is convected with the flow $\mathbf{u}(\mathbf{x},t)$ and does not influence the flow

Active heterogeneities: owing to differences in density and/or rheology, modify the flow

Stretching: flow type matters

The deformation of a material filament from $d\mathbf{X}$ to $d\mathbf{x}$ is given by

$$d\mathbf{x} = \mathbf{F} \cdot d\mathbf{X}$$

where $\mathbf{F}$ is the deformation tensor (which can be related to the velocity $\mathbf{u}$).

The magnitude of stretching is

$$\lambda = \lim_{|d\mathbf{X}| \to 0} = \frac{|d\mathbf{x}|}{|d\mathbf{X}|}$$

and the rate of stretching is

$$\frac{D(\ln \lambda)}{Dt} = \mathbf{E} : \mathbf{mm} \quad \text{with} \quad \mathbf{m} = d\mathbf{x}/|d\mathbf{x}|$$

with $\mathbf{E} = \frac{1}{2}[\nabla \mathbf{u} + (\nabla \mathbf{u})^T]$ is the stretching tensor.

Stretching: flow type matters

Lets consider **linear** 2D flows in the x-y plane

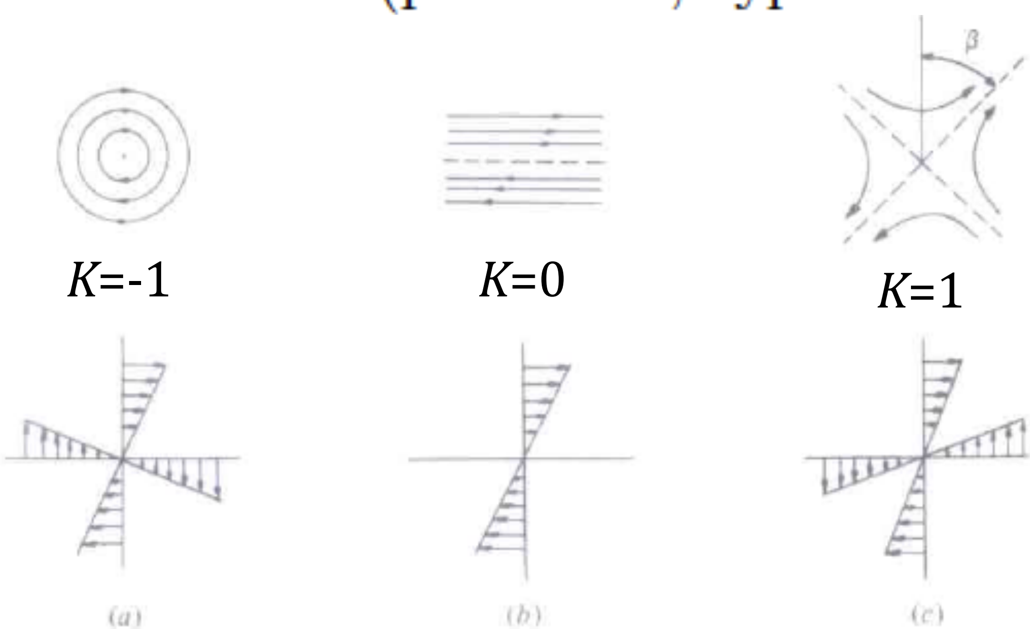
$$v_x = Gy \quad \text{and} \quad v_y = KGx$$

For long times, if $K = 0$ (simple shear)

$$\lambda \sim Gt$$

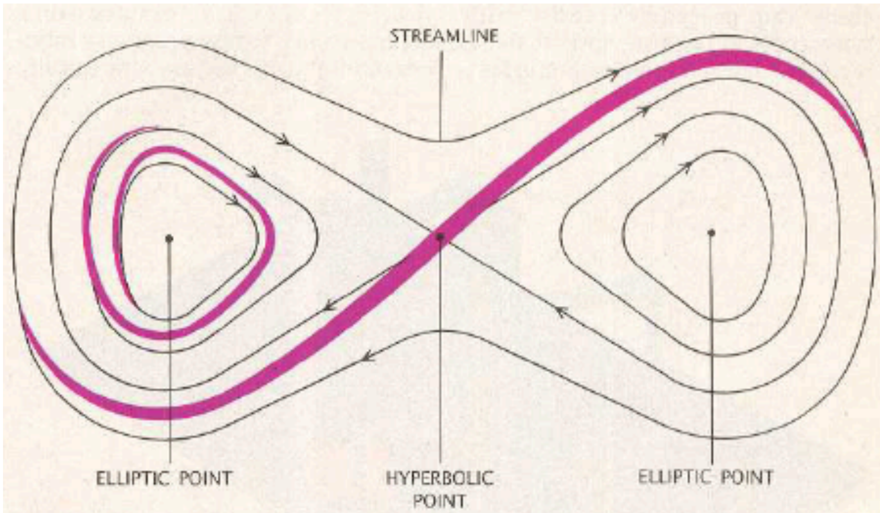
and if $K = 1$ (pure shear, hyperbolic flow)

$$\lambda \sim e^{Gt}$$



In a more complex flows, regions with pure shear (hyperbolic streamlines) will cause most of the *stretching*

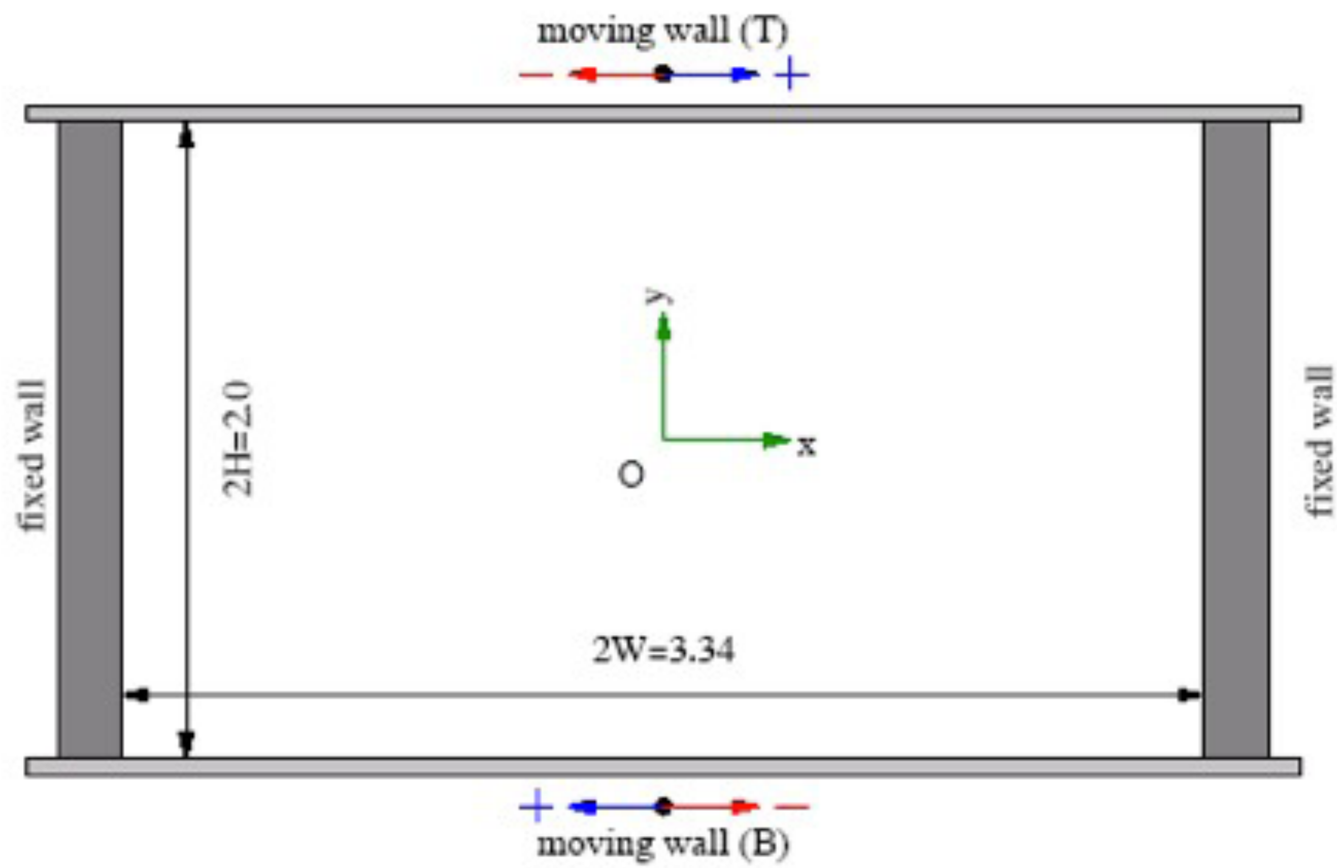
Two types of building blocks for flows: Elliptic and hyperbolic points



Ottino, *Scientific American* 1989

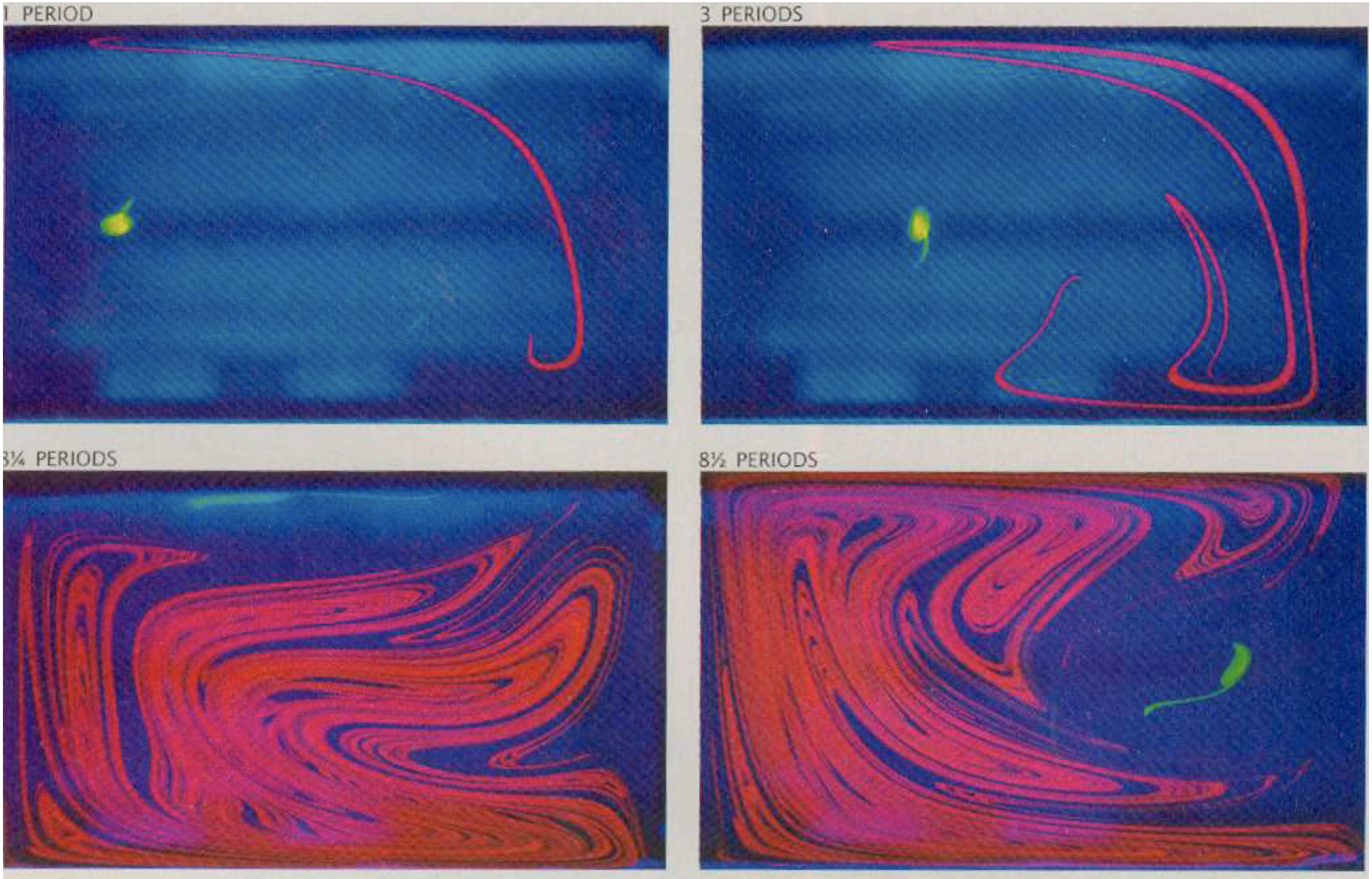
Steady two-dimensional flows are cannot mix well (no way to cross streamlines)

but, Aref (J Fluid Mech 1984) 2D time-periodic flows can mix effectively



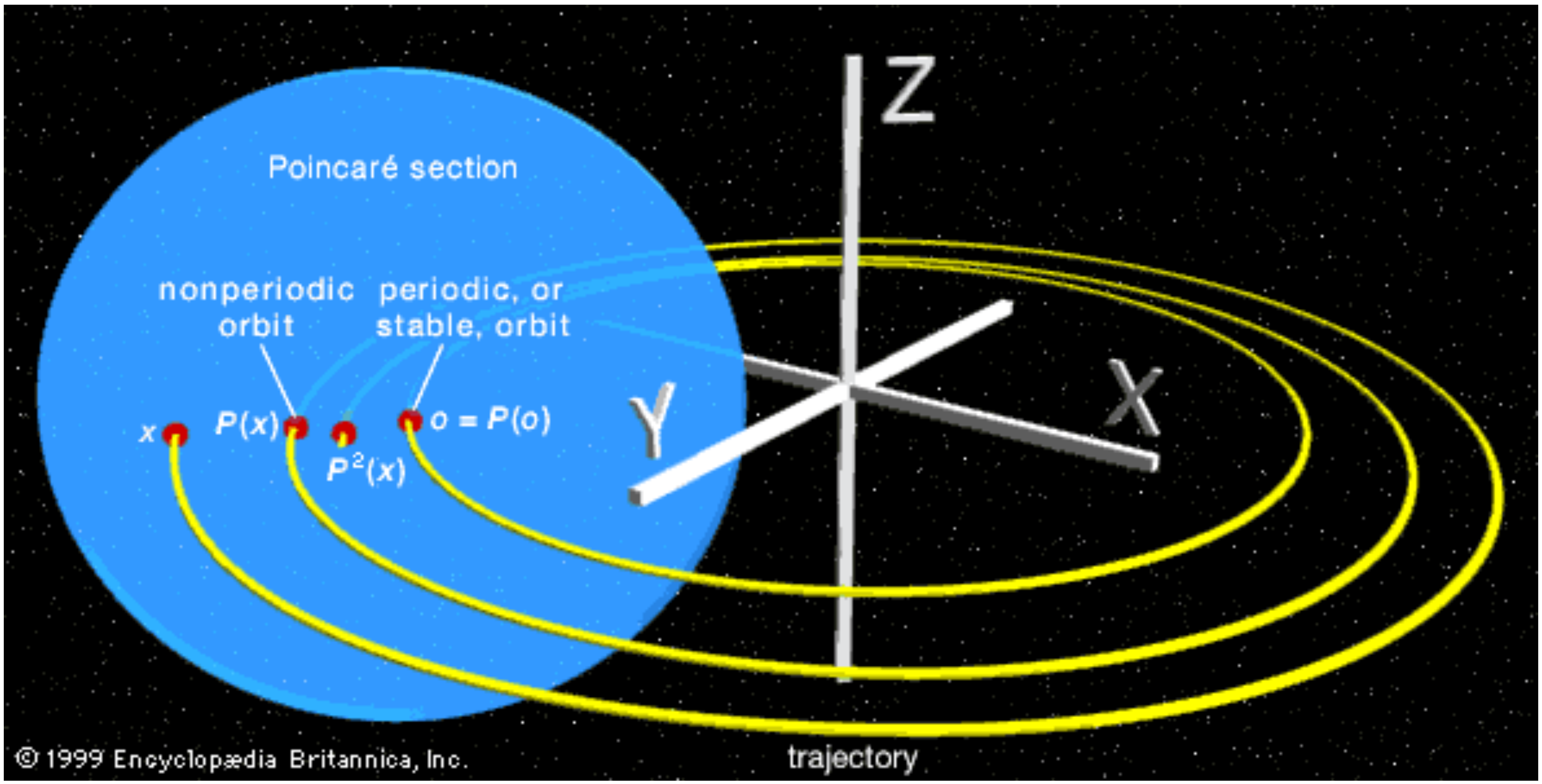
Dimensionless displacement: $D = \frac{VT}{2W}$, T is the duration of a period

Add time-dependence (periodic motion of boundaries)
well-mixed and not-well-mixed regions coexist



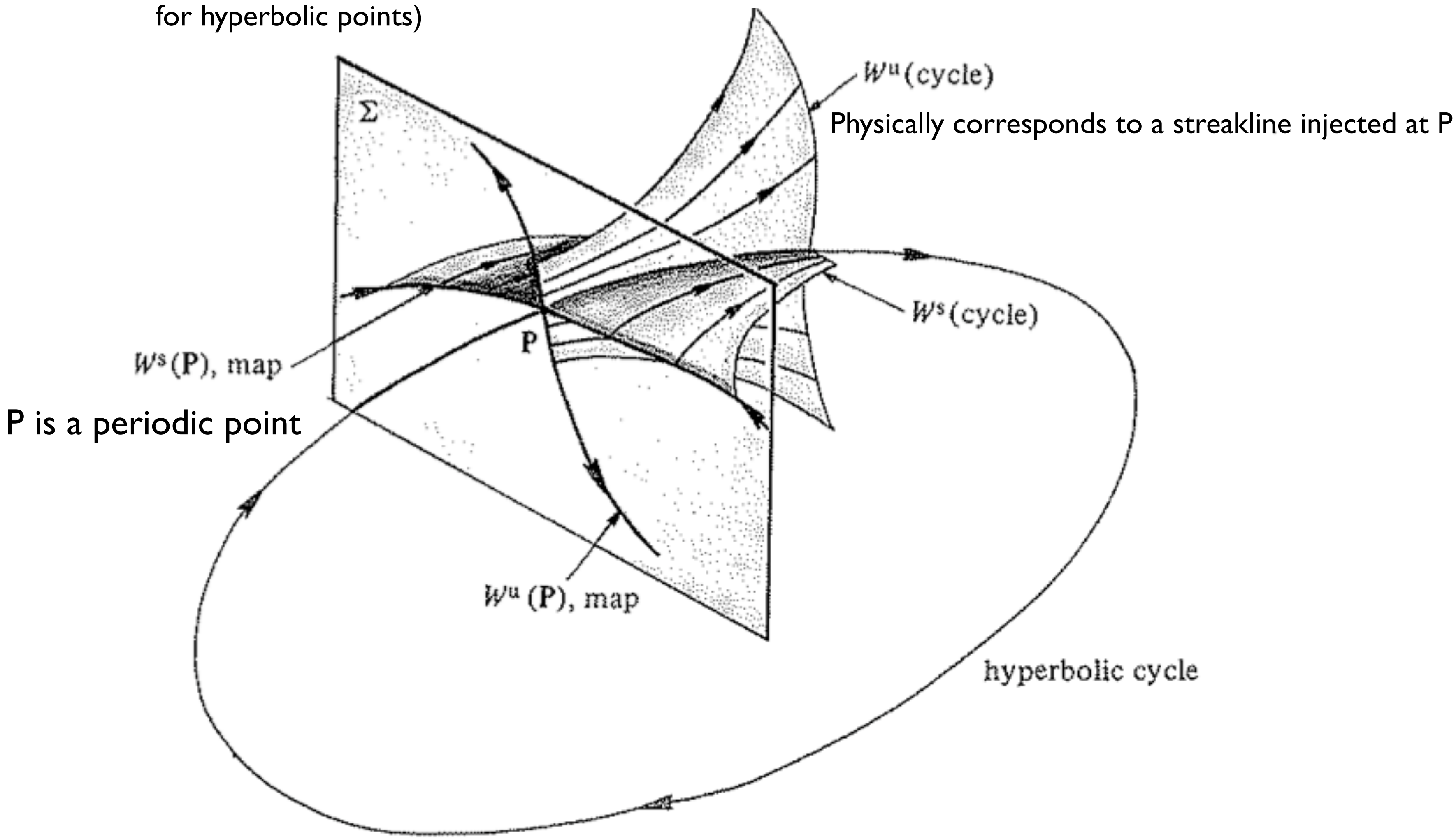
Poincare sections

(reduces dimensionality by converting flow into a map; convenient way to show the character of solutions for all possible initial conditions)

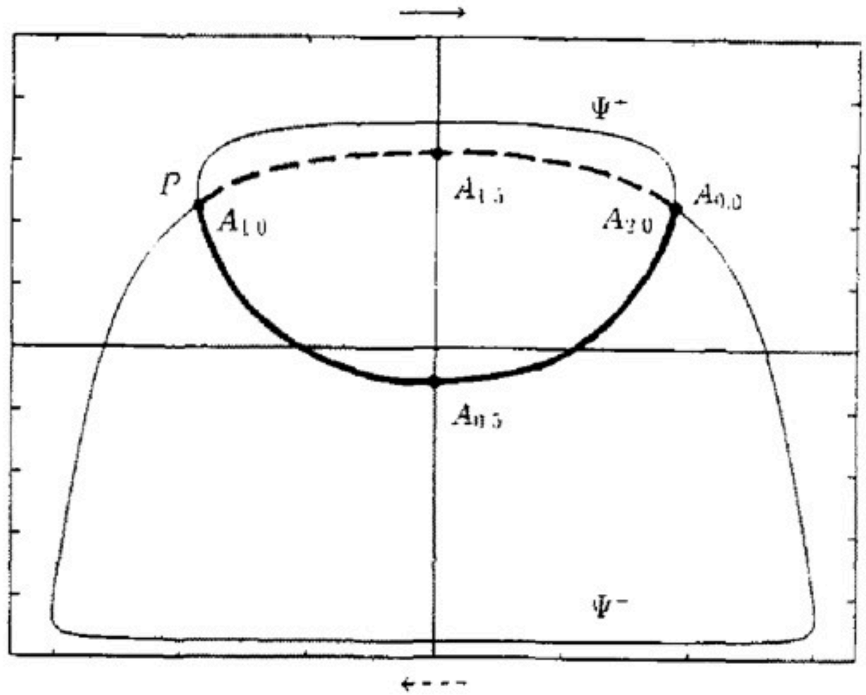


Poincare sections

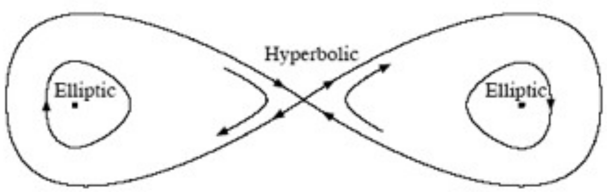
Periodic points can be hyperbolic or elliptic depending of the flow in its neighbourhood (net motion for elliptic point is rotation; contraction in one direction, stretching in another for hyperbolic points)

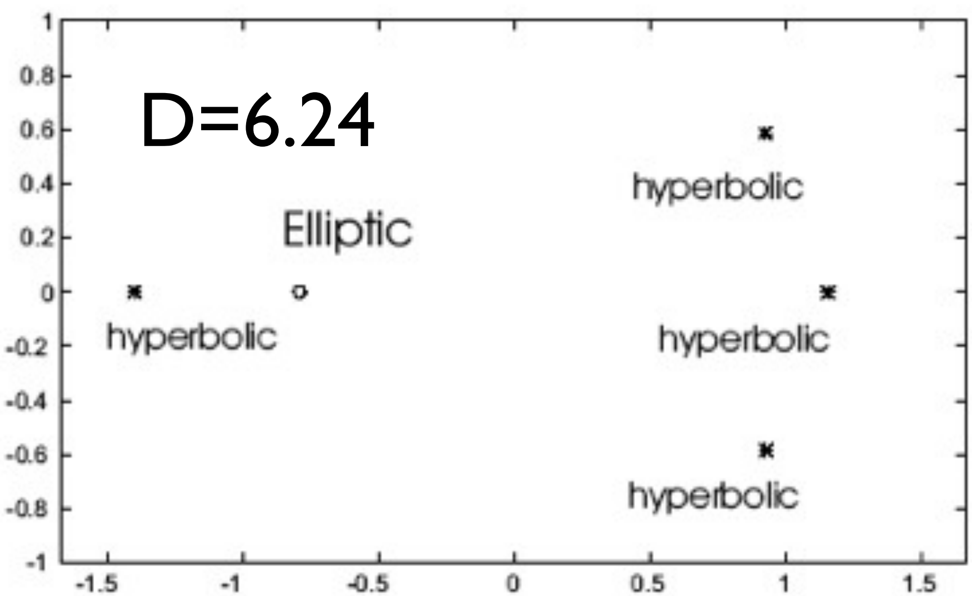


Good mixing requires that streamlines portraits at two successive times have crossing streamlines (instantaneous streamline portrait does not have to have saddle points)



