Geodynamics 4: Mixing

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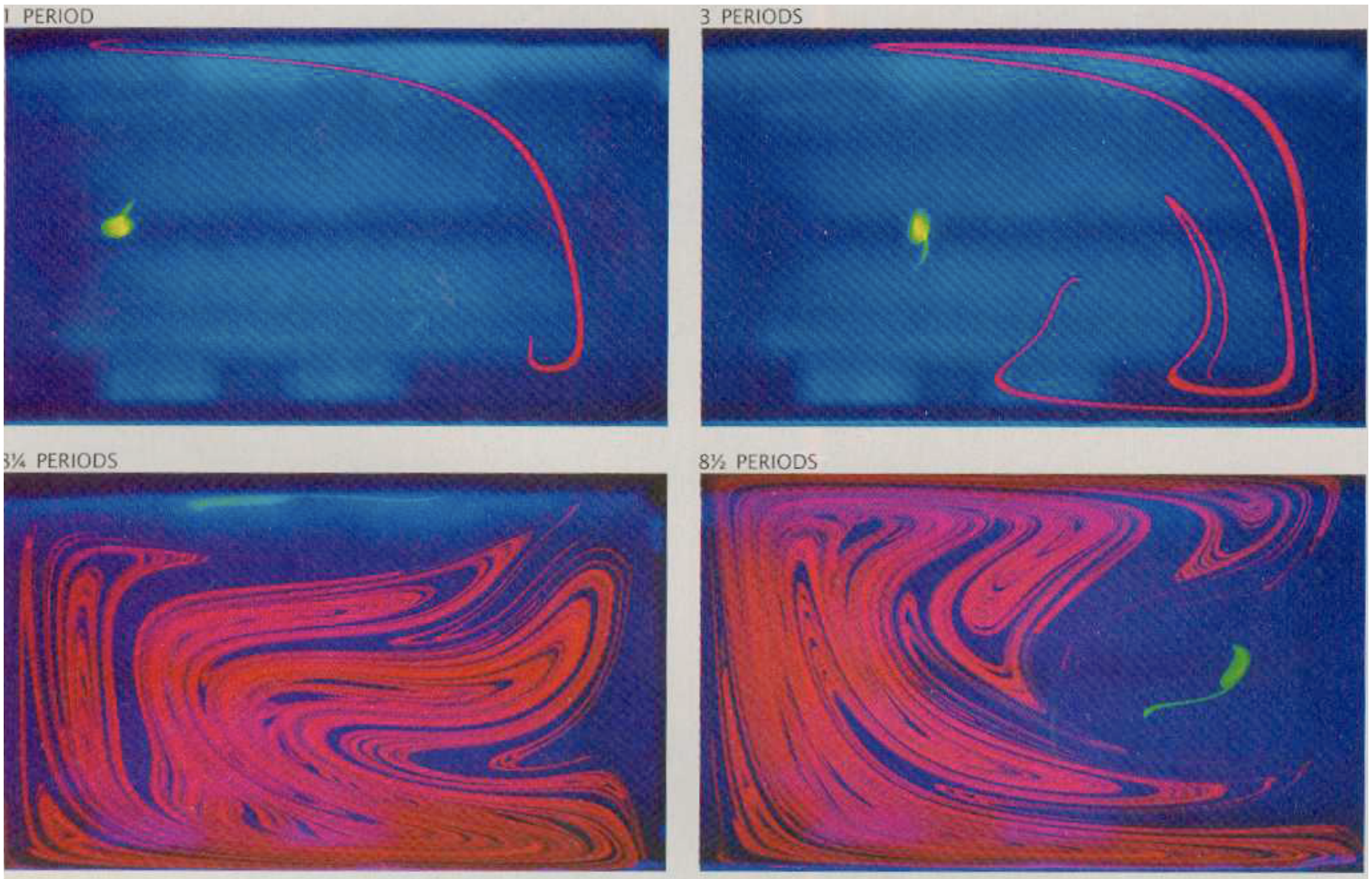


Ottino, The Kinematics of Mixing: Stretching, Chaos, and Transport, 1990

Outline:

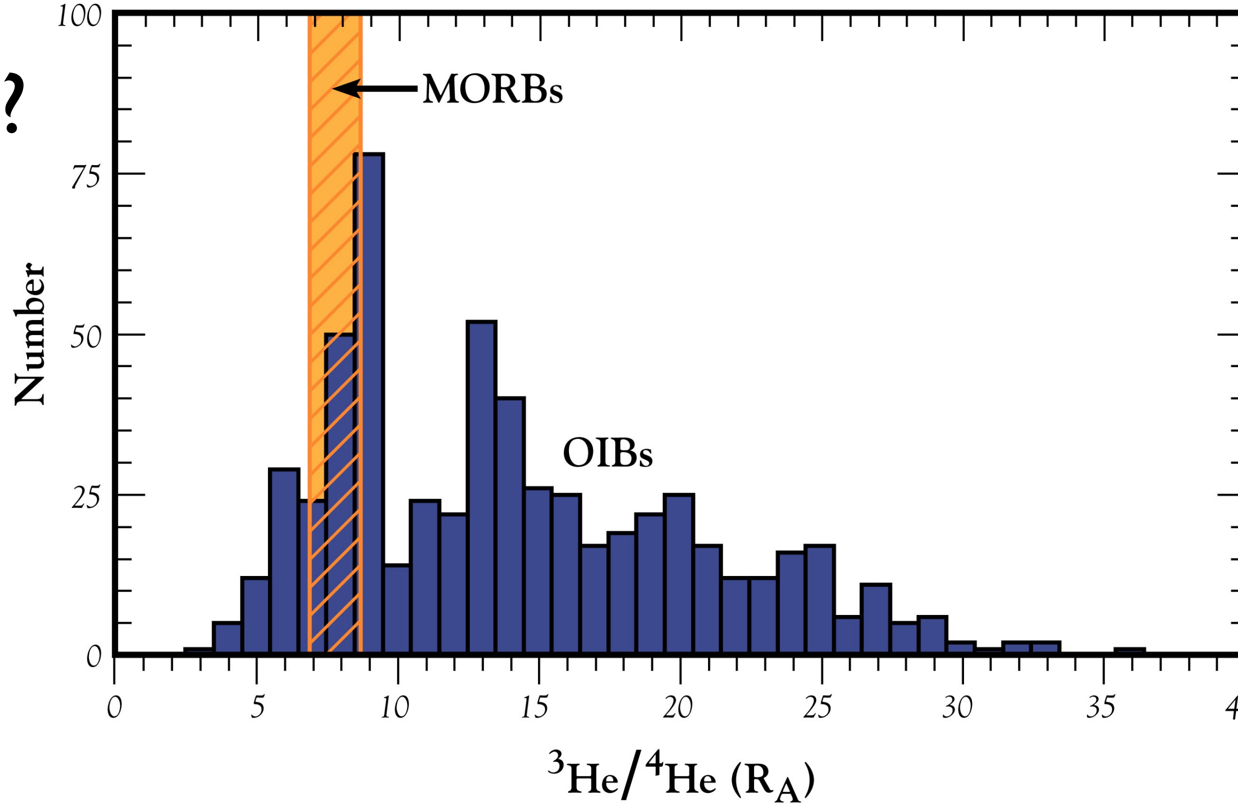
- Why think about/model mixing?
- Mechanics of mixing
- Mantle mixing
- Quantifying and visualizing mixing (and the challenges of 3D)
- Key points

Key points about mixing

- Stirring is controlled by: stretching and folding.
  - Creates a cascade of heterogeneity
- Mixing is complicated by:
  - Type of flow (turbulent vs. laminar)
  - 2D vs. 3D
  - Scale of interest
  - Time
- Mantle processes both create (melting) and destroy (mixing) heterogeneity.

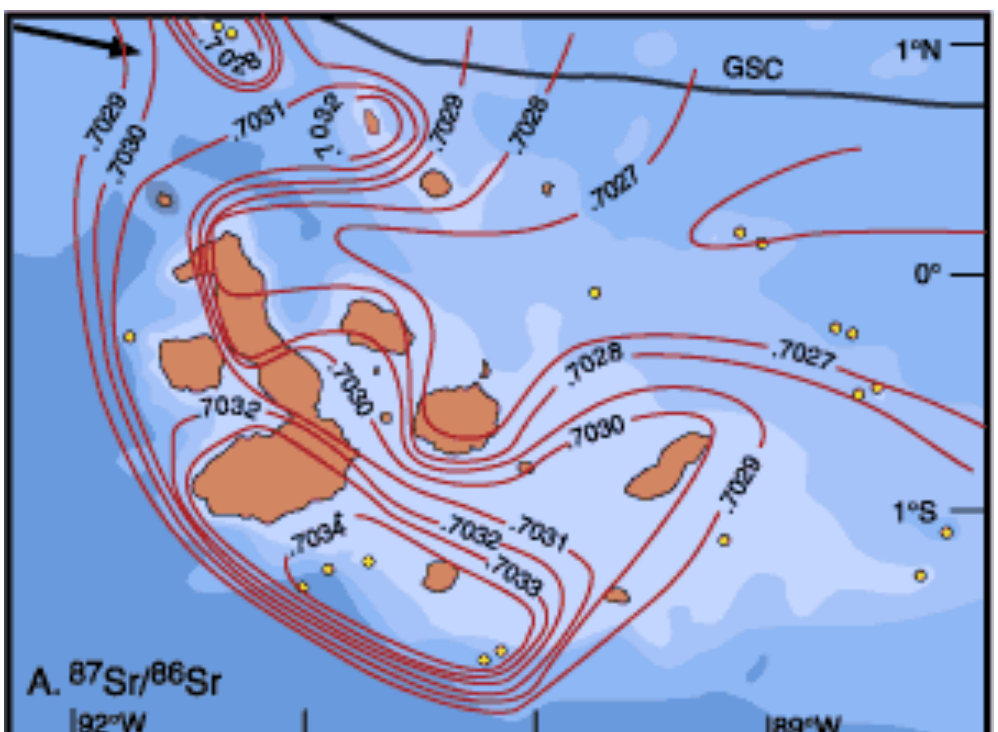
Why care about mixing?

Global scale: mantle contains both well-mixed regions and heterogeneity

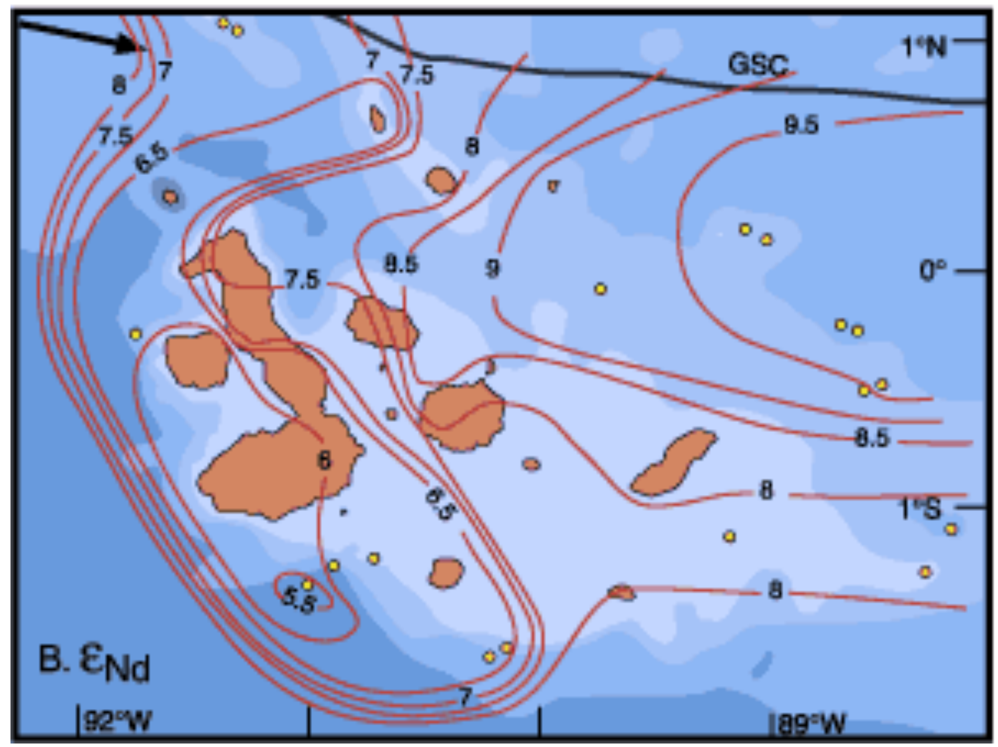


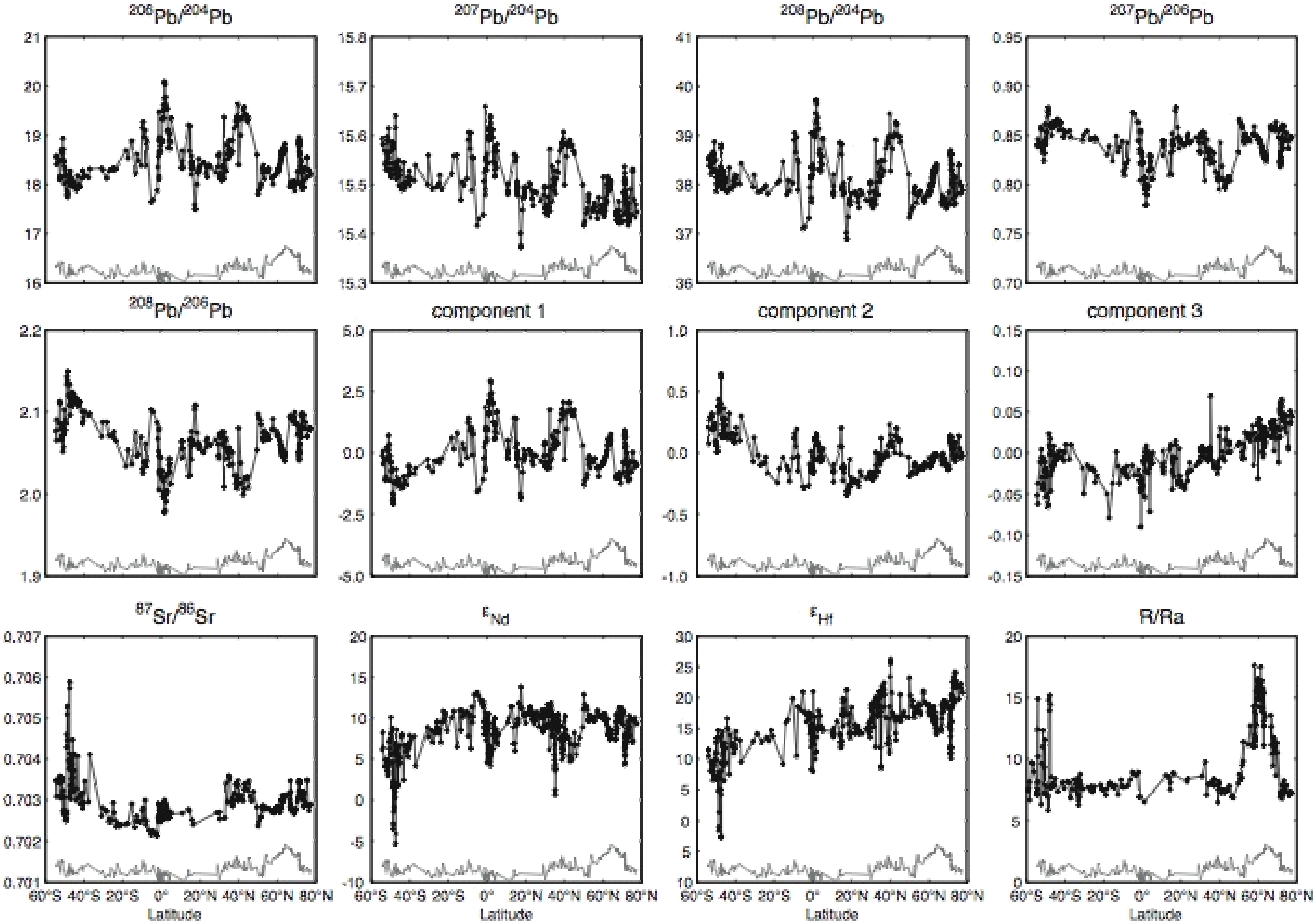
After Farley and Neroda, Ann. Rev. Earth Planet. Sci., 1998

Fine scale heterogeneity



Galapagos Islands, White et al. 1993





Heterogeneity along Mid-Atlantic Ridge

Agranier et.al (2005)

Scales of heterogeneity

Exposed peridotite

NATURE VOL. 323 11 SEPTEMBER 1986 ARTICLES 125

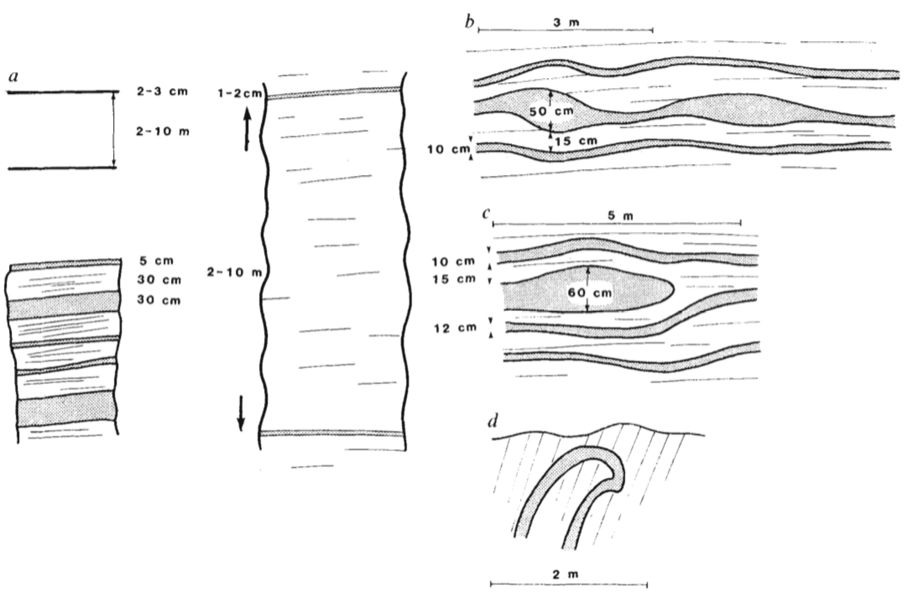


Fig. 2 Occurrences of pyroxenite layers in the Beni Bousera high-temperature peridotite. Grey, pyroxenite; white, lherzolite with foliation. *a*, Occurrences in an outcrop with no folding; *b*–*d*, occurrences with folding and boudinage.

