Massachusetts Institute of Technology

Rheology

Ulrich Faul

(Ian Jackson, Chris Cline, ANU)

Outline

fundamentals of defects

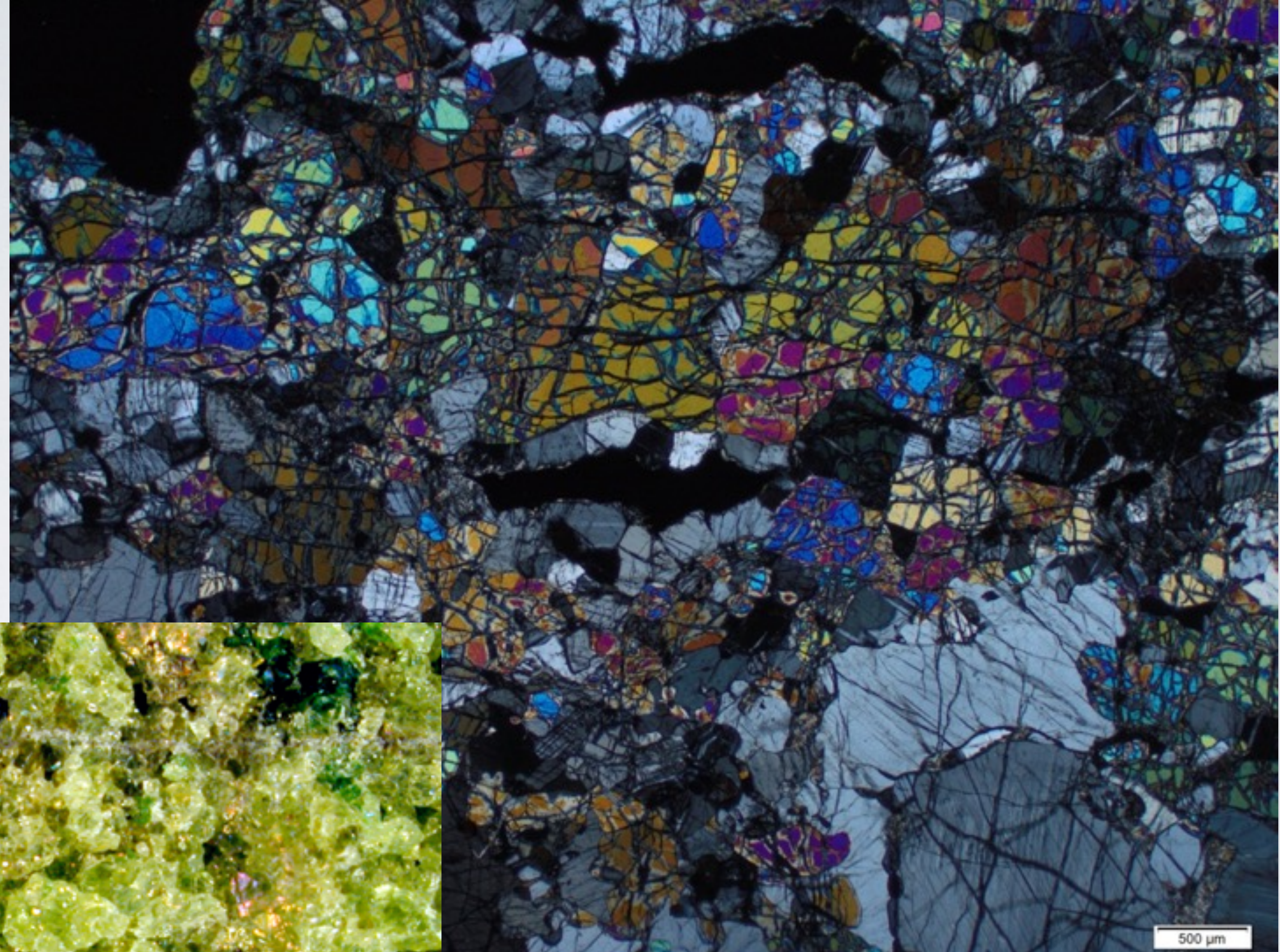
flow laws (constitutive equations)

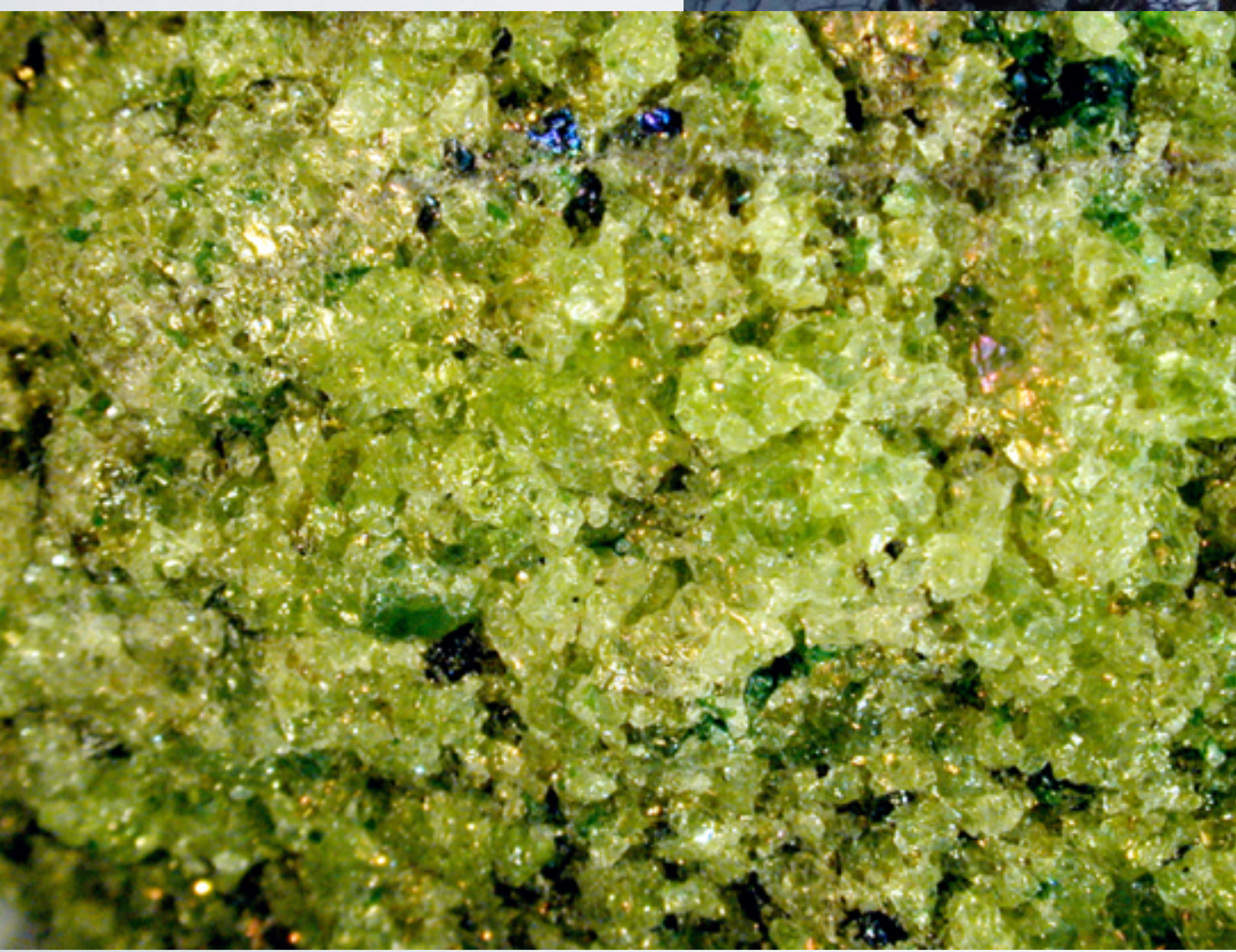
extrinsic defects: water

seismic properties

continuum of relaxation times

cross-polarized light image of a peridotite



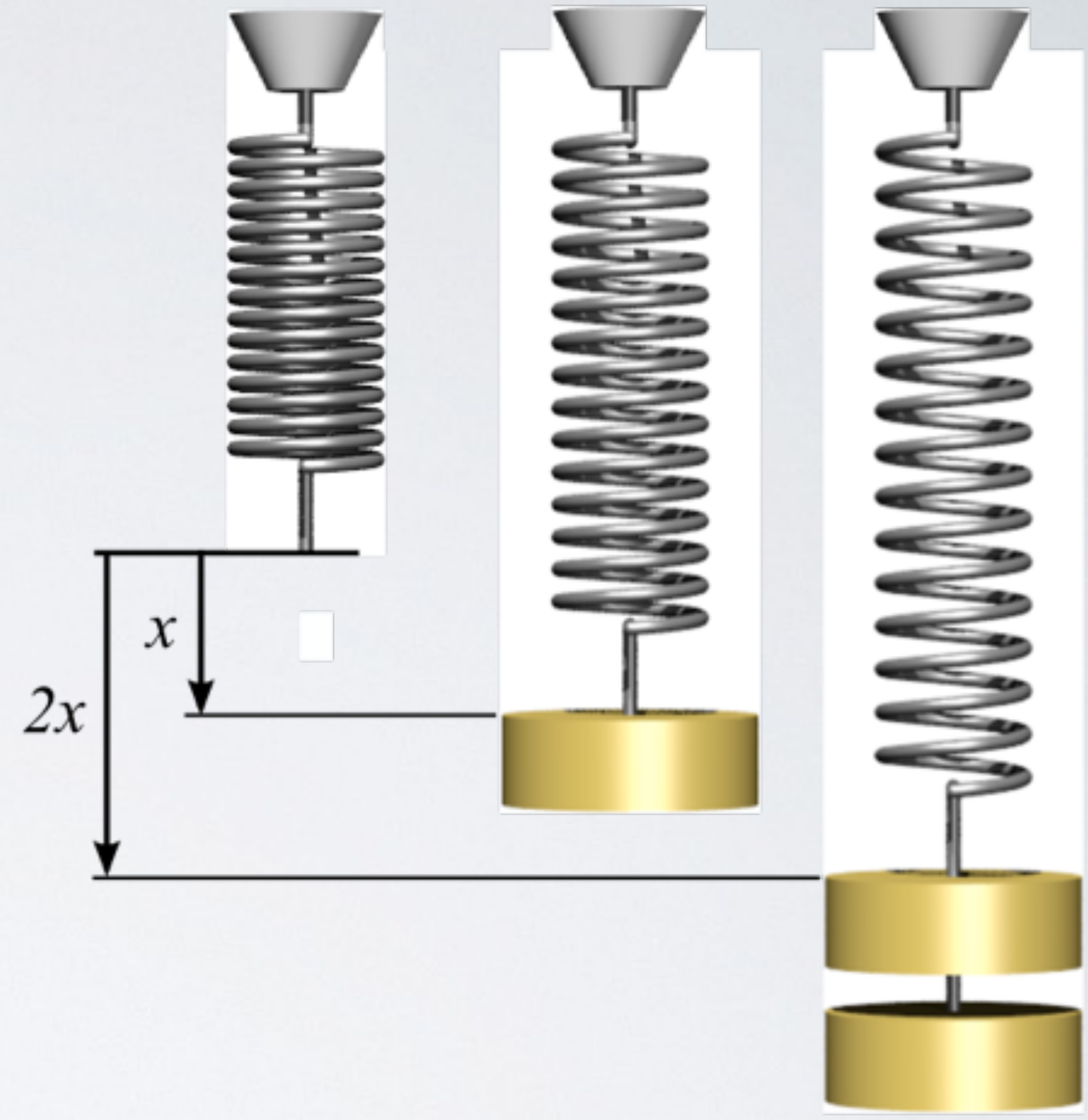


peridotite xenolith

Elastic behavior

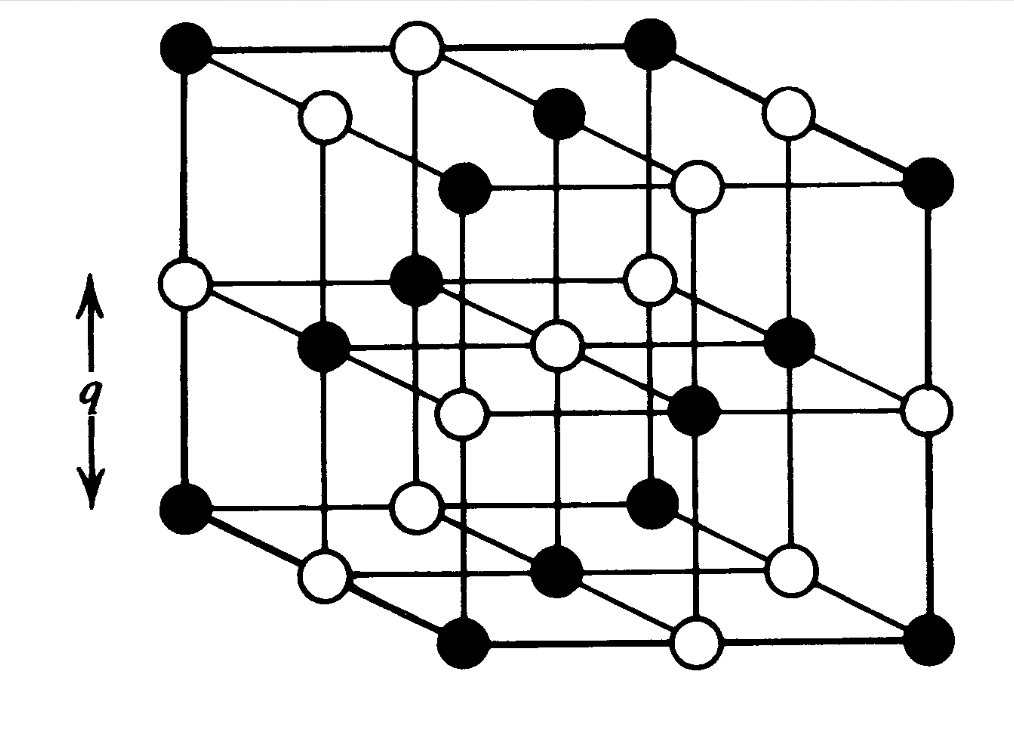
Deformation:
Force = spring const. x dist.