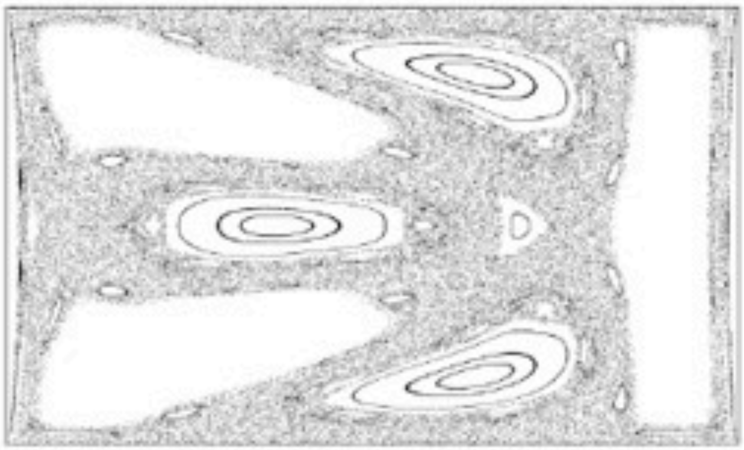
first-order periodic points



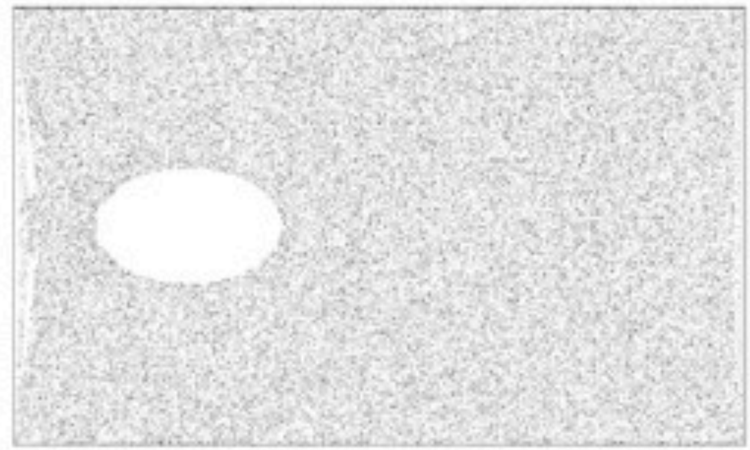


Poincare sections

(reduces dimensionality by converting flow into a map; convenient way to show the character of solutions for all possible initial conditions)

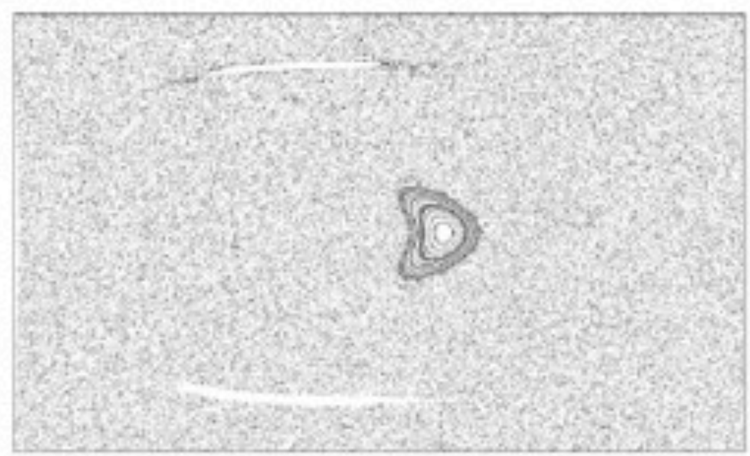


$D = 3$



$D = 6.24$

$D = 15$



$D = 20$

Stirring

Can produce complex structures AND unmixed islands

Under what circumstances does a deterministic flow widespread and efficient stretching of material surfaces (lines in 2D)?

(Mathematical) definition of chaotic flows

- The flow stretches and folds
- The trajectories of tracers are sensitive to initial conditions
- The flow has homoclinic and/or heteroclinic points
- The flow produces horsehoe maps

Mathematical characterization of stretching

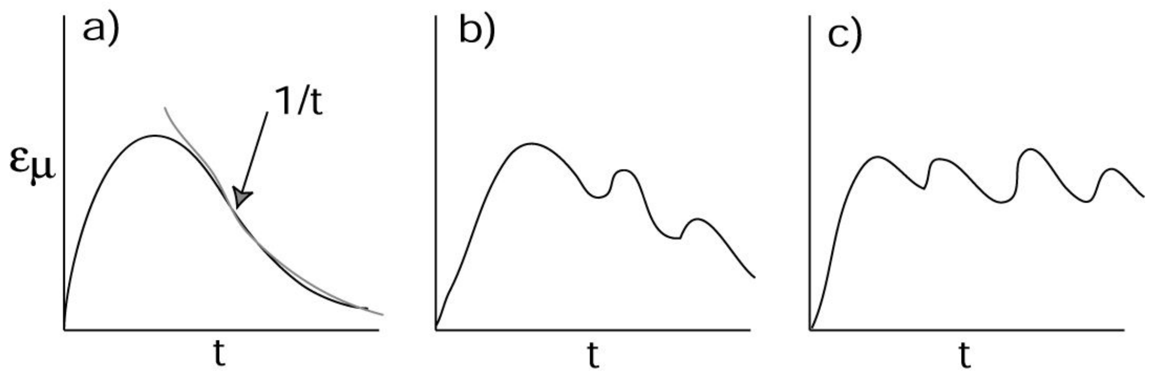
The magnitude of stretching is

$$\lambda = \lim_{|d\mathbf{X}|\to 0} = \frac{|d\mathbf{x}|}{|d\mathbf{X}|}$$

The stretching efficiency is

$$e_\lambda = \frac{D(\ln\lambda)/Dt}{(\mathbf{E}:\mathbf{E})^{1/2}} \leq 1$$

For simple shear, $e \to 0$ for large t (a random sequence of shears has a maximum of $e = 0.28$). For pure shear, $e \to 2/3$, but this requires an unbounded fluid. Hence, for good mixing, we need reorientation.

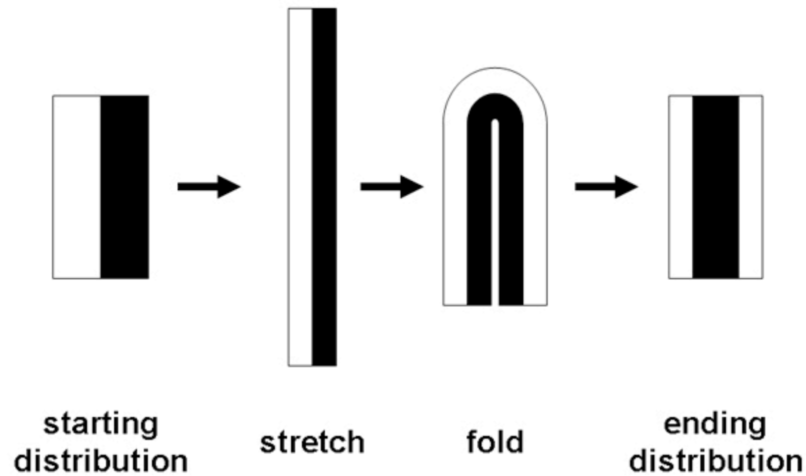


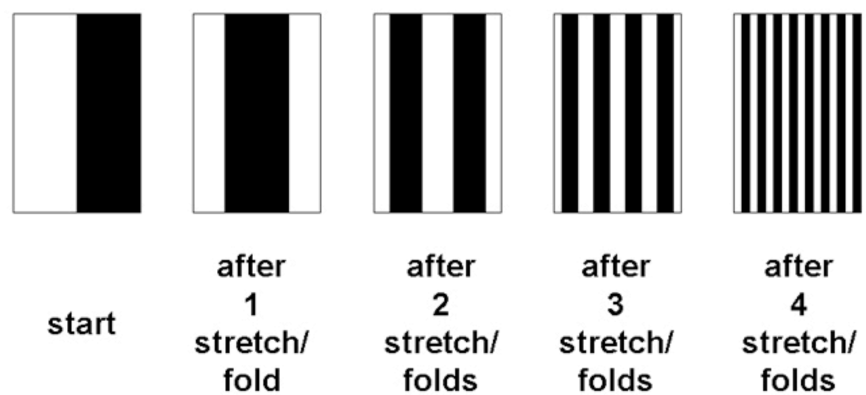
Another way to characterize mixing is with the Lyapunov exponents

$$\sigma = \lim_{|d\mathbf{X}|\to 0; t\to\infty} \left[\frac{1}{t}\ln\lambda\right]$$

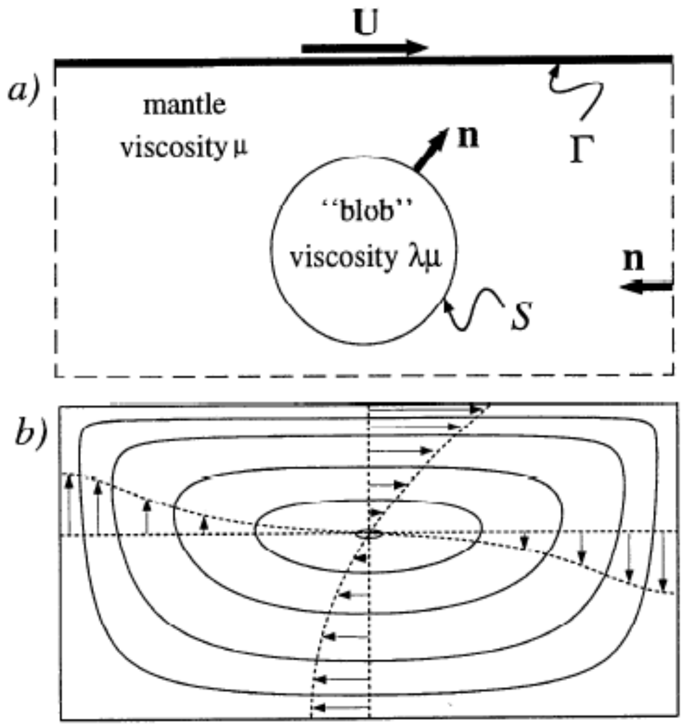
(not the same as e because $\mathbf{E}:\mathbf{E}$ varies in space and time). At a given point there is one σ in each direction and the sum is 0. Worry about the largest one.

Horseshoe maps



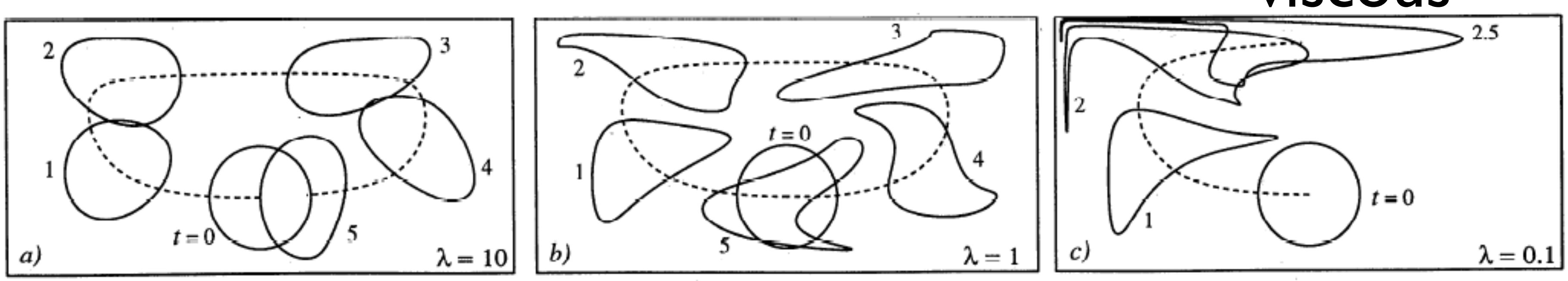


Flow must be capable of stretching and folding and returning it (stretched and folded) to its initial location – called a horseshoe map



Active heterogeneity: viscosity differences affect stretching

More viscous ⟵ Isoviscous ⟶ Less viscous

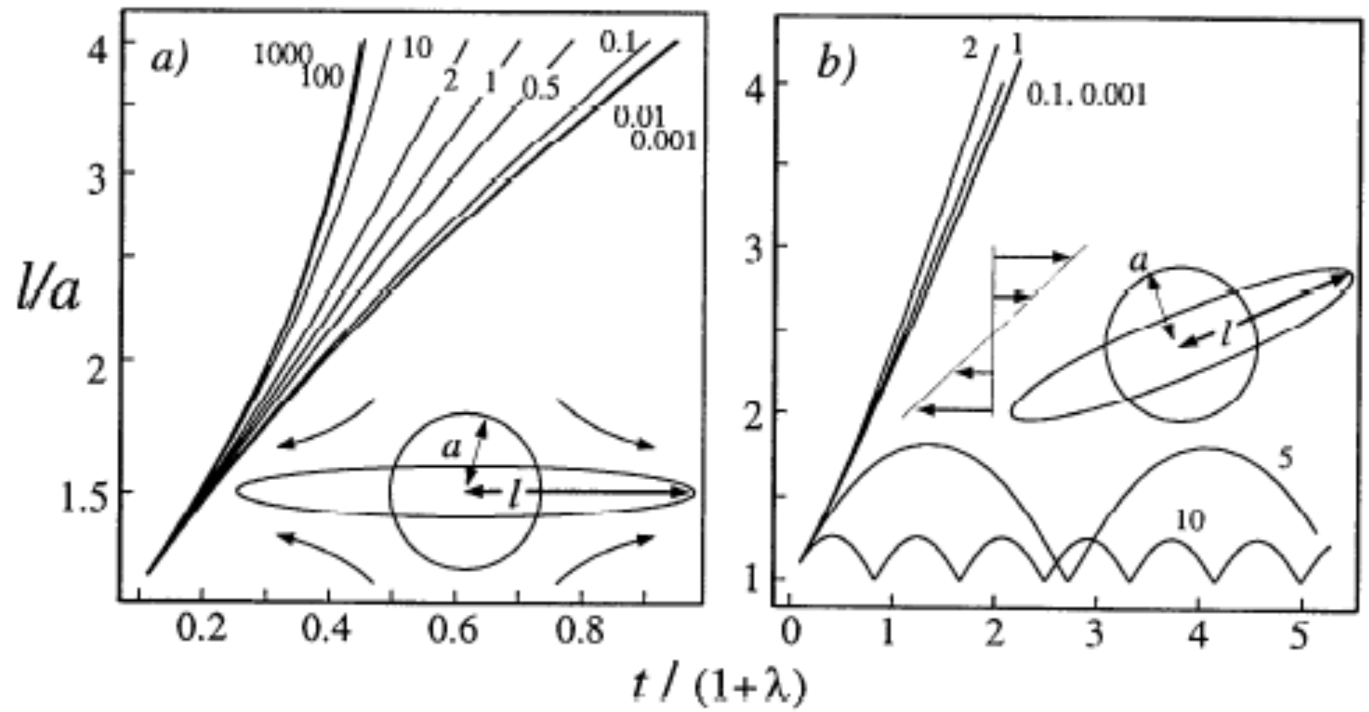


Manga, *GRL* 1996

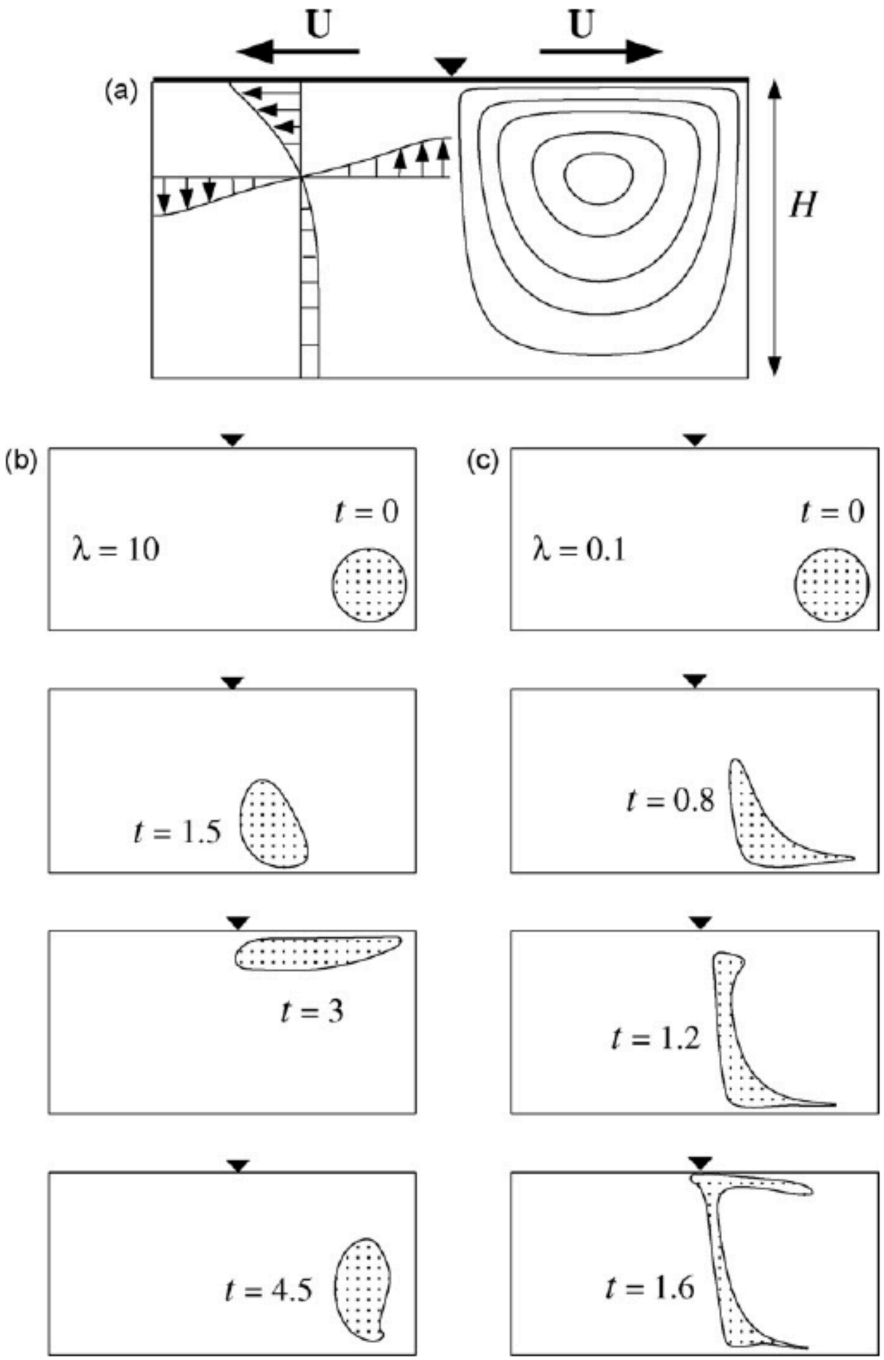
Active heterogeneity: viscosity differences affect stretching