Allègre and Turcotte (1986)

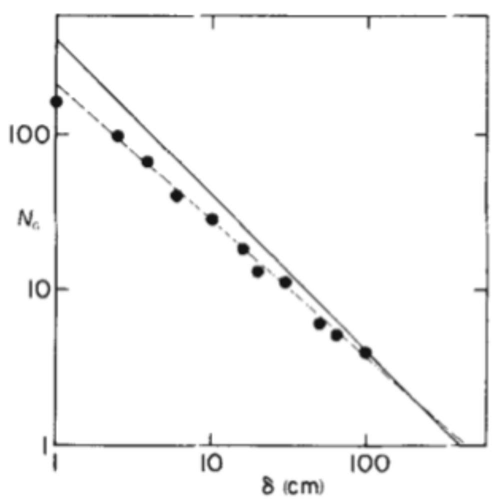
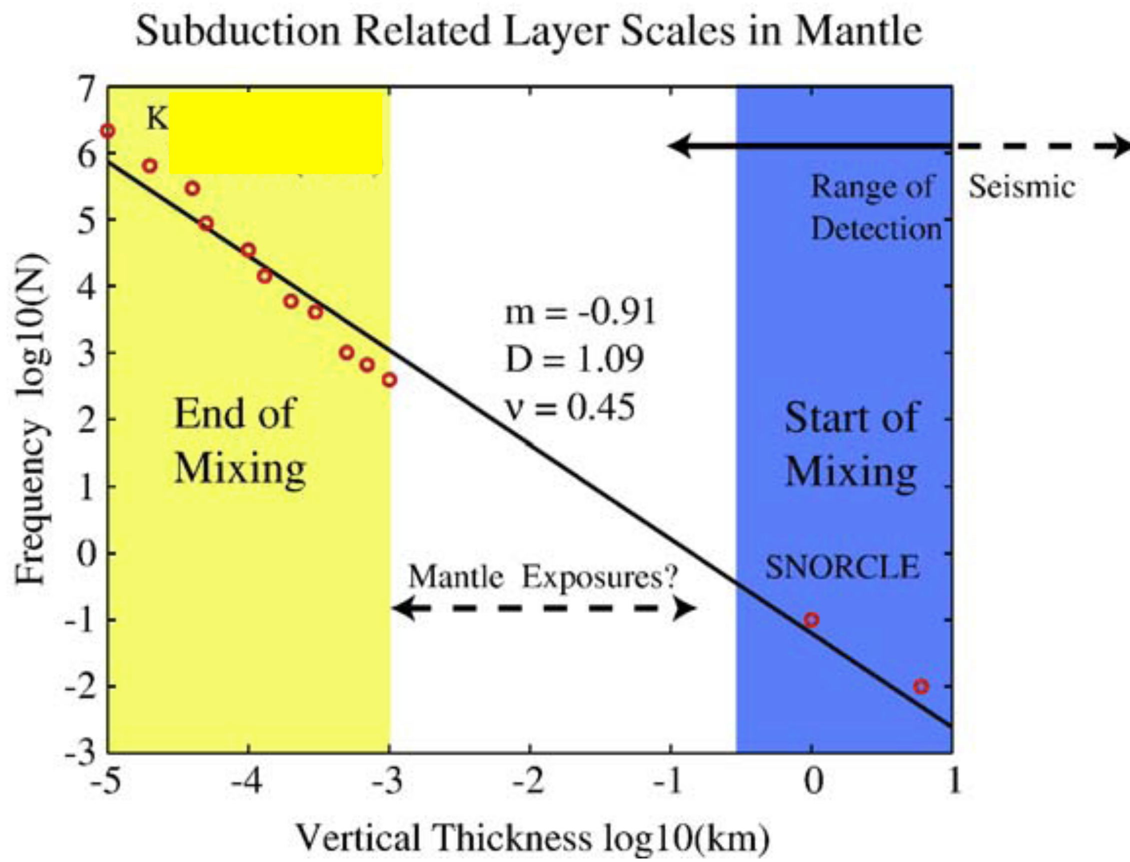


Fig. 3 Number of pyroxenite layers exposed at Beni Bousera with

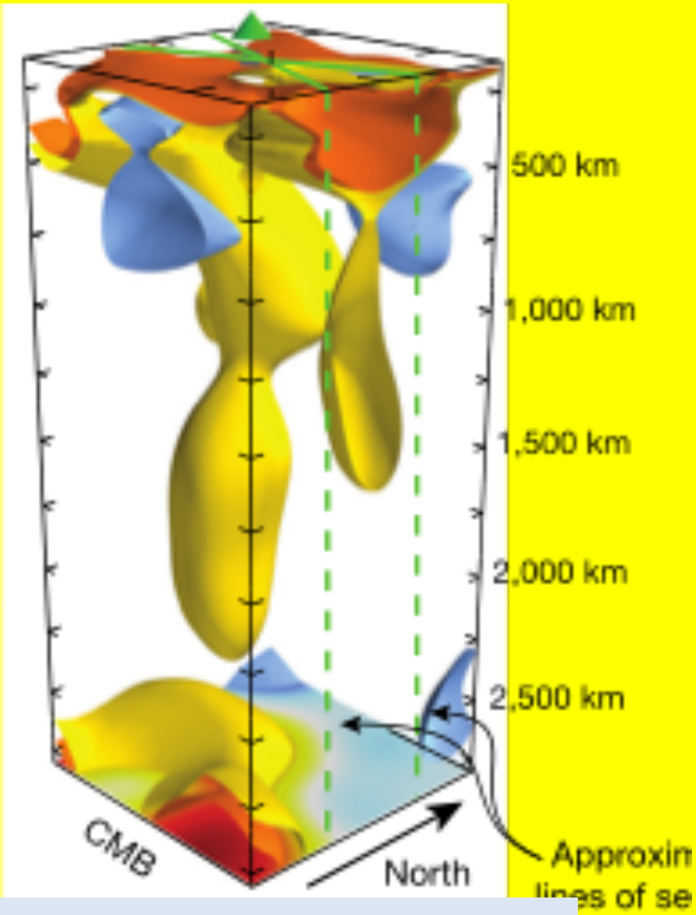


Levander et al. Tectonophysics 416 (2006) 167–185

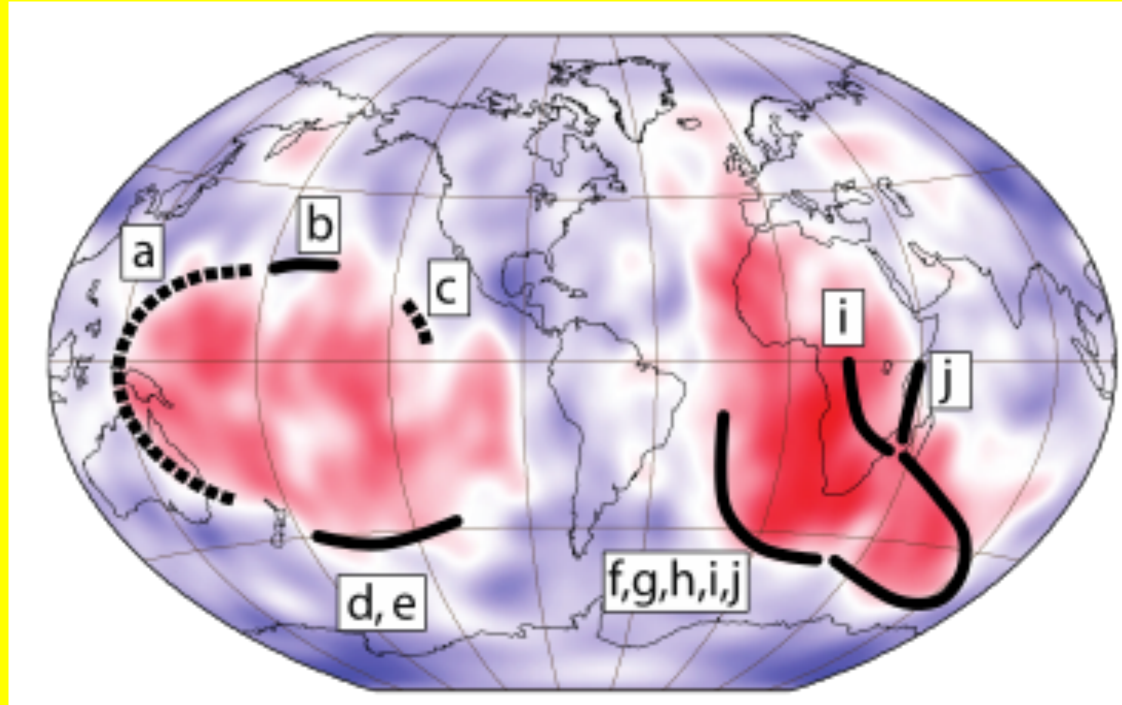
The scale of heterogeneity led Allègre and Turcotte (1986) to propose a 'marble cake' structure to the mantle

Image from epicurious.com

Origins of LLSVPs (content of this slide is borrowed from last week's CIDER talk by Quentin Williams)



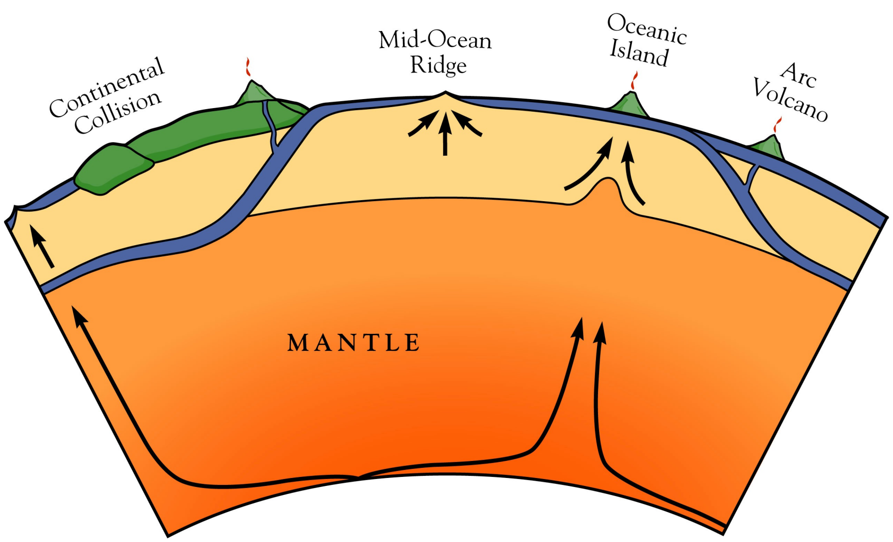
French and Romanowicz, Nature, 2015

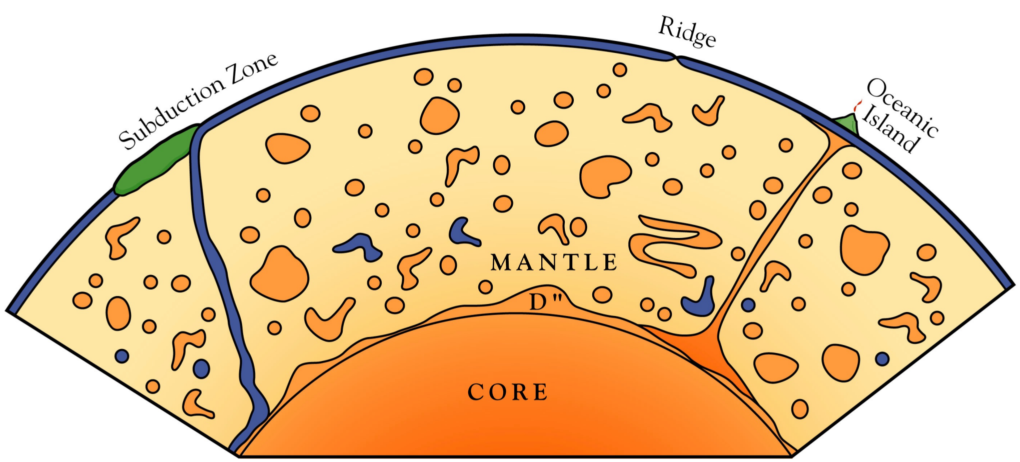


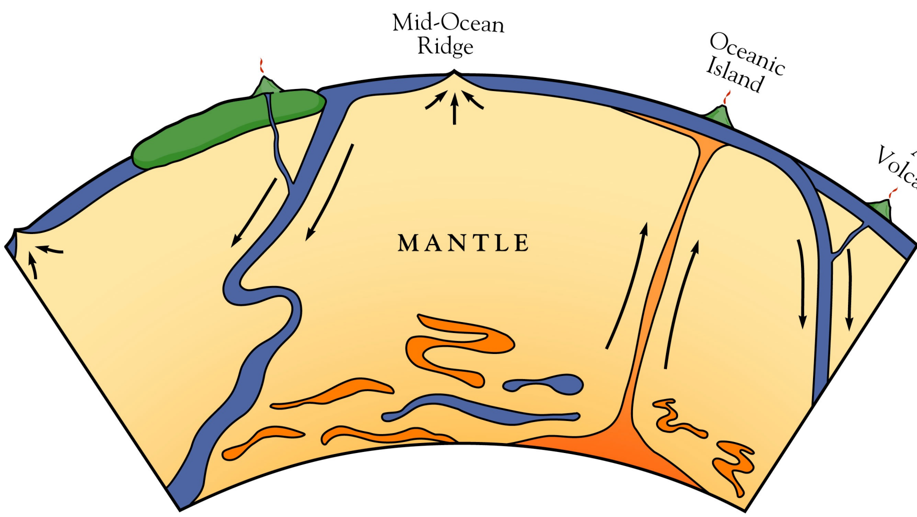
Garnero

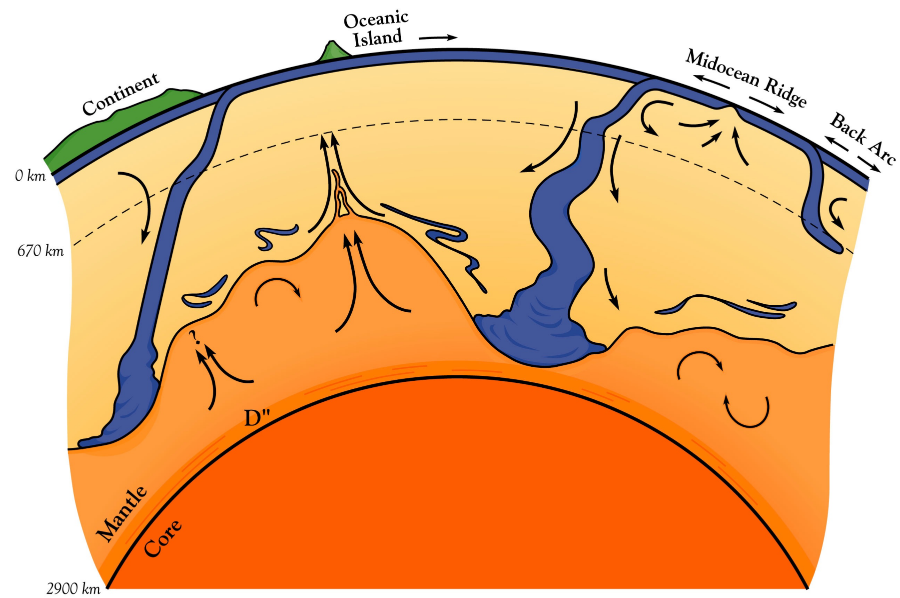
Outcome-- The Bridge has not been fully built between the composition, structure, formation and dynamics of LLSVPs: Ongoing CIDER Prospects

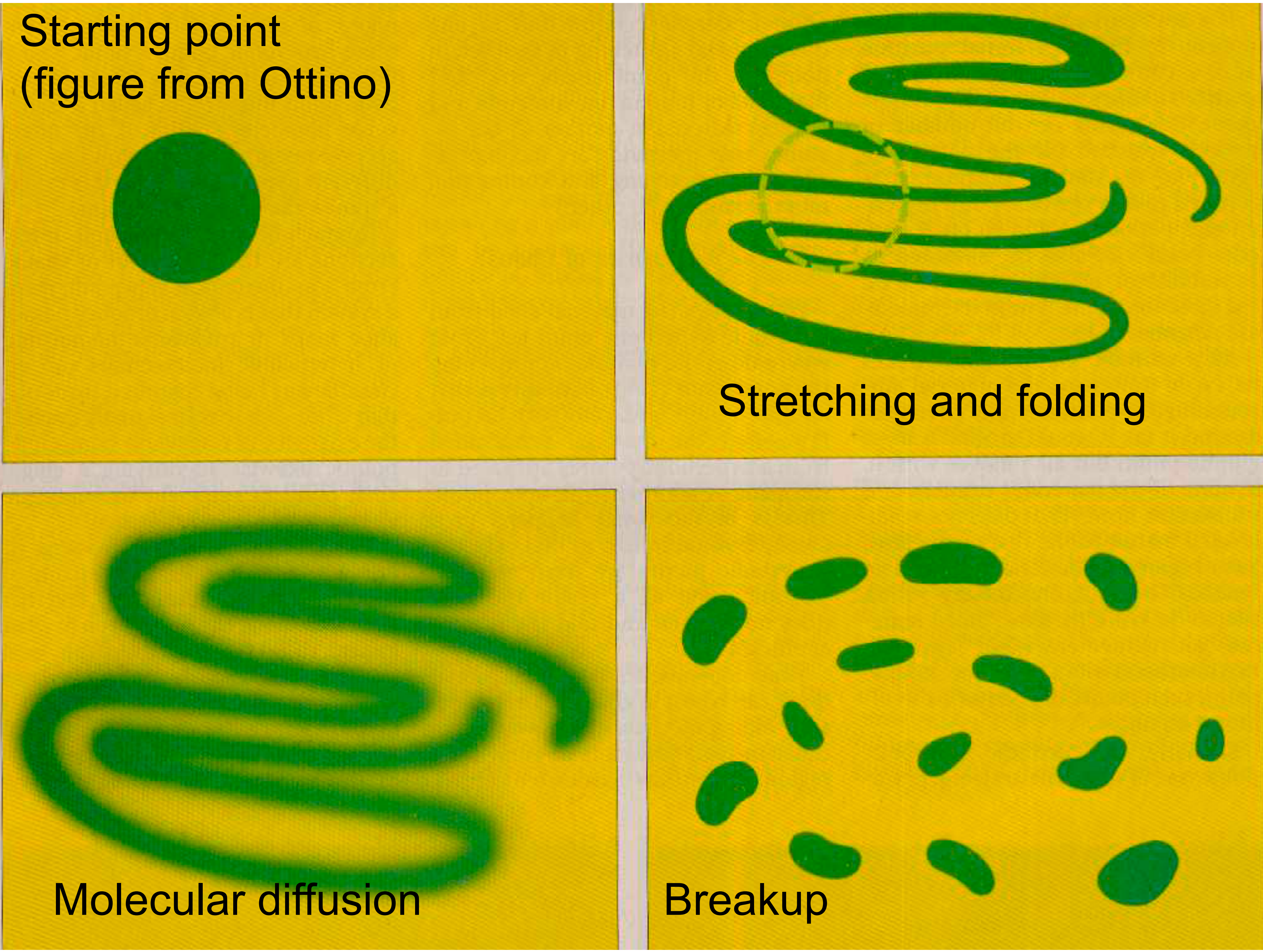
What configuration gives this combination of signatures?