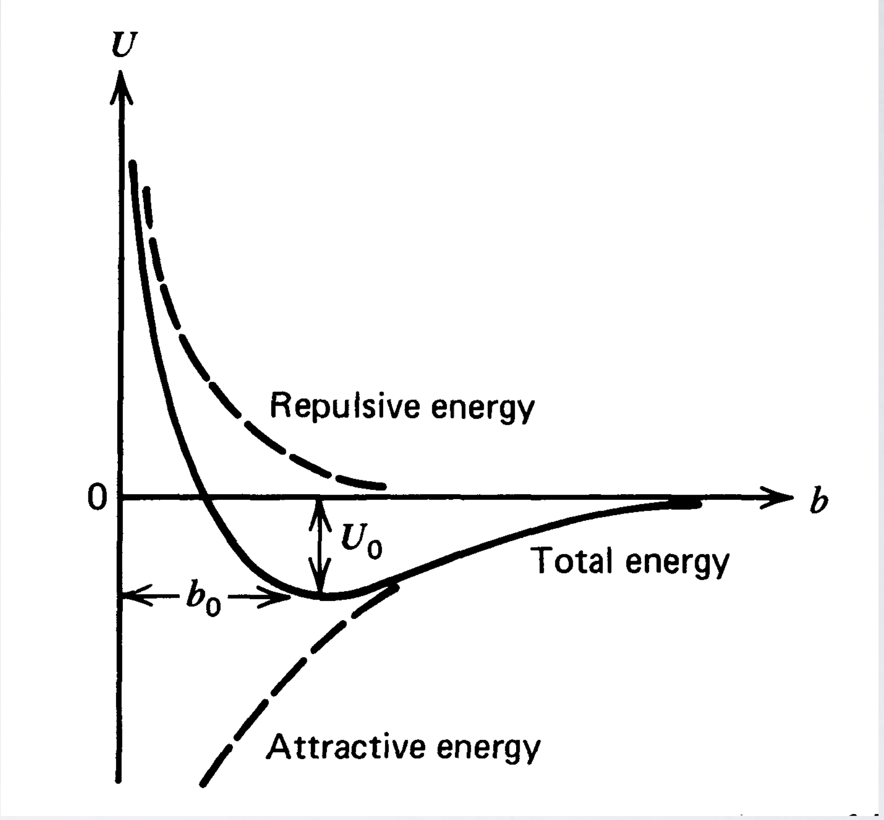
F = k x
(Hooke's law)



strain: instantaneous - recoverable

Elastic behavior: Solids





Unique equilibrium position of atoms in crystal lattice
displacement from that position requires force: elastic moduli
e.g. $\sigma = E\,\varepsilon$
E Young's modulus (tensile deformation, linear strain)
K bulk modulus (uniform compression)
G shear modulus (rigidity, shear deformation)
Elastic moduli are of orders 10s of GPa

Deformation:

stress = modulus x strain

$\sigma = G\,\varepsilon$

Ice:

$G \sim 4$ GPa = 4×10^{9}Pa,

$\varepsilon \sim 1$

stress ~ modulus

driving flow of ice: gravity

Pressure (stress) = density x g x thickness

= 1000 kg/m^3 x 10 m/s^2 x 1000 m = 10^7 Pa

moduli of rock-forming minerals of order of 100 GPa
convective stresses ~ 1 - 0.1 MPa

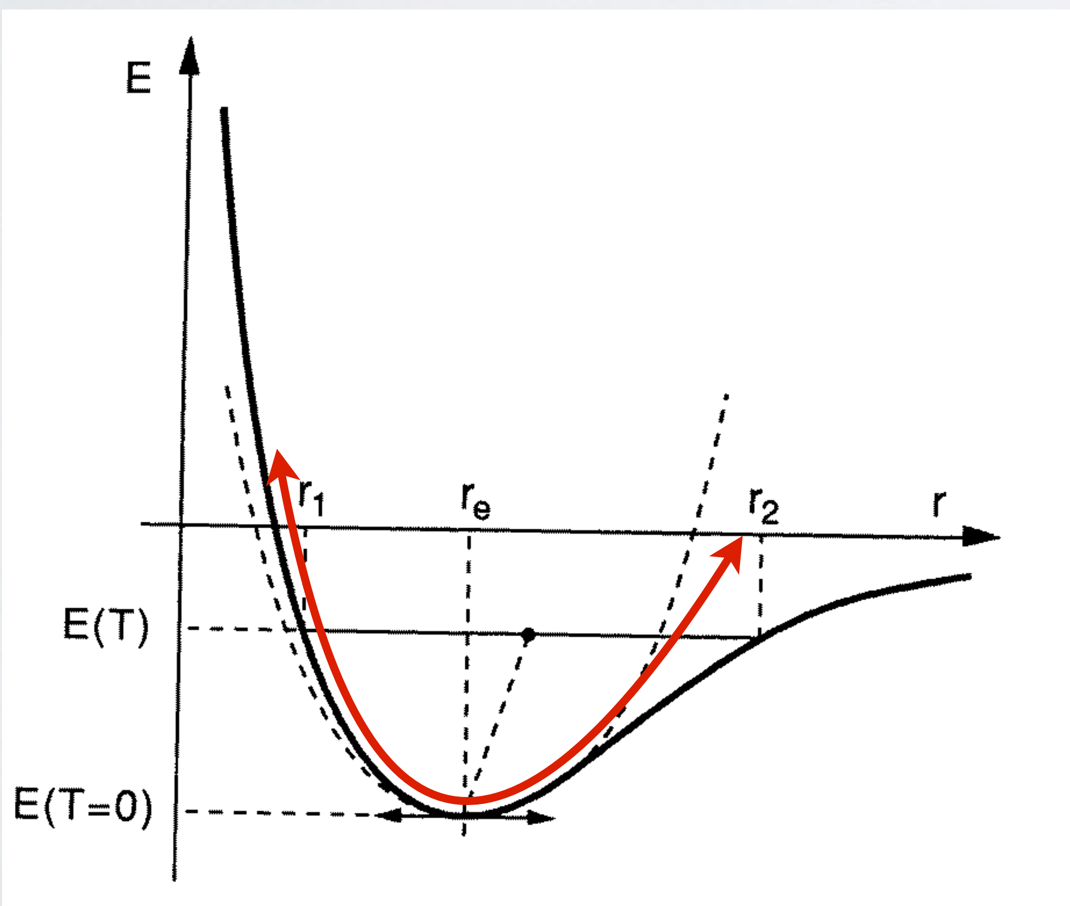
need to modify perfect elastic moduli: defects!

Why do we have defects?

Thermodynamics *(Fundamental state functions)*

Internal energy U: Energy content of a system, the sum of the potential energy stored in interatomic bonding (electrostatic energy) plus the kinetic energy of atomic vibrations.

Temperature



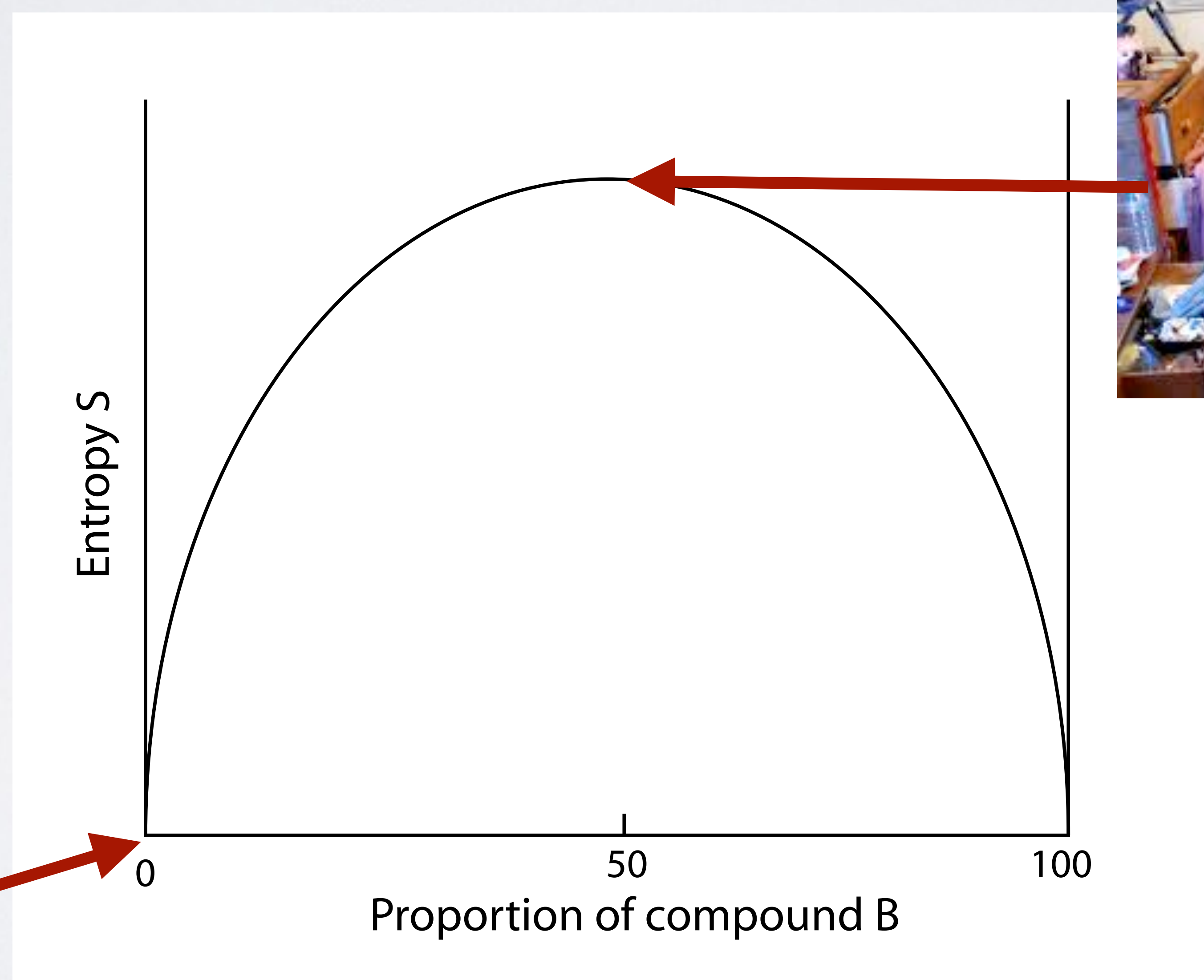
In the Earth need to account for pressure:

Enthalpy: $H = U + PV$

Why do we have defects?

Entropy S: measure of the state of disorder in a system.