exponential stretching

linear stretching



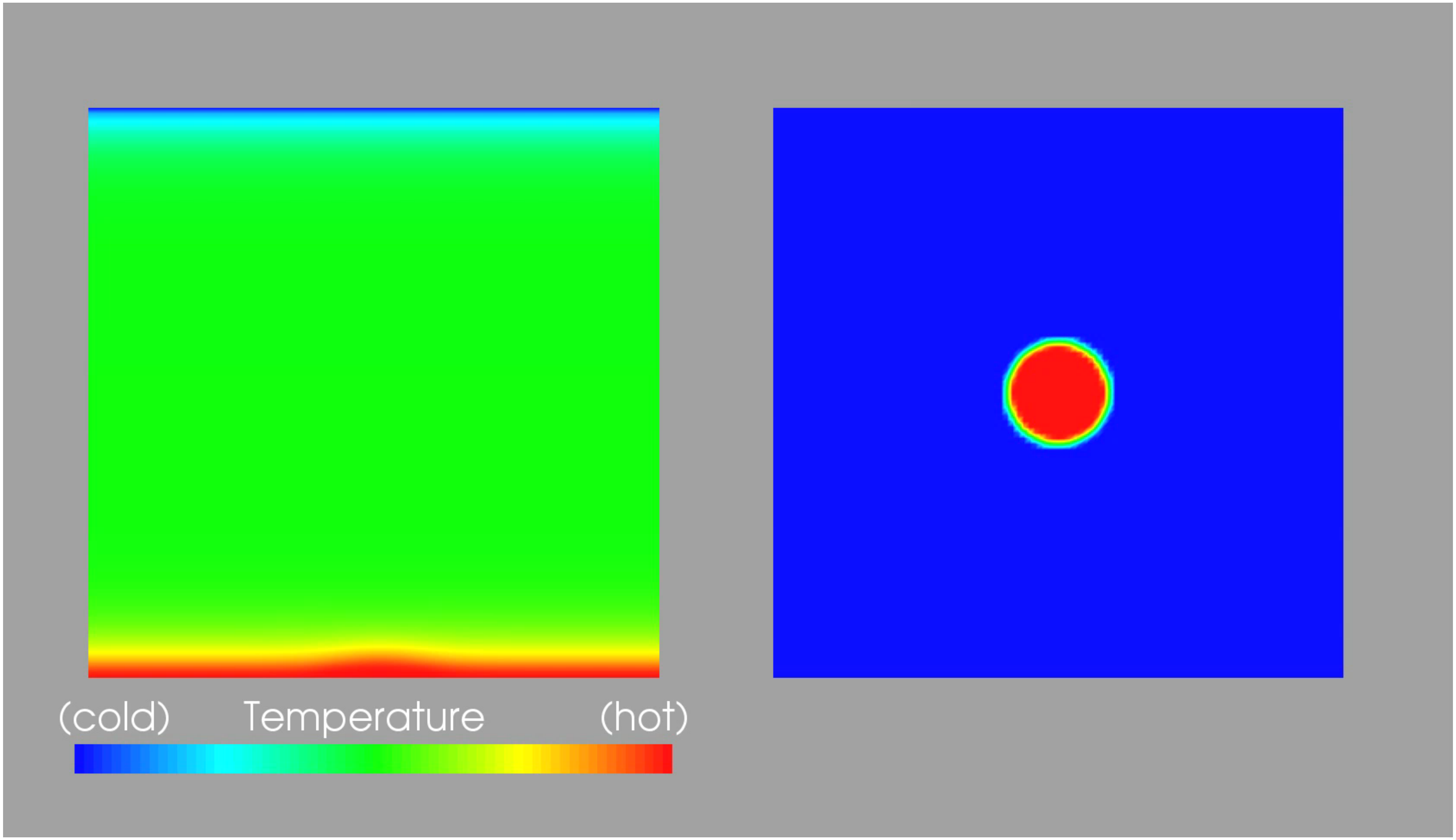
Manga, *GRL* 1996



Active heterogeneity: viscosity differences affect stretching and hence flow

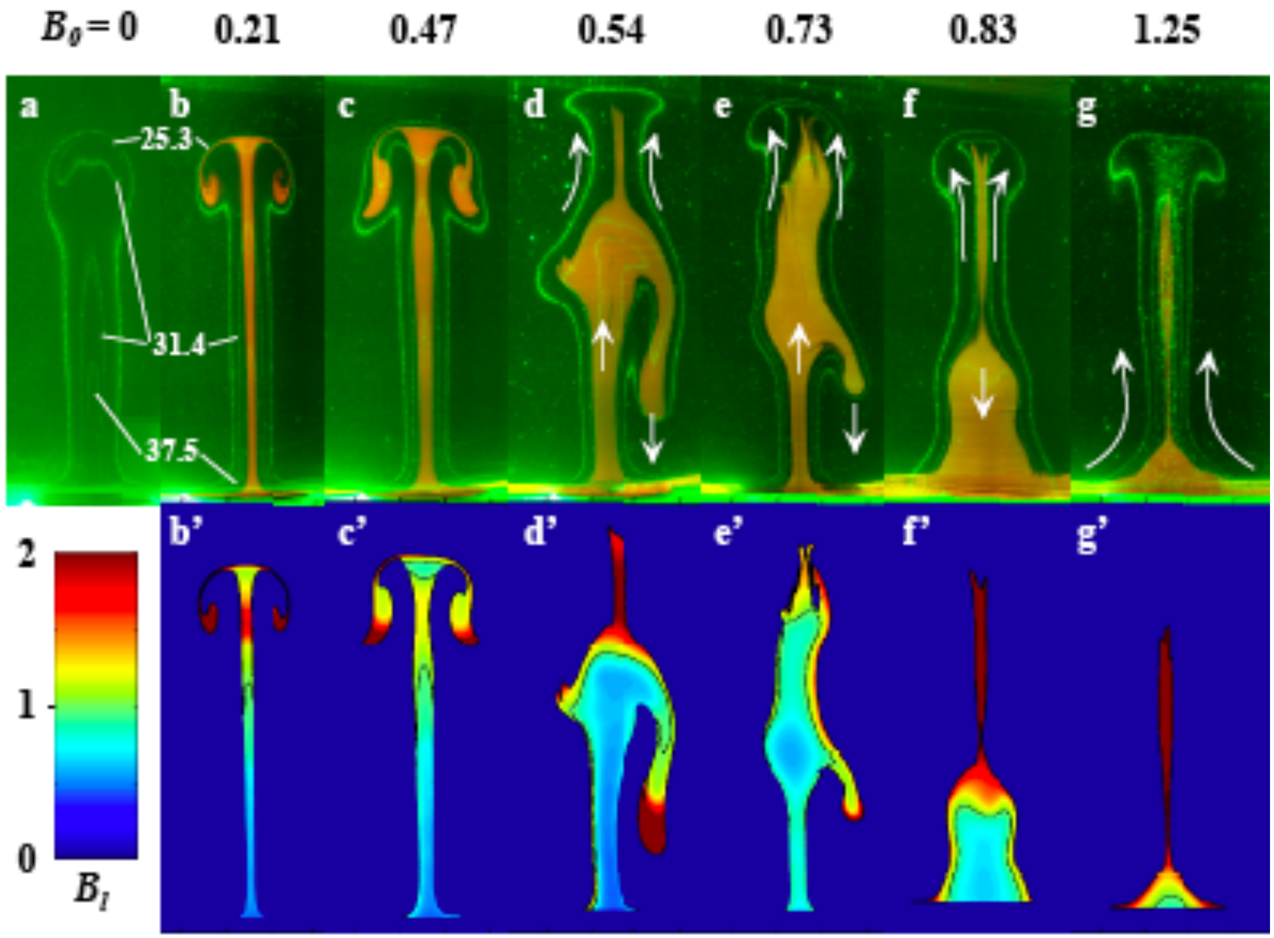
Manga, *PEPI* 2010

Active heterogeneity: viscosity differences affect stretching and hence flow



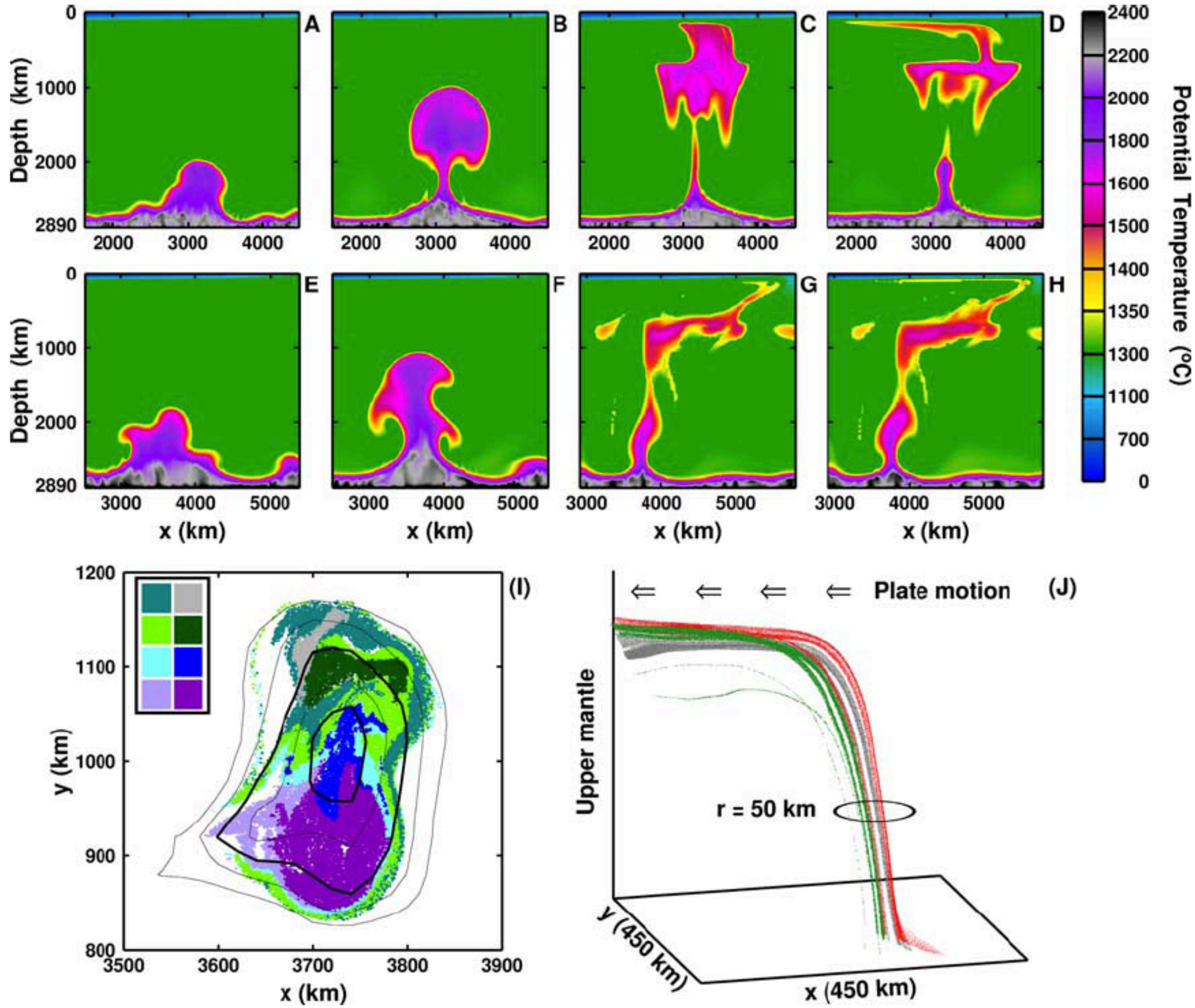
From Henri Samuel

Active heterogeneity
density differences influences velocity field
(and, of course, amount entrained)



$$B_0 = \Delta\rho_{\mathrm{Xeff}} / \rho\alpha\Delta T_{\mathrm{eff}}$$

Kumagai et al., *GRL* 2008



Farnetani and Samuel, *GRL* 2005

GI Taylor movie

Mixing in 3D

- Arnold (C R Acad Sci Paris Ser A 1965) showed that 3D steady flows can have chaotic streamlines)
- Steady, isoviscous thermal convection in a spherical shell, however, is not chaotic (Schmalzl et al. JGR 1996)
- Plate motion changes this story . . .

Mixing associated with plate motion Poloidal vs toroidal flow

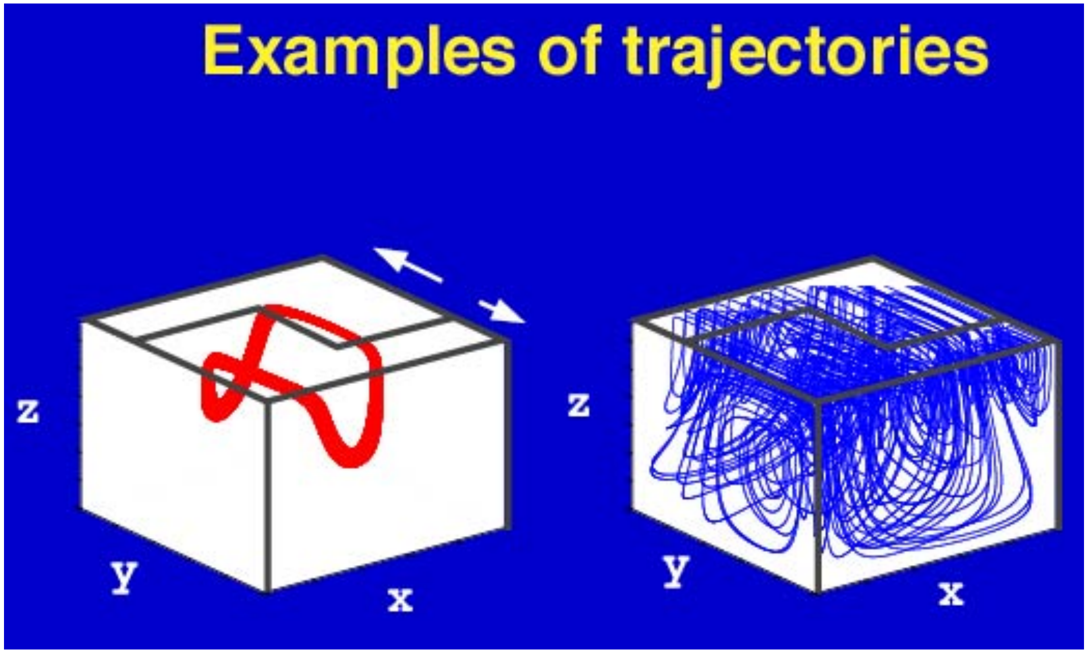
- Poloidal flow: no vertical (radial) vorticity
- Toroidal flow: rotations in horizontal (confined to spherical shells) plane

Surface manifestations

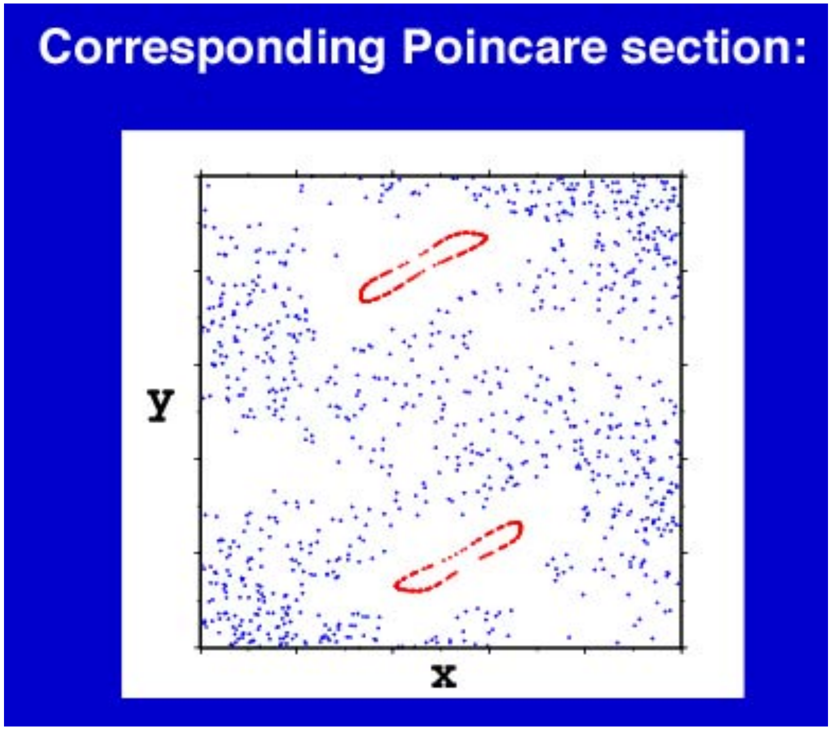
Poloidal motion: ridges and trenches

Toroidal motion: transform boundaries

Roughly equal in magnitude

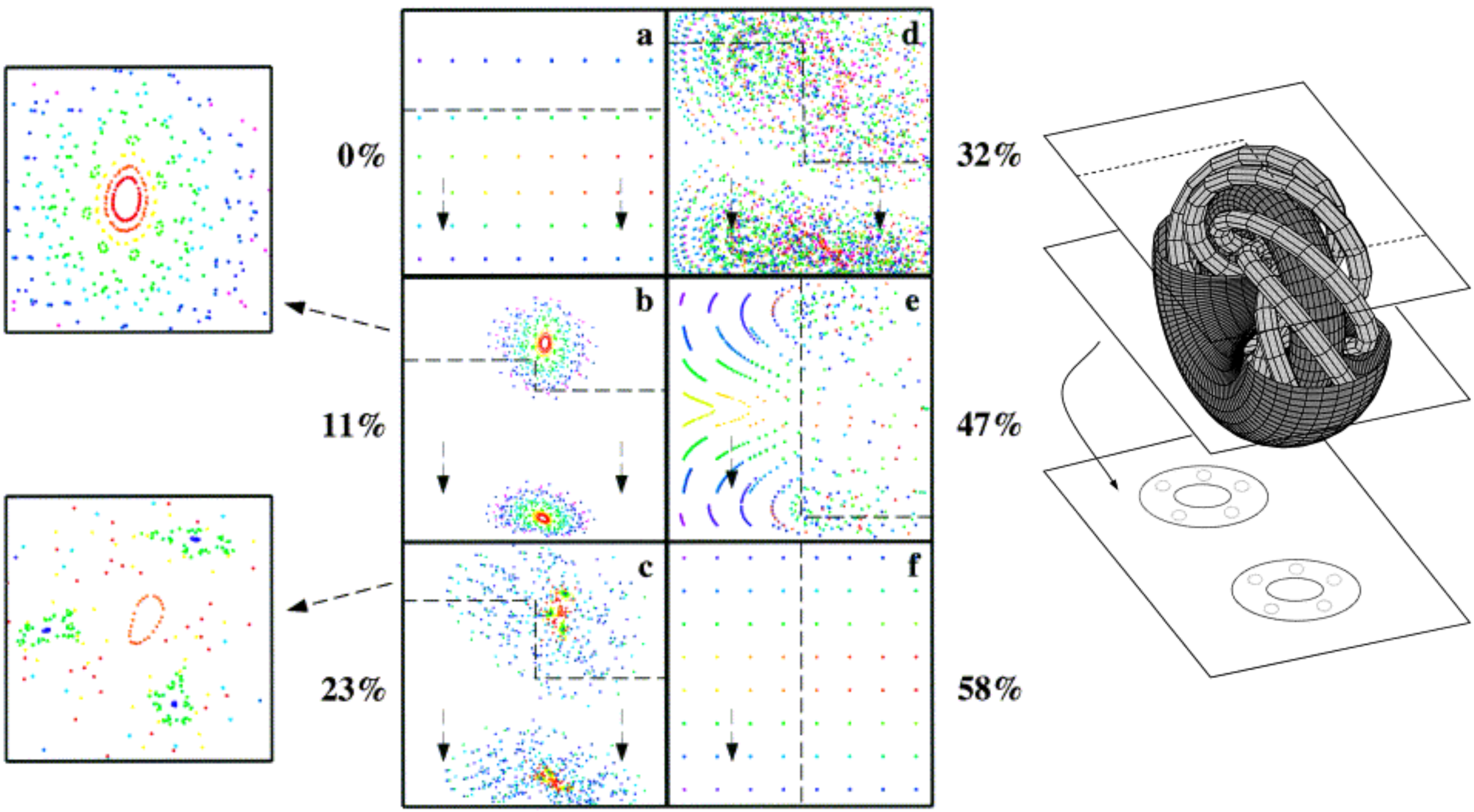


Ferrachat and Ricard, *JGR* 2001

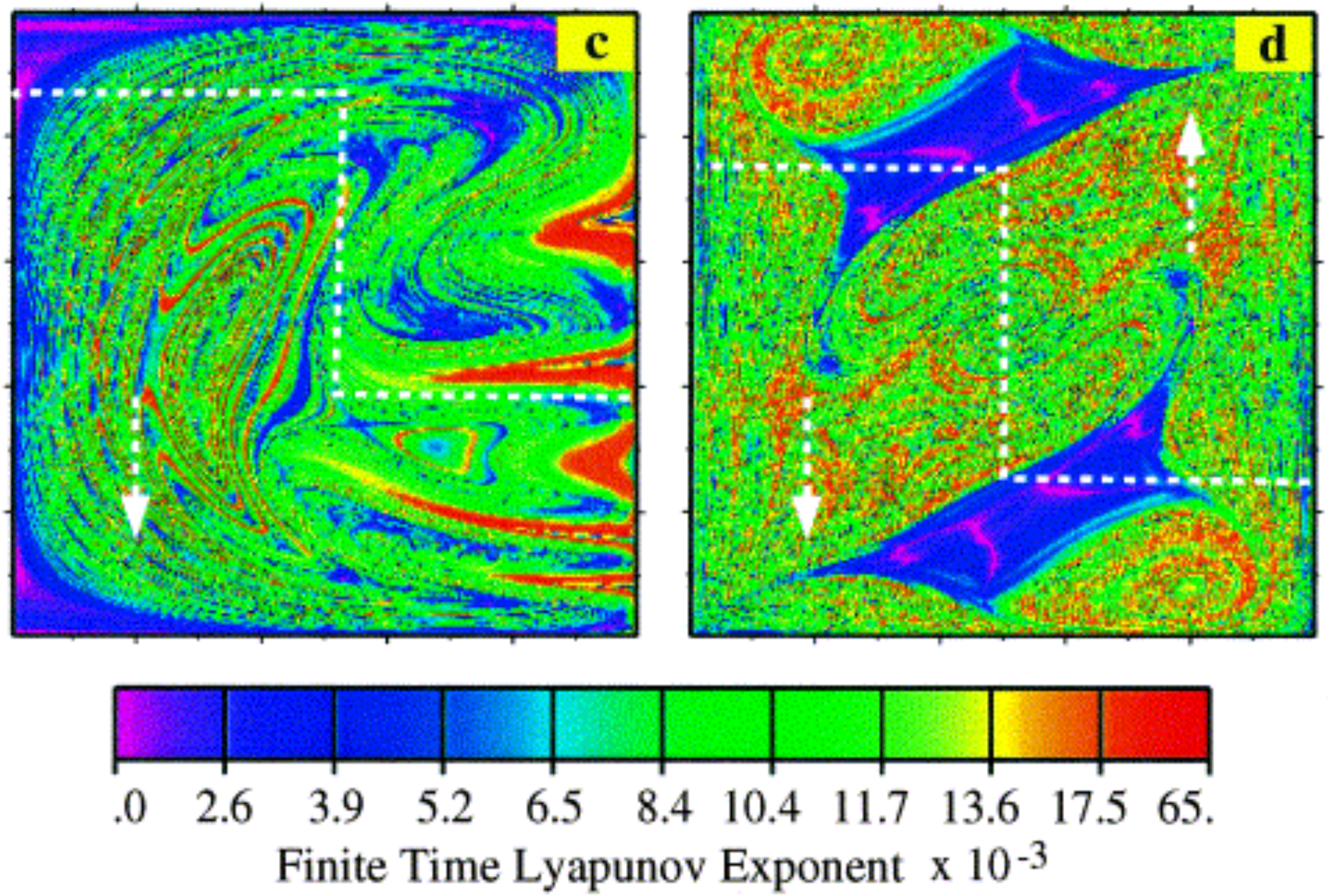


Chaotic trajectories in steady-state plate driven flows

Why? Hyperbolic points do the stretching, toroidal motion does the reorientation

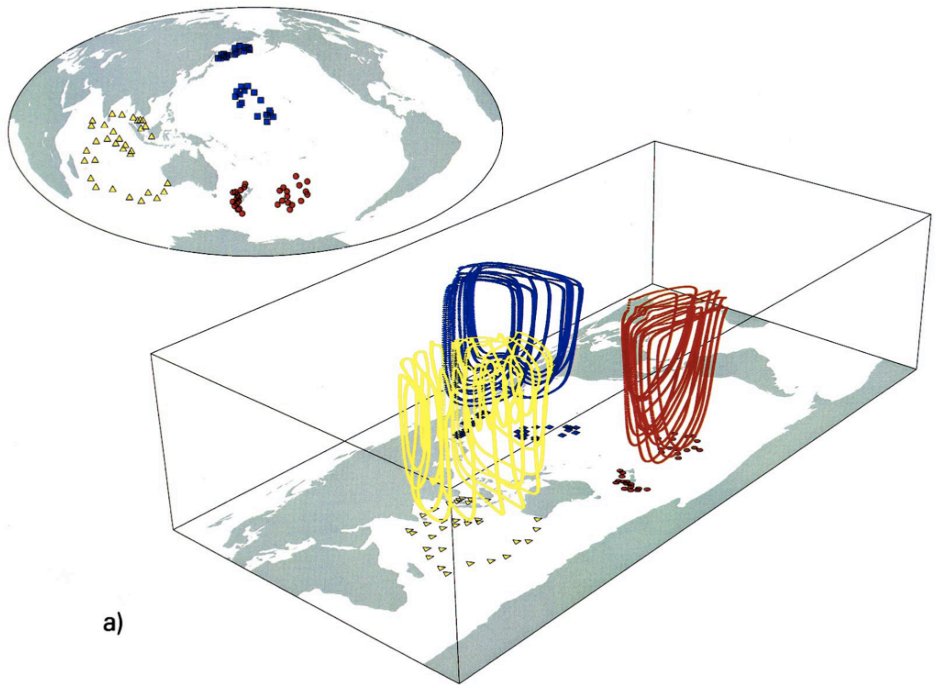


Ferrachat and Ricard, *JGR* 2001

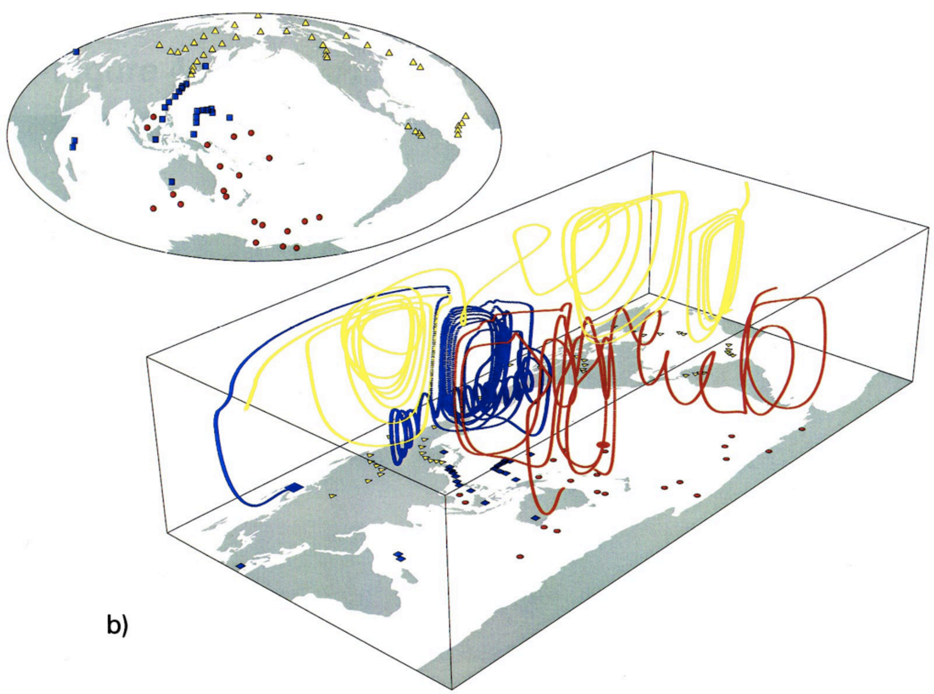


Lyapunov exponents σ estimated by tracking tracers:
Both chaotic and laminar mixing are observed