Starting point
(figure from Ottino)
Stretching and folding
Molecular diffusion
Breakup

Some terminology

Stirring: stretching and folding of material surfaces to reduce length scales

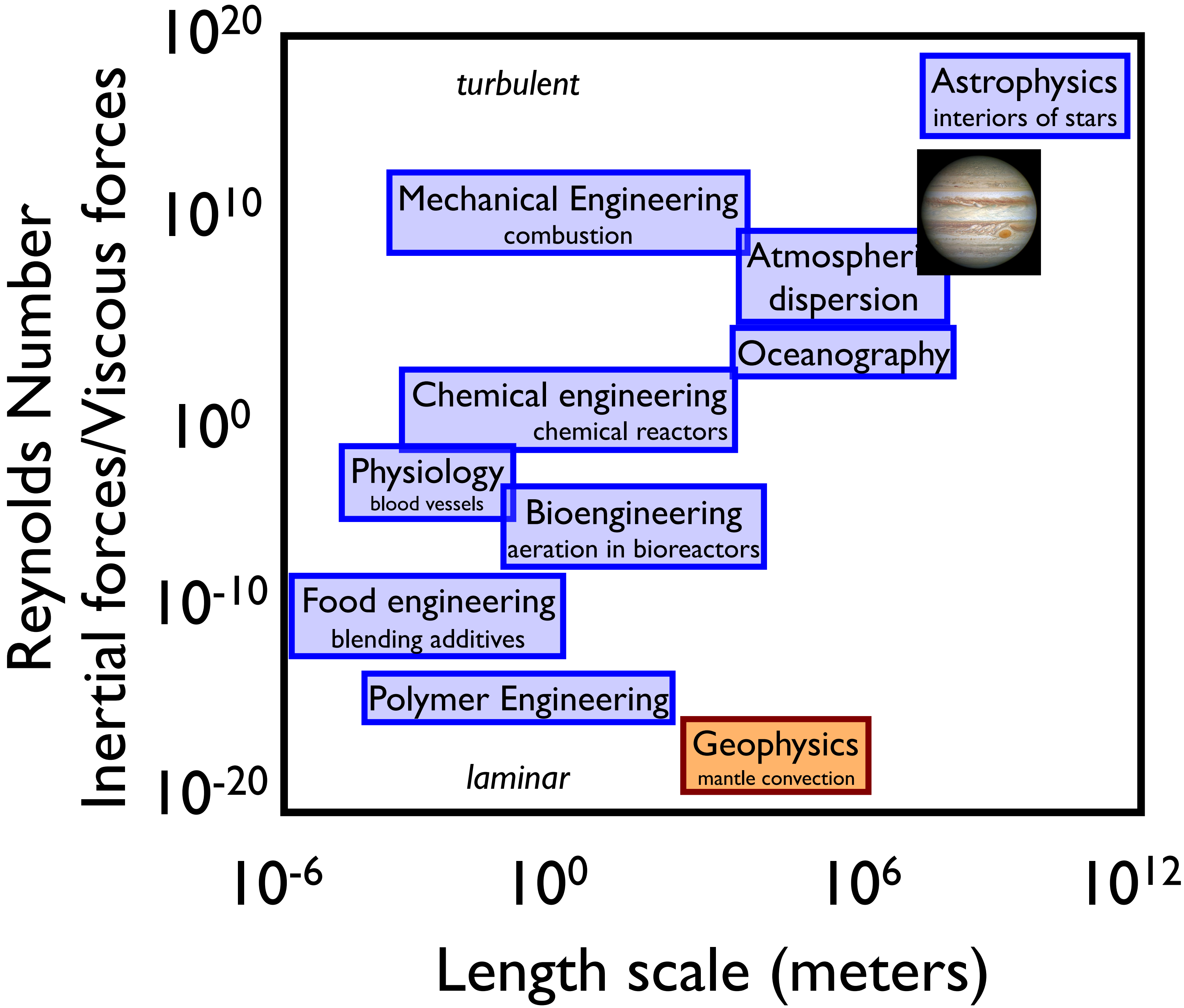
Mixing: homogenization by stirring and diffusion

Tracer: infinitesimal marker advected with the flow u(x,t)

Passive: Tracer does not influence the flow

Active: Tracer introduces differences such as density, rheology, others; presence of tracers **modify** the flow

Some mixing scales

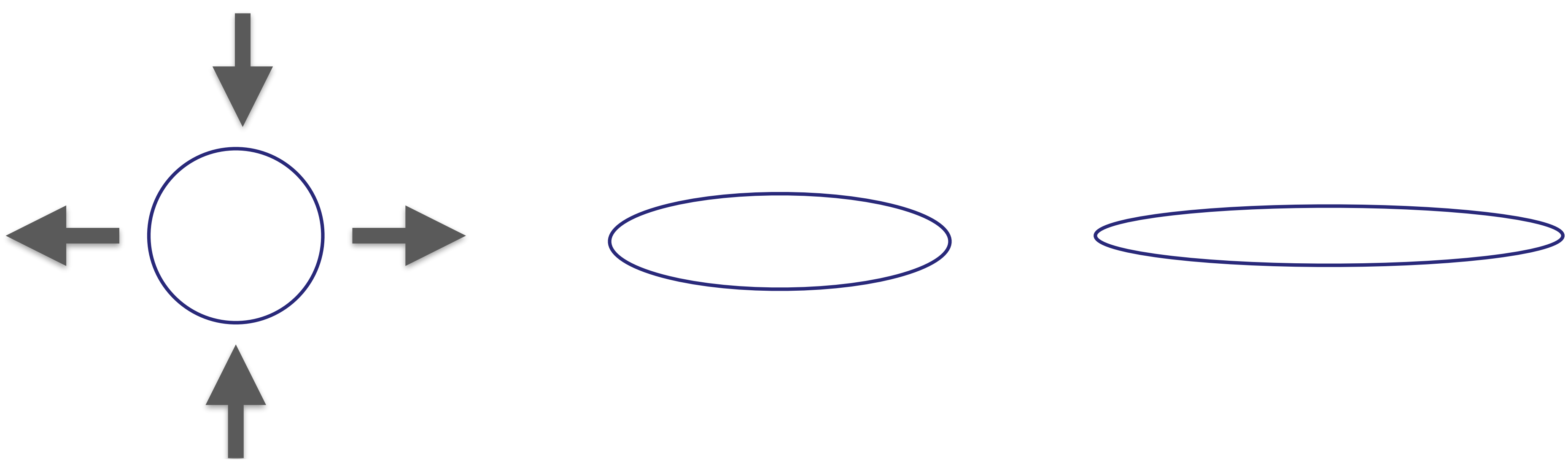


clip from fluid dynamics film series - 13:09

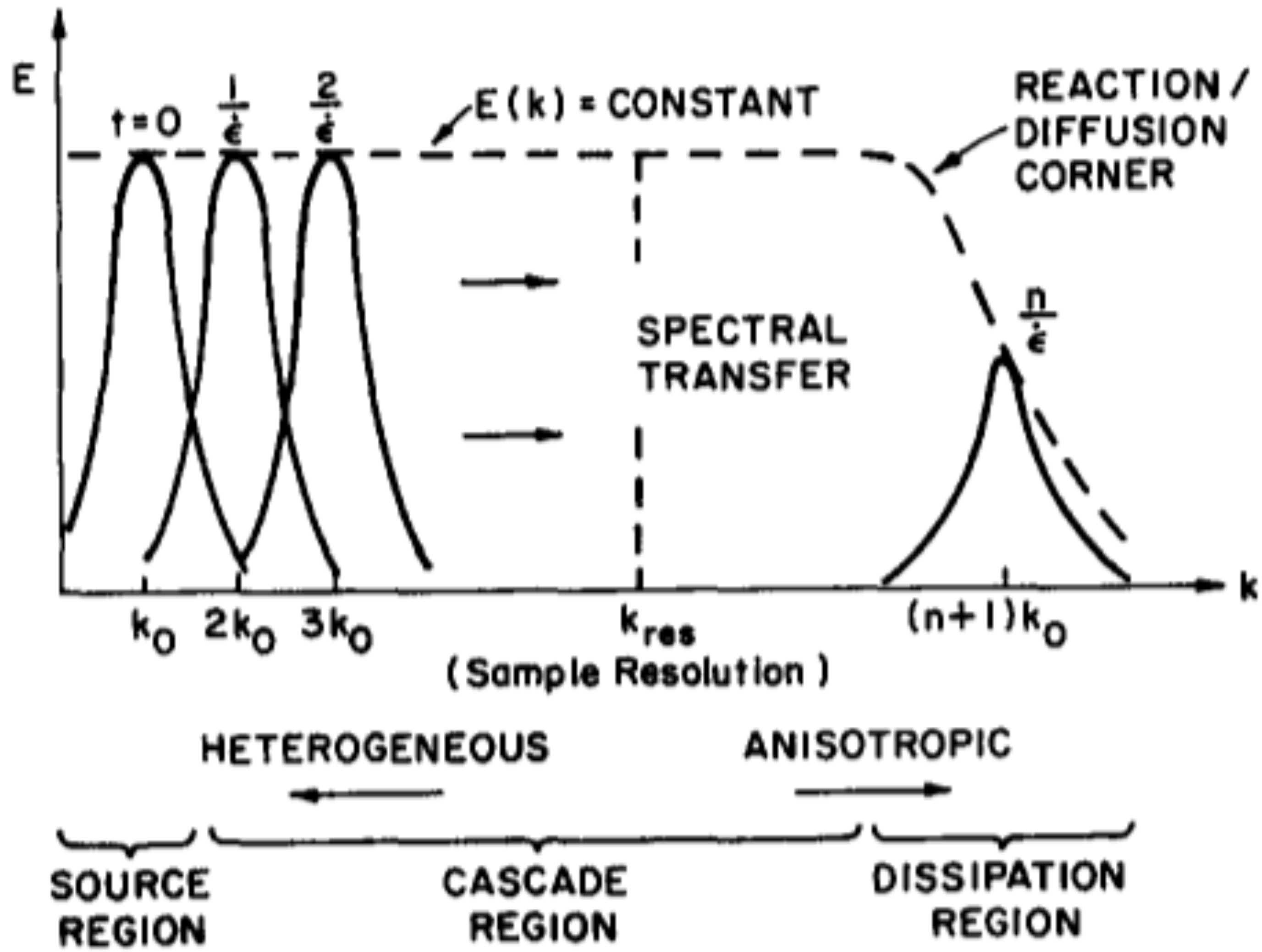
Modes and timescales

Simple shear: thinning is linear with time (~ strain rate x time)

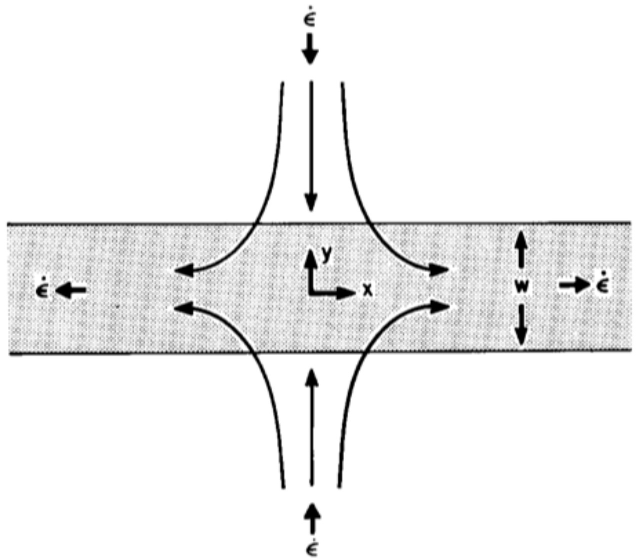
Pure shear: exponential thinning (~ exp (strain rate x time))

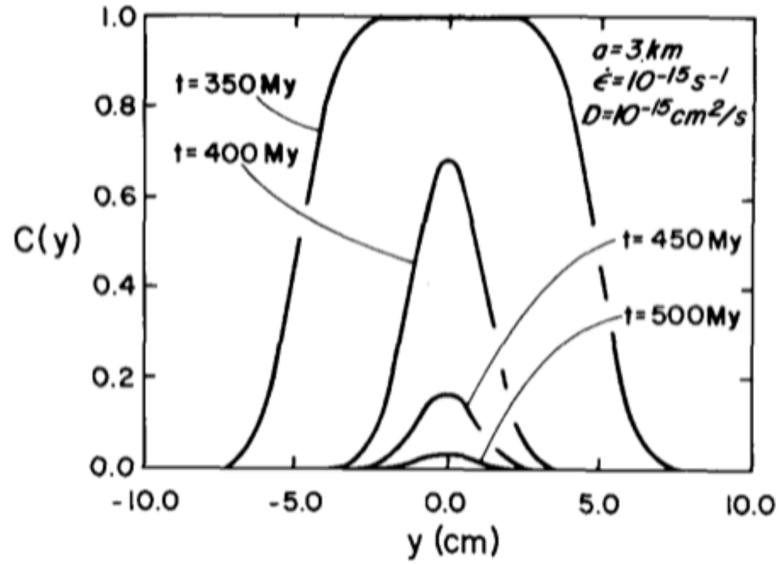


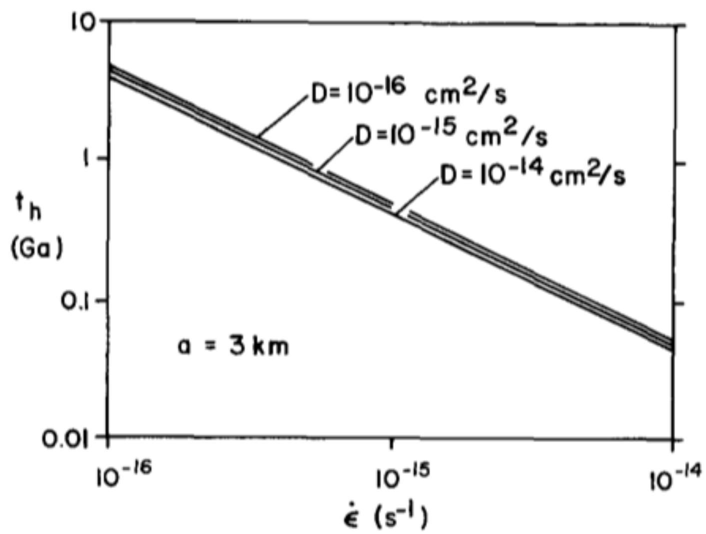
Convective mixing and the fine structure of mantle heterogeneity,
Peter Olson, David A. Yuen and Derick Balsiger
Physics of the Earth and Planetary Interiors, 36 (1984) 291—304

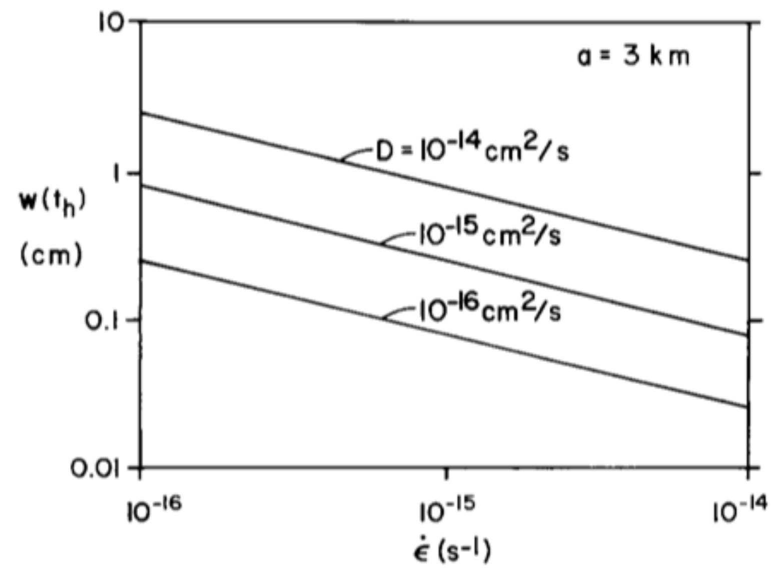


What about chemical diffusion?