Example: Configurational entropy:
Entropy is at a maximum for XB = XA



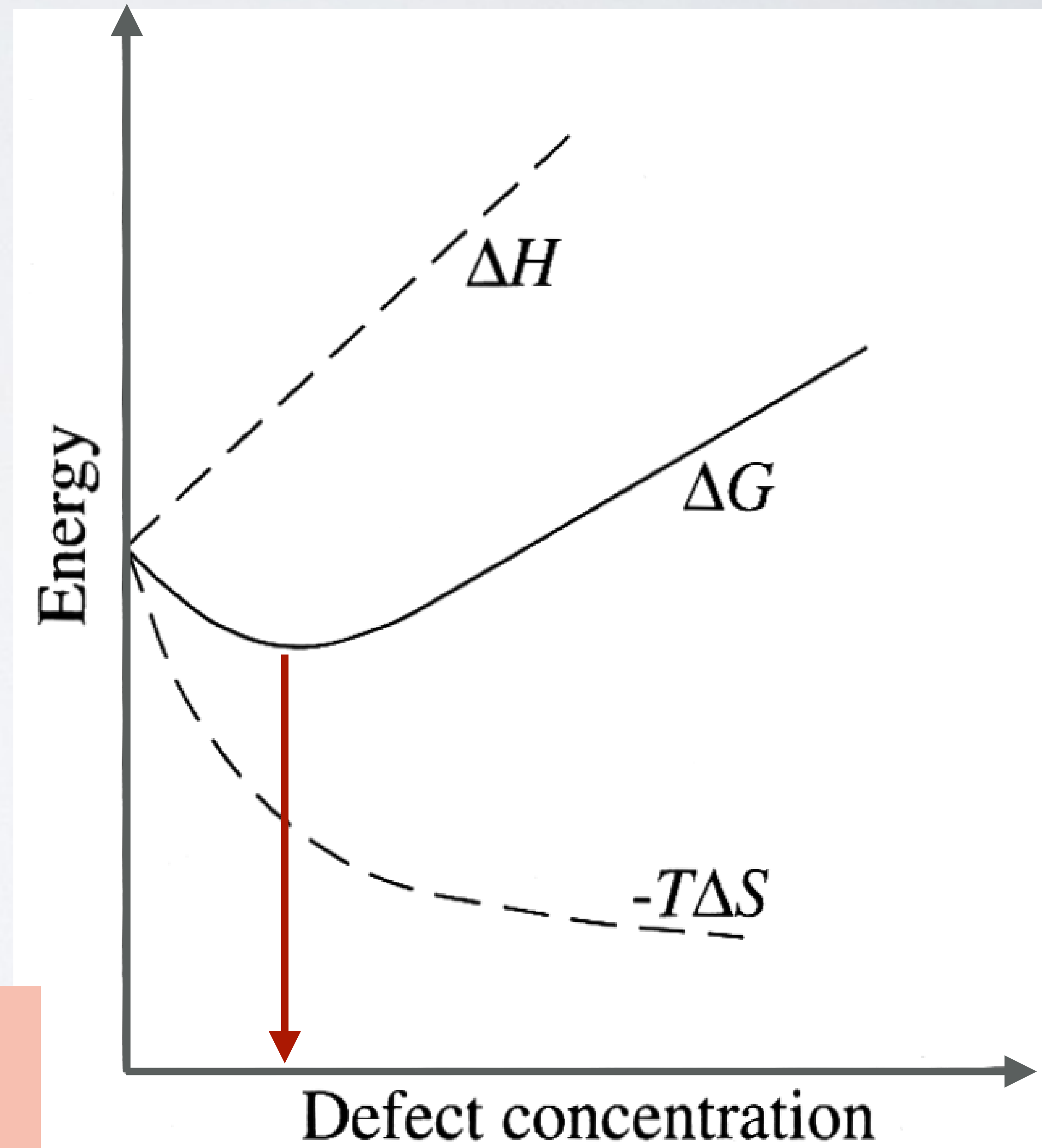
Thermodynamically why do we have defects?

Creation of point defect requires energy:
local distortion of lattice + imperfect satisfaction of bonding
☛ enthalpy H increases.

But: point defect increases disorder in an otherwise perfect crystal:
entropy S increases.

Gibbs free energy: G = H -TS

For small defect concentrations entropy increase is *greater* than enthalpy increase, for larger defect concentrations enthalpy increase dominates:

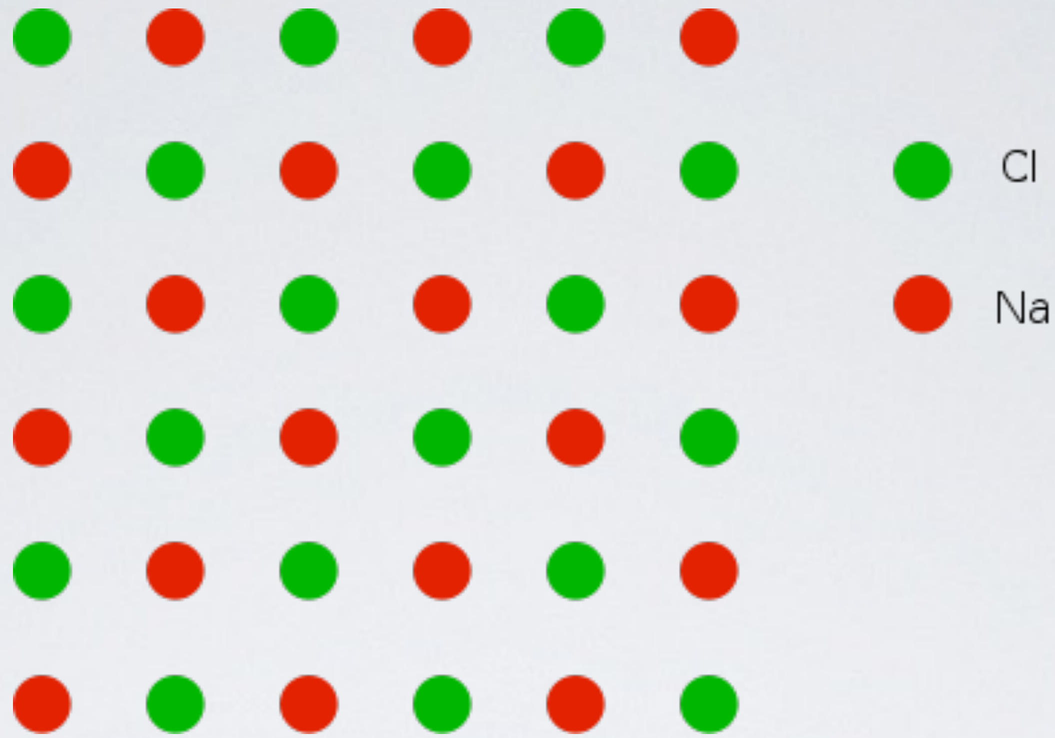


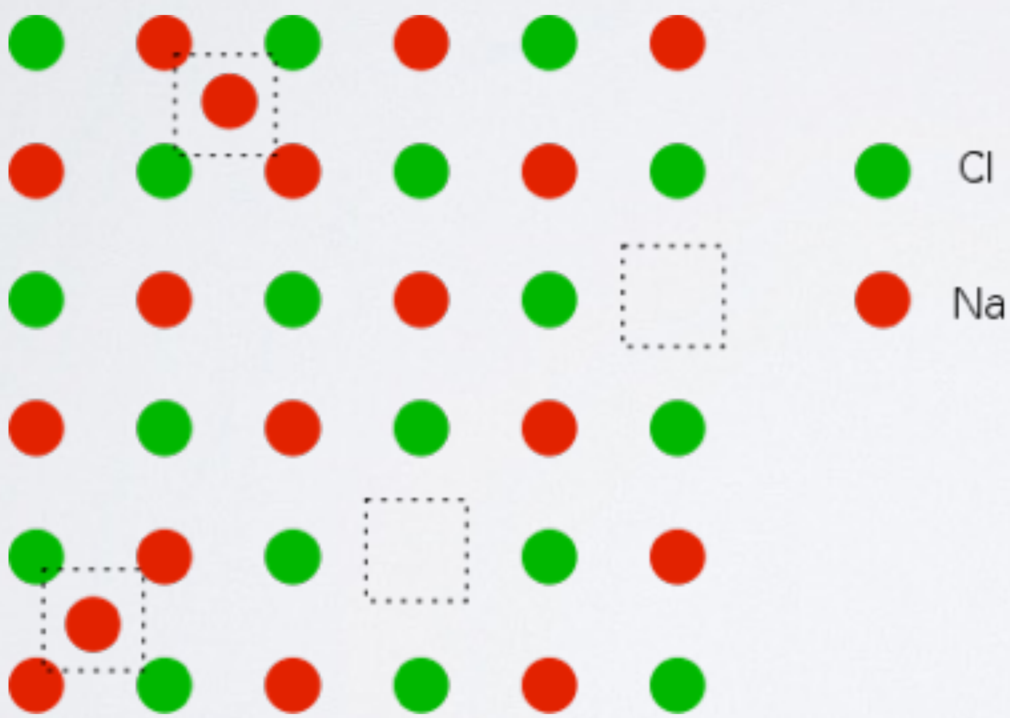
Minimum of Gibbs free energy at some finite concentration of point defects!

Defects!

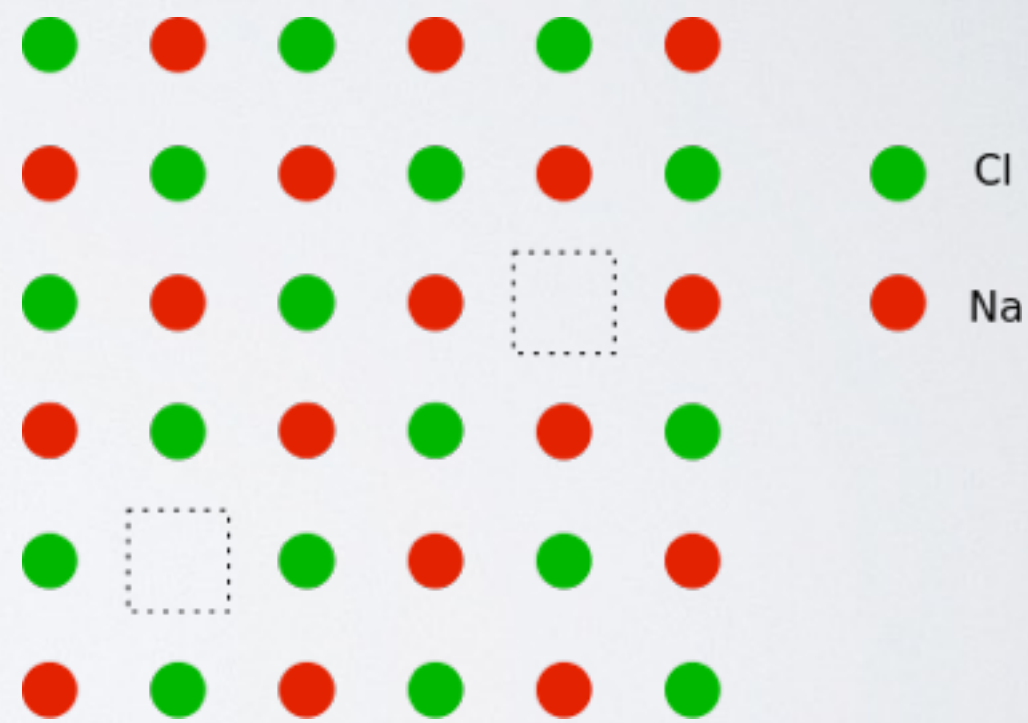
(point defects)