Ferrachat and Ricard, *JGR* 2001



a)



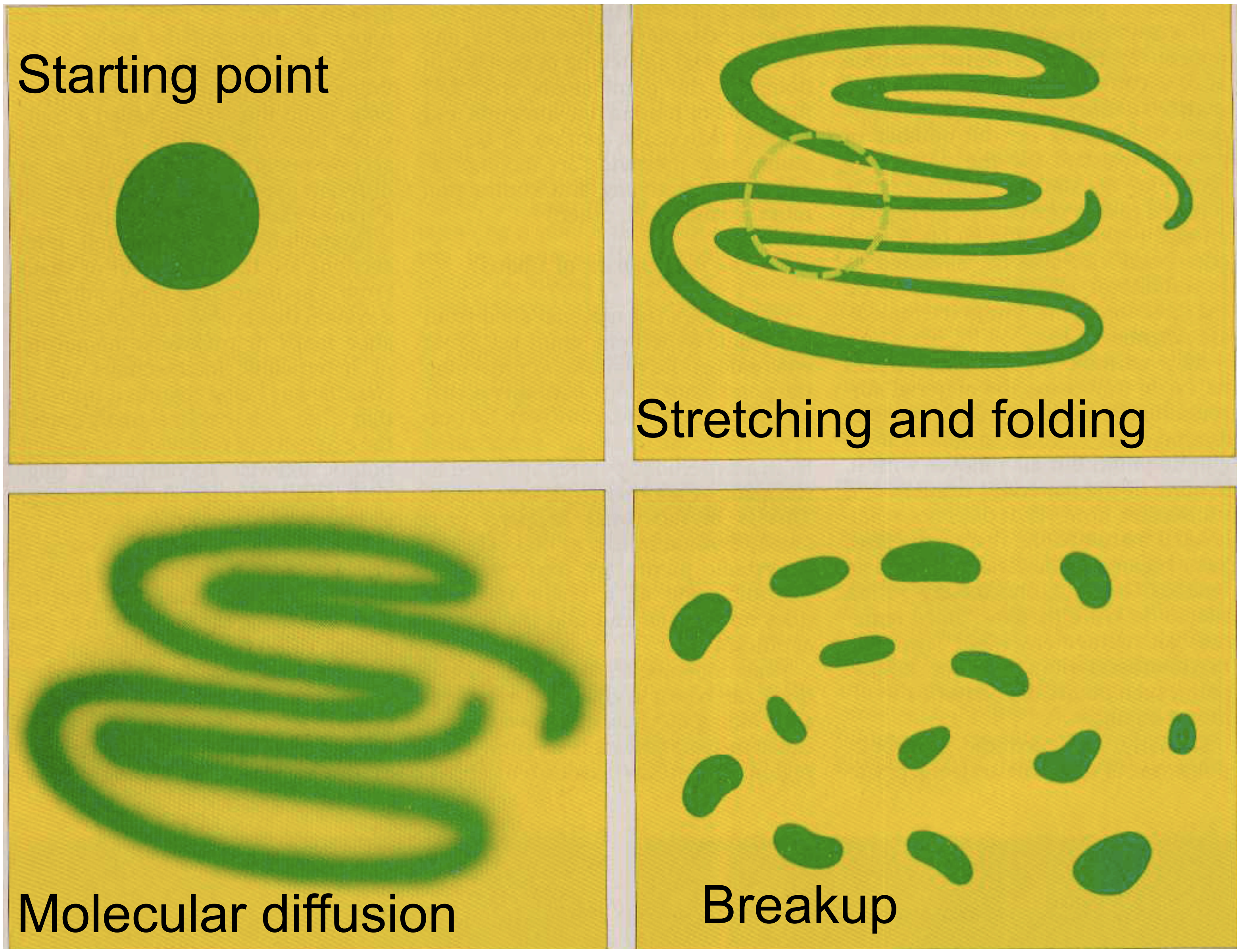
b)

With plate motion, well mixed and poorly mixed regions

Take steady flow driven present day plate motion and trace particles for 4 Ga

van Keken and Zhong, *EPSL* 1999

How does mixing occur?



Ottino, *Scientific American* 1989

What about chemical diffusion?

$$\frac{DC}{Dt} = \kappa \nabla^2 C$$

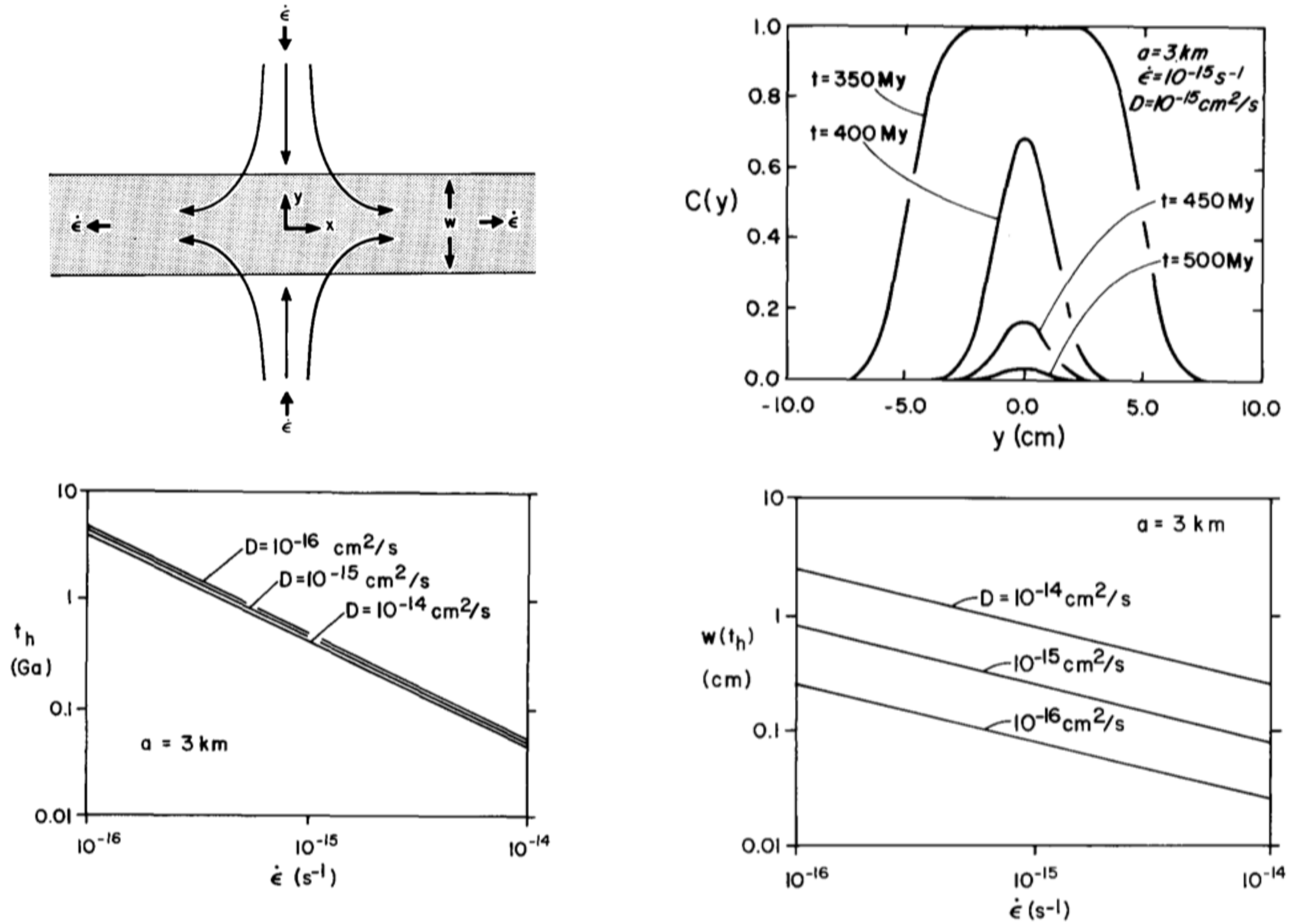
$$O(\Delta C / t) \qquad O(\kappa \Delta C / \delta^2)$$

$$\delta \sim \sqrt{\kappa t}$$

Diffusivities are 10^{-18} – 10^{-20} m^2/s in the mantle
10^{-11} m^2/s in magmas

In 4 Ga, diffusion over < 1 m in the mantle
In 30 ka, diffusion over 1 m in magmas

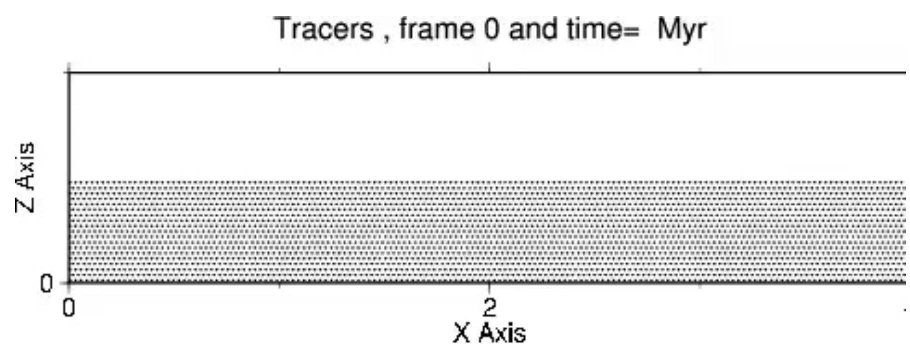
What about chemical diffusion?



Kellogg and Turcotte, *EPSL* 1987

Some ways to analyze mixing in models of the mantle

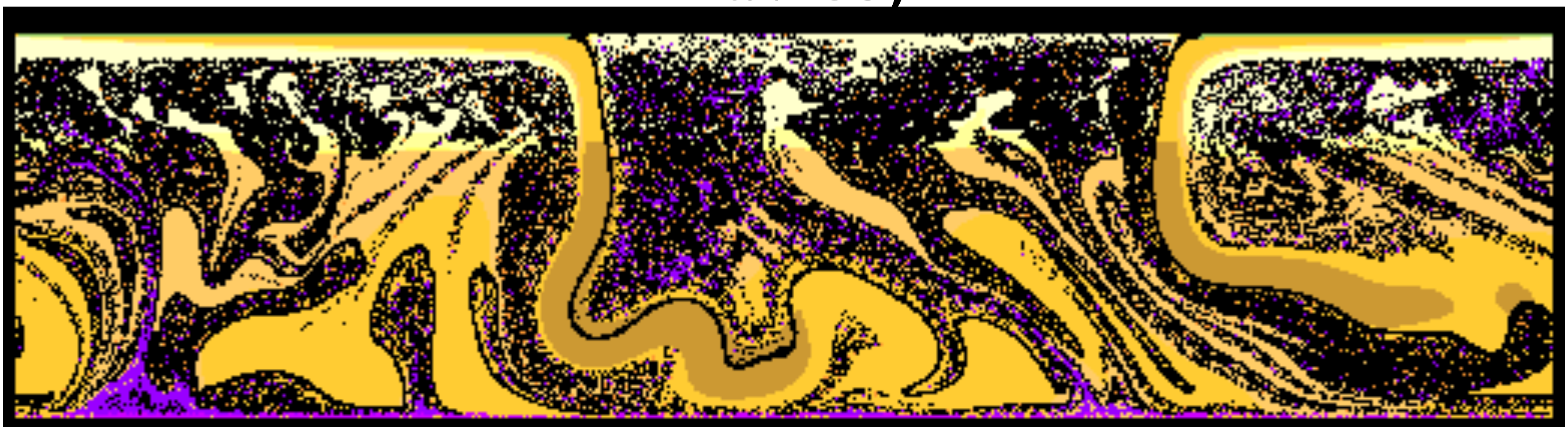
- Dispersal of heterogeneities (visually or using statistical methods)
- Compute derived isotopic signatures



4 Ga of processing mantle at ridges (Geoff Davies)

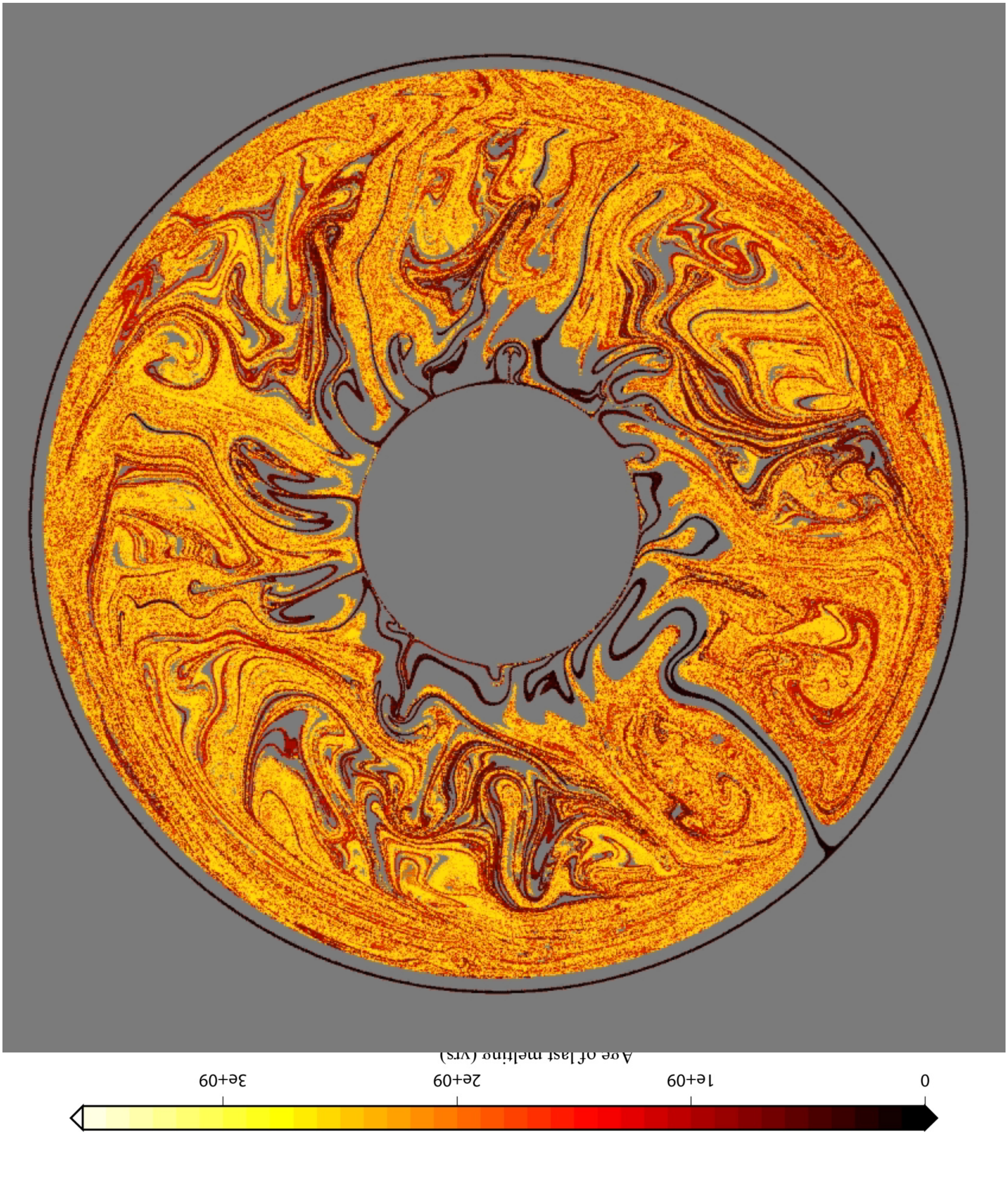
250,000 tracer particles (initially orange)
Crust (stuff melted below ridges) in black
Crust that gets within 20 km of the CMB in purple
Darkness scales with viscosity

Stirring and segregation (Geoff Davies)



Tracers are more dense than surroundings
Segregation of depleted mantle from crust

Brandenburg et al., *EPSL* 2008



Characterization of structure

"Important to distinguish between mixing measure and the process producing mixing . . . The measure should be selected according to the application, and the measurements should be related to the fluid mechanics." Ottino, *Kinematics of mixing* 1989

- e and σ characterize effectiveness of a given *flow* at stirring
- Other measures can be used to characterize observed *structures* (e.g., spectral analysis, fractal analysis)
- Easiest: striation thickness *s* (1D)
- Characterization of structure can be used to distinguish between mixing processes

Evidence for length scale reduction in the mantle (?), recorded in an exposed peridotite

NATURE VOL. 323 11 SEPTEMBER 1986 — ARTICLES — 125

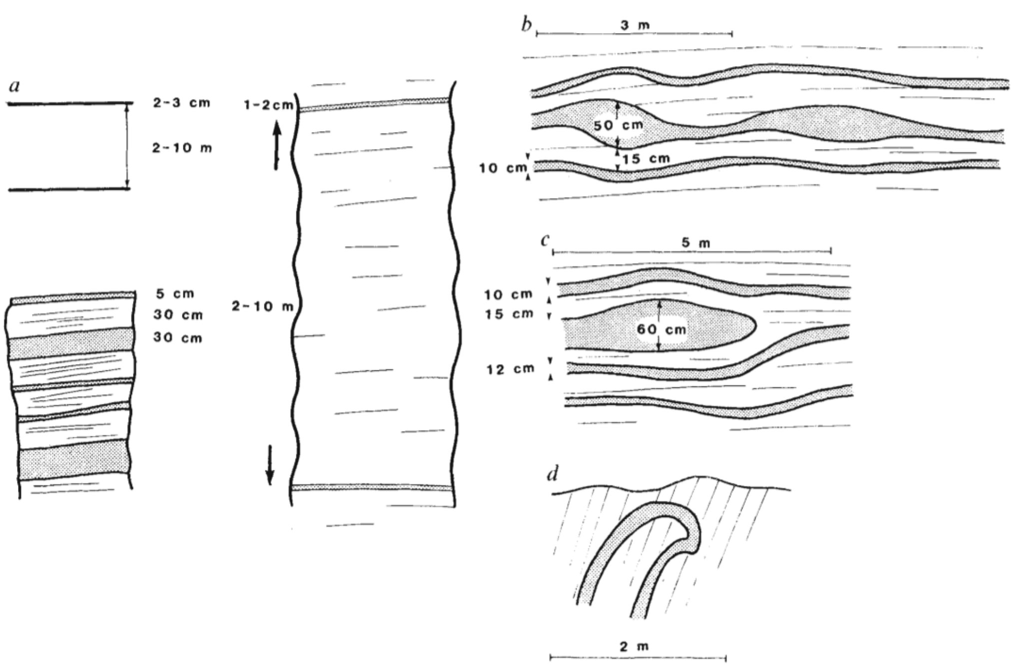


Fig. 2 Occurrences of pyroxenite layers in the Beni Bousera high-temperature peridotite. Grey, pyroxenite; white, lherzolite with foliation. *a*, Occurrences in an outcrop with no folding; *b*–*d*, occurrences with folding and boudinage.

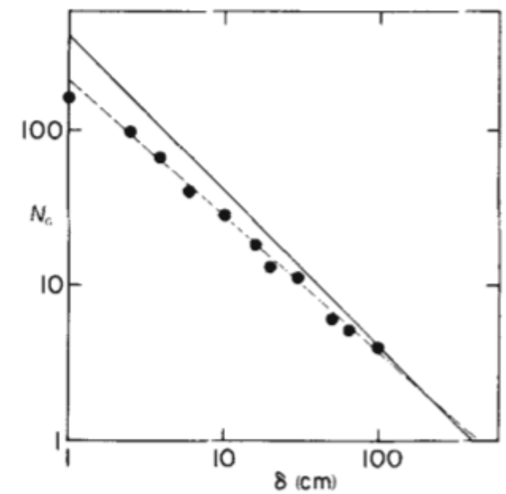


Fig. 3 Number of pyroxenite layers exposed at Beni Bousera with a thickness greater than δ, N_c, as a function of δ. Points, observations; dashed line, $N_c \propto \delta^{-0.87}$; solid line, $N_c \propto \delta^{-1}$.

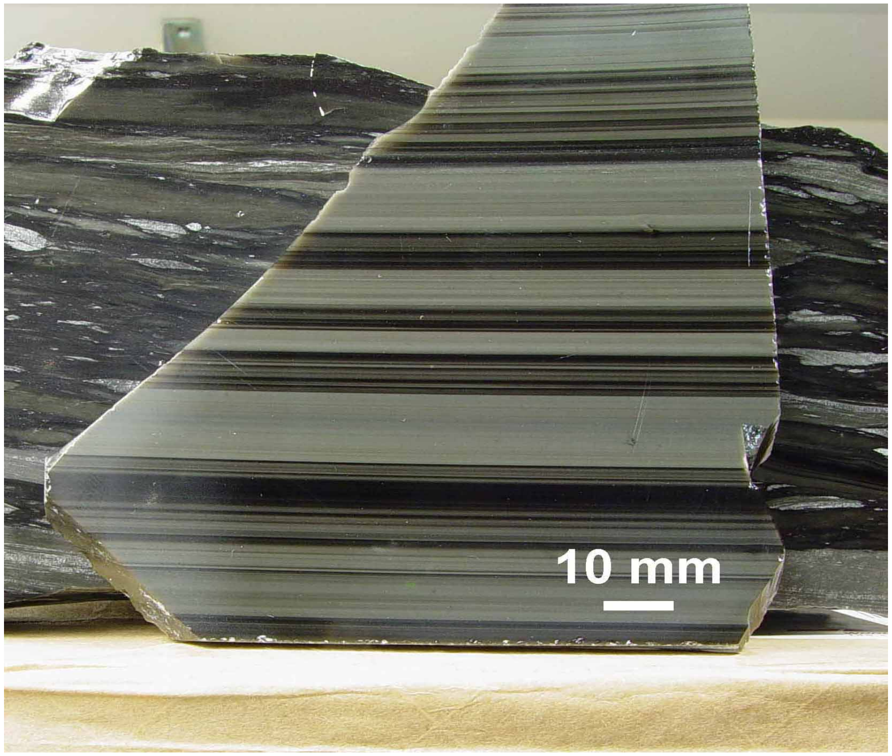
Allègre and Turcotte, *Nature* 1986

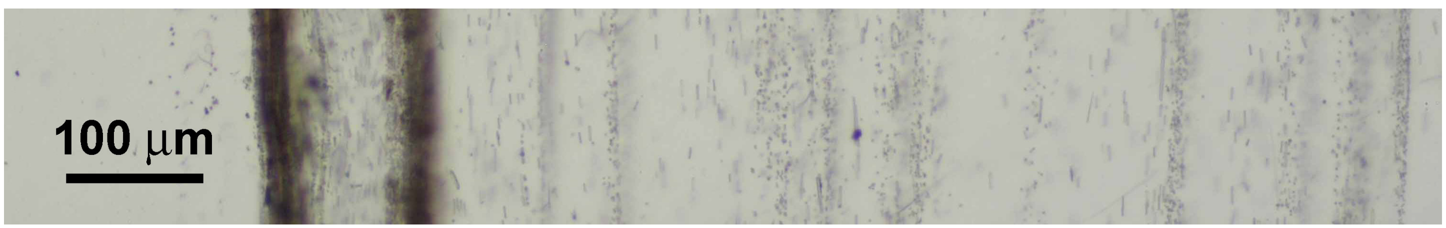
The scale of heterogeneity led Allègre and Turcotte (1986) to support their 'marble cake' structure to the mantle

Static.ifood.tv

Easier to see in magmas . .

Obsidian is banded at all scales





Do these bands (in some cases) record how the obsidian deformed?

Ideas for how to make these bands?