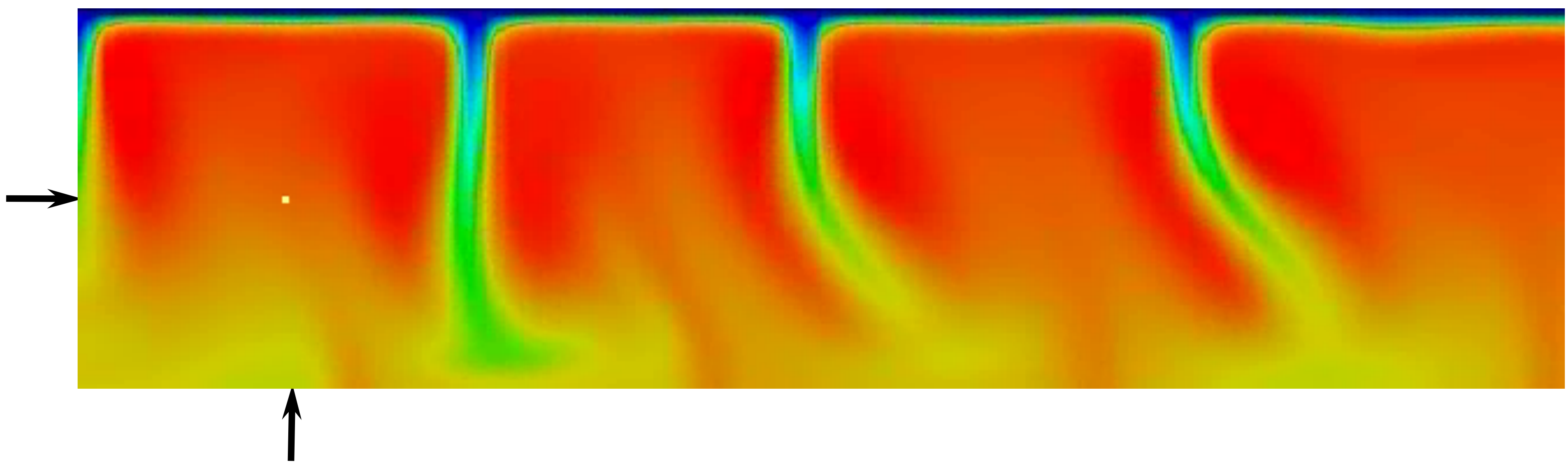






Kellogg and Turcotte, 1987 EPSL

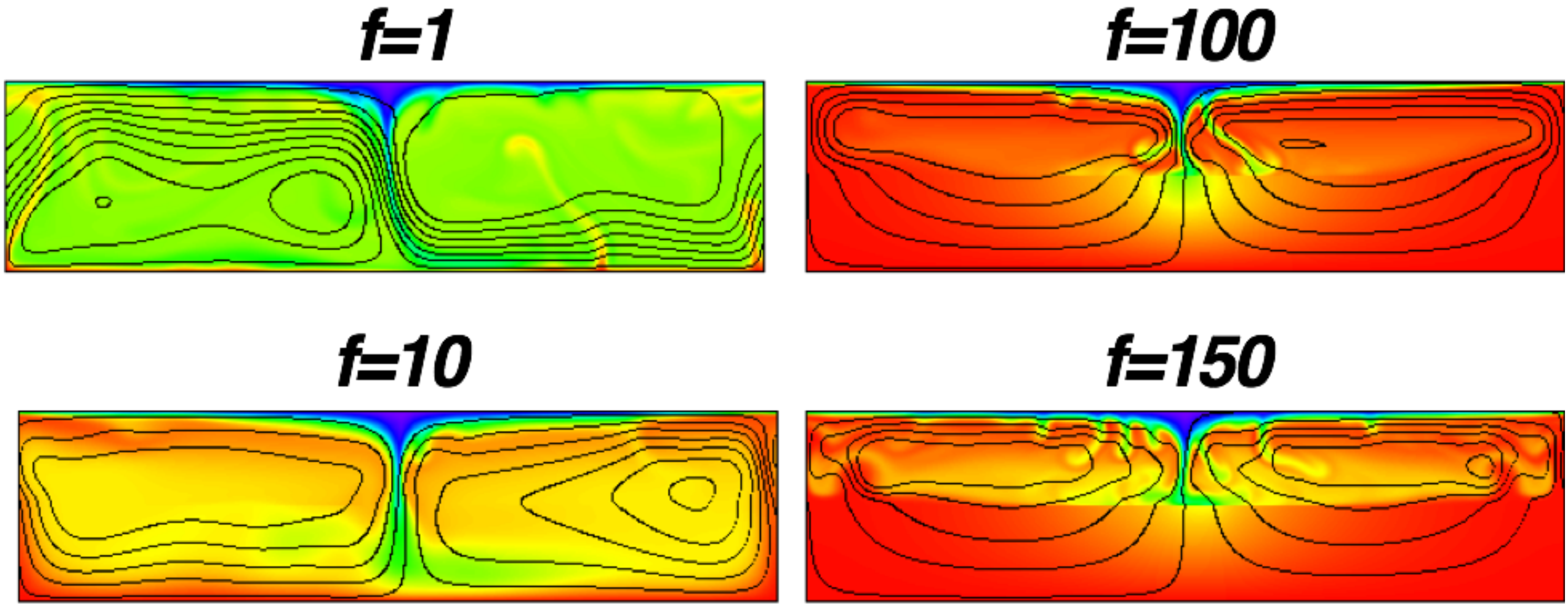
Example of mixing in 2-D internally heated convection

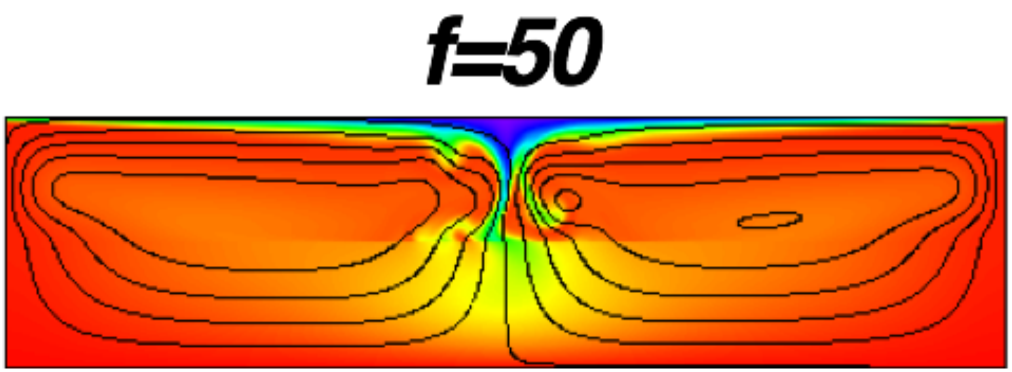


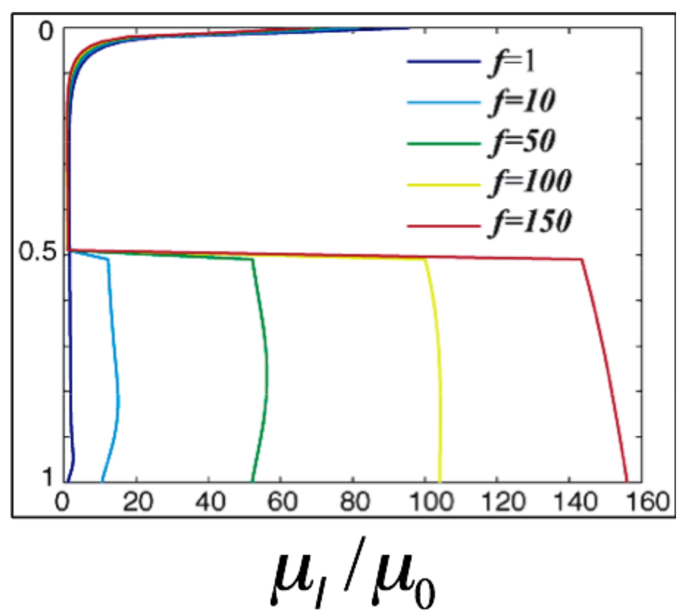
300,000 particles
Starting position

Some ways to analyze mixing in models of the mantle

- Large scale mixing
  - Introduce tracers into the model and monitor dispersal of tracers using box-counting or your favorite statistical method.
- Fine-scale mixing
  - Compute the time-averaged distance between **initially proximal tracers** as a measure of stretching (Lyapunov exponent)
  - Treat each tracer as **strain marker** and compute stretching for each tracer
- Compute derived properties such isotopic signatures

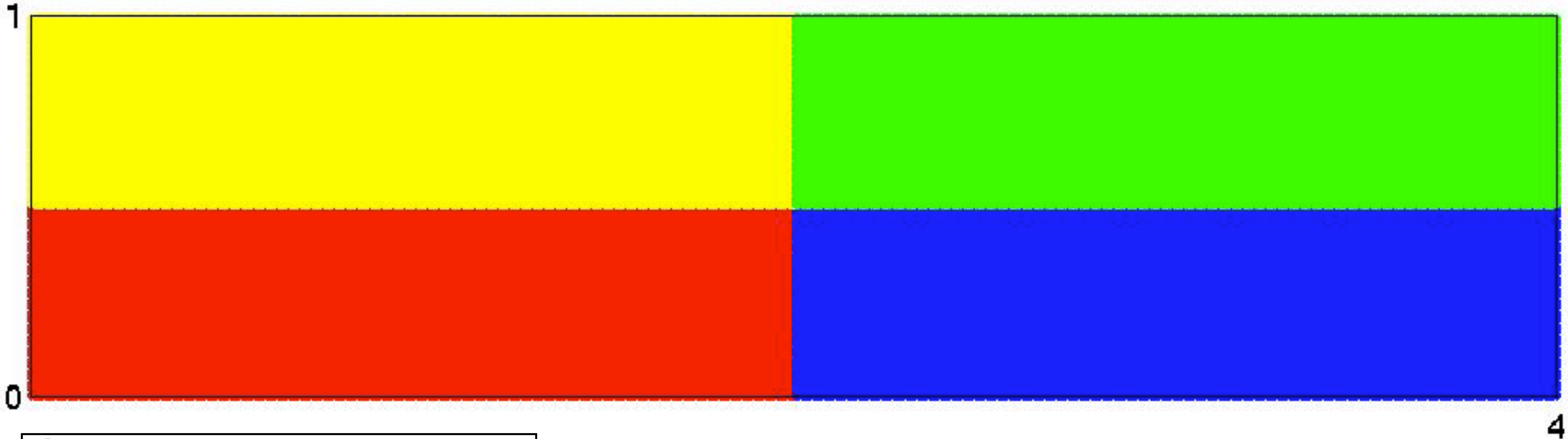


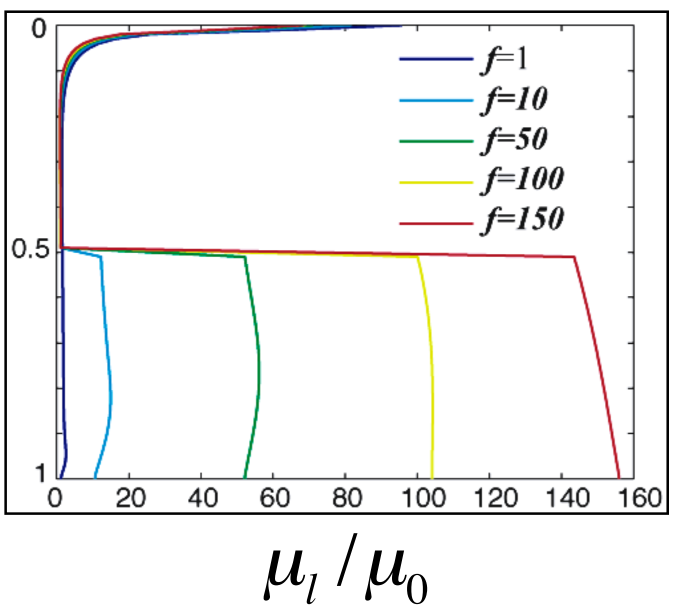




Naliboff and Kellogg

f = 1
(reference model)