intrinsic defects





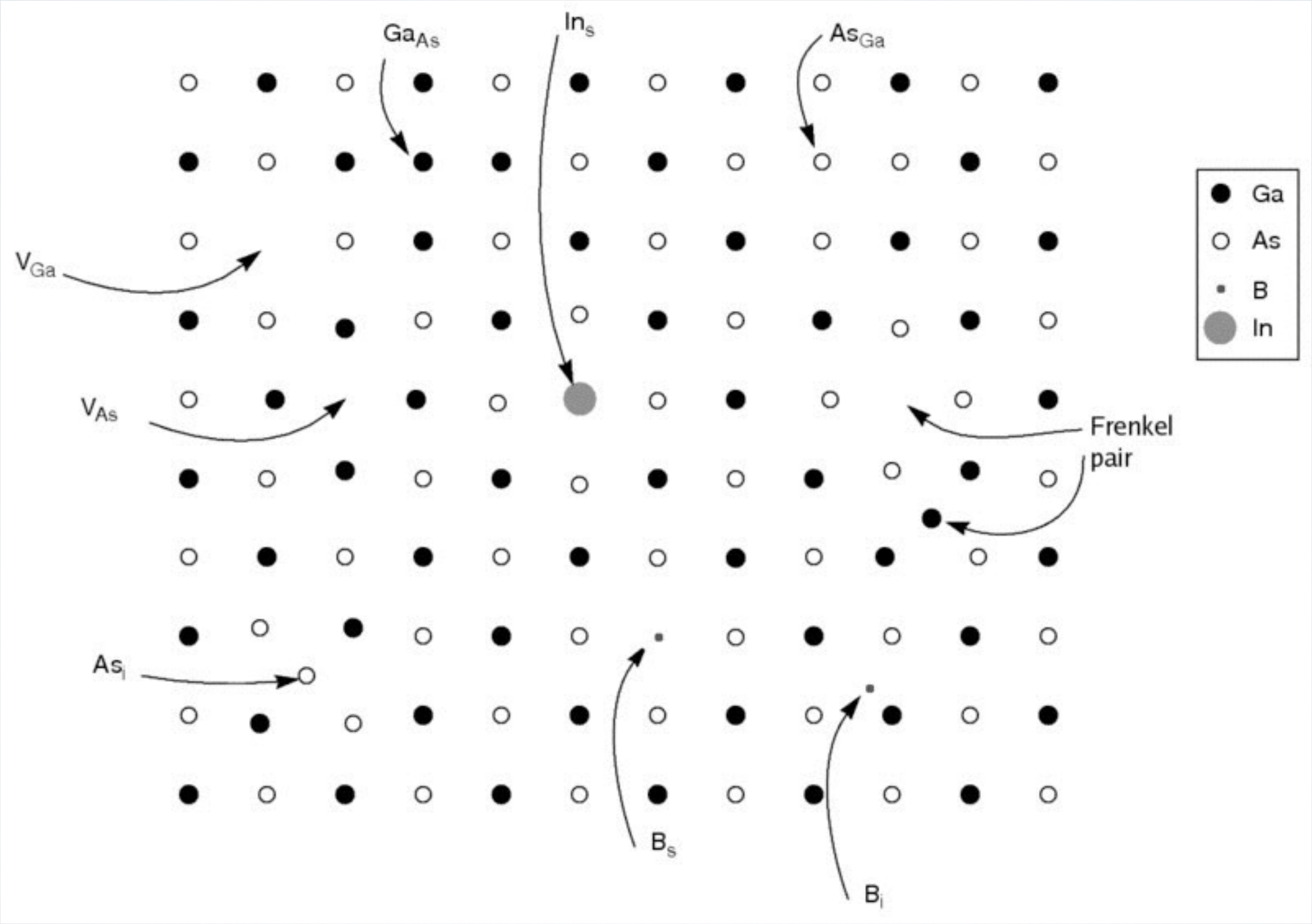
Frenkel



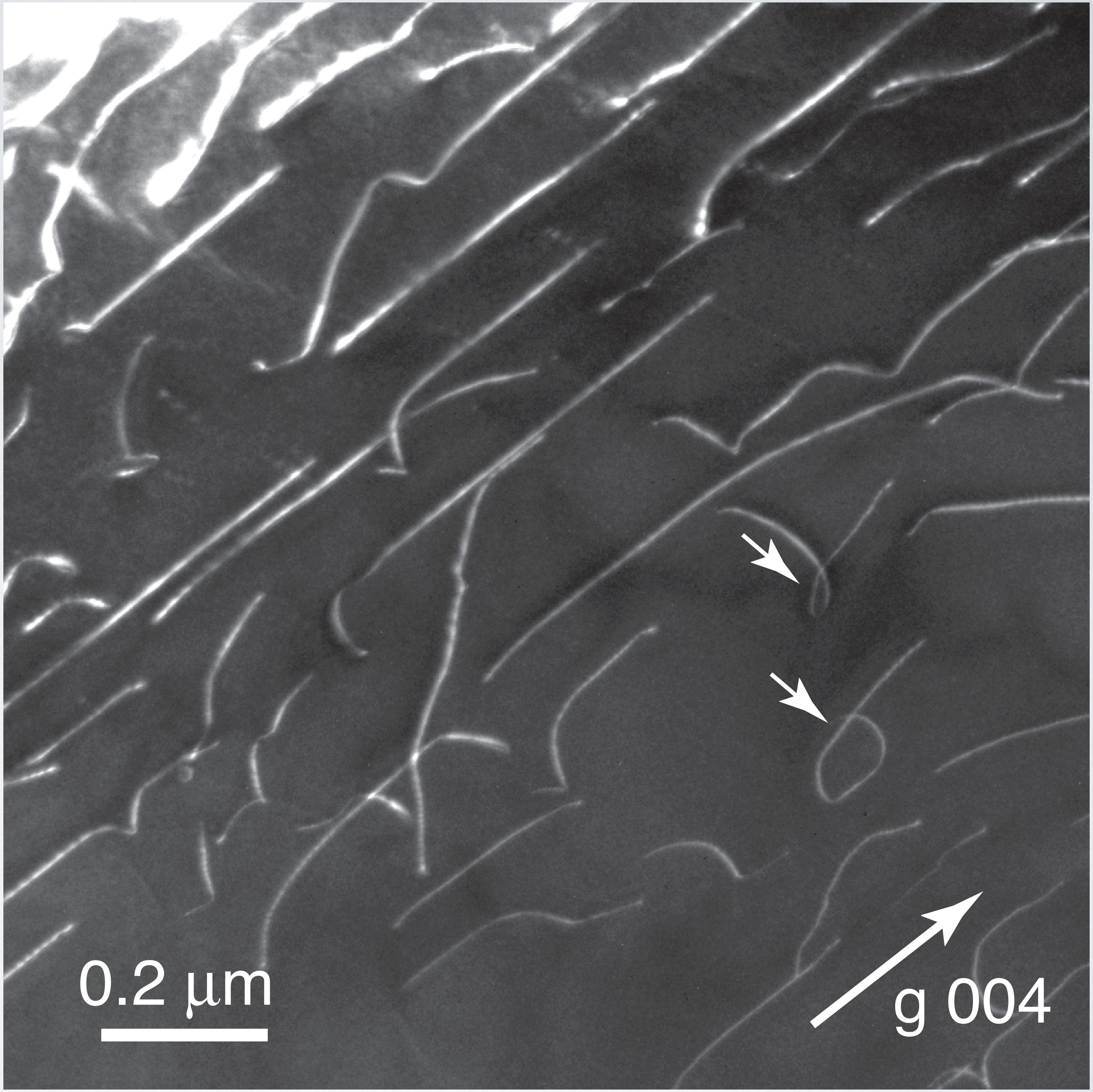
Schottky

charge balance needs to be maintained

defects can also be impurity atoms: extrinsic defects!

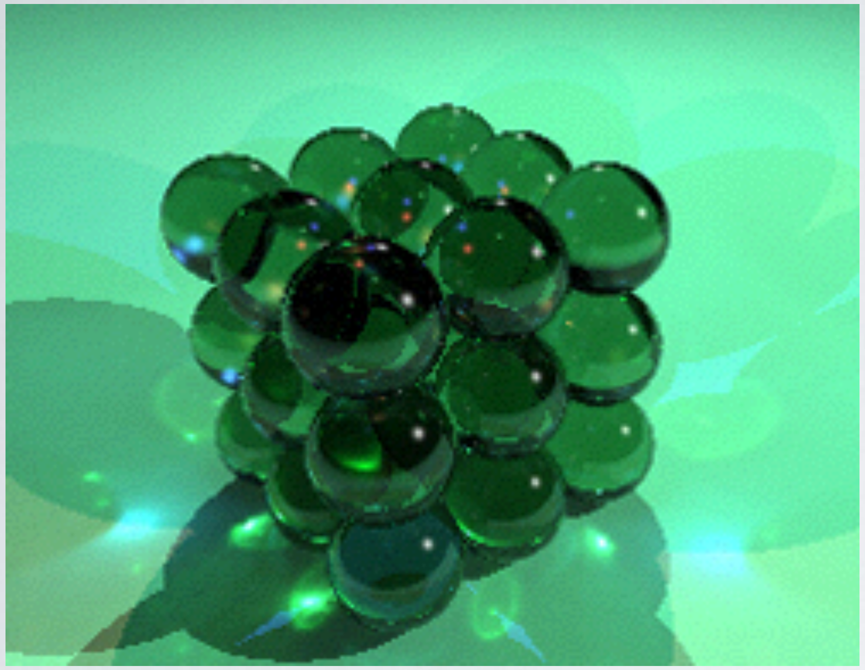


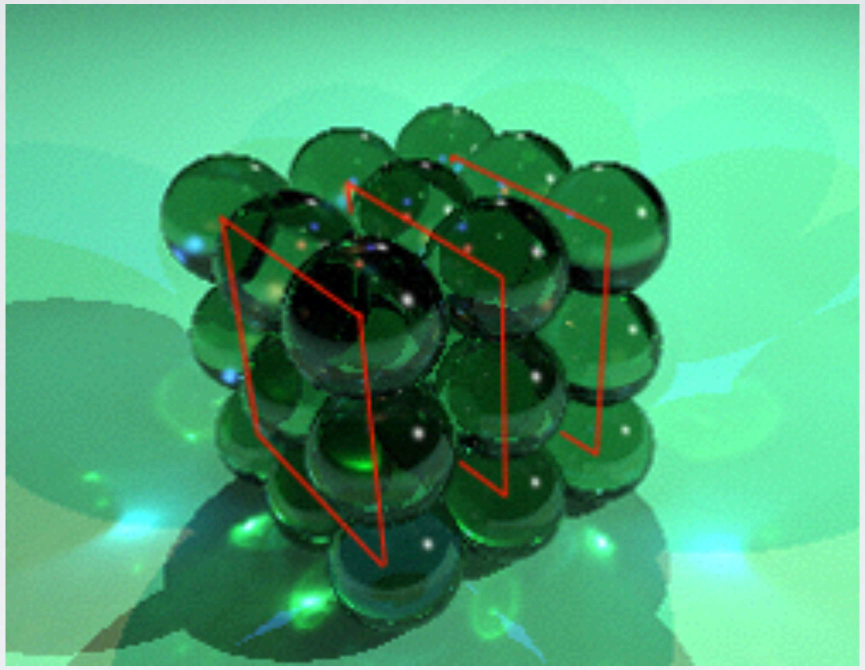
Dislocations
-line defects

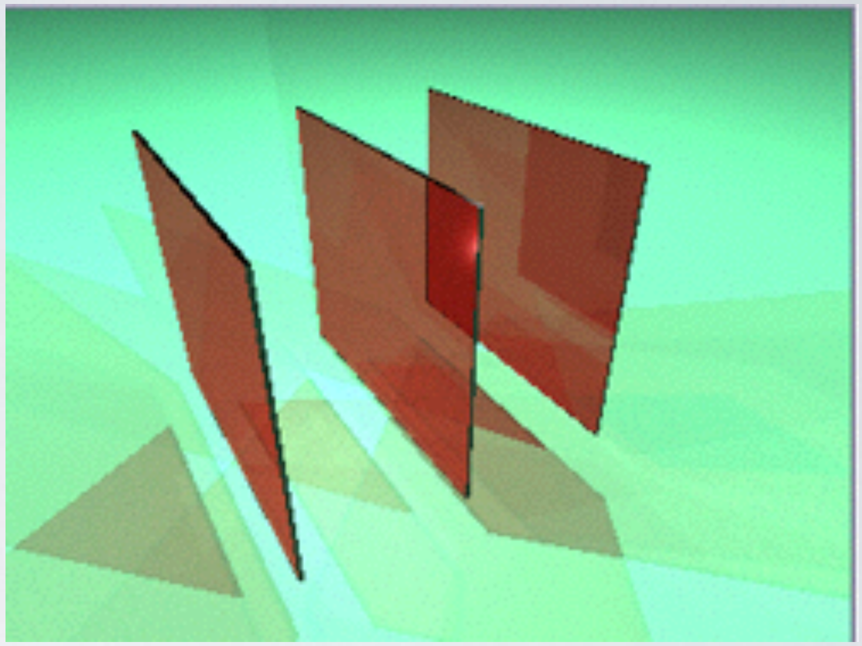


Transmission electron microscope image of screw dislocations in olivine

What is a dislocation?