Terminology

Scale invariance: Attributes do not changes if lengths are changes (no specific scale can be identified - all scales are equally important)

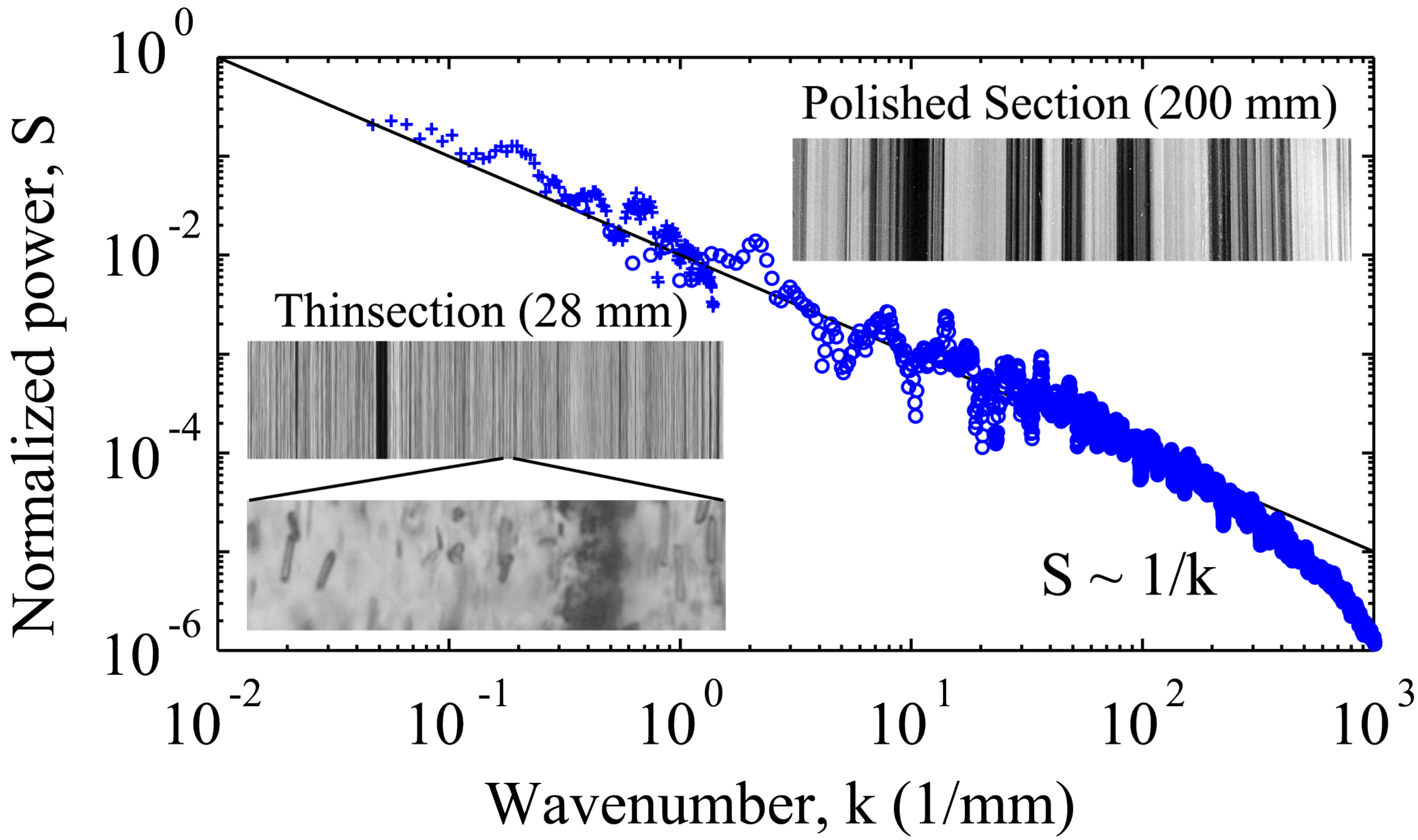
Fractal: A fractal is generally "a rough or fragmented geometric shape that can be split into parts, each of which is (at least approximately) a reduced-size copy of the whole,"[1] a property called self-similarity. Roots of mathematical interest on fractals can be traced back to the late 19th Century, the term however was coined by Benoît Mandelbrot in 1975 and was derived from the Latin fractus meaning "broken" or "fractured."

Multifractal: A single exponent is insufficient, and a continuous spectrum of exponents is needed; around any point, there is a local power law and the “singularity distribution” describes its variation

Multiplicative: recursive process that produce interdependencies in different scales, results in multifractal properties

From Wikipedia

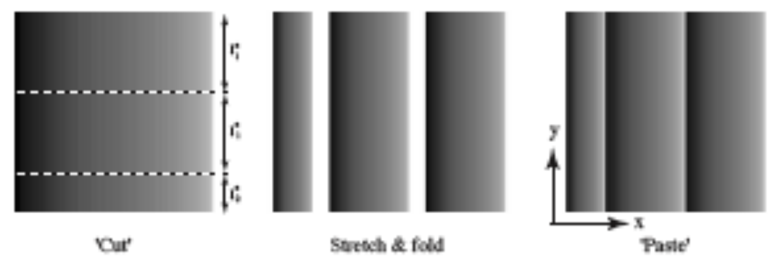
Power spectrum: Scale invariant banding

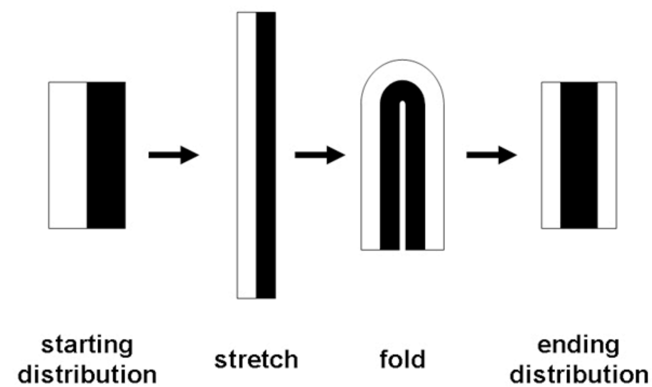


Band widths are scale invarient over 4 orders of magnitude

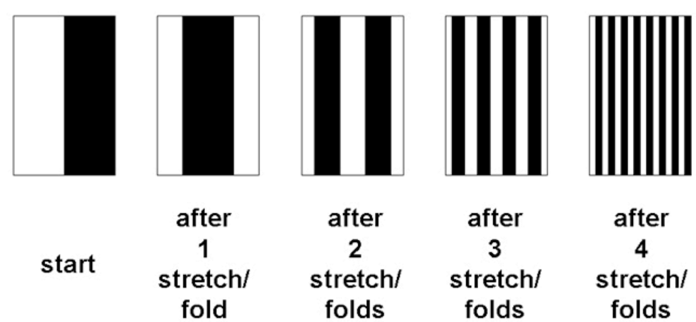
model for mixing by convection

Baker's map

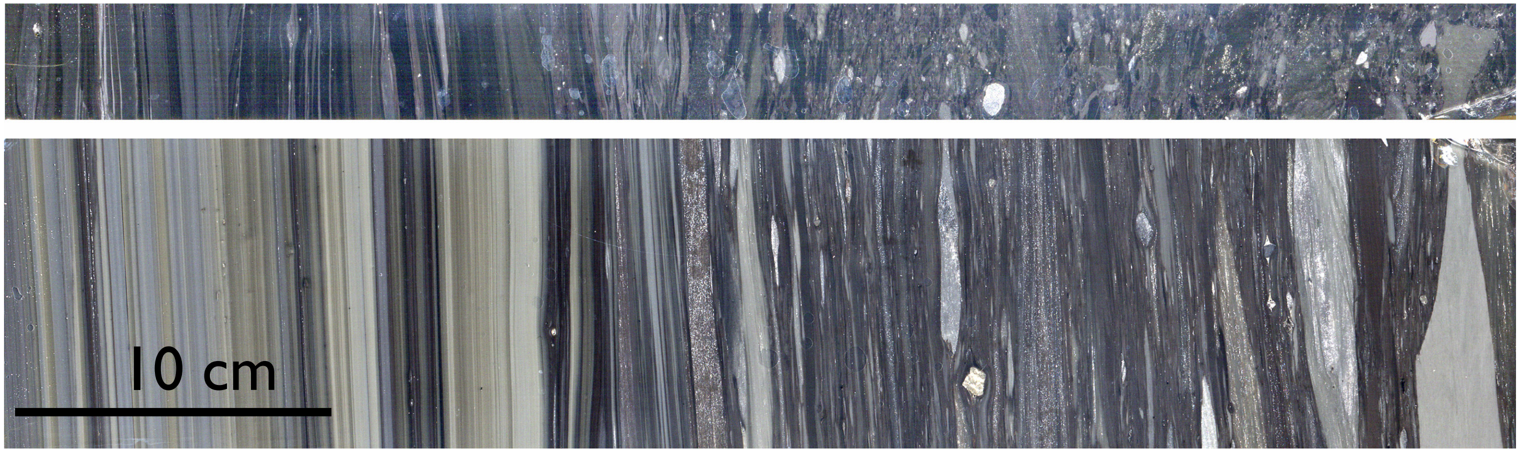




Horseshoe maps

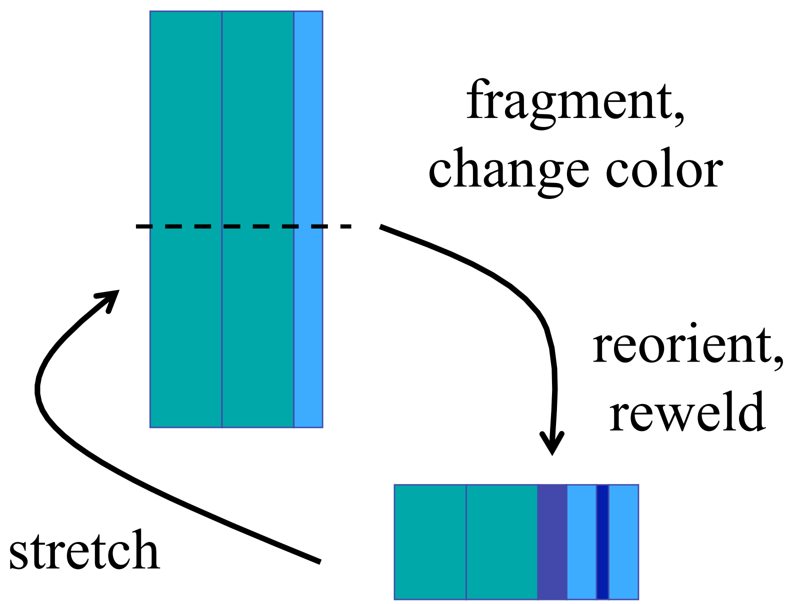


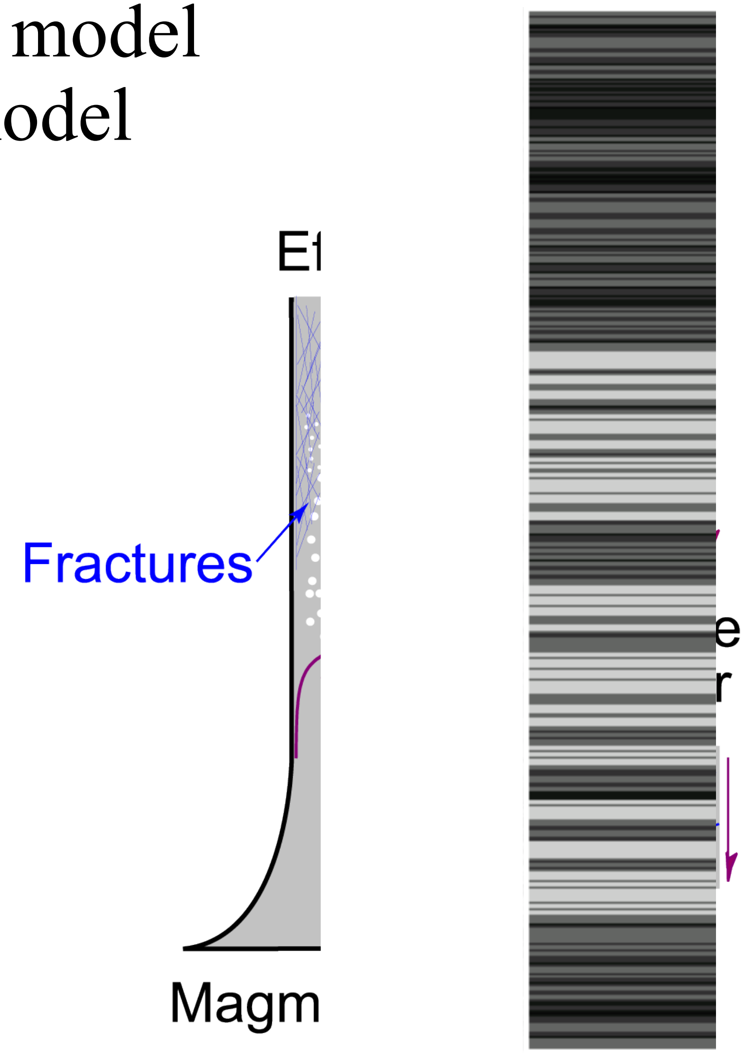
Brecciation, rewelding and deformation



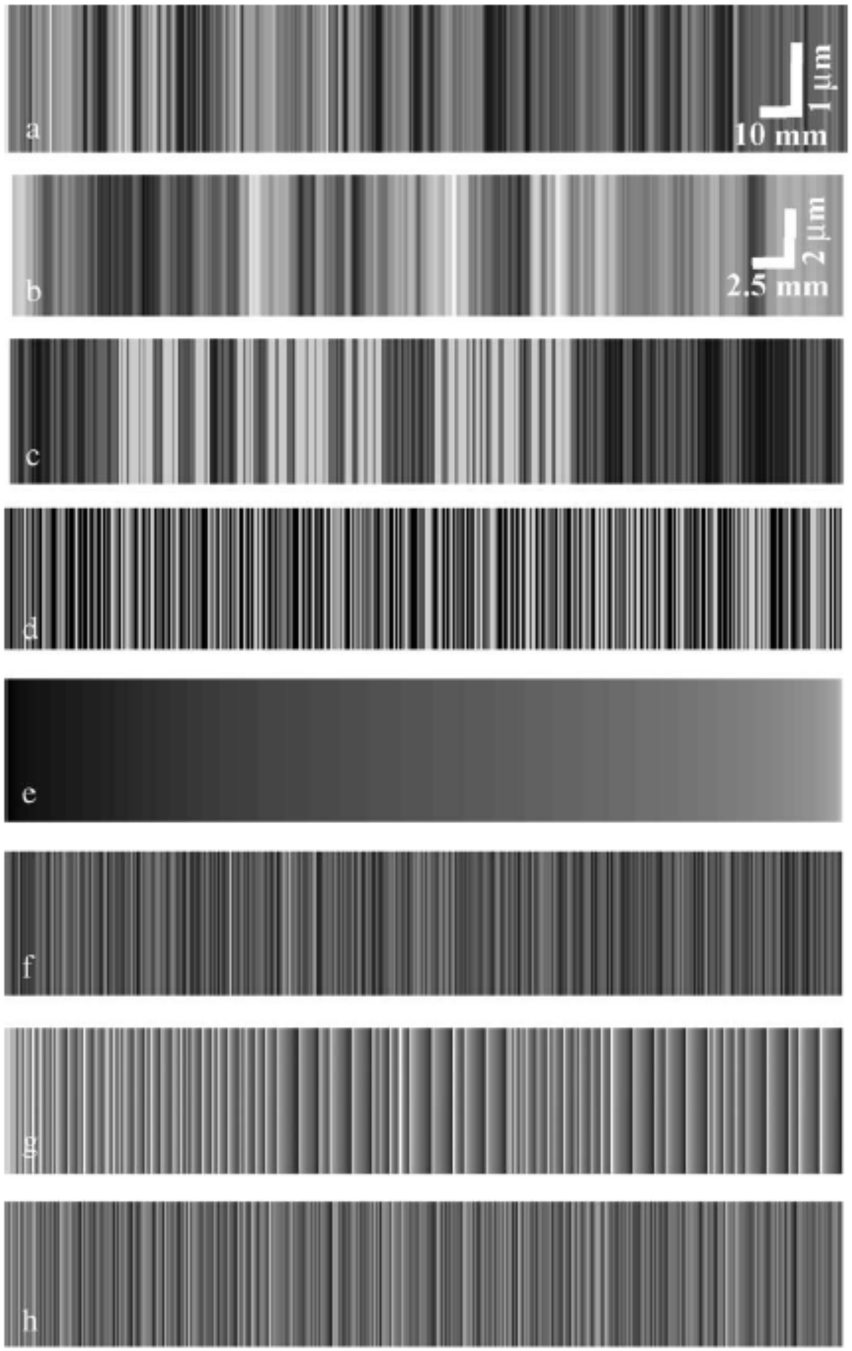
Cantor set

A representative model
Cantor model





Bands consistent with repeated brecciation, reorientation of fragments, welding (stick back together) and stretching (reproduce power law and multifractal characteristic of bands)



BGM (200 mm)

MI (55 mm)

Cantor set

B&W Cantor set

BGM color sorted

Random redistribution

4 step Baker

6 step Baker

Table 1
Comparison of obsidian samples

Record	MF[a]	$(S \sim k^{-1})$[b]	MP[c]	Implications
Big Glass Mountain (BGM)	Y	Y	Y	Concurrent microlite growth and deformation into bands
Mayor Island (MI)	Y	Y	Y	Concurrent formation of variable vesicularity and deformation
Cantor (MC)	Y	Y	Y	Concurrent development of heterogeneity and deformation
Cantor binary	N	N	N	No binomial measure
BGM randomized	N	N	N	Decoupled microlite growth and deformation into bands
Baker's map	Y	N	N	Decoupled microlite growth and deformation into bands

Cantor map with hypothesis tests.
[a] Multifractal.
[b] S is spectral power and k is wavenumber.
[c] Multiplicative process.

Baker's map should describe convective stirring

Sampling filter

decreasing sampling volume

increasing time

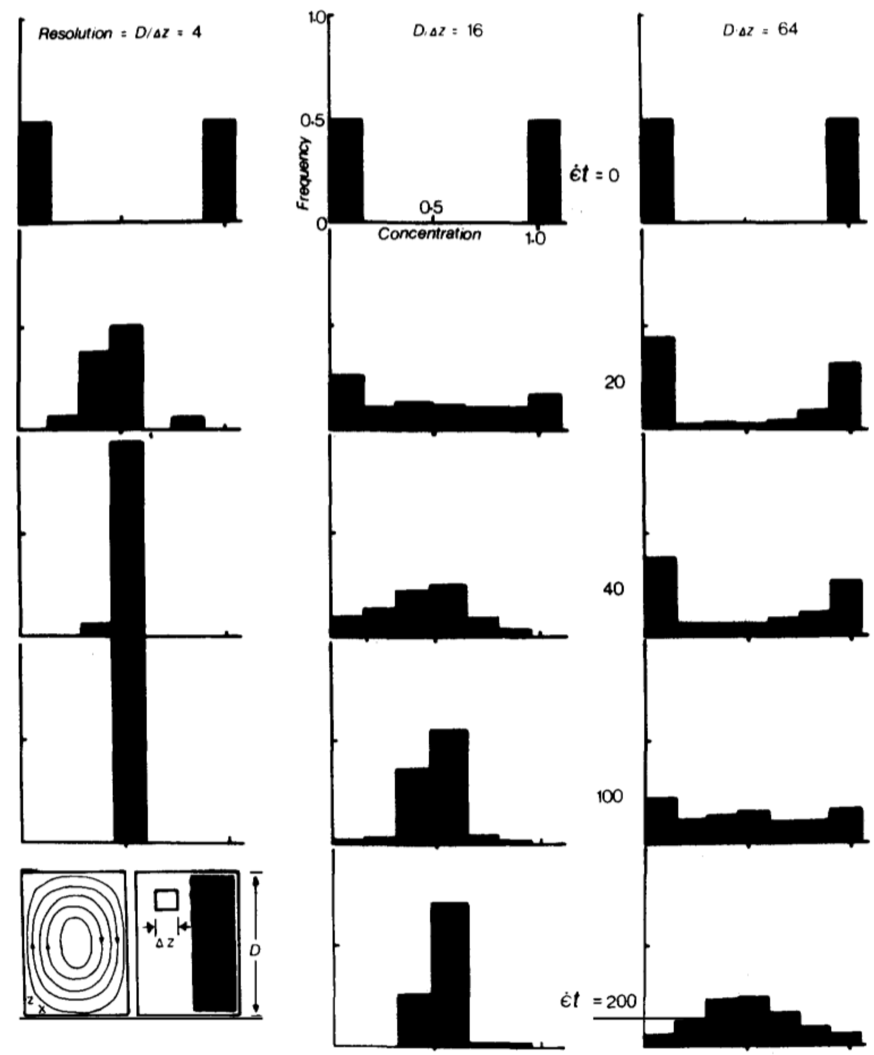
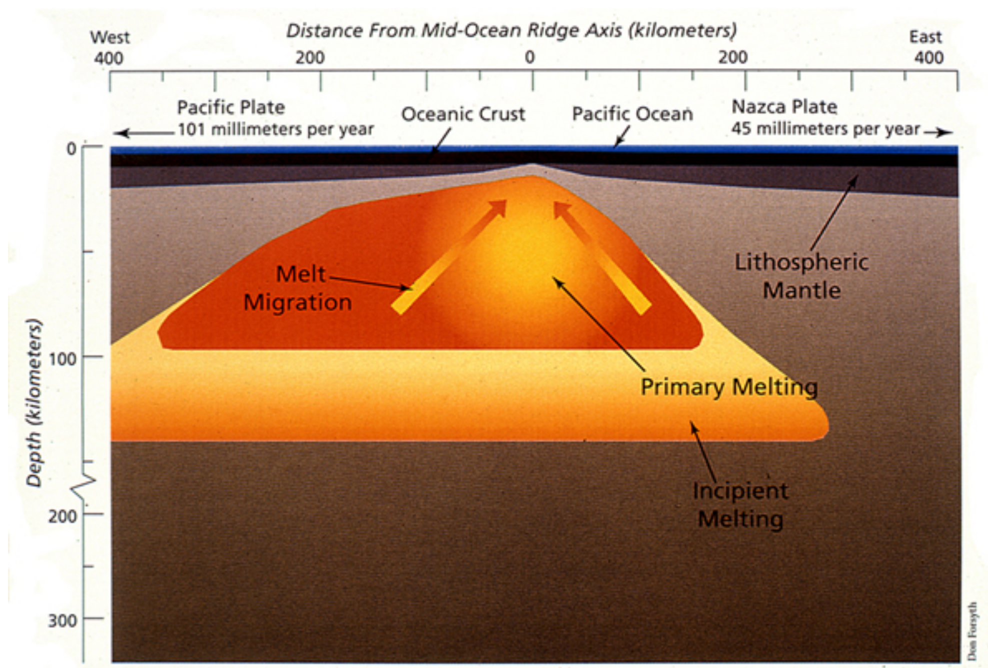


Fig. 7. Histograms of anomalous concentration versus time and sample resolution for a large scale heterogeneity in Bénard convection. The insert shows the initial distribution.

Olson et al., *PEPI* 1984

Convection is a source and sink of heterogeneity

- Melting at ridges
- Fluid migration and melting at subduction zones
- Melting at mid mantle phase transitions?
- Melting at the base of the mantle
- Chemical reactions between the mantle and core?