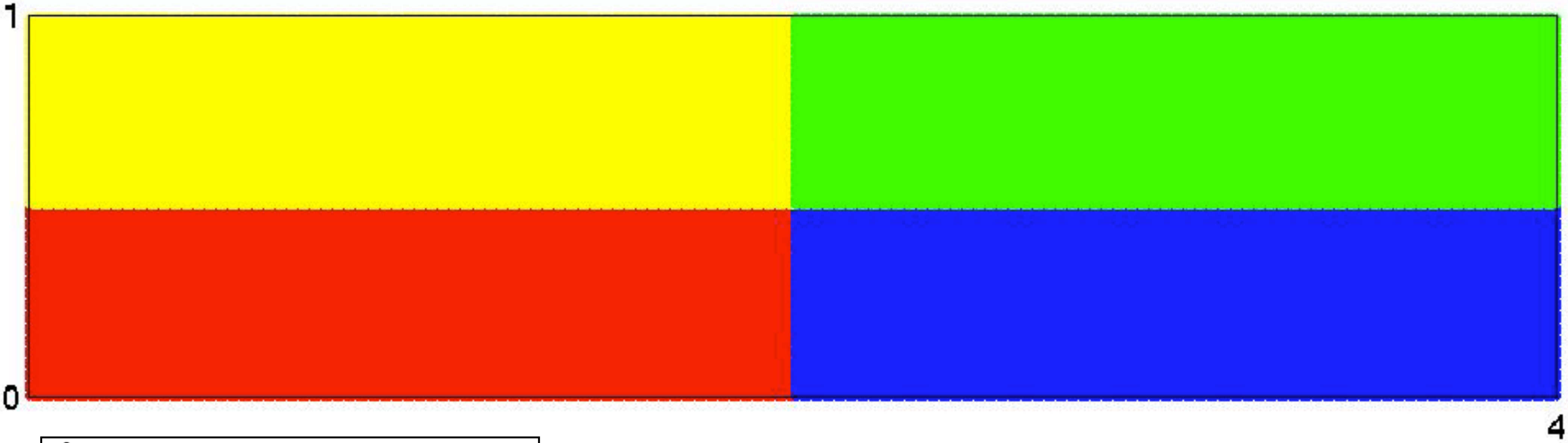


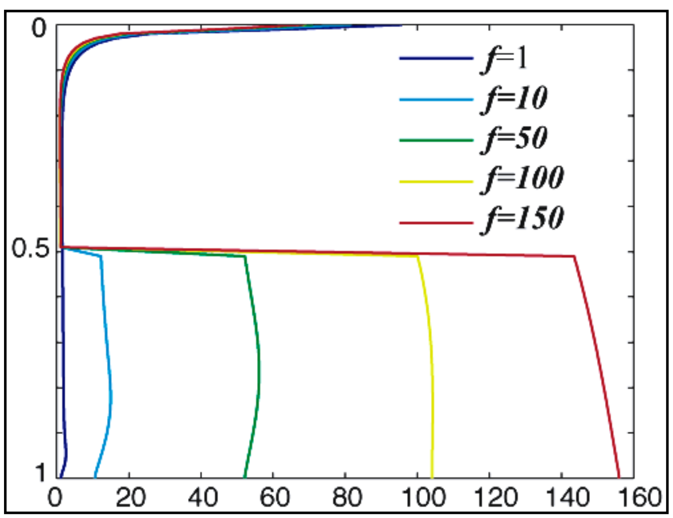


Naliboff and Kellogg

Movie1p

f = 50



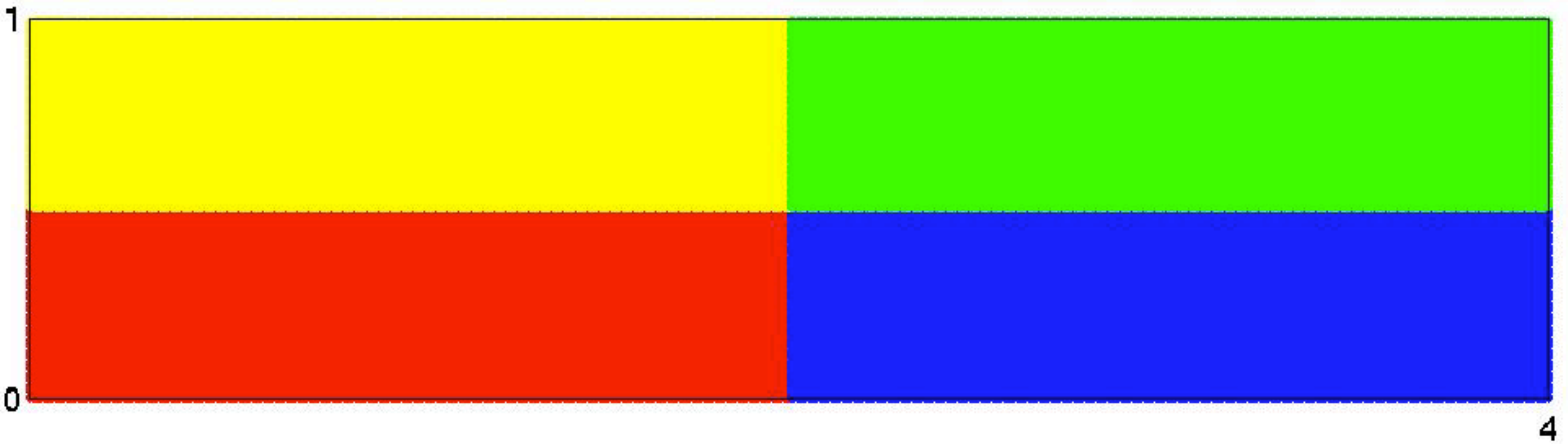


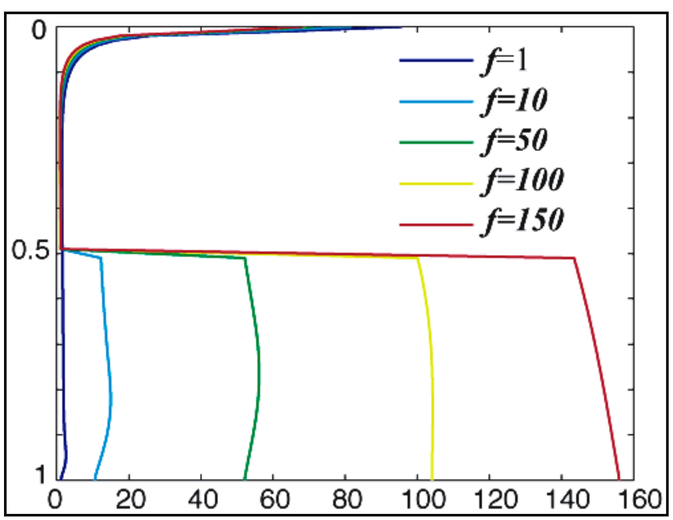
μ_l / μ_0

Naliboff and Kellogg

Movie3p.mov

f = 100





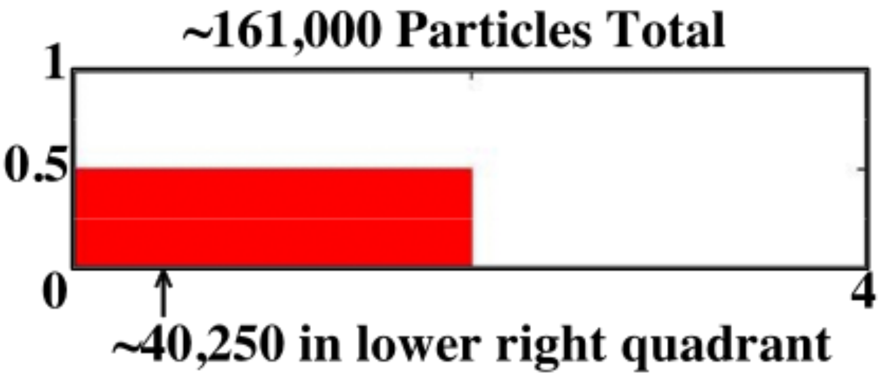
μ_l / μ_0

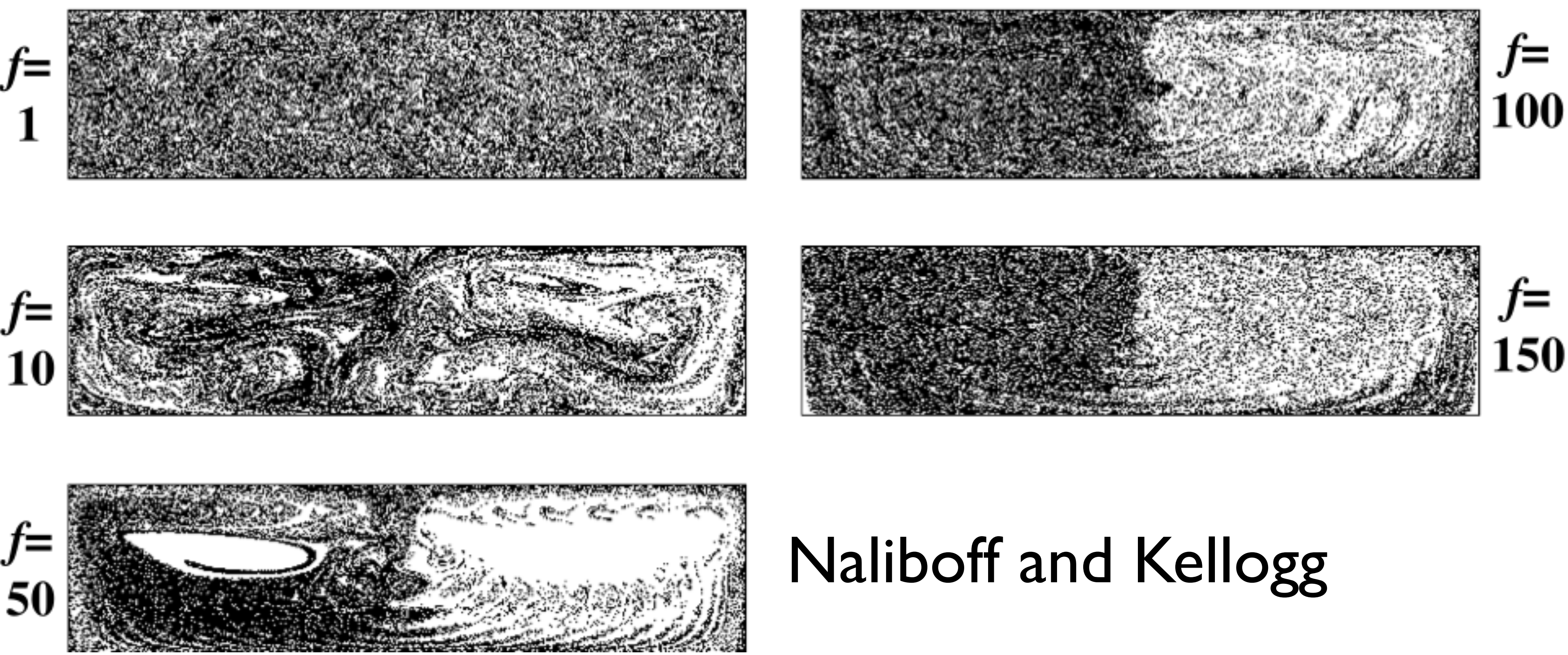
Naliboff and Kellogg

4pi.mov

Particle Distributions after 4 Ga

Starting particle distribution at steady-state conditions



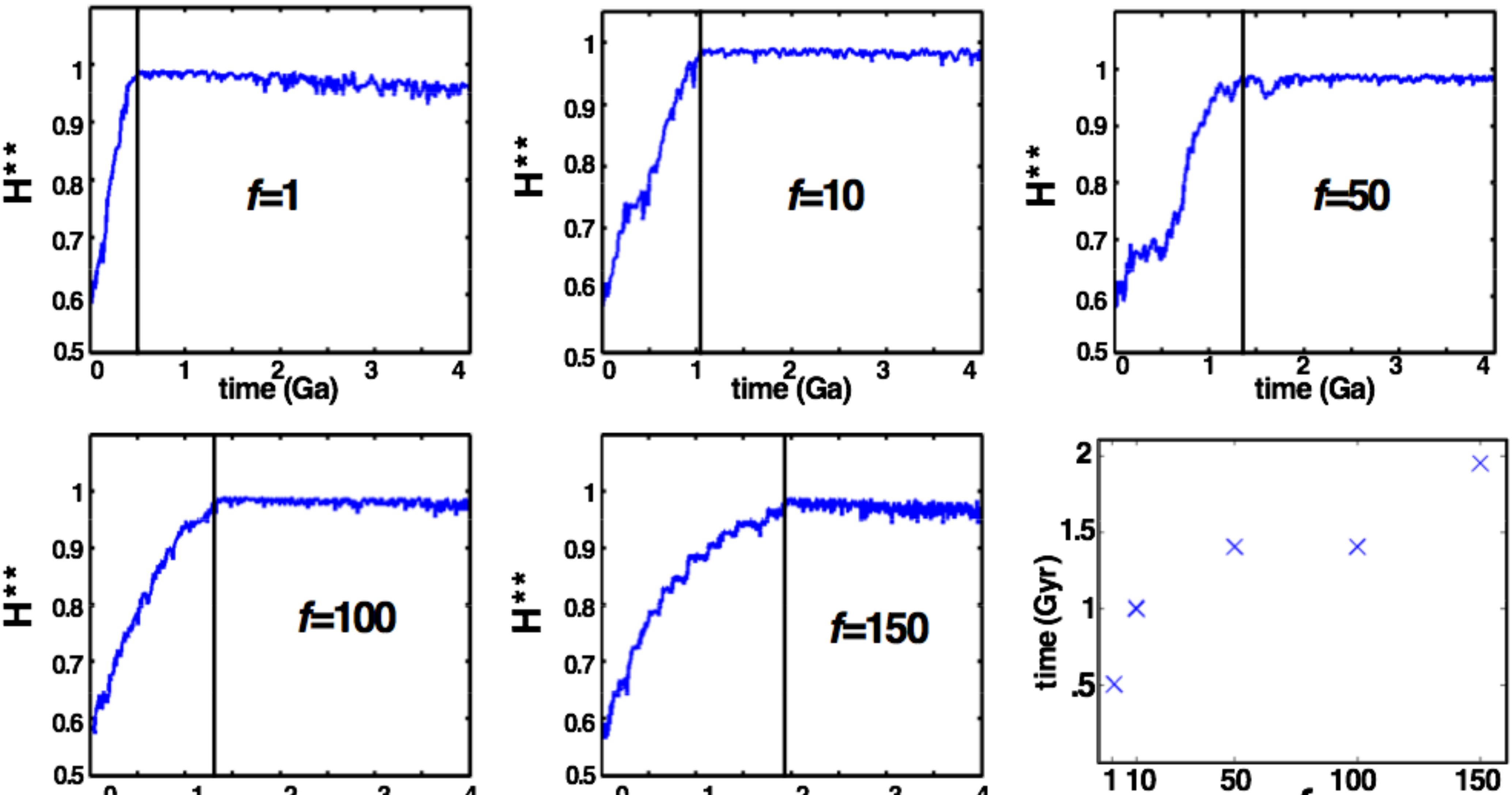


Naliboff and Kellogg

Configurational Entropy

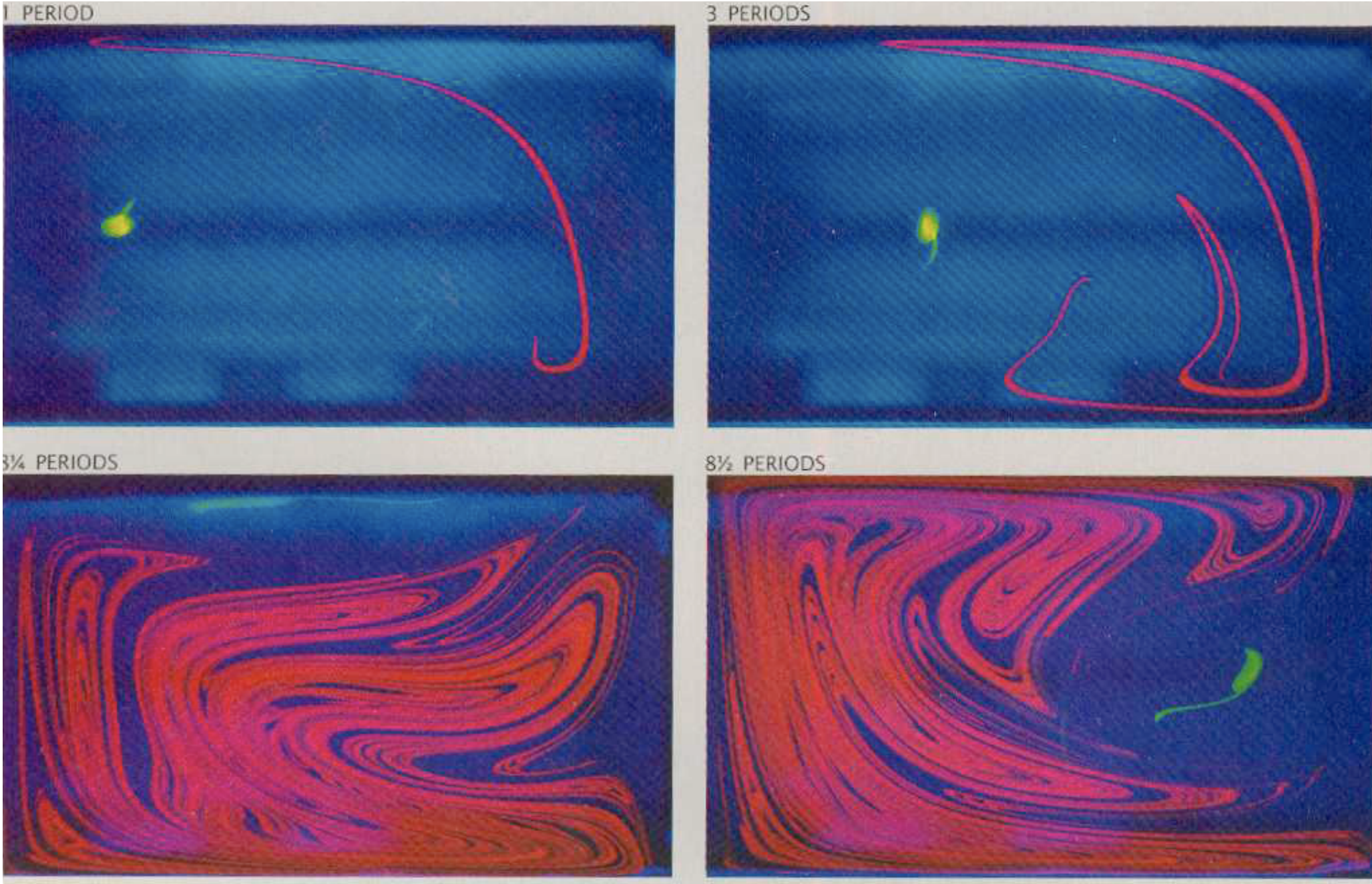
Calculating the configurational entropy of particles ***with starting positions above the increase in viscosity thermal conductivity*** - varies as function of the distribution of particles in space.

Method defined by Goltz and Bose (2004) and Turcotte (2001)

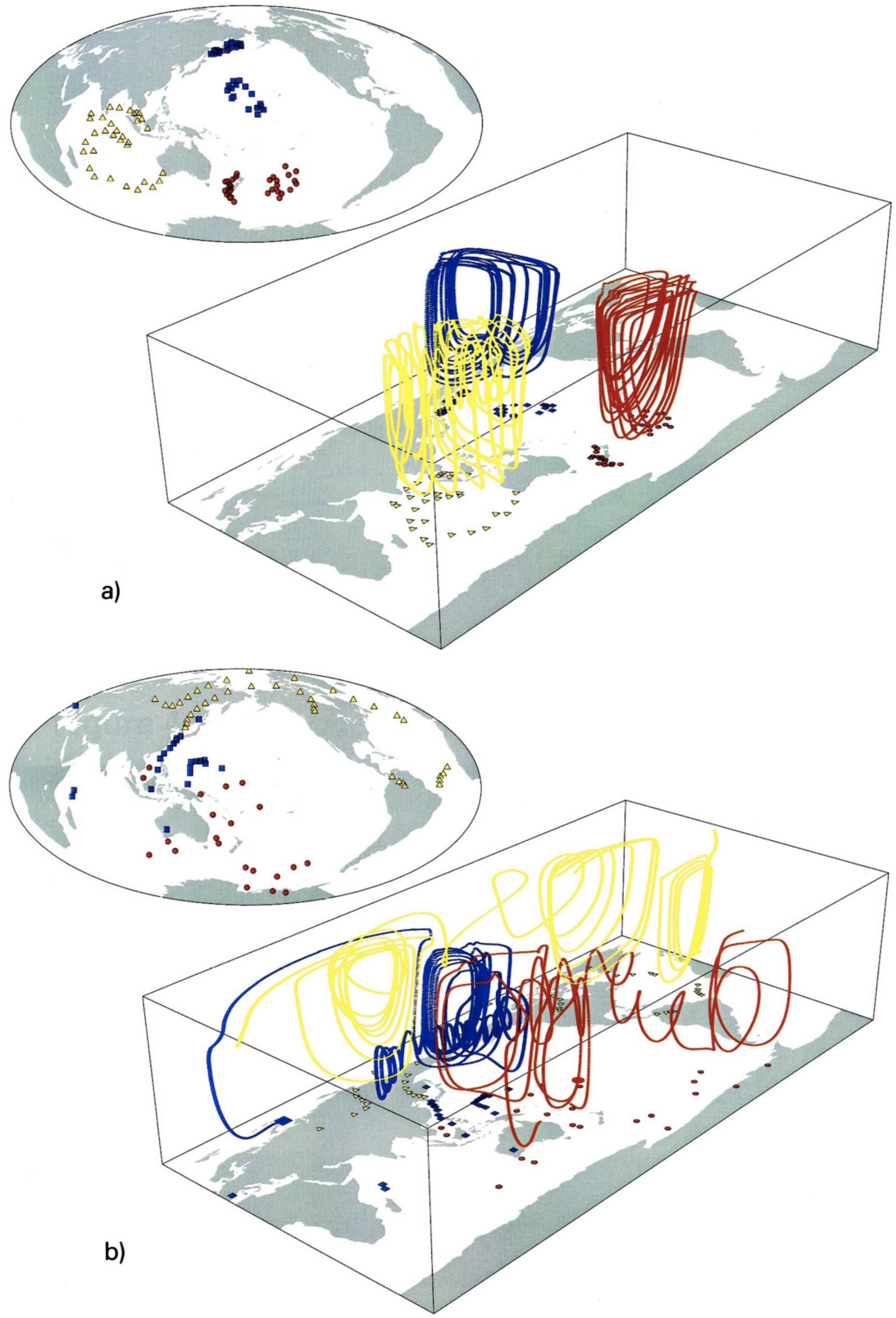


Naliboff and Kellogg

Ottino: Kinematics of Mixing



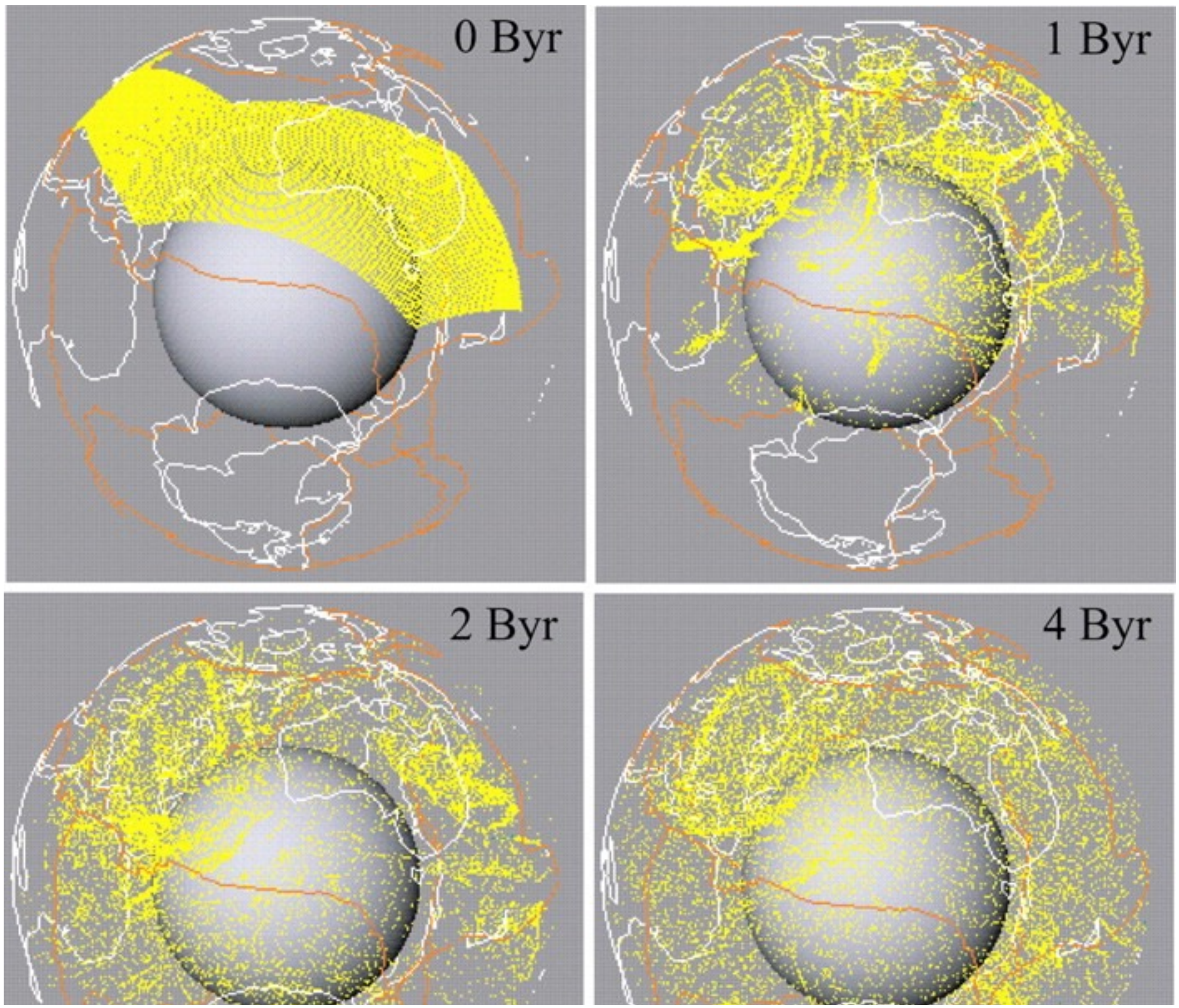
Mixing in a 3D spherical model of present-day mantle convection
Peter van Keken and Shijie Zhong