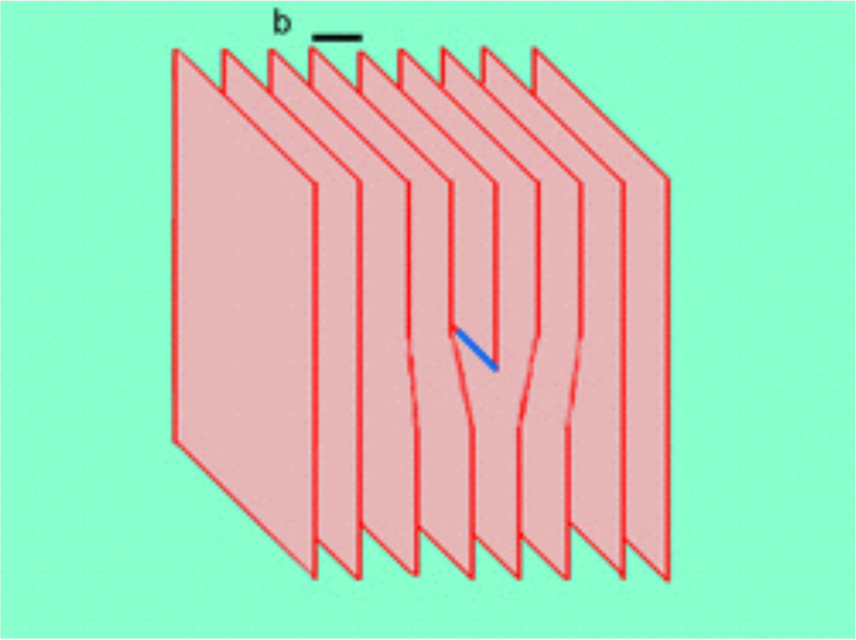






Lattice planes in a cubic crystal

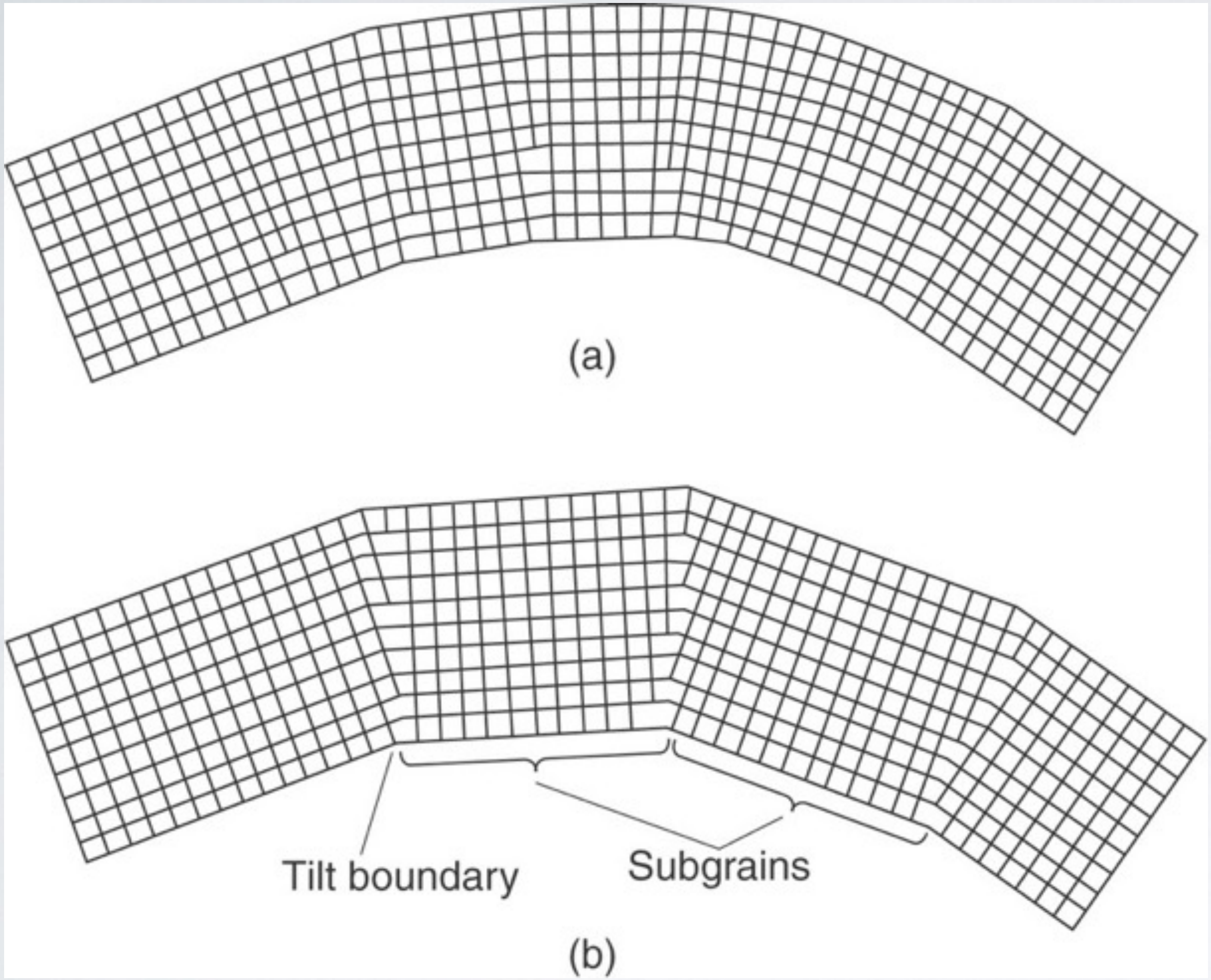
Insert an extra half plane

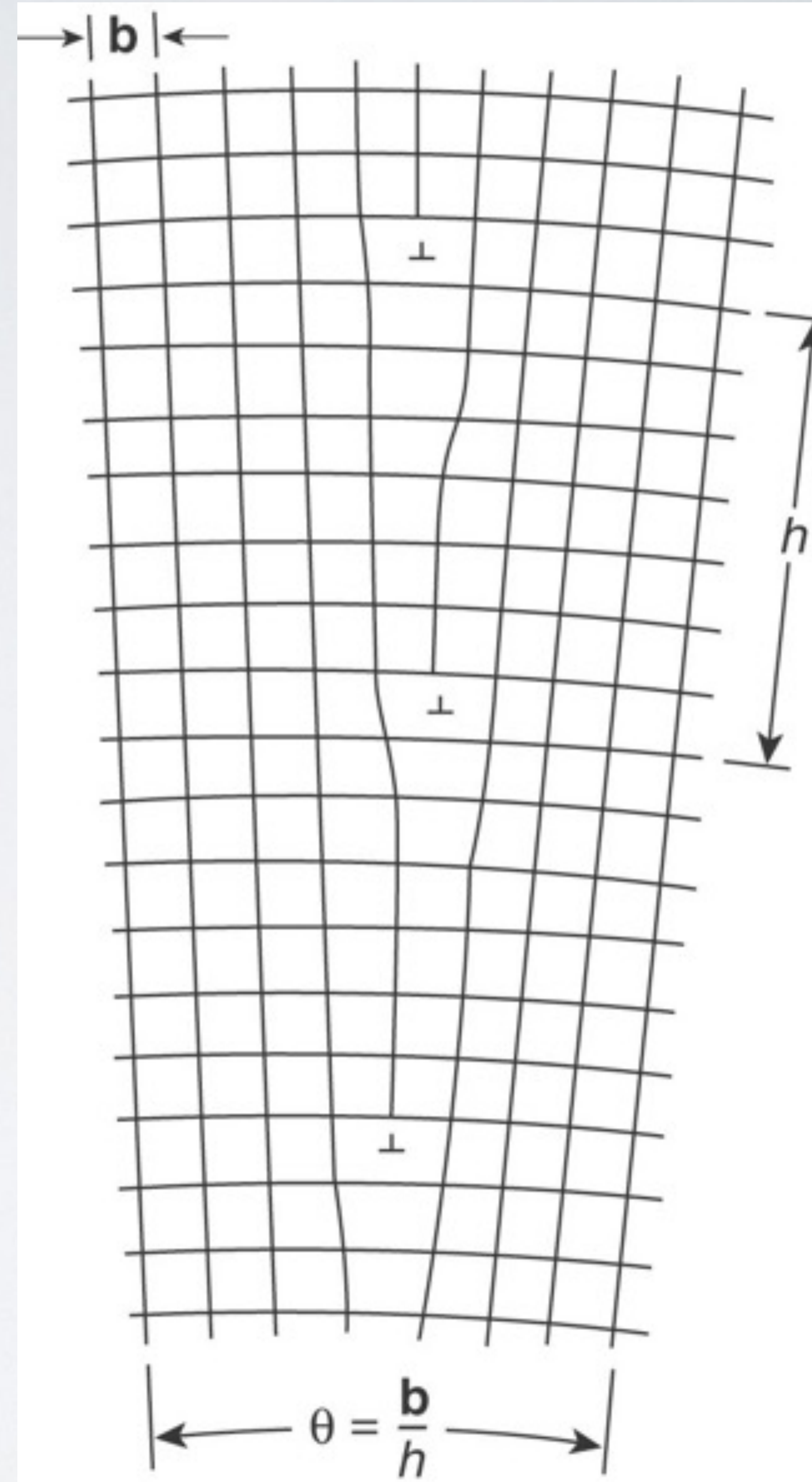


Edge dislocation

From dislocations to grain boundaries

Arrays of dislocations: subgrain boundaries



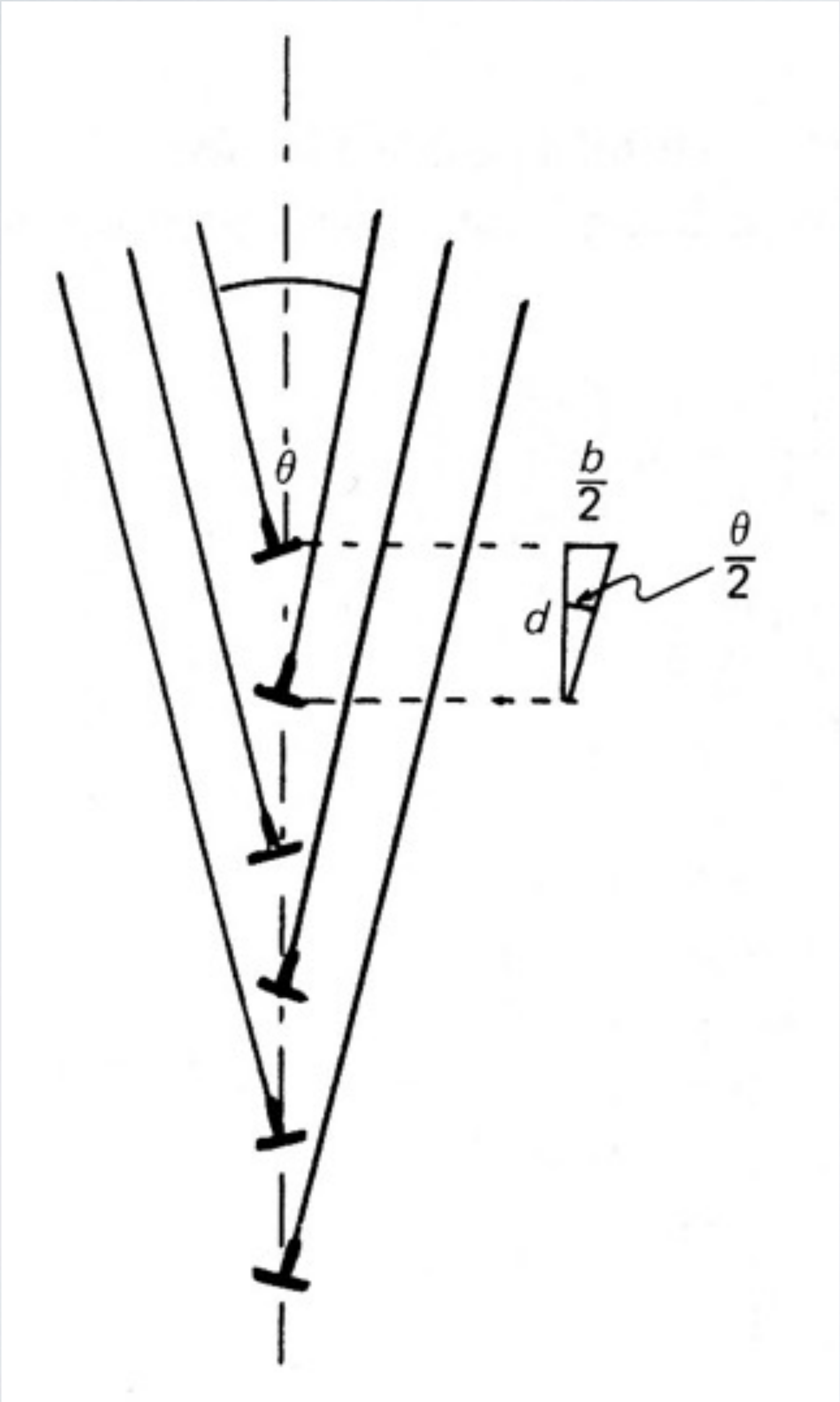


grain is defined by having a misorientation to the neighbor > 10°

subgrain boundaries in quartz

undulose extinction in quartz

Grain boundaries



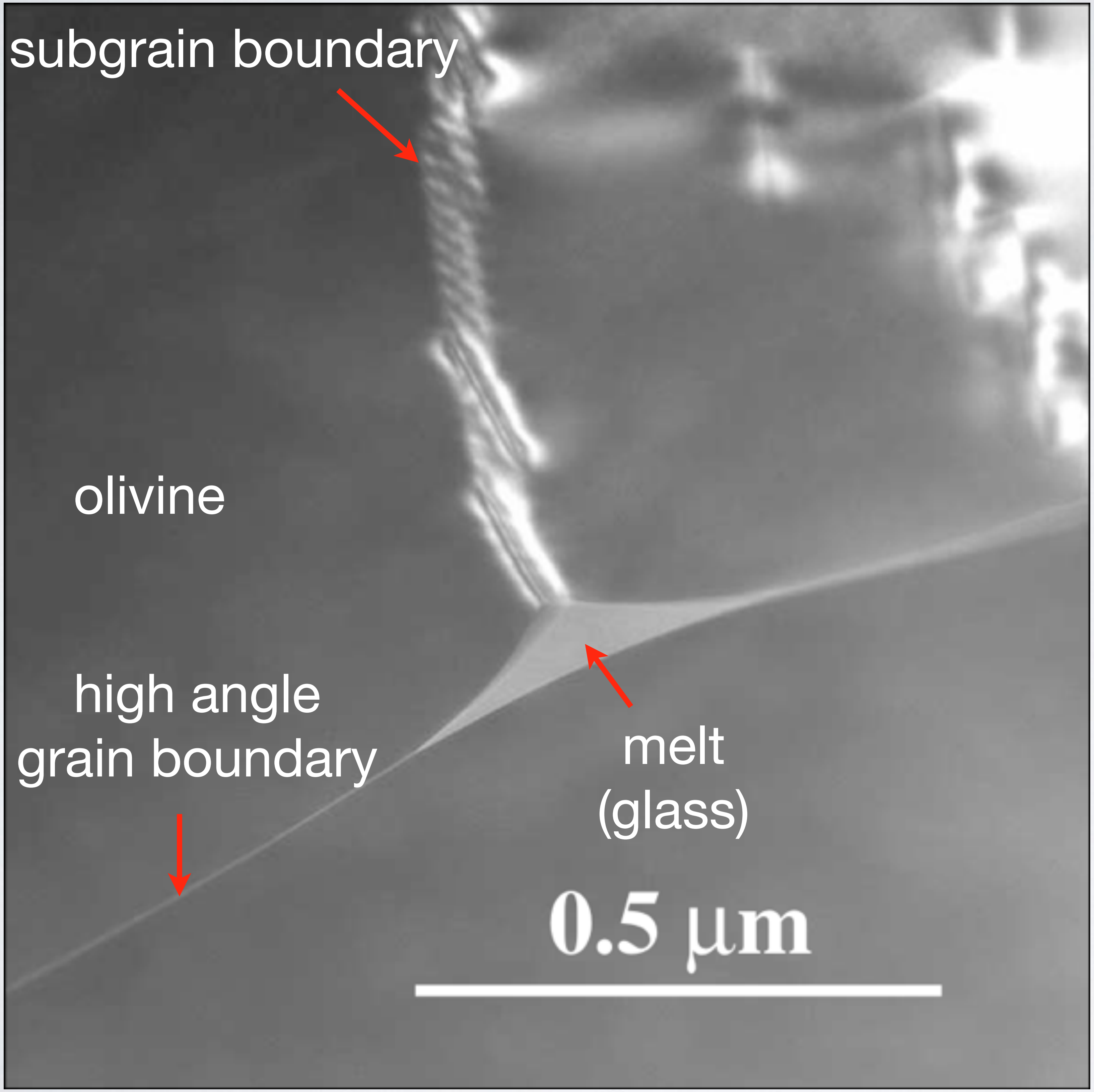
Why look at grain boundaries?