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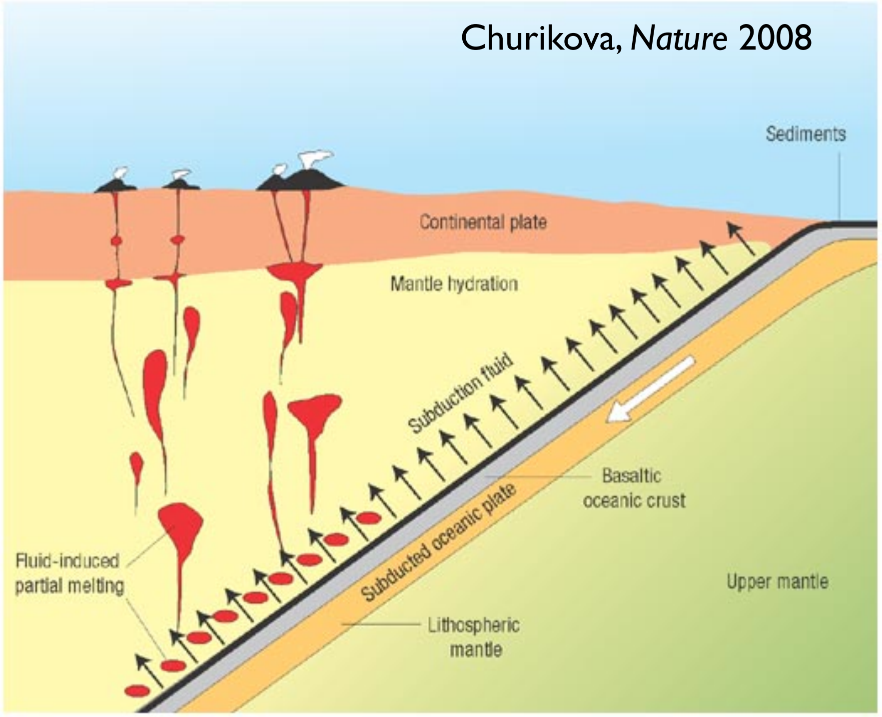

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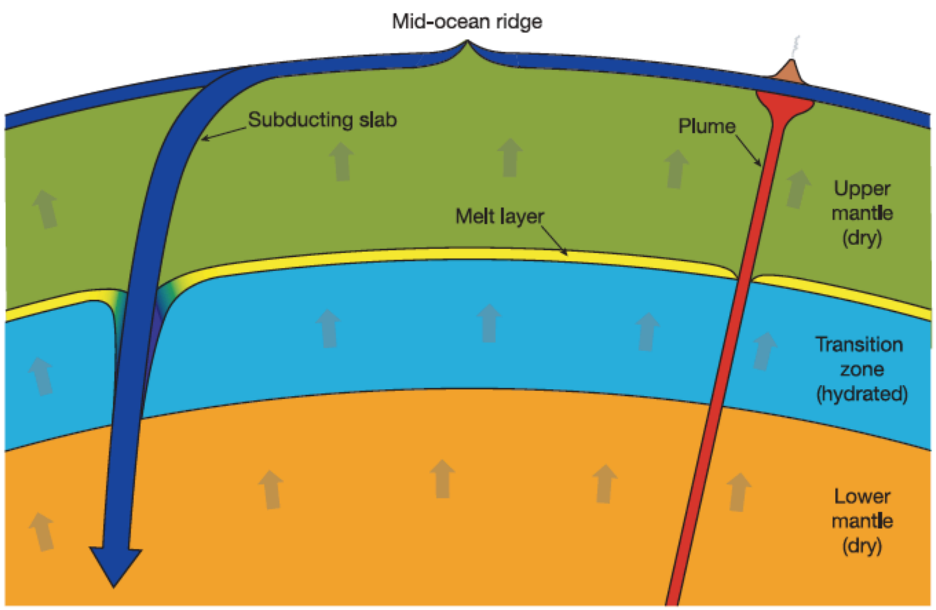
Bercovici and Karato, *Nature* 2003



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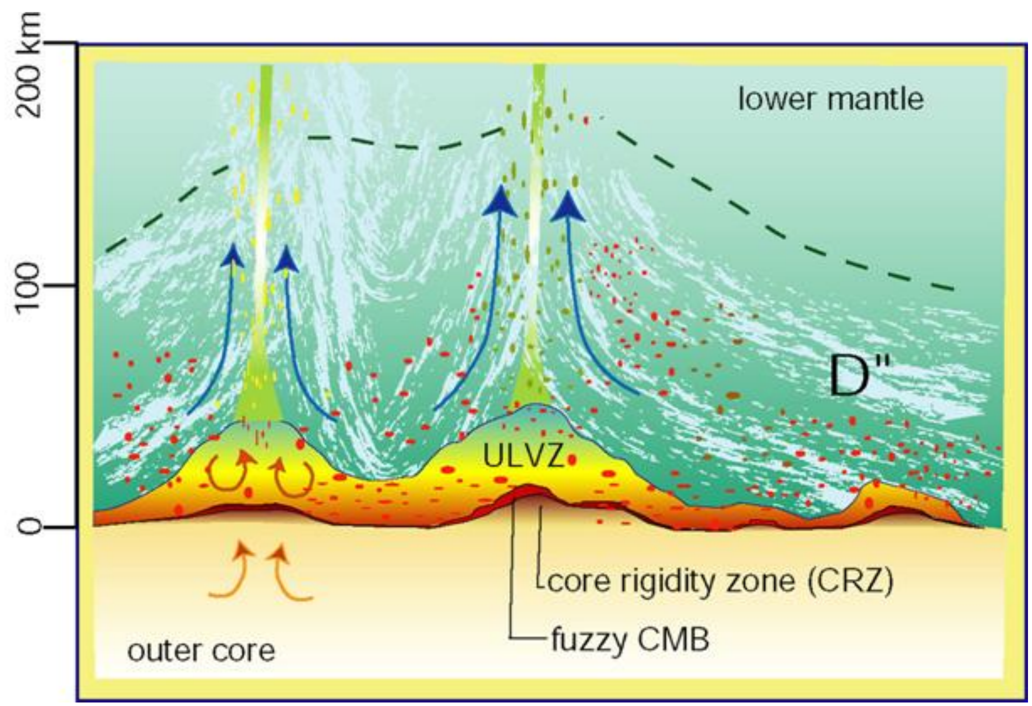

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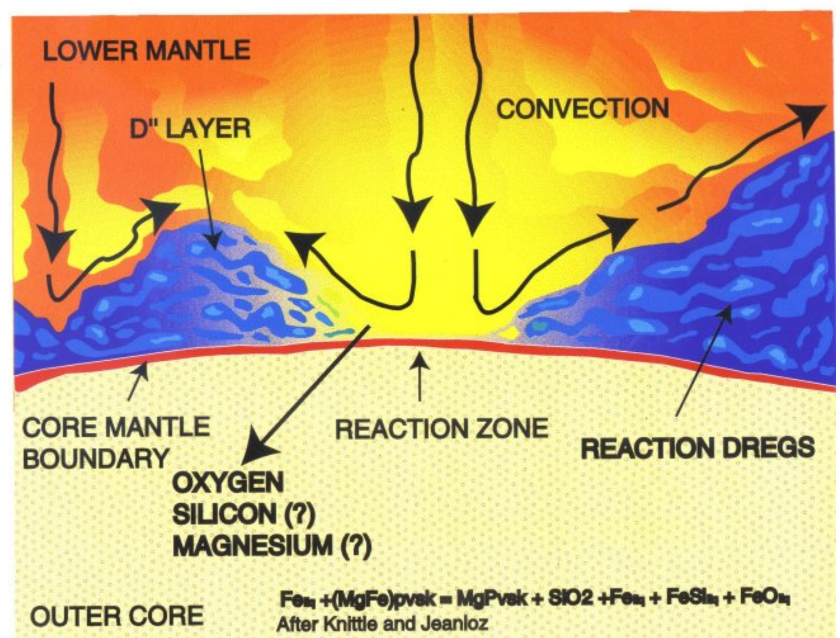
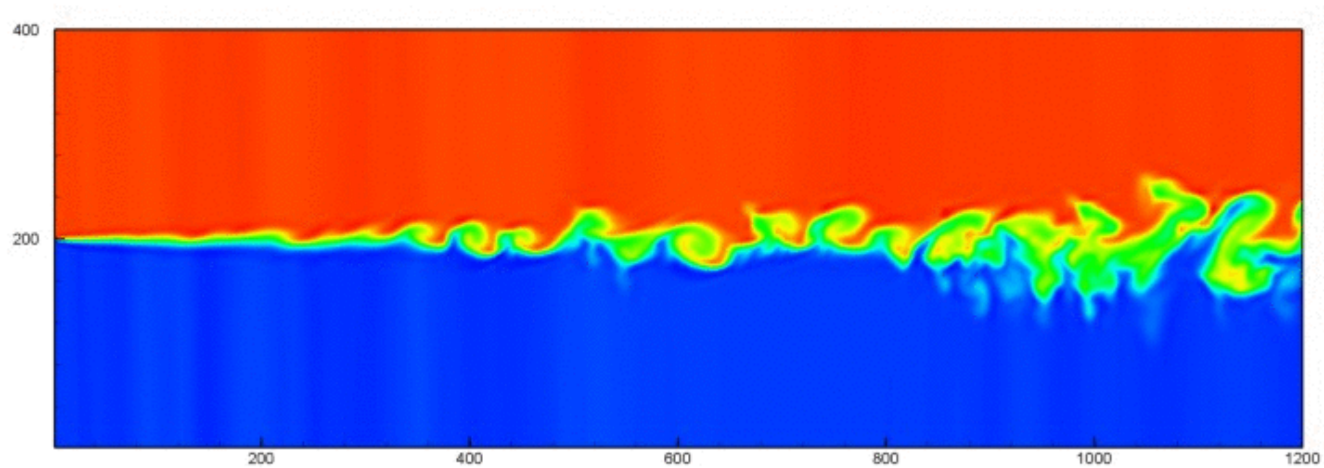
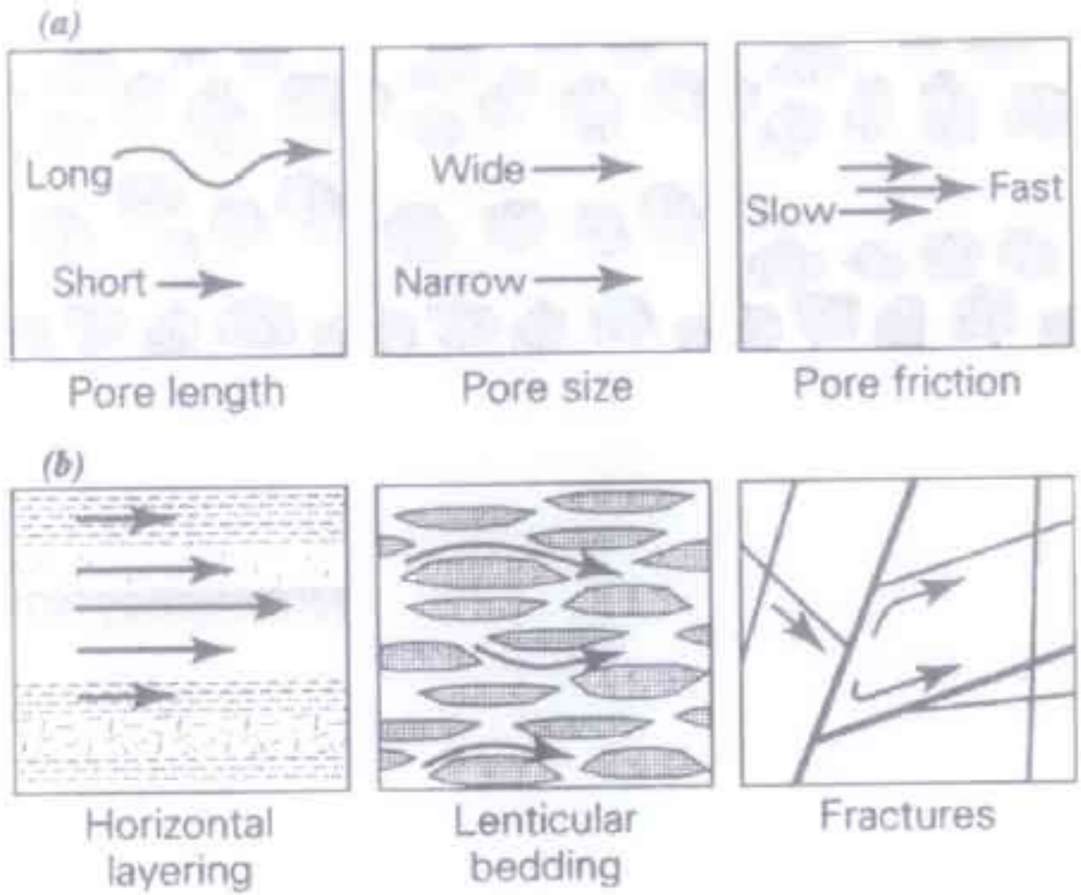


Turbulent mixing

- Energy transfer from large to small scales
- Intermittency in space and time
- Velocity is a complicated function of time



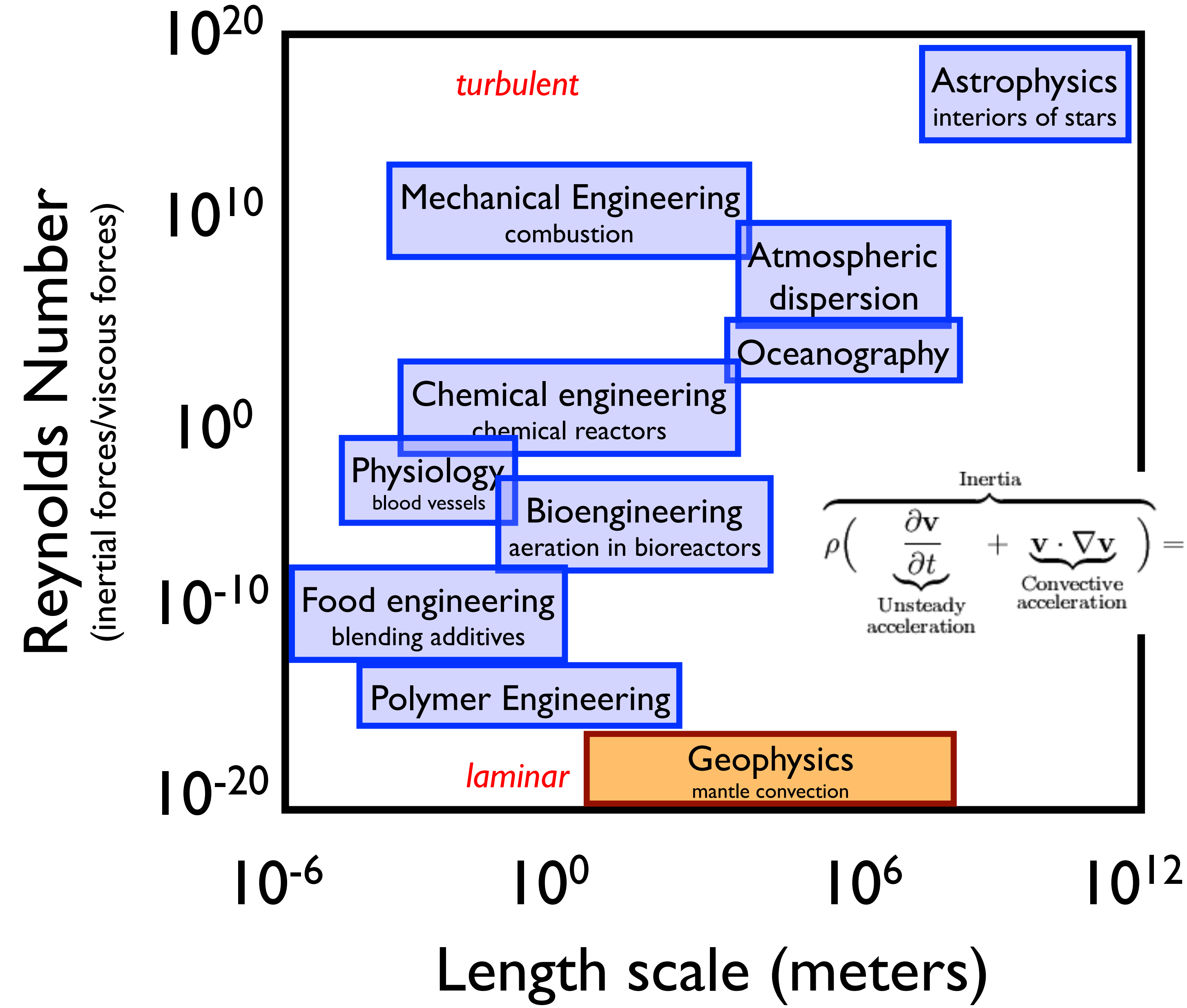
Dispersion in porous materials



From Ingebritsen

- Complexity in flow paths, spatial variation in velocity greatly enhance mixing (dispersion)

Some mixing scales



$$\overbrace{\rho\Big(\underbrace{\frac{\partial \mathbf{v}}{\partial t}}_{\text{Unsteady acceleration}} + \underbrace{\mathbf{v}\cdot\nabla\mathbf{v}}_{\text{Convective acceleration}}\Big)}^{\text{Inertia}} = \underbrace{-\nabla p}_{\text{Pressure gradient}} + \underbrace{\mu\nabla^2\mathbf{v}}_{\text{Viscosity}} + \underbrace{\mathbf{f}}_{\text{Other forces}}$$

“Virtually everyone agrees that mixing is complicated”

“However there is no agreement as to the source of the complications . . . What makes mixing complex? Usually realistic mixing problems have been regarded as nearly intractable from a modeling viewpoint owing to the complexity of the flow fields. Also in many problems of interest the fluids themselves are rheologically complex . . . . .

Mixing problems have been attacked traditionally on a case by case basis. However . . . merging of kinematics with dynamical systems and chaos are providing a paradigm for the analysis of mixing from a rather general viewpoint.”

Ottino, *Ann Rev Fluid Mech* 1990

Main points

- Flow type matters
- Time dependence matters
- Properties of heterogeneity matter (active heterogeneity is different from passive tracers)
- Mixing will depend on history of Earth and properties of interior (all of which have uncertainty), hence a stochastic approach may be useful
- Convection both creates and destroys heterogeneity

Backup slides

Mixing – simplest analysis
(time, no spatial dimensions)

Mid–ocean ridge mass flux

“processing
zone”
depth $\approx$ d

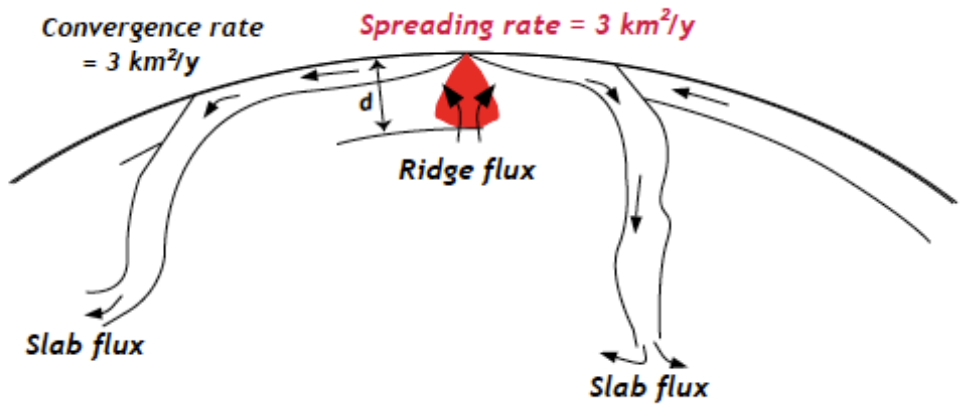


Plate creation rate	$a \approx 3$ km^2/y
Mass flux in zone	$\dot{M}_r = ad\rho$
Mass of upper mantle	$M_{um} = 1 \cdot 10^{24}$ kg

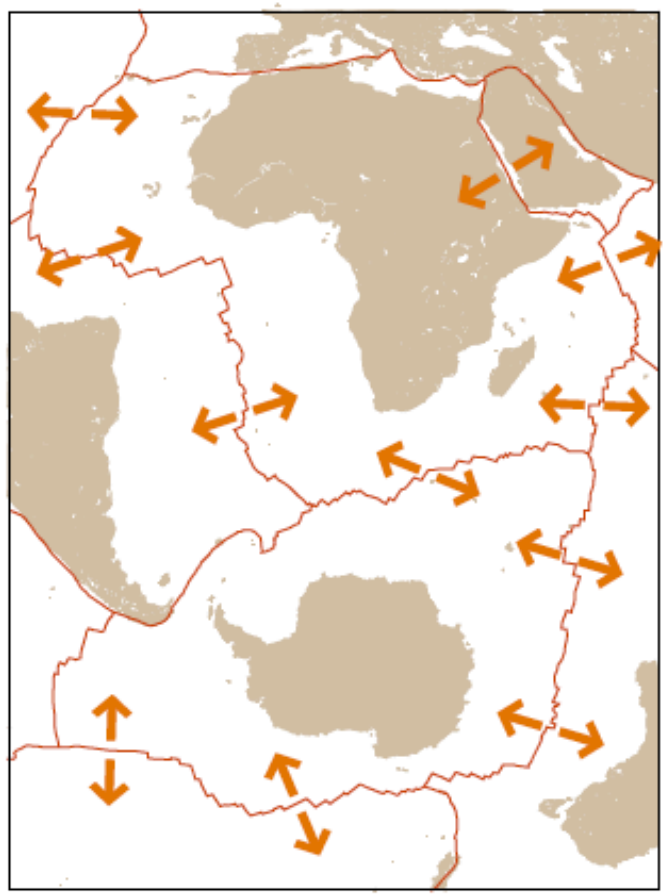
From Rick O’Connell, Harvard

“Turnover” time: $\tau_{um} = M_{um}/\dot{M}_r$

depth	τ_{um} (upper mantle)	τ_{em} (entire mantle)
50 km	2 Gy	8 Gy
100 km	1 Gy	4 Gy

Ridge migration over mantle

Ridges move
relative to
one another



Total ridge length	$L \approx 56,000$ km
Ridge migration rate	$u \geq 2$ cm/y
Surface area	$A = 5 \cdot 10^{14}\text{m}^2$

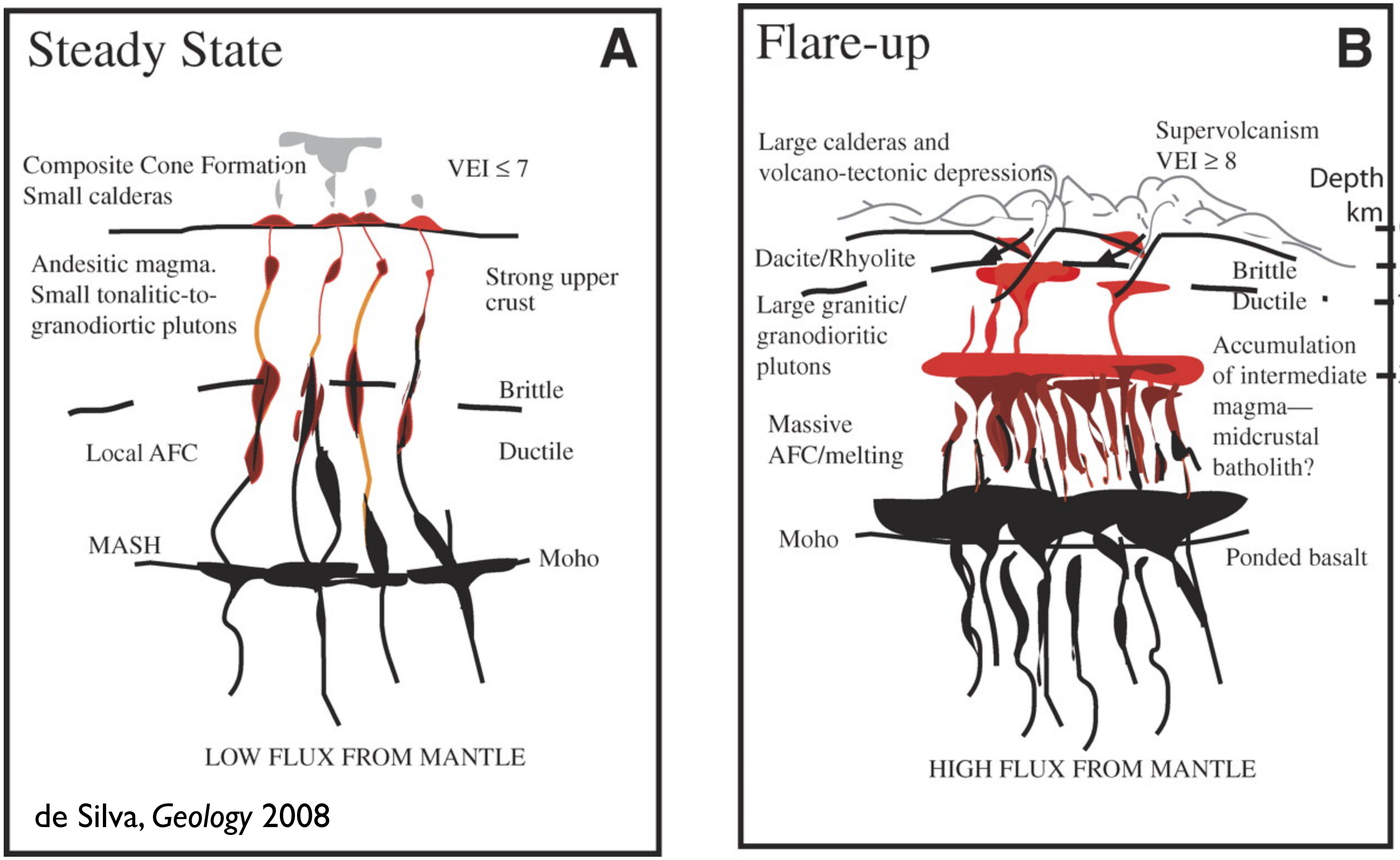
Ridges sweep over Earth's surface with time scale:

"Passover" time, whole Earth $\frac{A}{Lu} \leq 500$ My

From Rick O'Connell, Harvard

The magmatic and volcanic filter

Convection (again) creates and destroys heterogeneity; heterogeneity preserved from the crystal scale (< 1 mm) to the batholith scale (> 10 km)



For MORBs, recent review by Rubin et al., *Nat Geosci* 2009

Summary

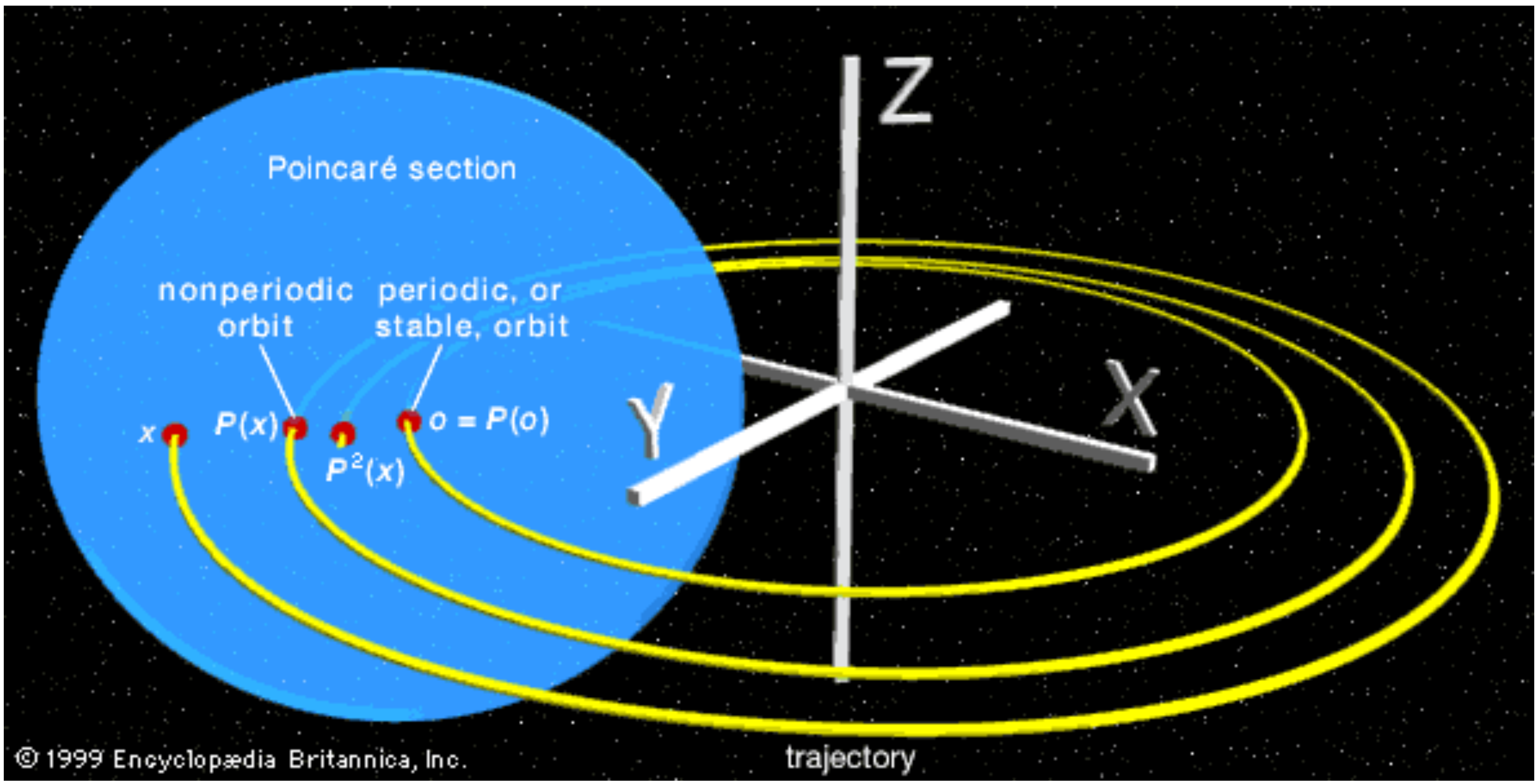
The mantle is not homogeneous

Ideas for preserving heterogeneity
(taken van Keken et al, *Ann Rev Earth Planet Sci* 2002)

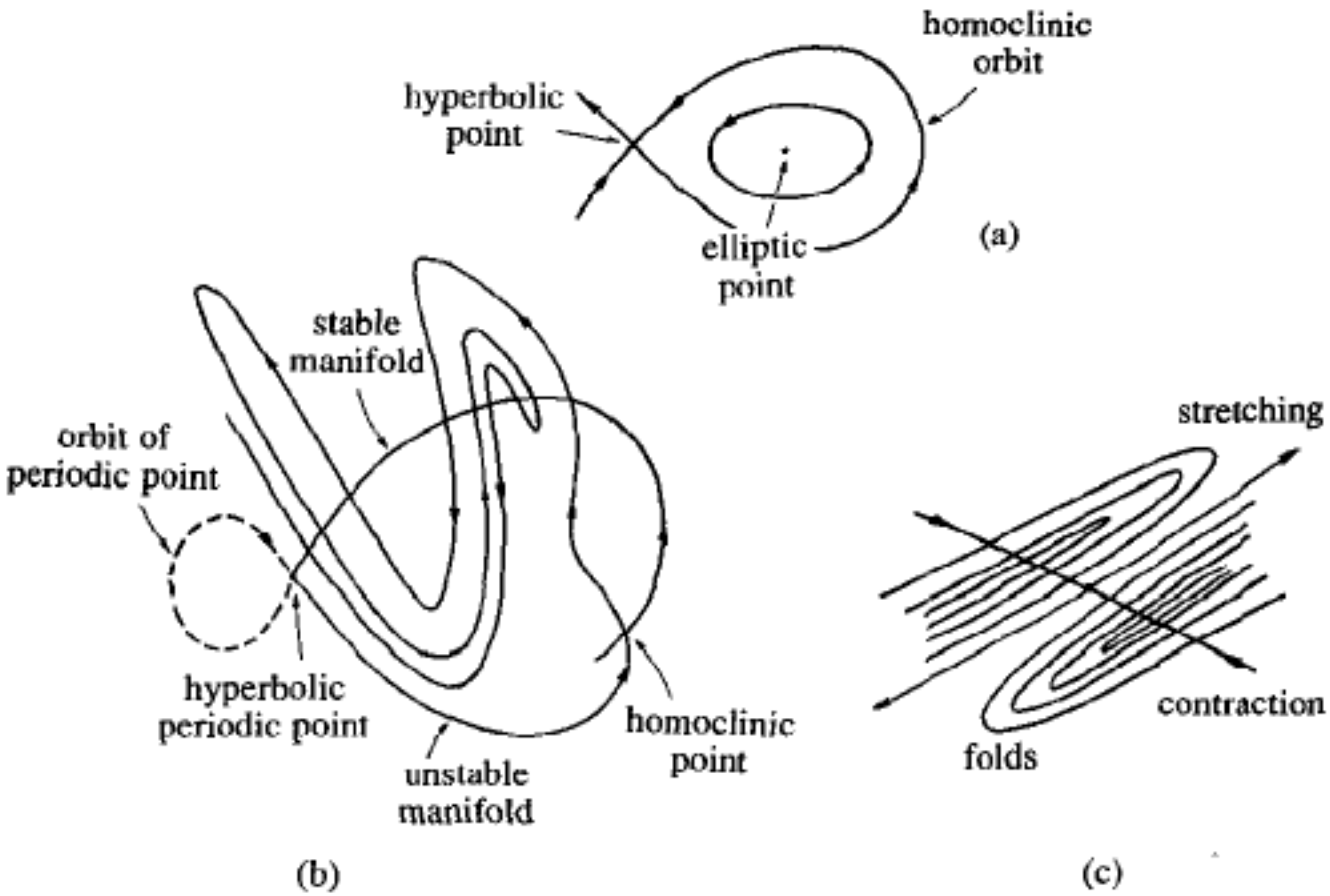
- Convective layering of the mantle
- Distributed high viscosity blobs
- Ocean crust recycling and storage
- Storage within and exchange with the core

Poincare sections

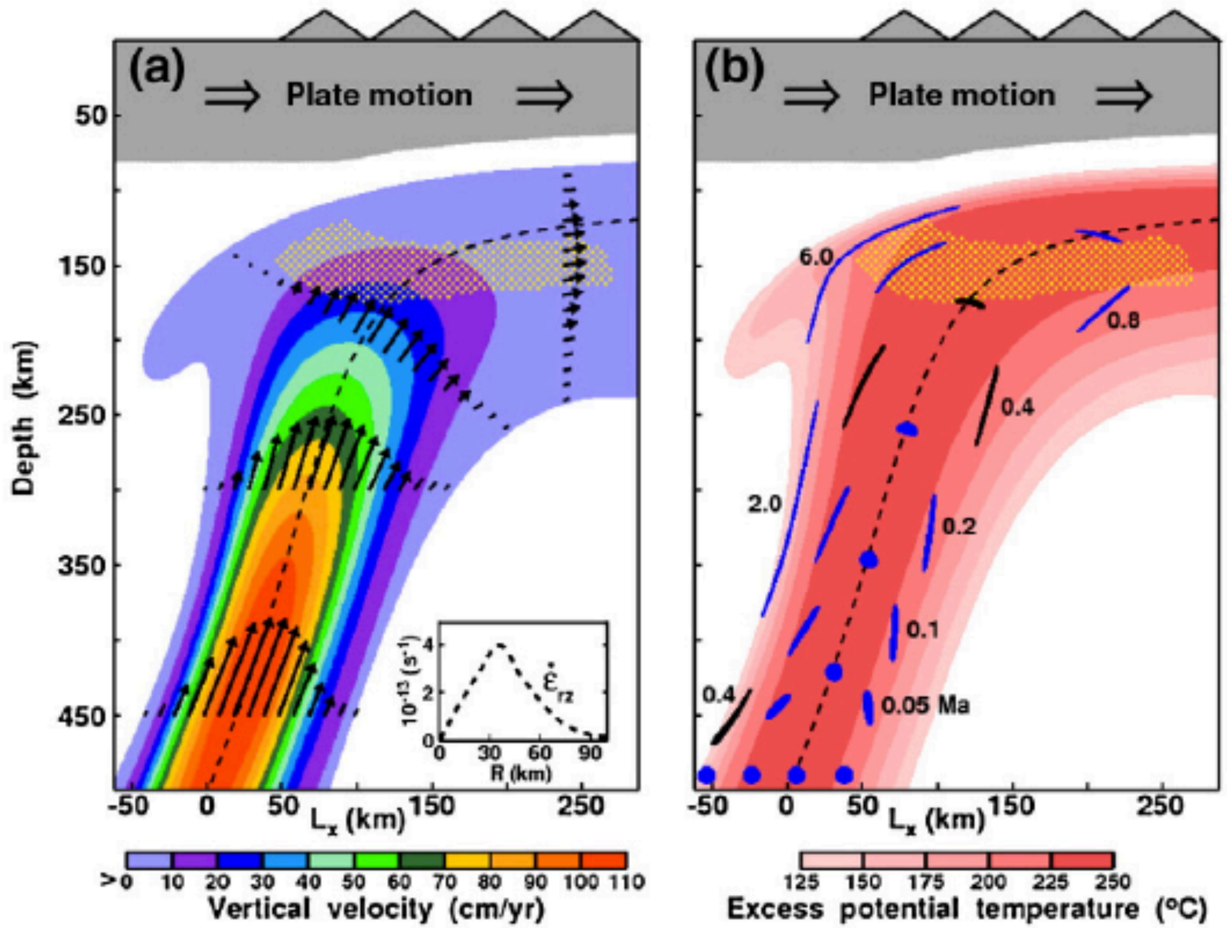
(reduces dimensionality by converting flow into a map; convenient way to show the character of solutions for all possible initial conditions)



If unstable and stable manifolds intersect, jump from one to the other, leading to sensitivity to initial conditions



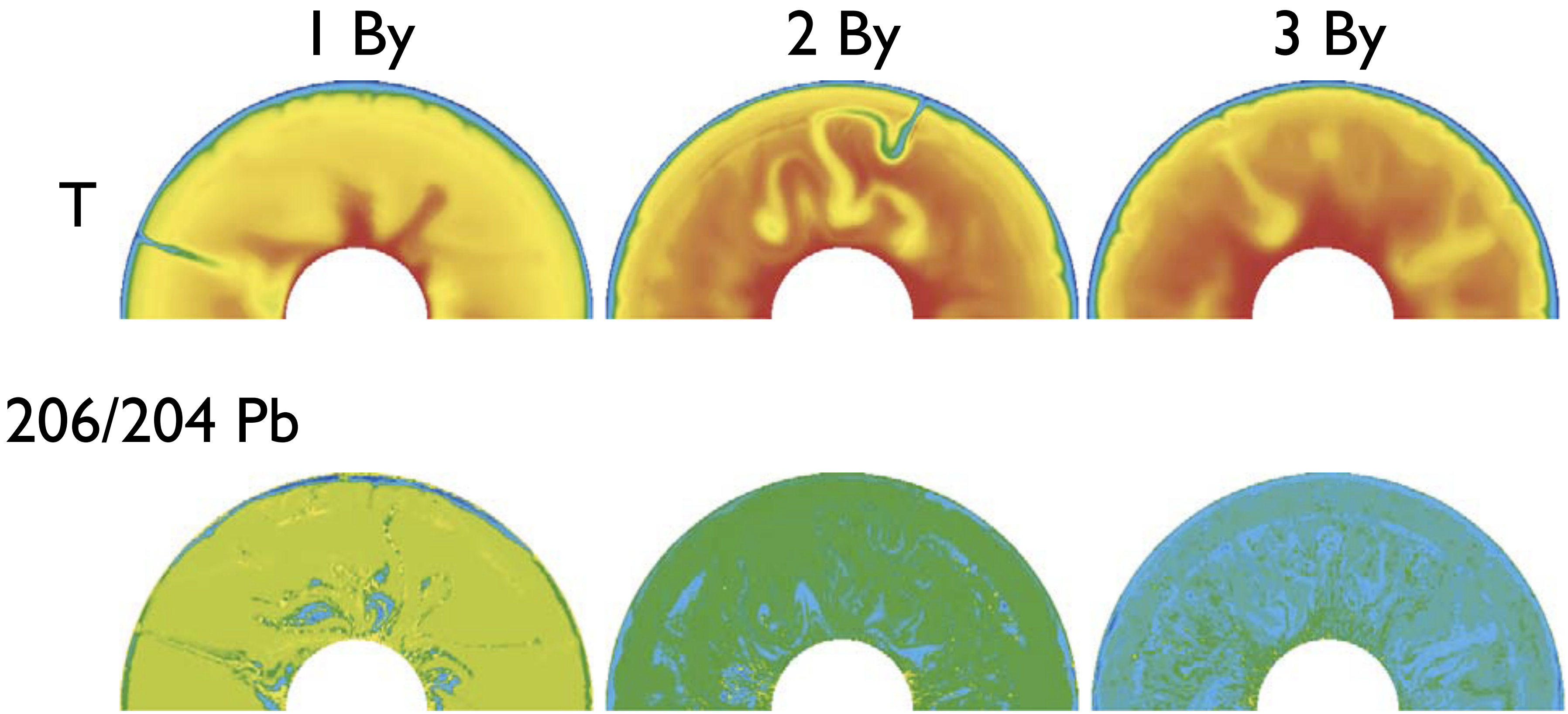
Stretching in plumes



Farnetani and Hofmann, *EPSL* 2010

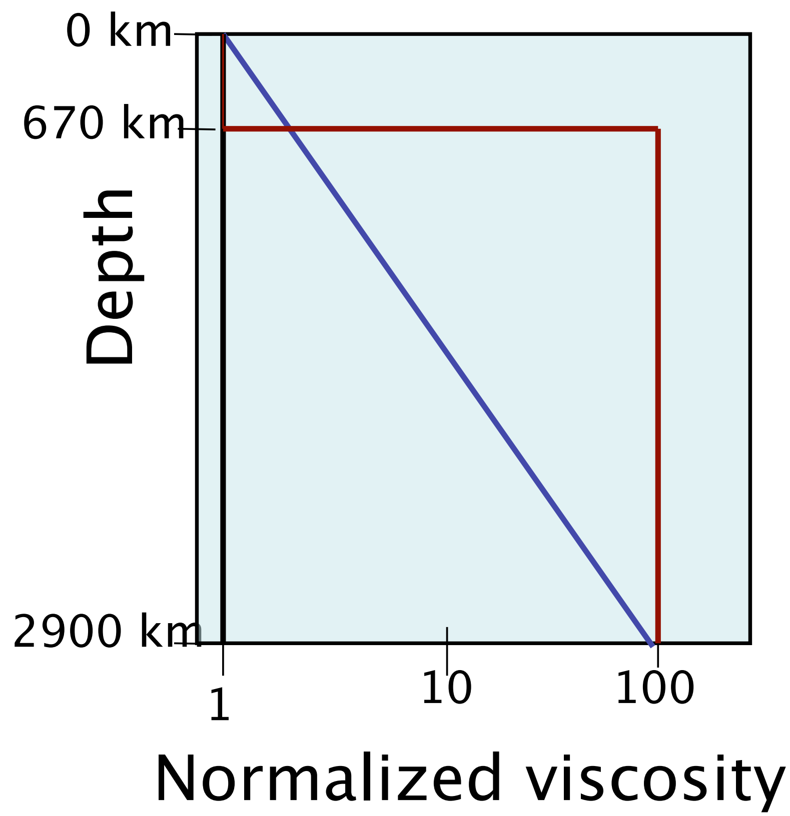
Computing isotopic signatures

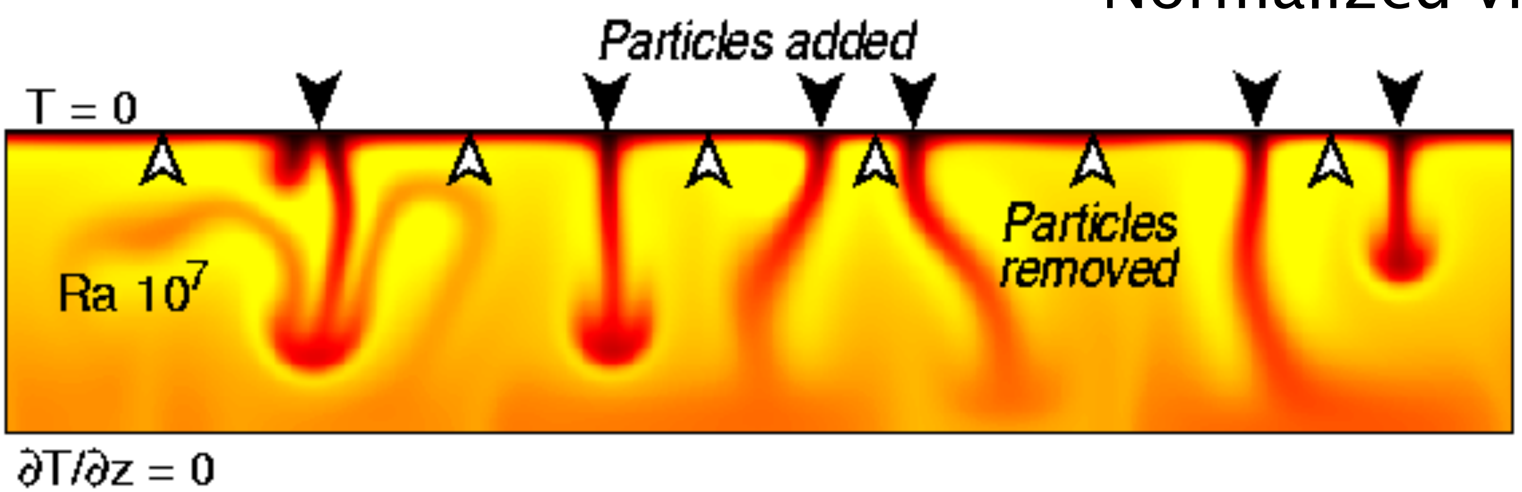
Evolution of U-Pb and Sm-Nd systems in numerical models of mantle convection and plate tectonics
Shunxing Xie and Paul J. Tackley, J. Geophys. Research, 109, B11204, 2004



Hunt and Kellogg 2000

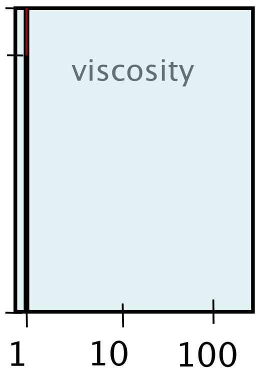
Mixing in 2-D with particles
- Added at subduction zones
- Removed at mid-ocean ridges



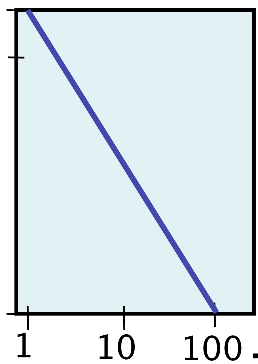


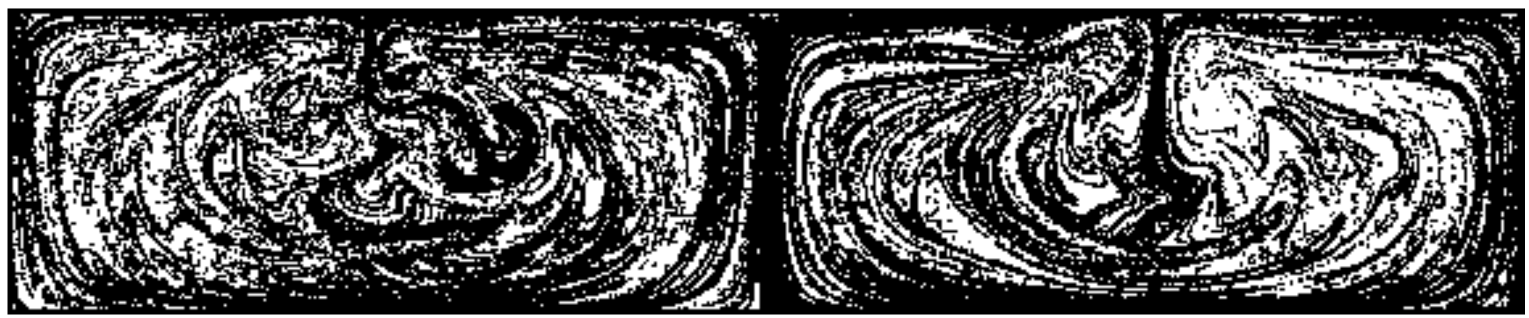
Hunt & Kellogg - effect of viscosity on mixing