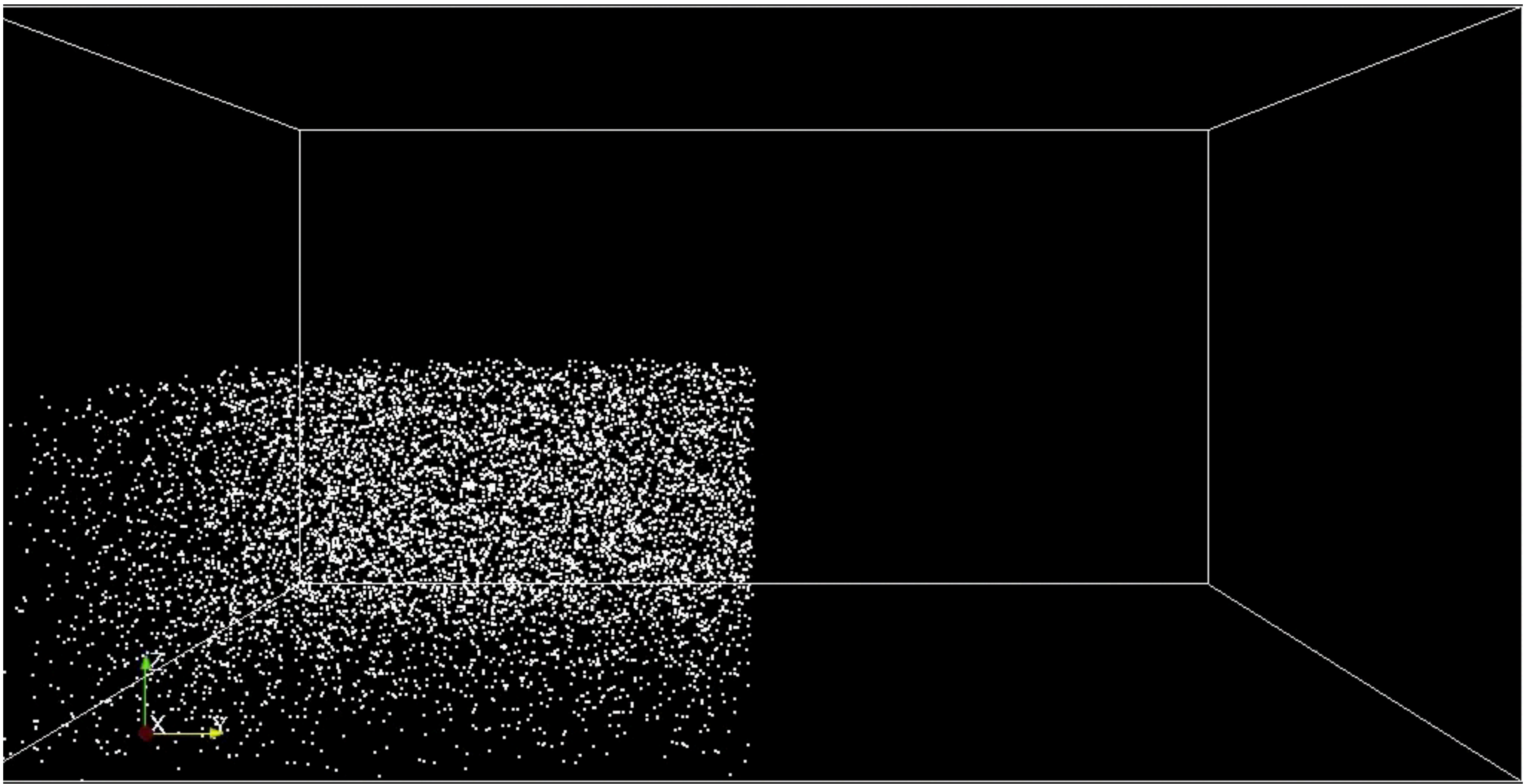
Mixing driven by modern plate flow

Van Keken and Zhong, 1999

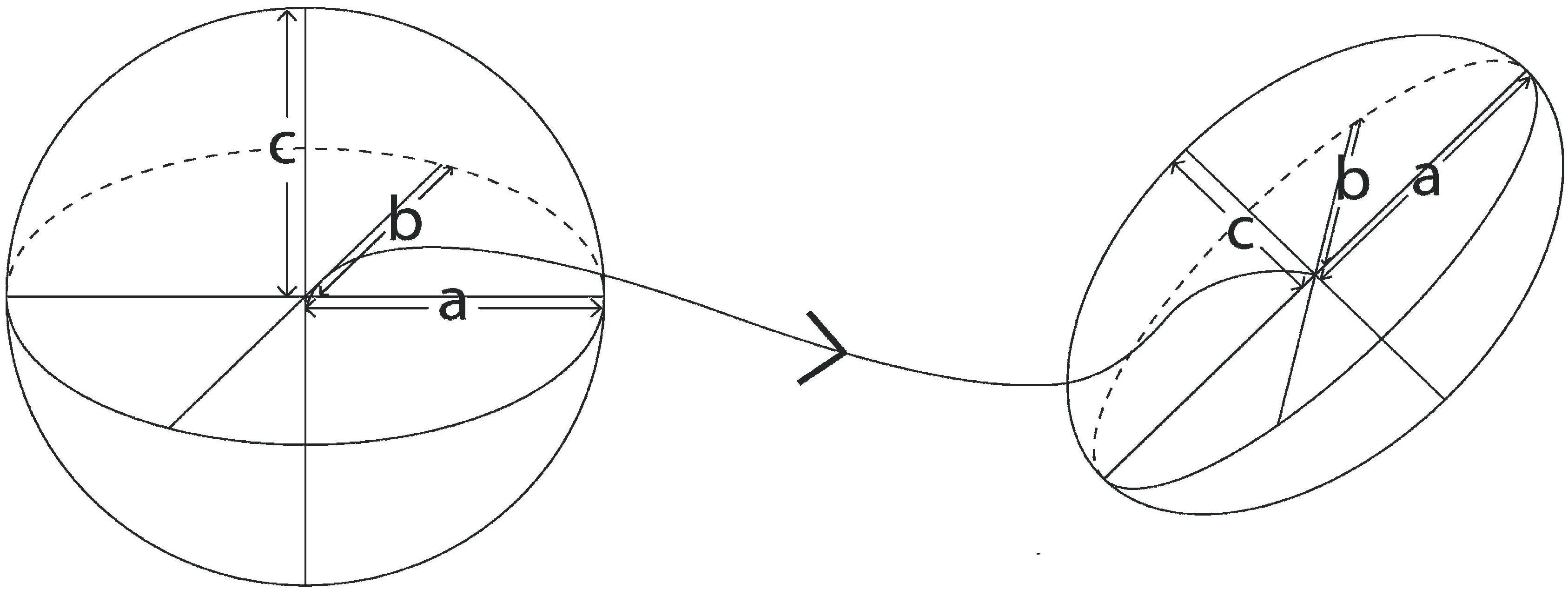
see also Van Keken et al Annual Reviews of Earth & Planetary Sci. 2002



Ra = 300,000 (time varying)



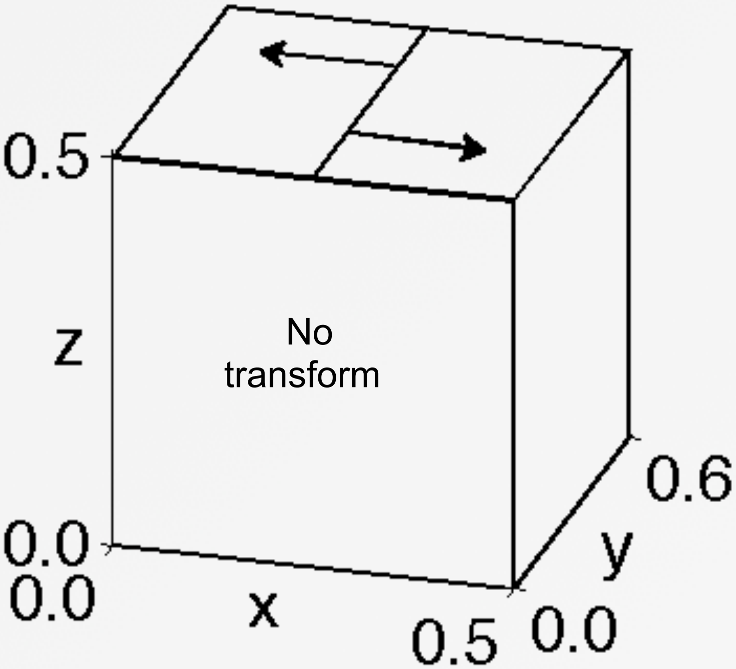
Model using Aspect



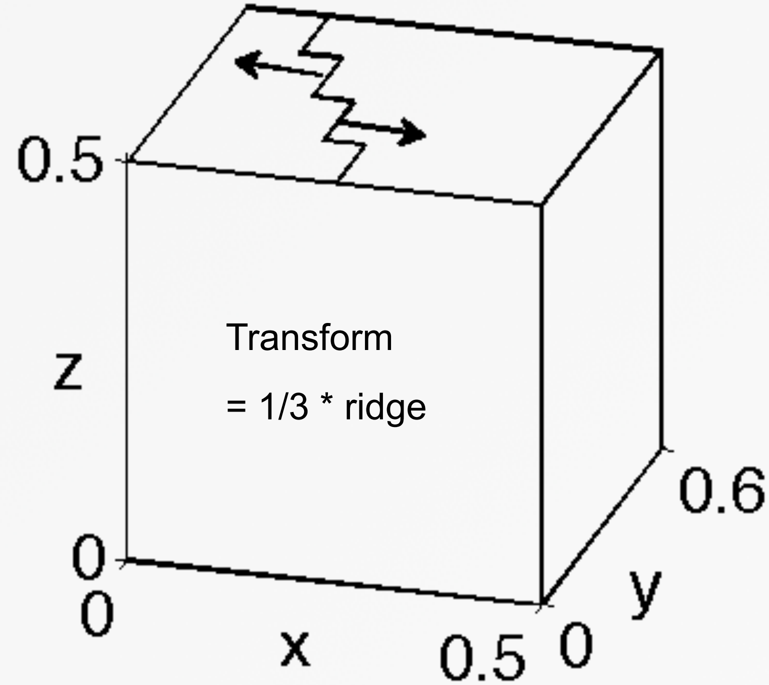
- Introduce infinitesimal strain ellipsoids = tracers
- Calculate the stretching in 3 orthogonal directions as tracer is advected by the flow.
- S = a(t)/a(0) is the measure of stretching.
- Method is adapted from structural geology