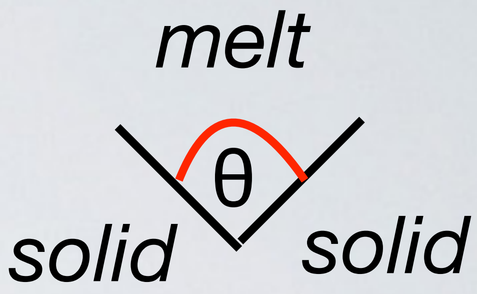
1. GB as crystalline defect present in all rocks.
2. GB affect
- diffusion
- deformation
- seismic properties
- electrical conductivity....

GB: 2-D lattice defect that introduces a change in lattice orientation

Lattice misorientation given by θ

Visualization of types of grain boundaries



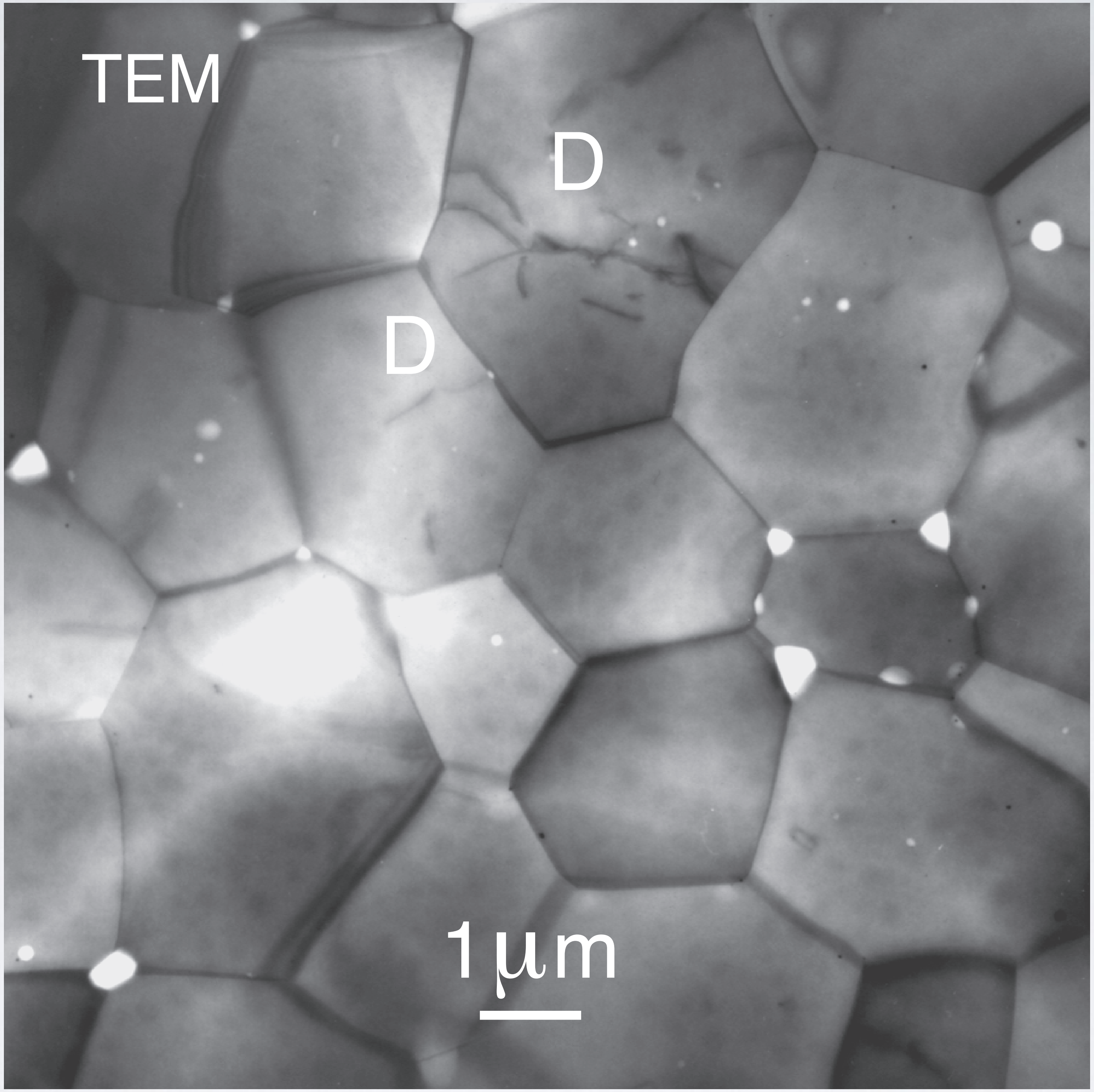


$\cos \theta/2 = \gamma_{ss}/2\gamma_{sl}$

γ_{ss} = solid - solid surface energy

γ_{sl} = solid - liquid surface energy

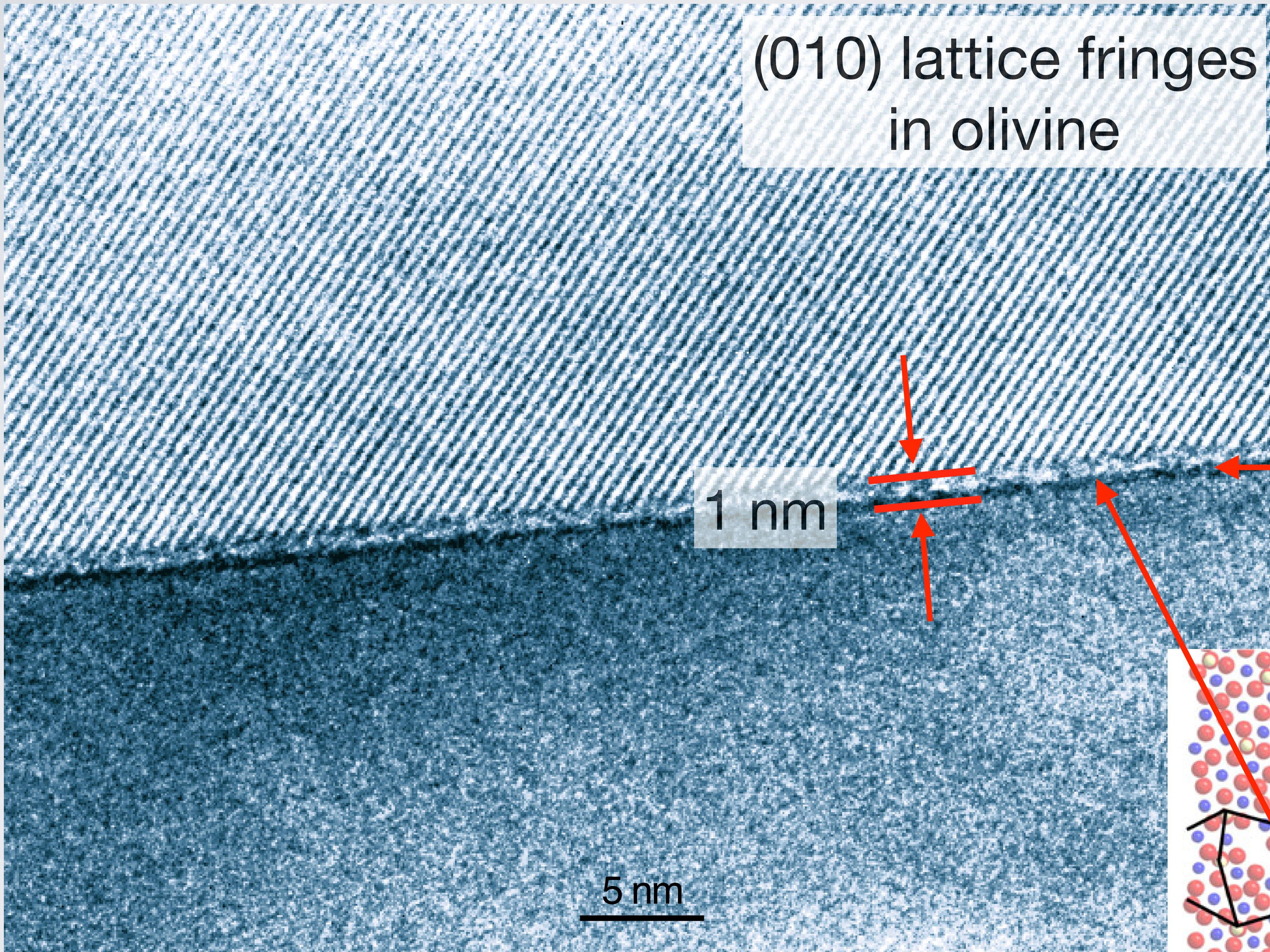
Grain boundaries: melt-free polycrystalline olivine