Constant viscosity

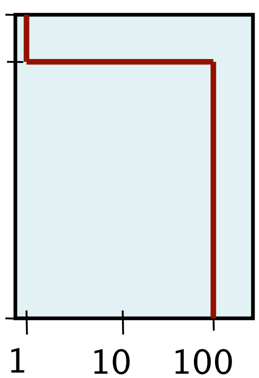


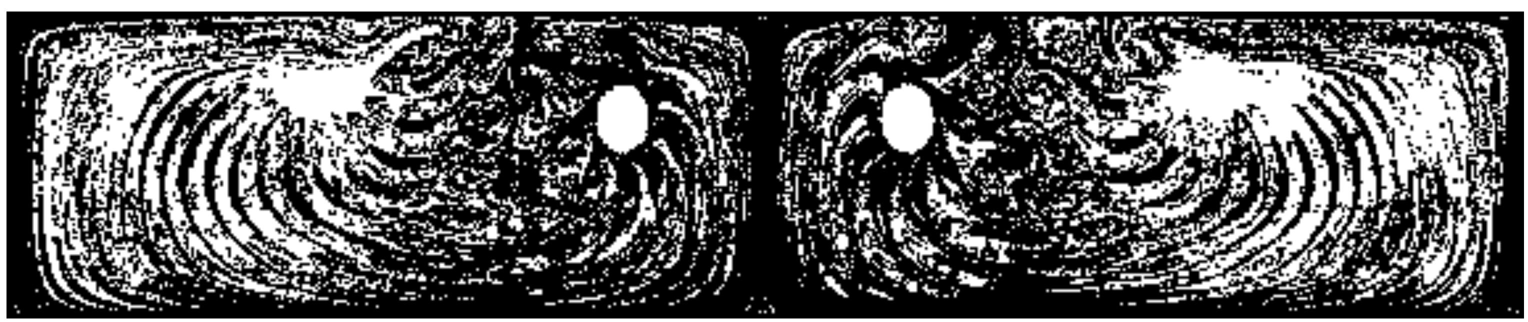
Pressure-dependent viscosity: smooth increase



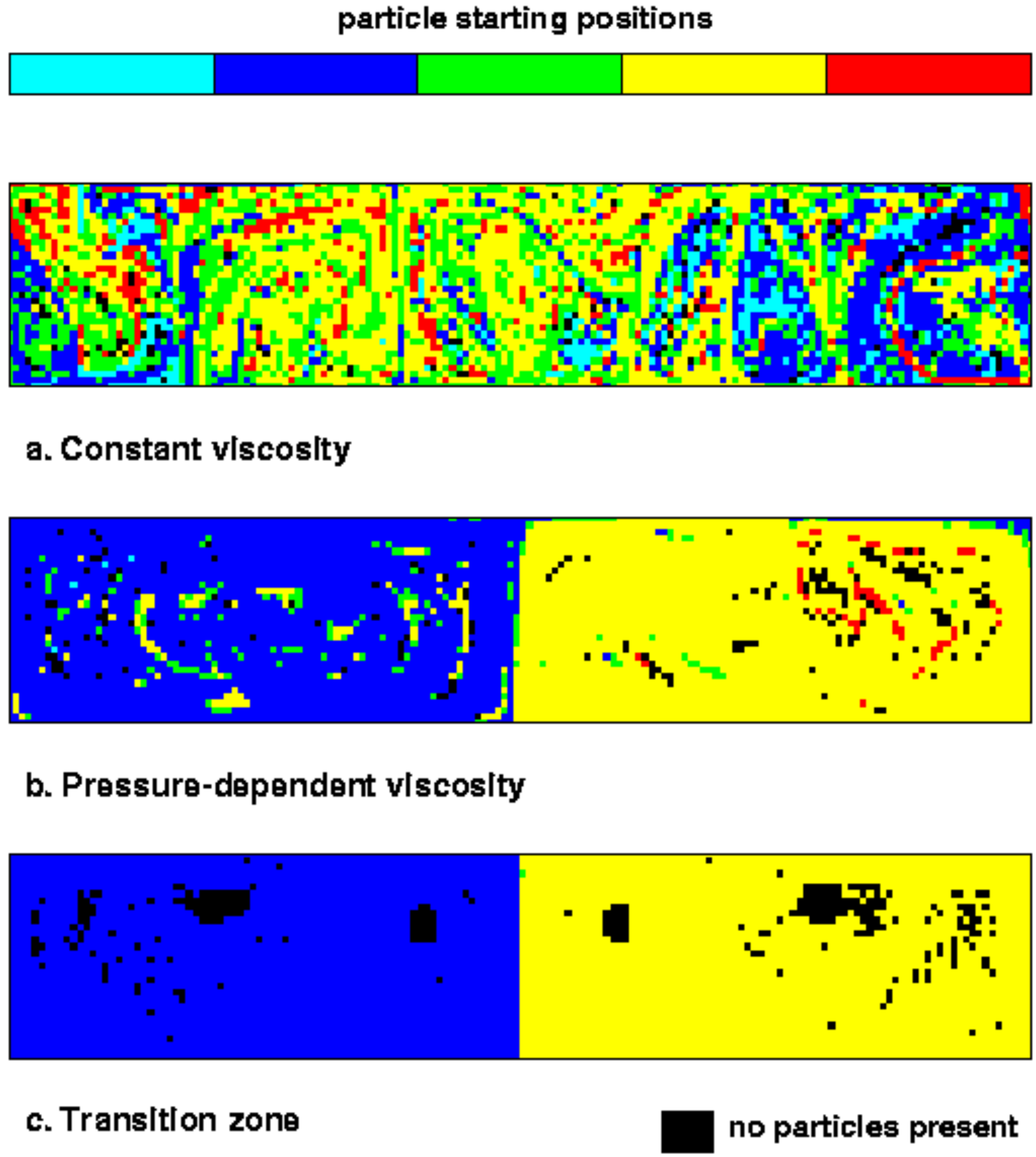


Transition zone viscosity: Jump at 670 km



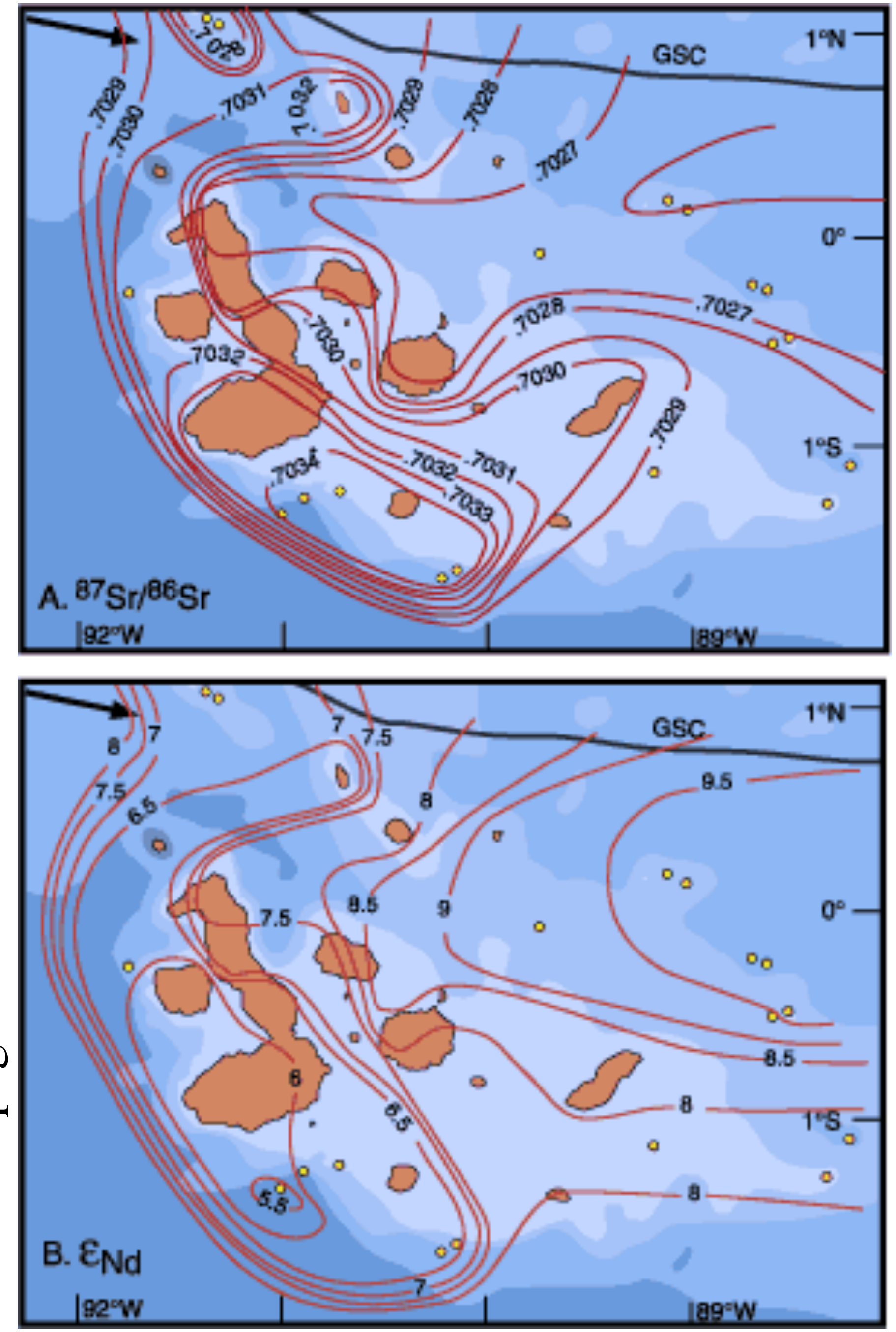


Initial location of particles (Hunt and Kellogg model)



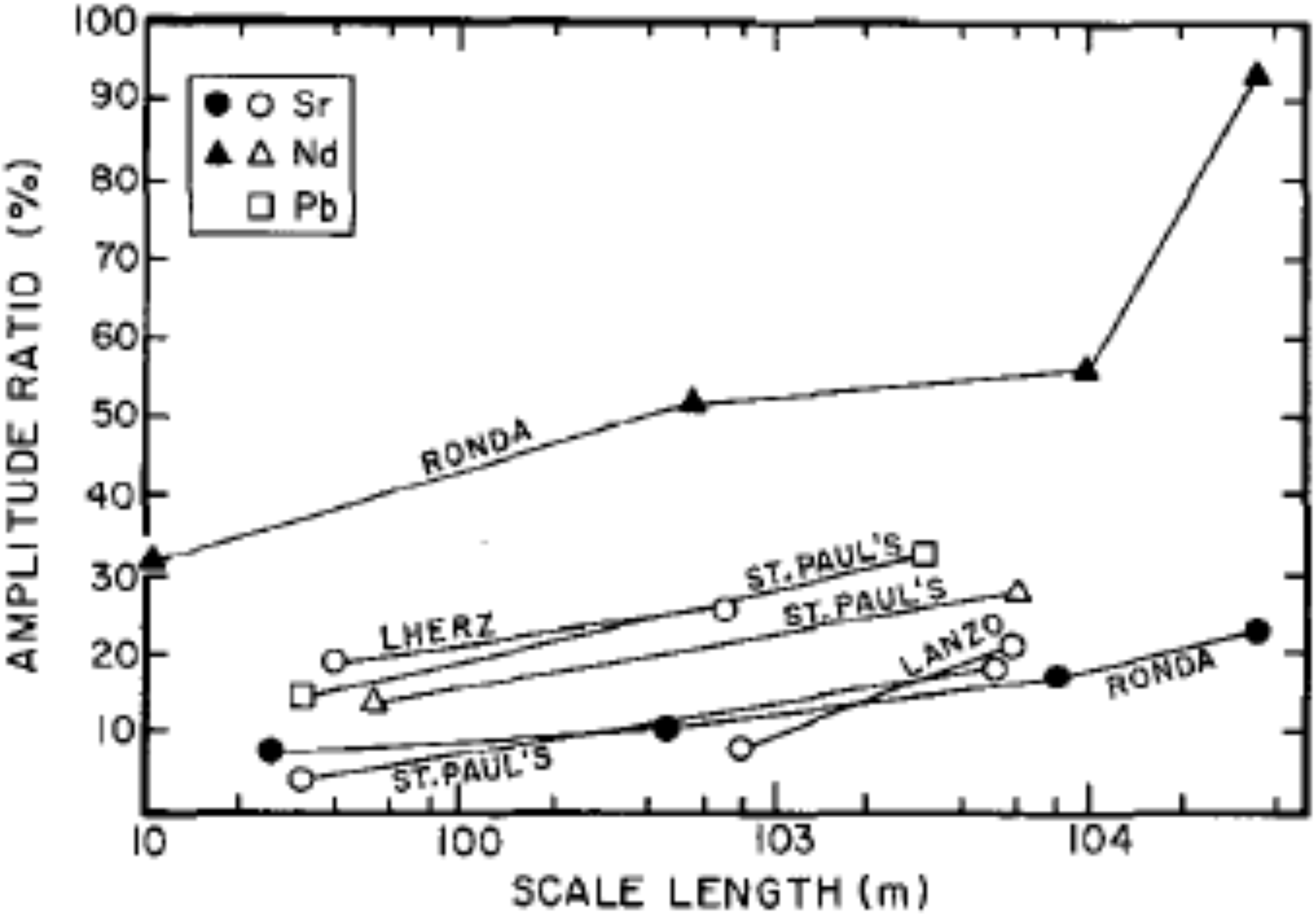
Fine-scale variations
in the Galapagos

Galapagos Islands



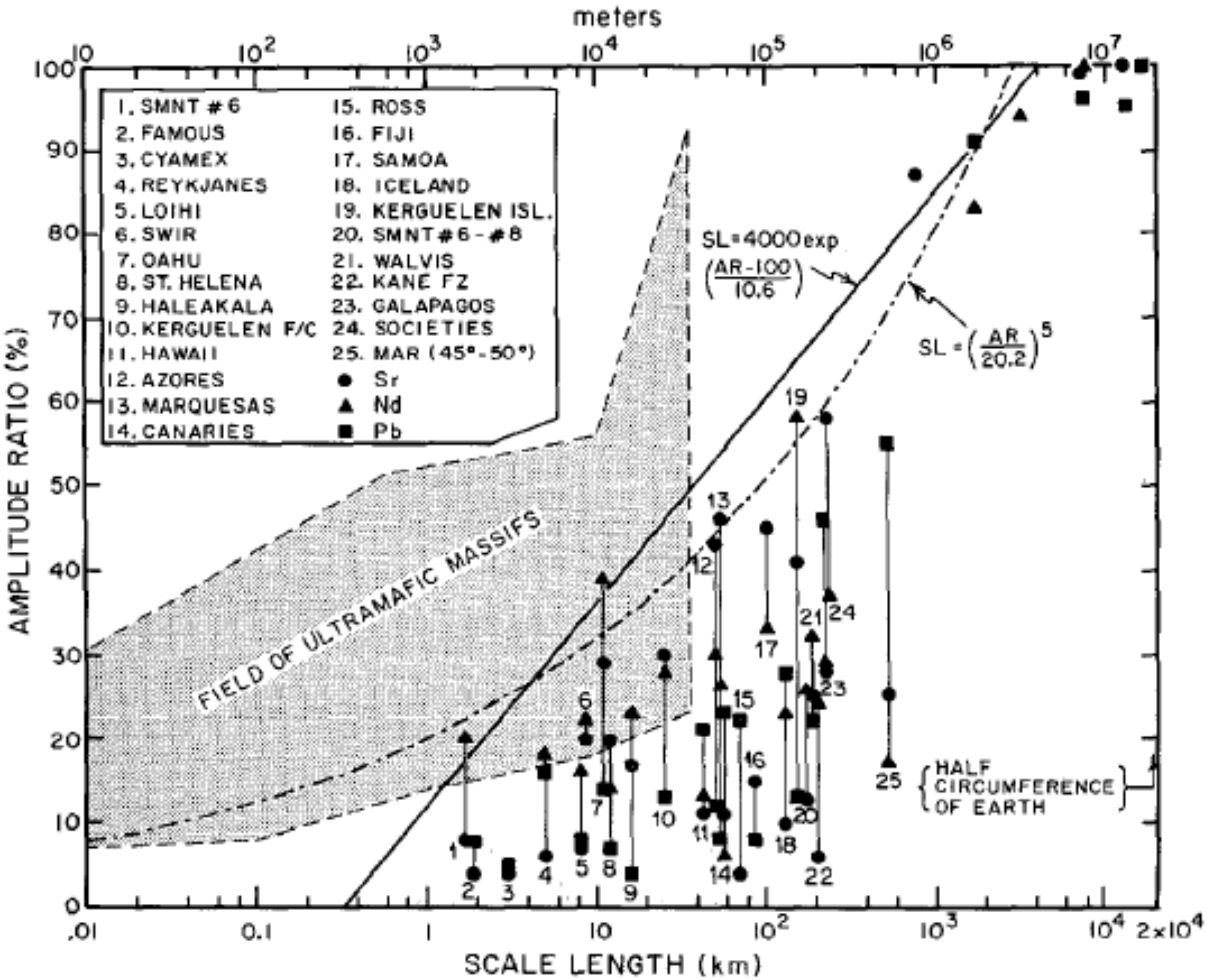
Harpp and White *G*[3] 2001

Scales of geochemical heterogeneity



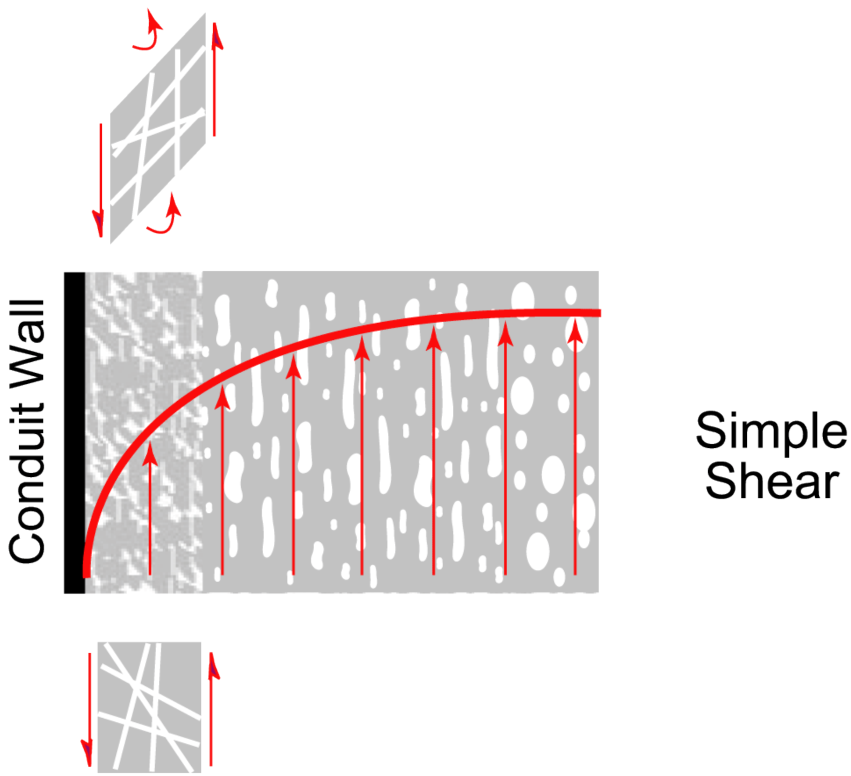
Zindler and Hart, *AREPS* 1986

Scale of geochemical heterogeneity



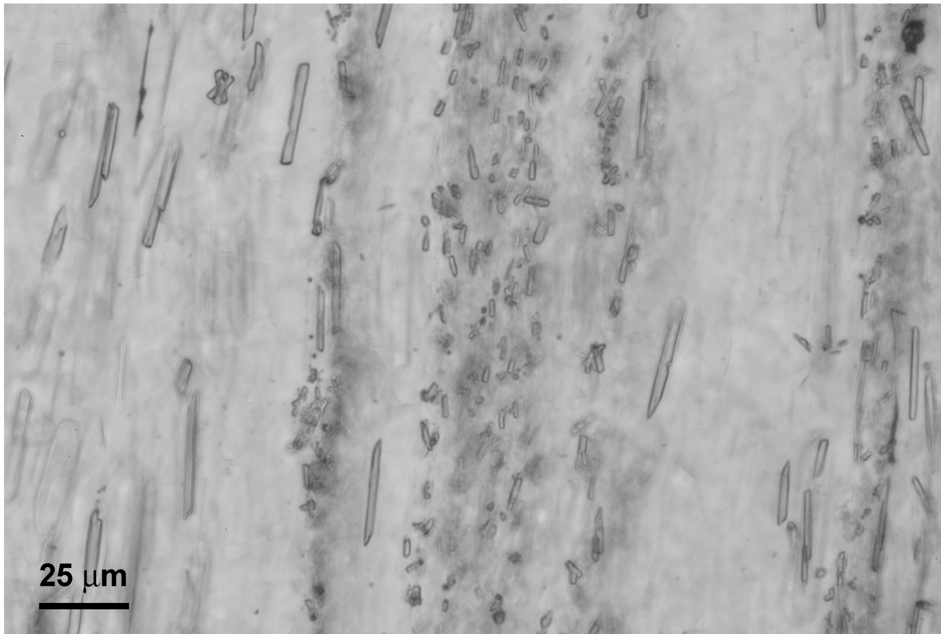
Zindler and Hart, *AREPS* 1986

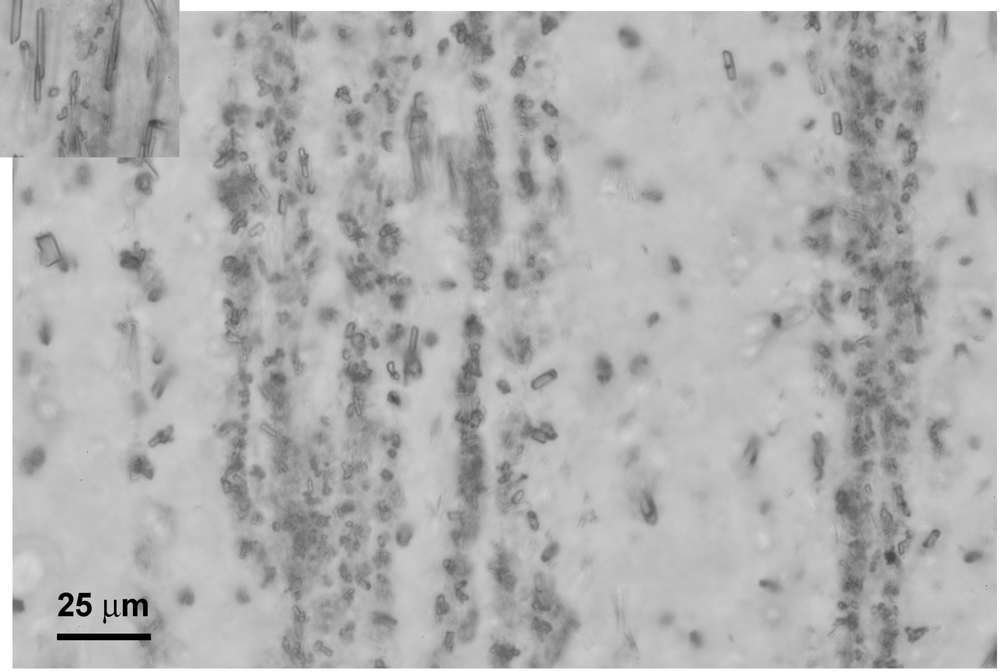
Simple shear . . . .



. . . . rotation and stretching

Repeated fragmentation and dispersal of cpx?





Open questions

- Are there large undegassed or unsampled regions of the mantle?
- What is the mass (species and amounts) flux across the CMB ?
- Is the present structure of the mantle representative of that in the past?
- Can we test the BMO hypothesis?
- Physical nature and origin of geochemical reservoirs?