Simple plate-driven flow

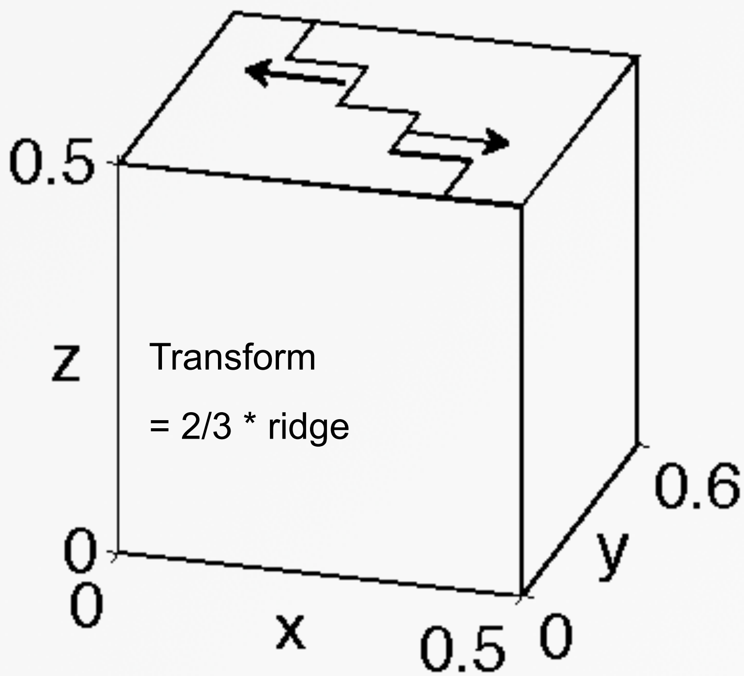
Model 1



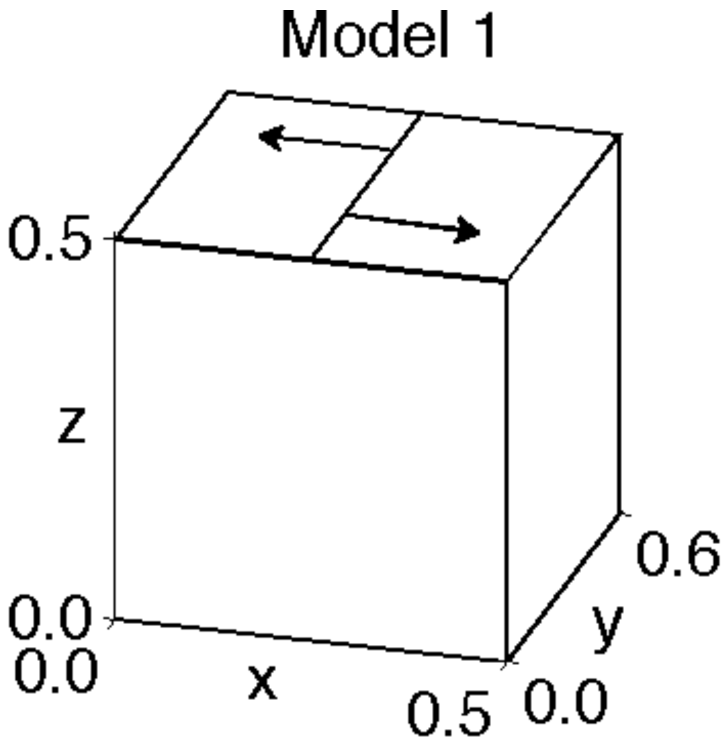
Model 2

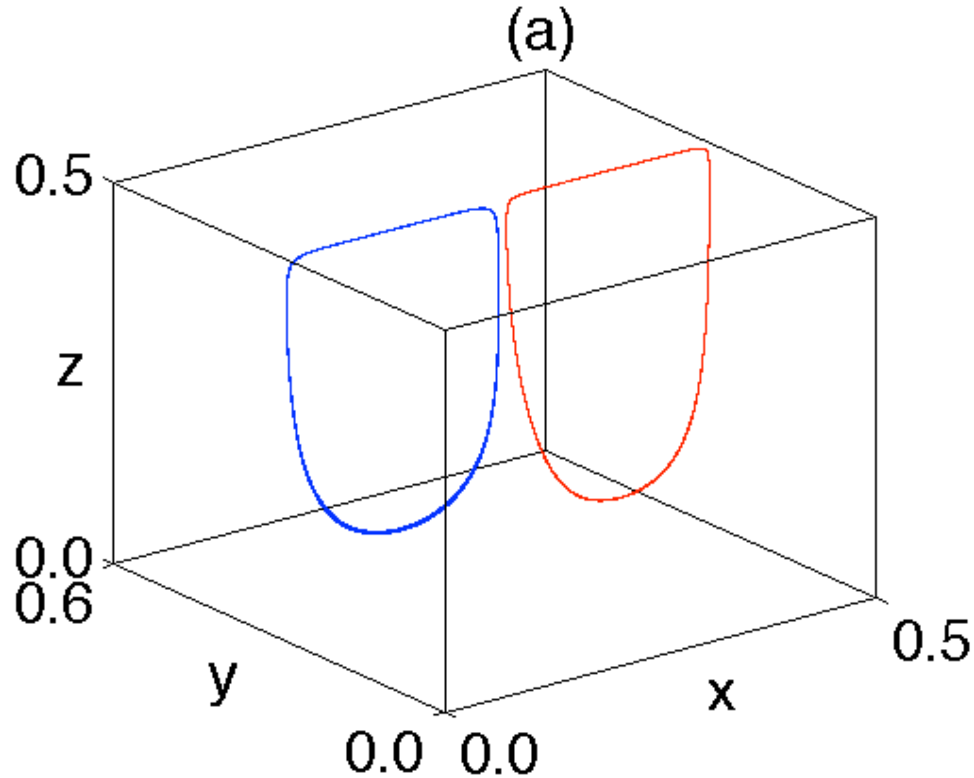


Model 3



C.S. Natarajan &
L.H. Kellogg





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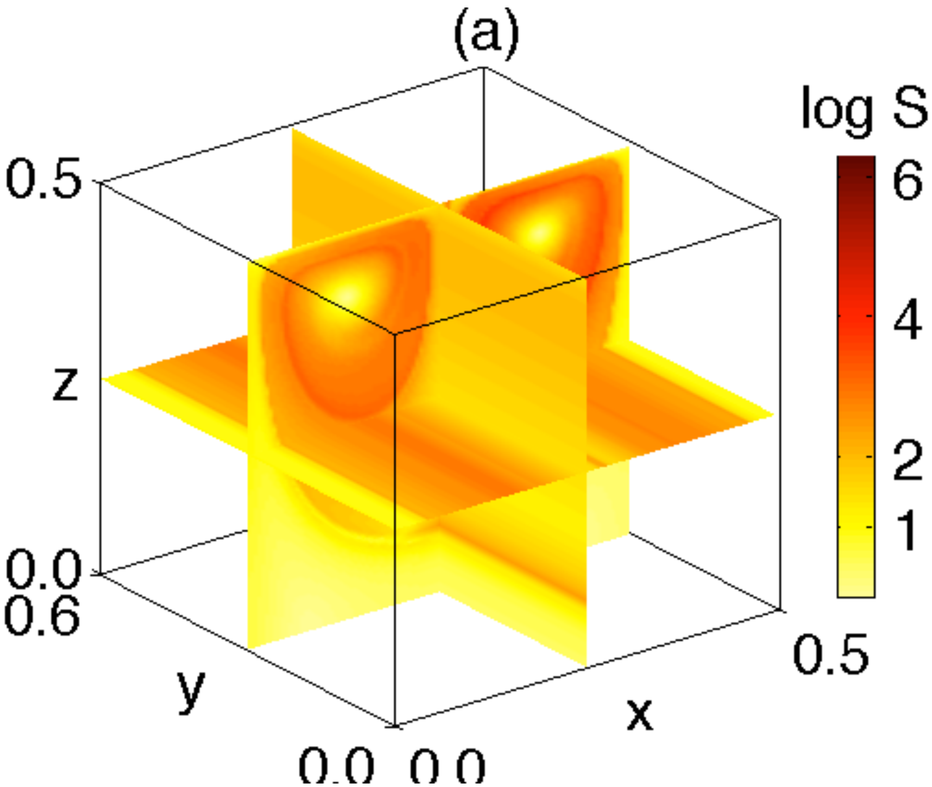
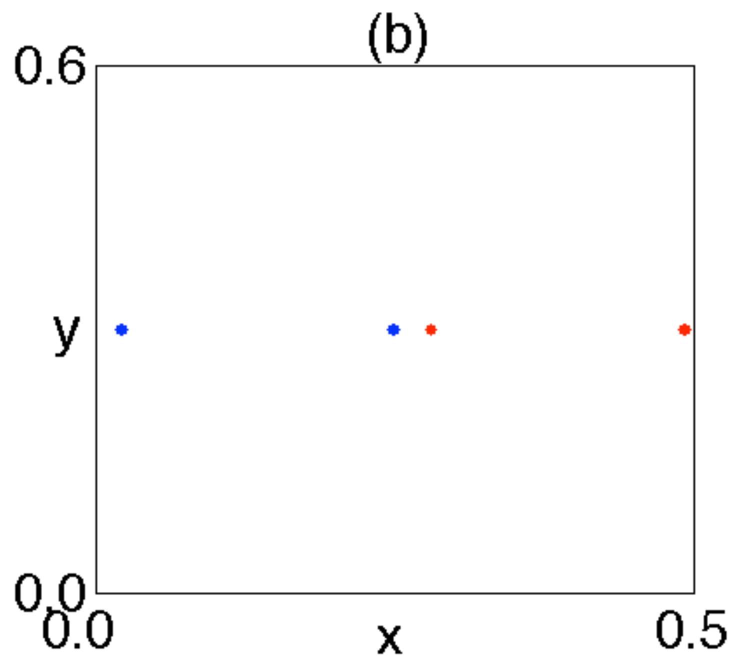
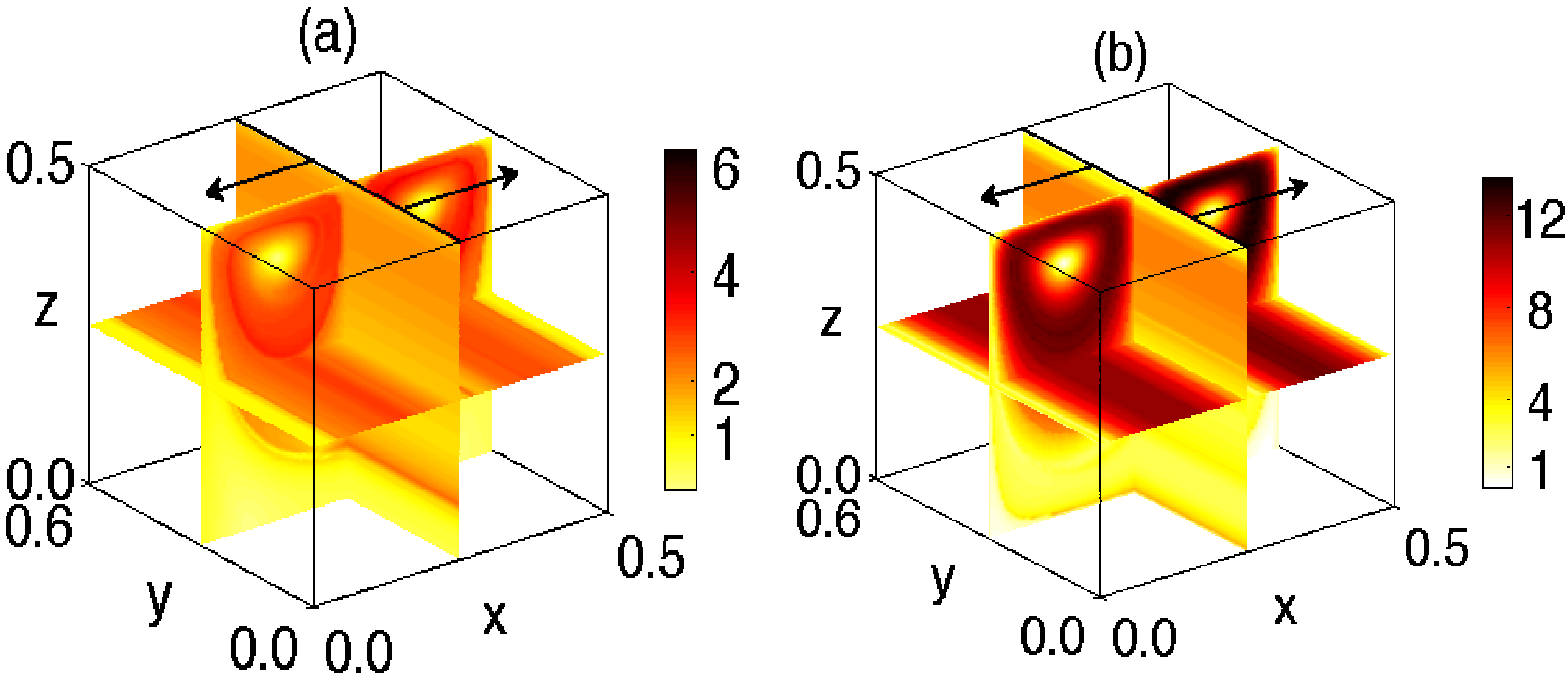




Stretching in flow driven by a simple ridge

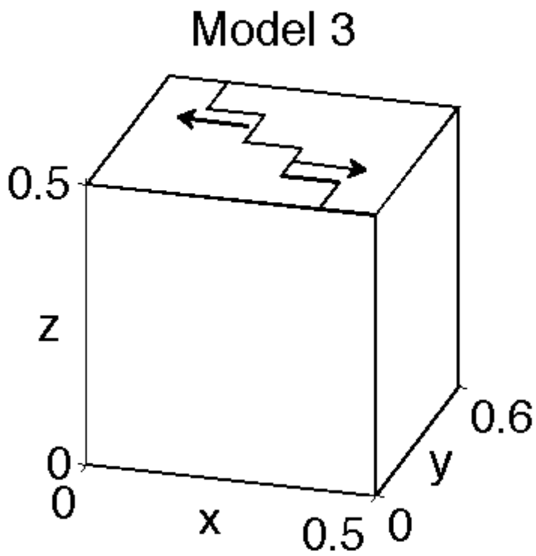


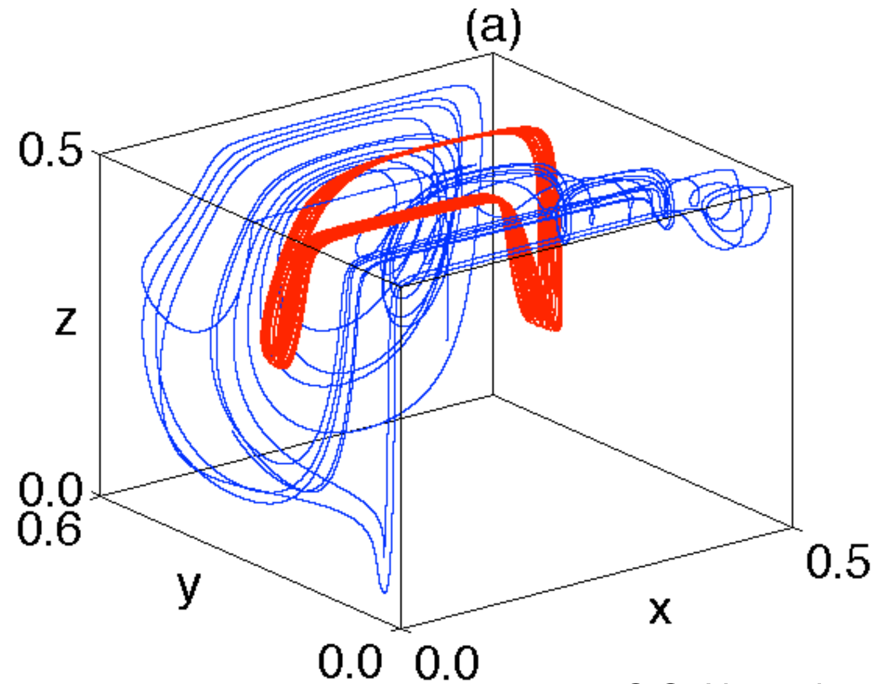
8 transit times

32 transit times

C.S. Natarajan &
L.H. Kellogg

Add transform faults





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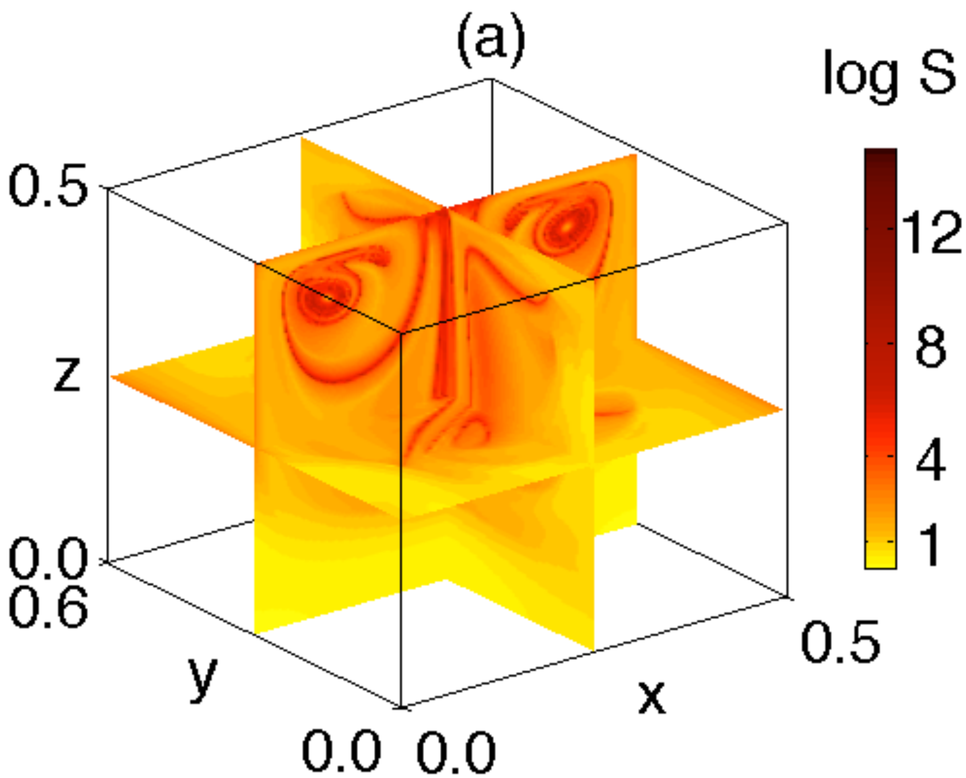
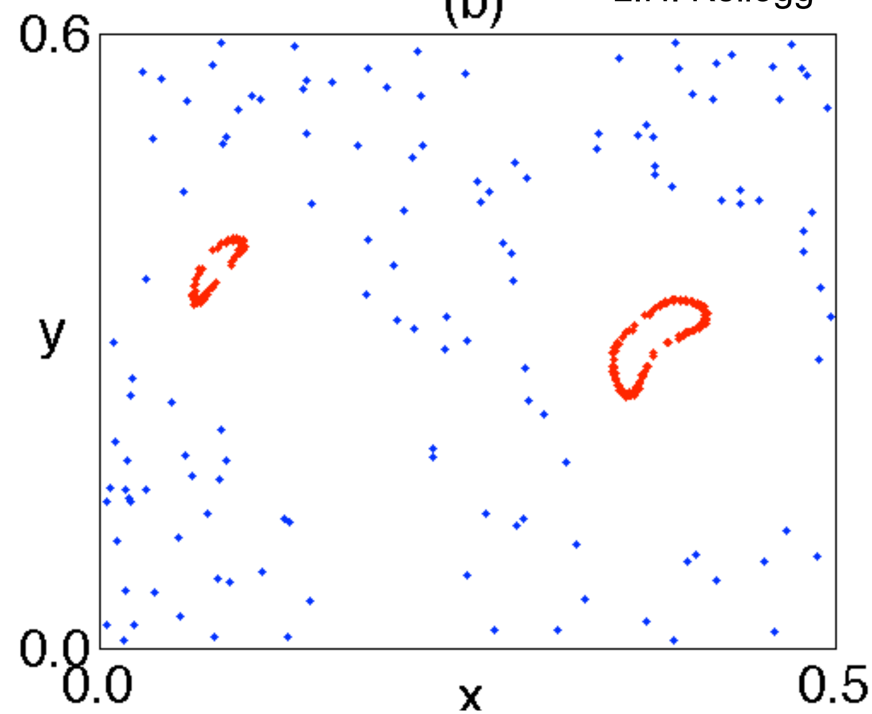
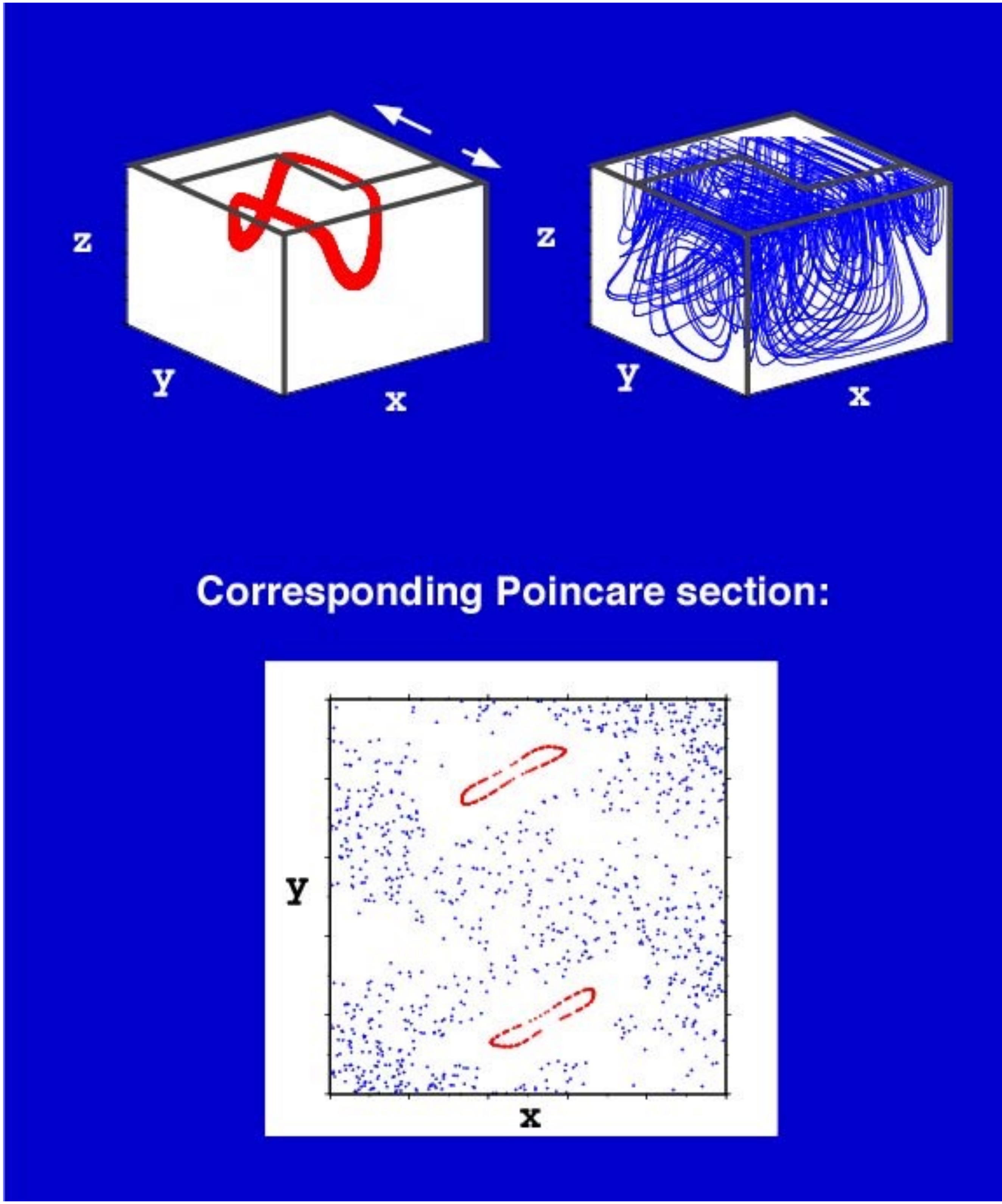
L.H. Kellogg