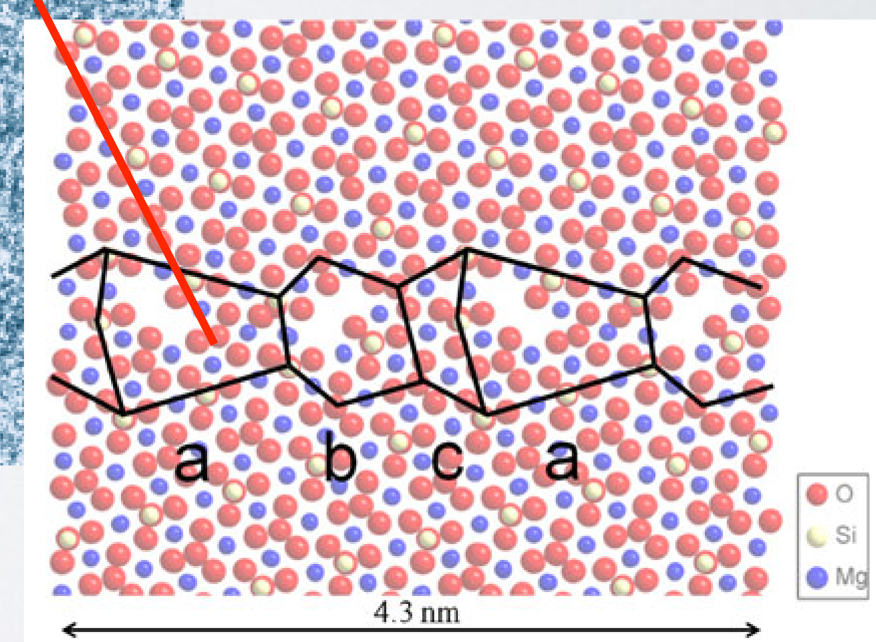
Jackson et al., 2002

defects: grain boundaries, dislocations

High resolution image of olivine grain boundaries



Faul et al., 2004



Adjaoud et al., 2012

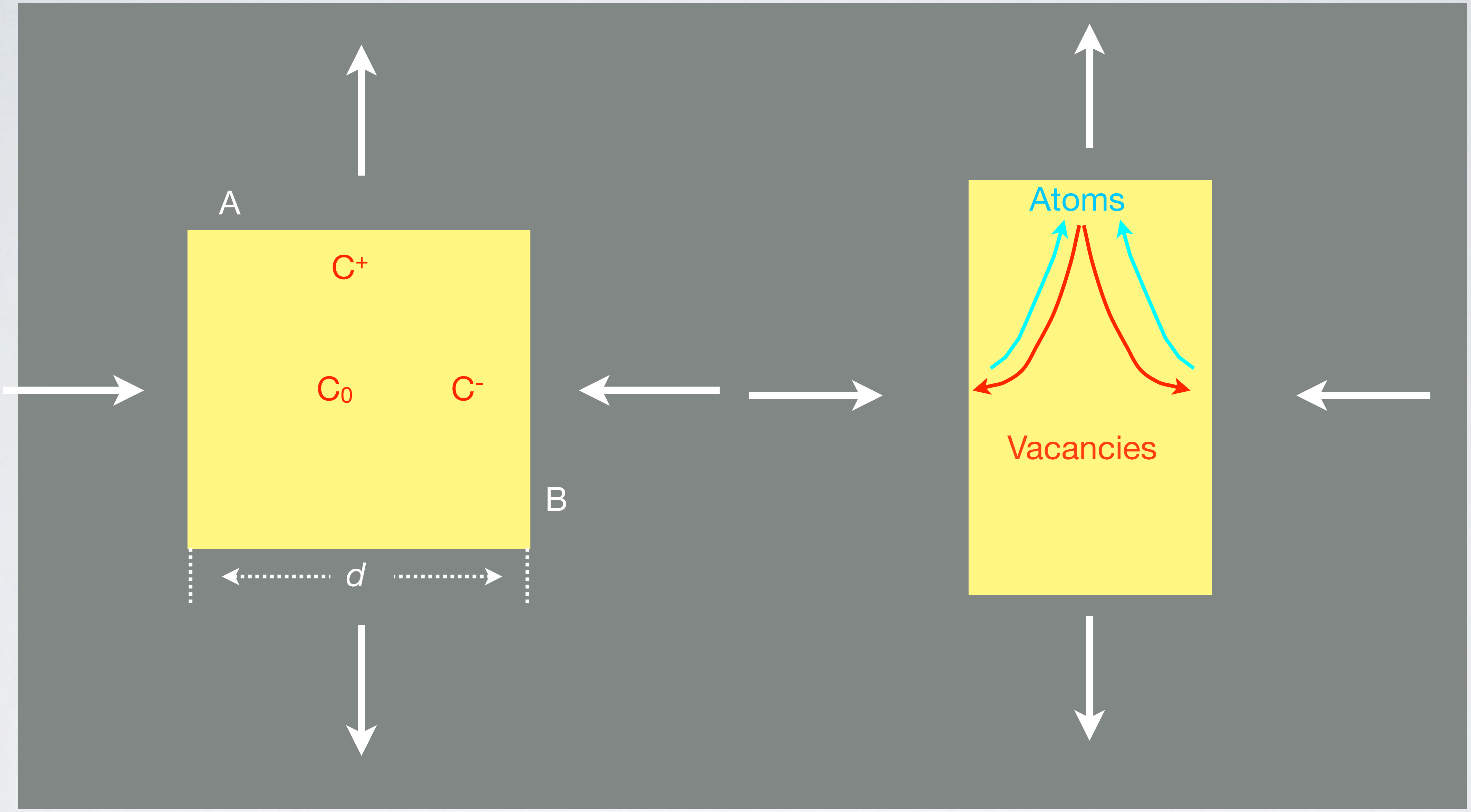
Deformation - flow laws

‚first principles‘ derivations - physical model

constitutive equations relating strain rate to stress

Poirier, Creep of Crystals, 1985

Application of a differential stress:
Pure shear deformation of a single crystal (Nabarro- Herring creep)



Compressive stress at face B reduces the number of vacancies.
Tensile stress at face A increases the number of vacancies.

differential stress -> concentration gradients -> diffusion

concentration gradient face A - face B: length scale (grain size d)

diffusion ~ diffusivity (atomic species, crystal structure), **temperature**

flow law (constitutive equation, strain rate $\dot{\varepsilon}$ as a function of stress σ):

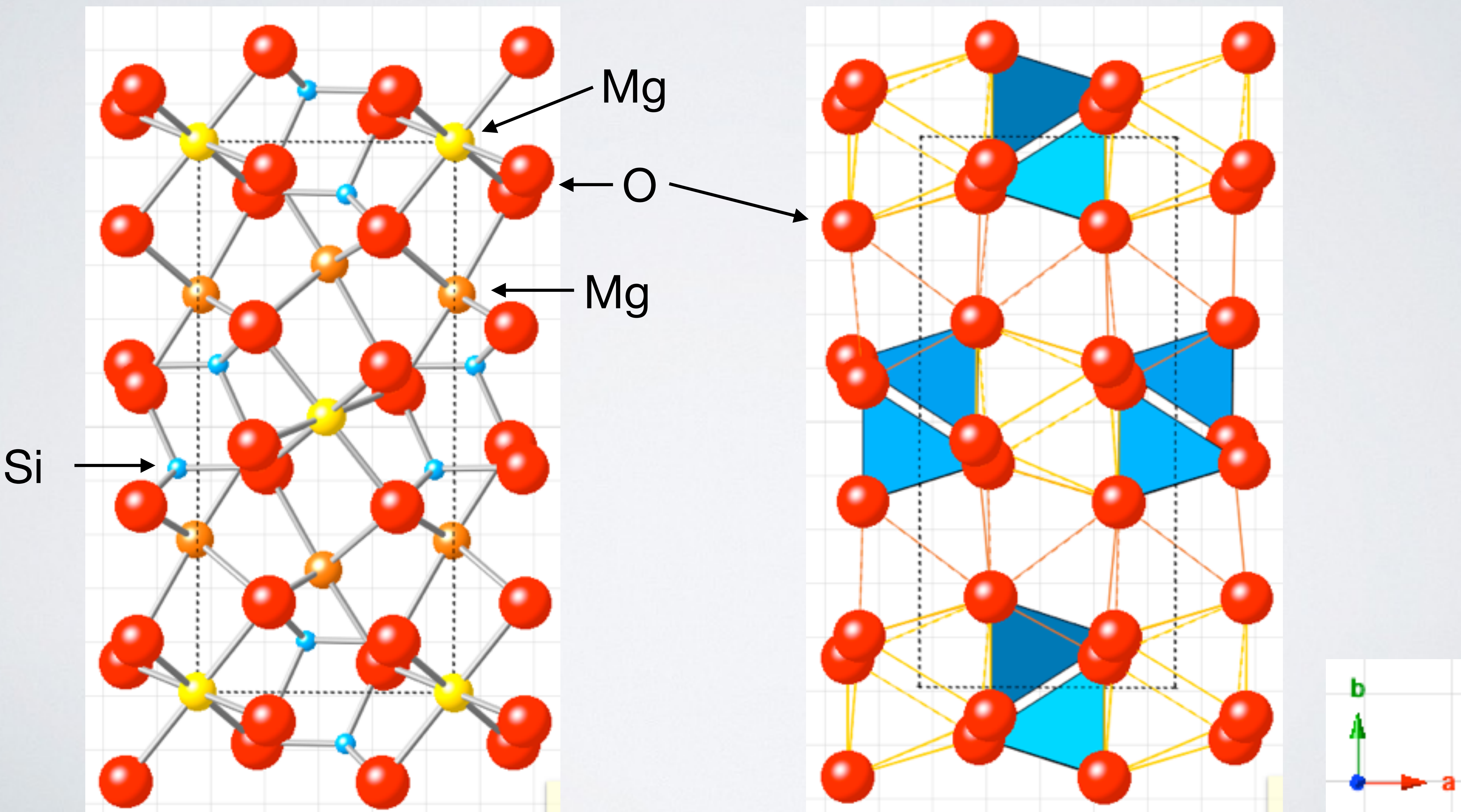
$$d\varepsilon/dt = \dot{\varepsilon} = A\,\sigma\,d^{-2}\exp[-(E+PV^*)/RT]$$

A constant (experimentally determined), *d* grain size,
E activation energy, *V** activation volume

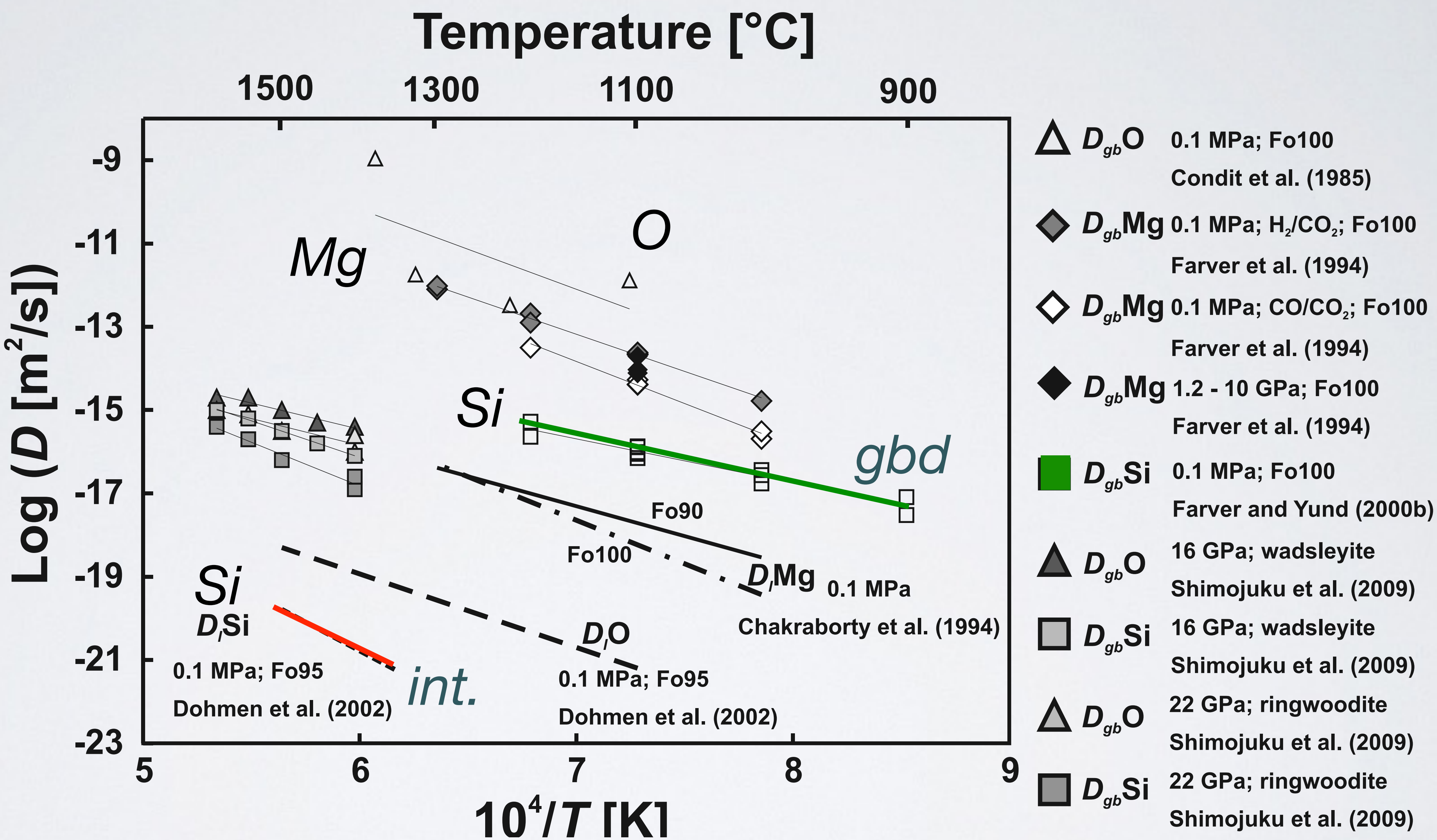
$$d\varepsilon/dt = A\,D_{SD}\,\Omega\,\sigma/kTd^2$$

Olivine (Mg_2SiO_4)

- Tetrahedrally coordinated Si, octahedrally coordinated M sites.
- Si-O bonds shortest and strongest.
- Large spacing of (010) planes and close spacing of (100) planes.