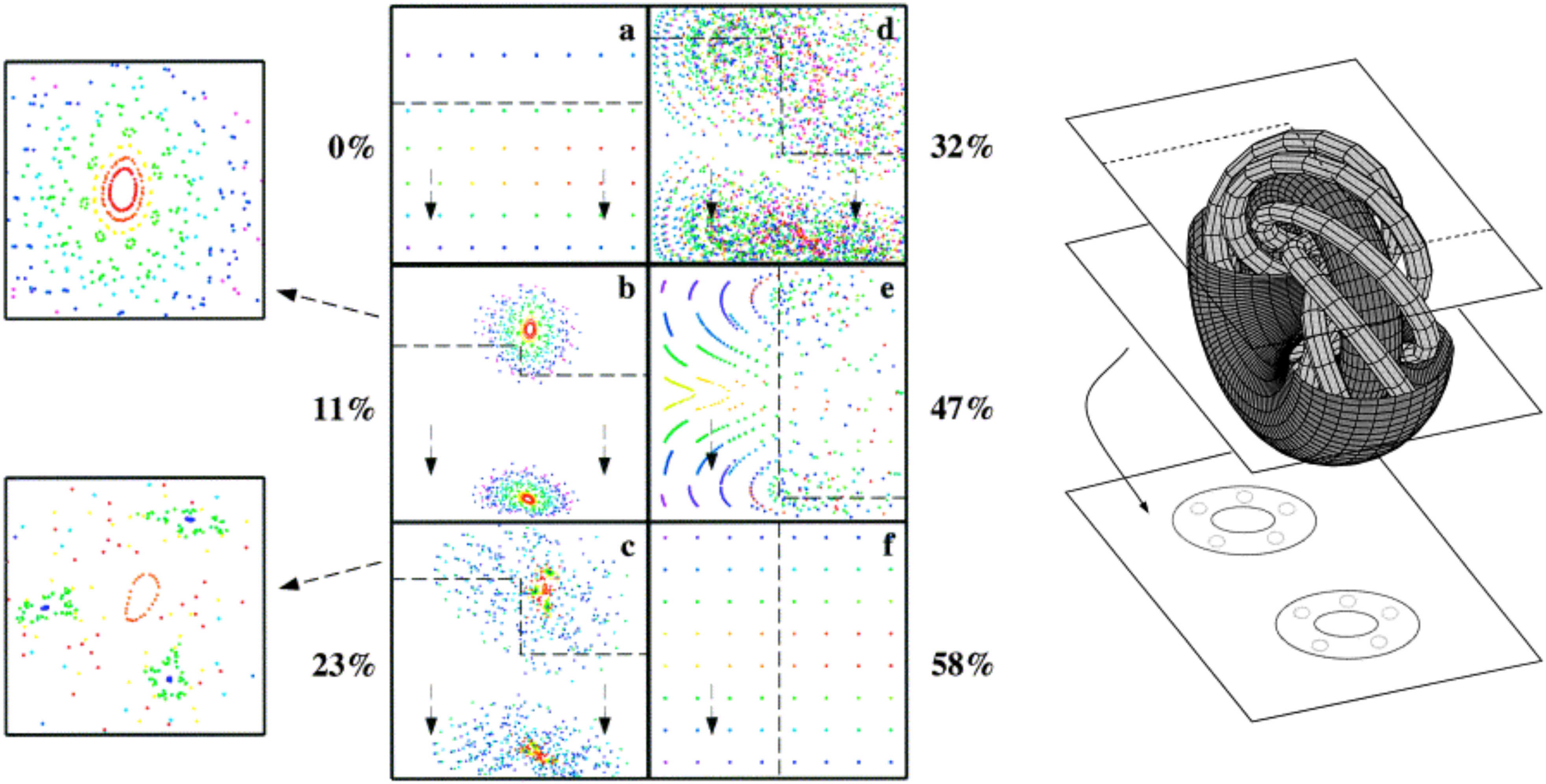




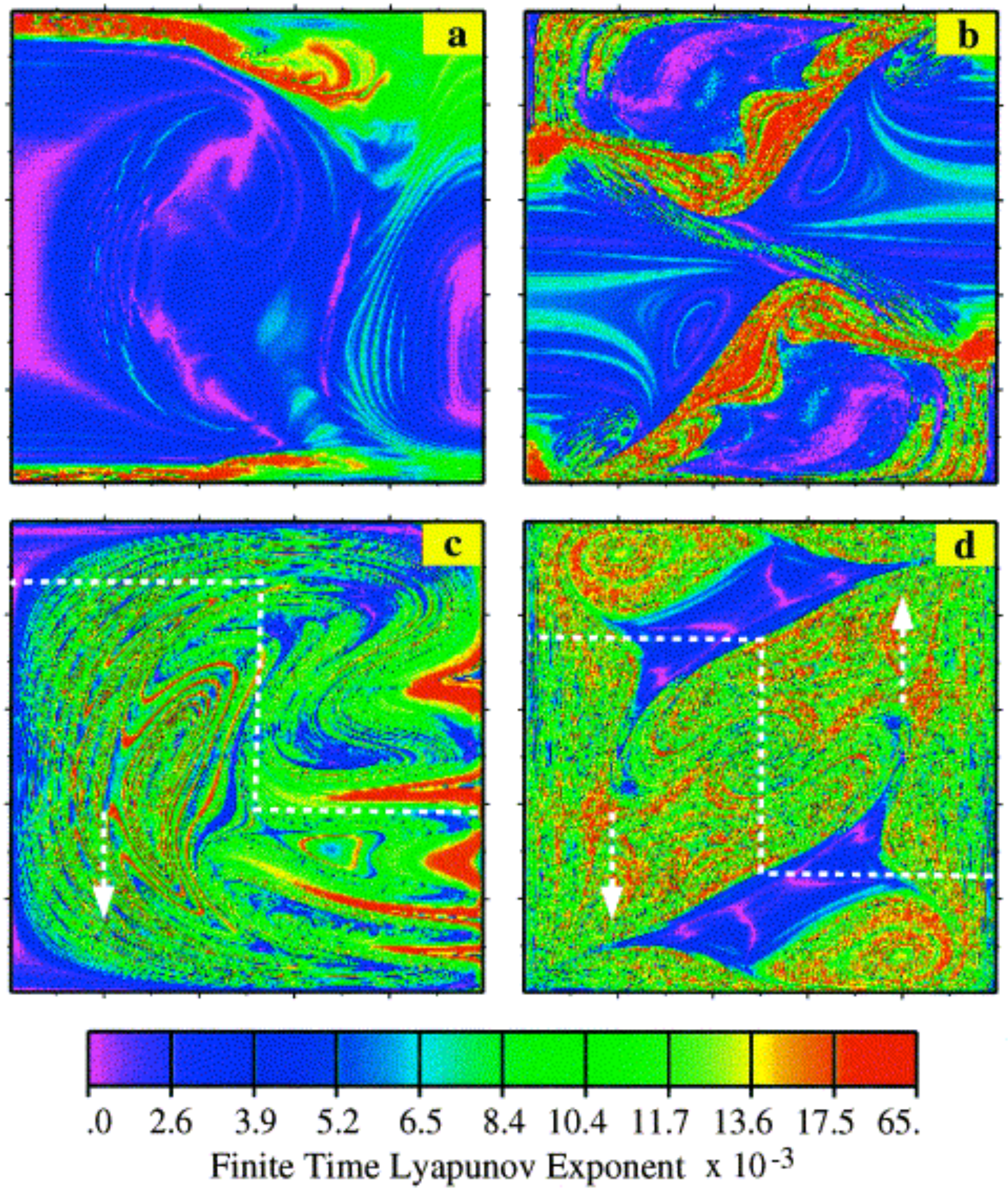
Ferrachat & Ricard, Mixing in 3-D plate driven flows

Chaotic trajectories occur even in steady-state flows





50 Poincare sections - Ferrachat & Ricard 2001



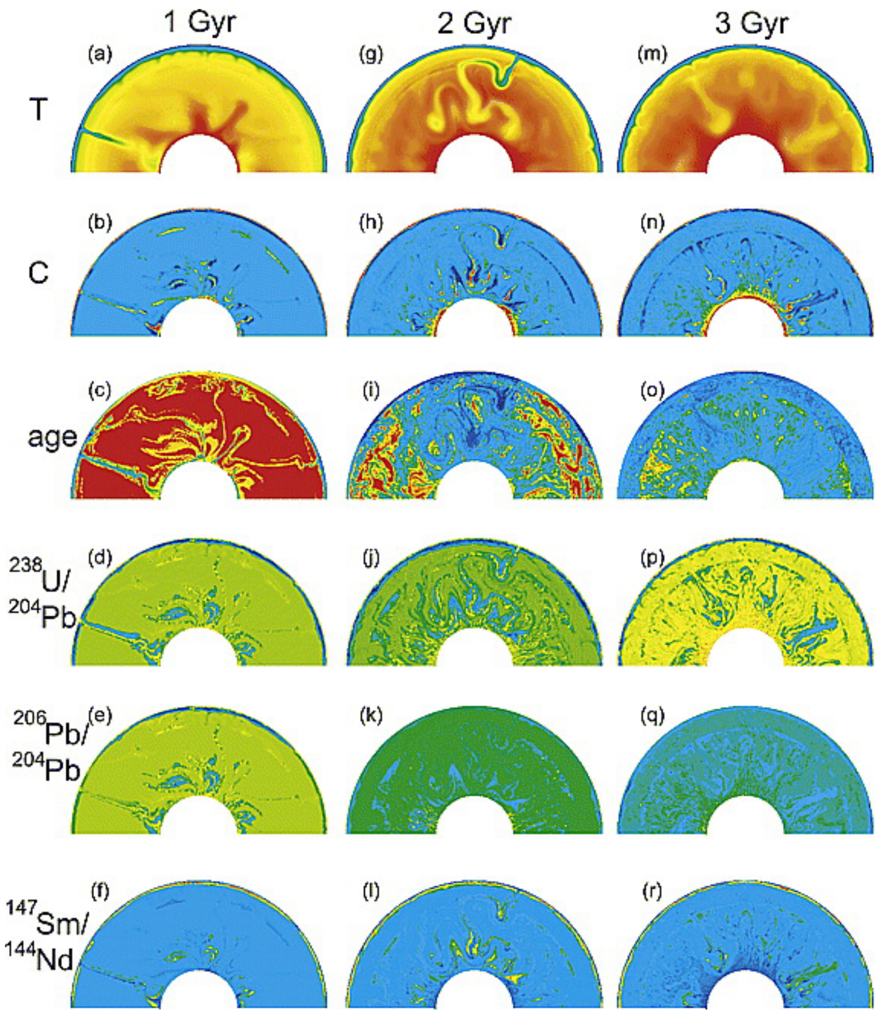
Lyapunov exponents estimated by tracking tracers:
Both chaotic and laminar mixing are observed

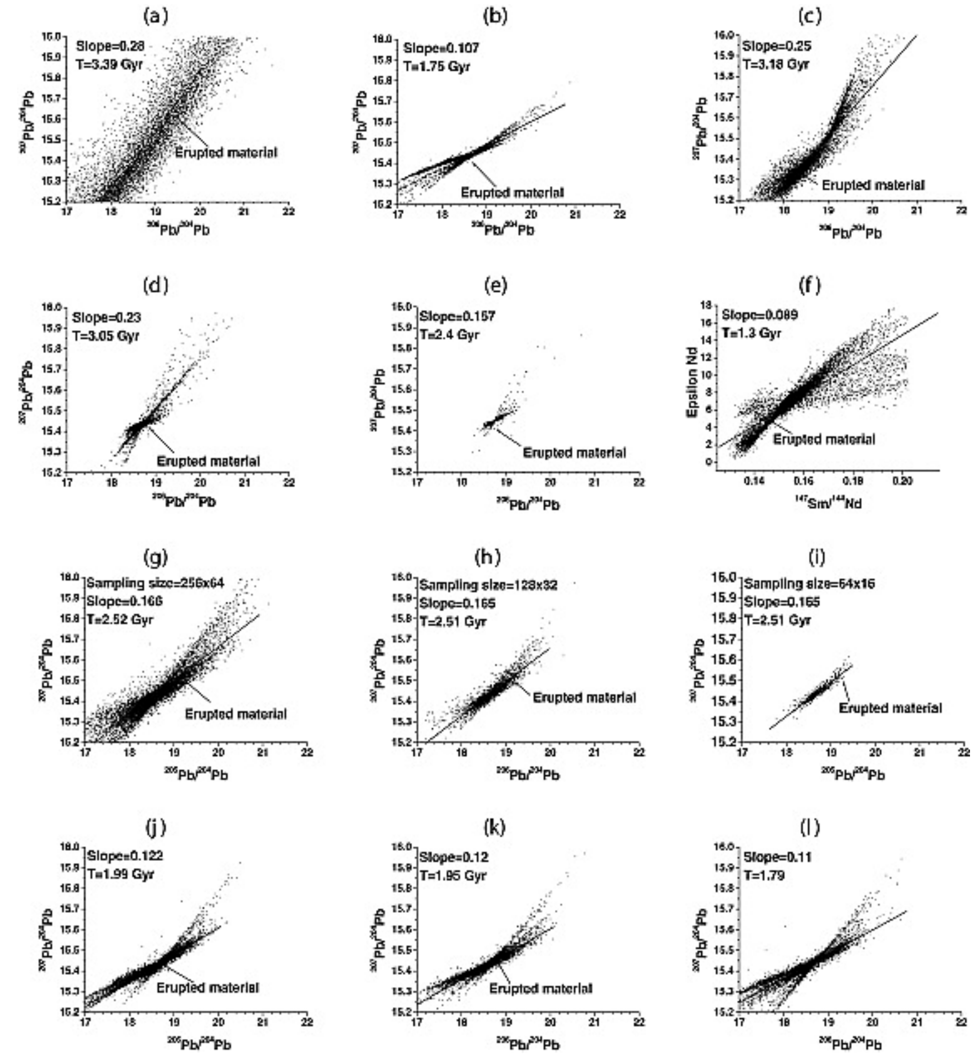
Ferrachat & Ricard 2001

Computing predicted isotopic signatures

Shunxing Xie and Paul J. Tackley 2004

Evolution of U−Pb and Sm−Nd systems in numerical models of mantle convection and plate tectonics

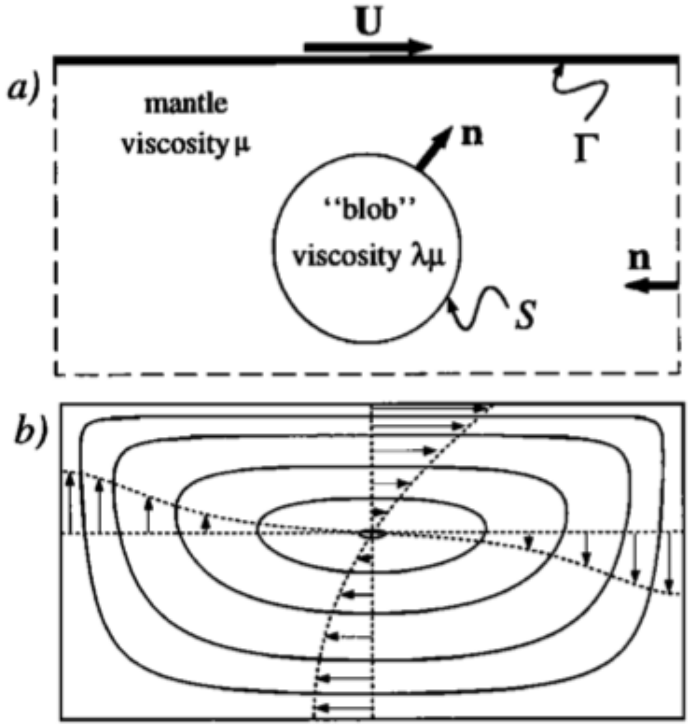




Journal of Geophysical Research: Solid Earth

Volume 109, Issue B11, B11204, 18 NOV 2004 DOI: 10.1029/2004JB003176

http://onlinelibrary.wiley.com/doi/10.1029/2004JB003176/full#jgrb14189-fig-0004

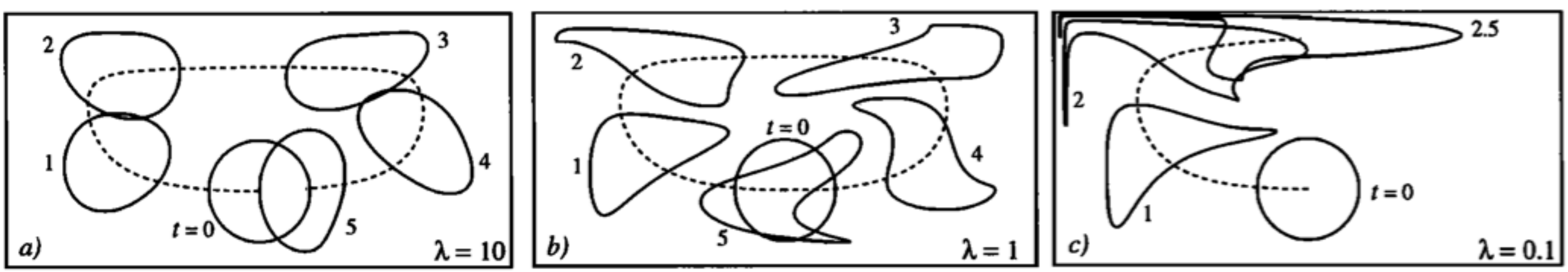


The role of viscosity contrasts

Mixing of heterogeneities in the mantle: Effect of viscosity differences

Michael Manga

GEOPHYSICAL RESEARCH LETTERS
VOL. 23, NO. 4, PAGES 403-406, FEBRUARY 15, 1996



More viscous ← Isoviscous → Less viscous

Key points about mixing

- Stirring is controlled by: stretching and folding.
  - Creates a cascade of heterogeneity
- Mixing is complicated by:
  - Type of flow (turbulent vs. laminar)
  - 2D vs. 3D
  - Scale of interest
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- Mantle processes both create (melting) and destroy (mixing) heterogeneity.