Diffusion is rate controlling: slowest species along its fastest path

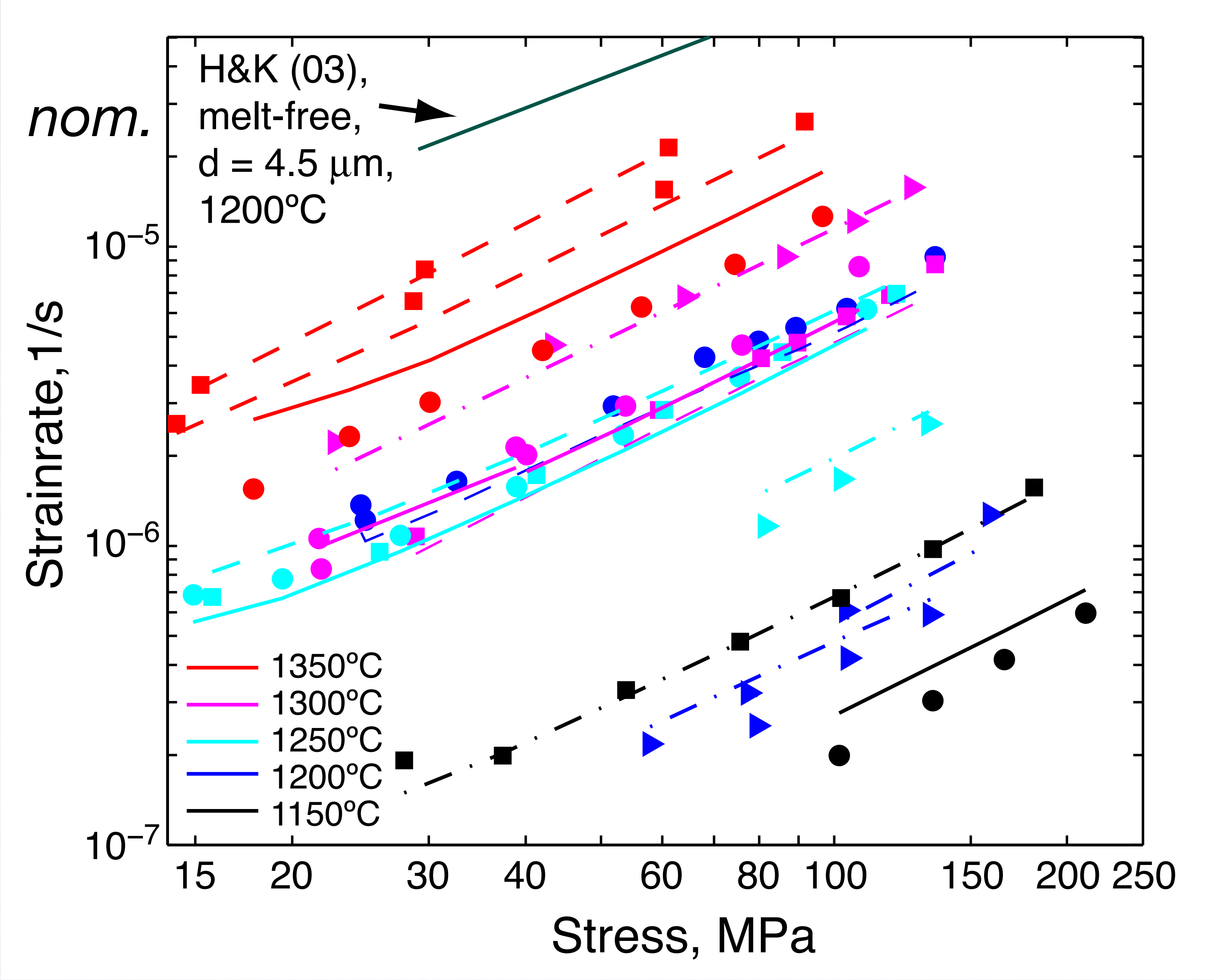


Dohmen & Milke, 2010

flow law for grain boundary diffusion

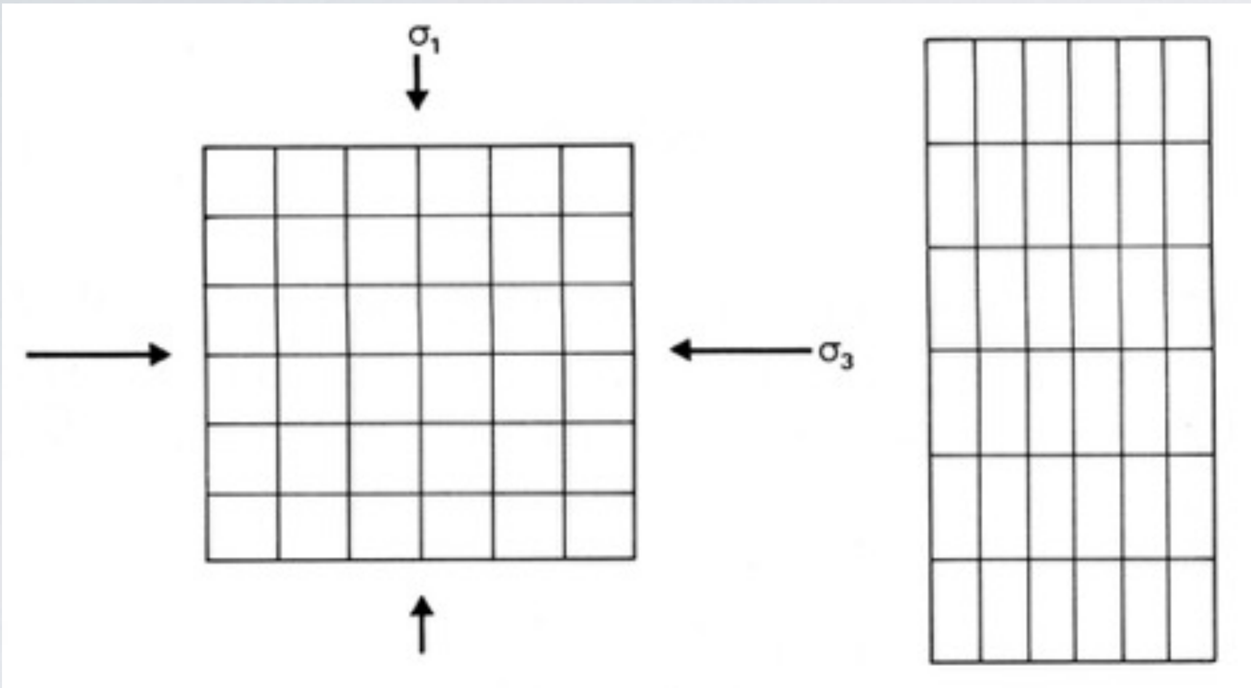
$$\dot{\varepsilon} = A\,\sigma\, d^{-3} \exp[-(E+PV^*)/RT]$$

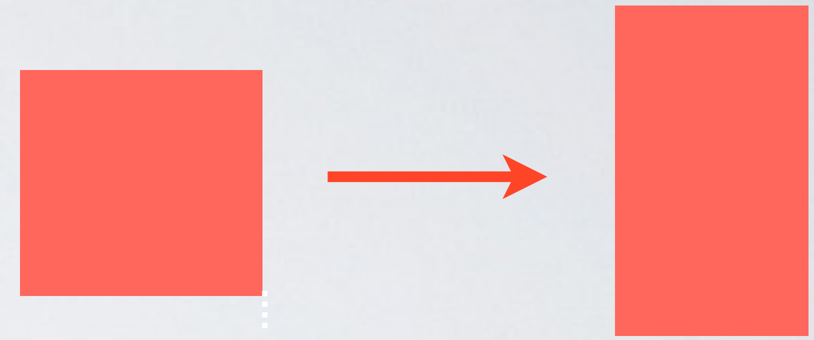
but *A, E, V** are also specific to diffusion mechanism



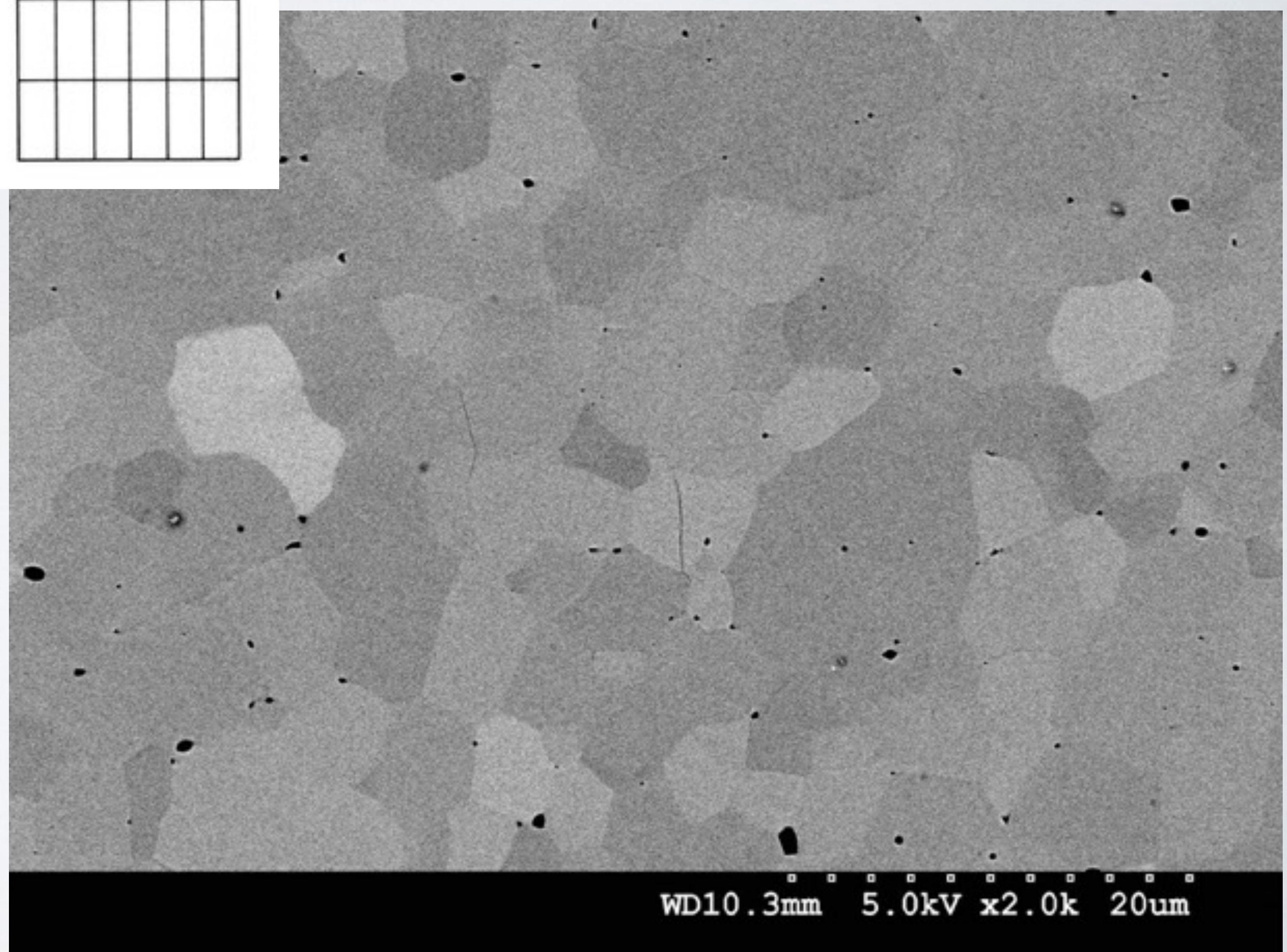
Faul and Jackson, 2007

But: “Pure” diffusion creep would lead to shape change of grains:

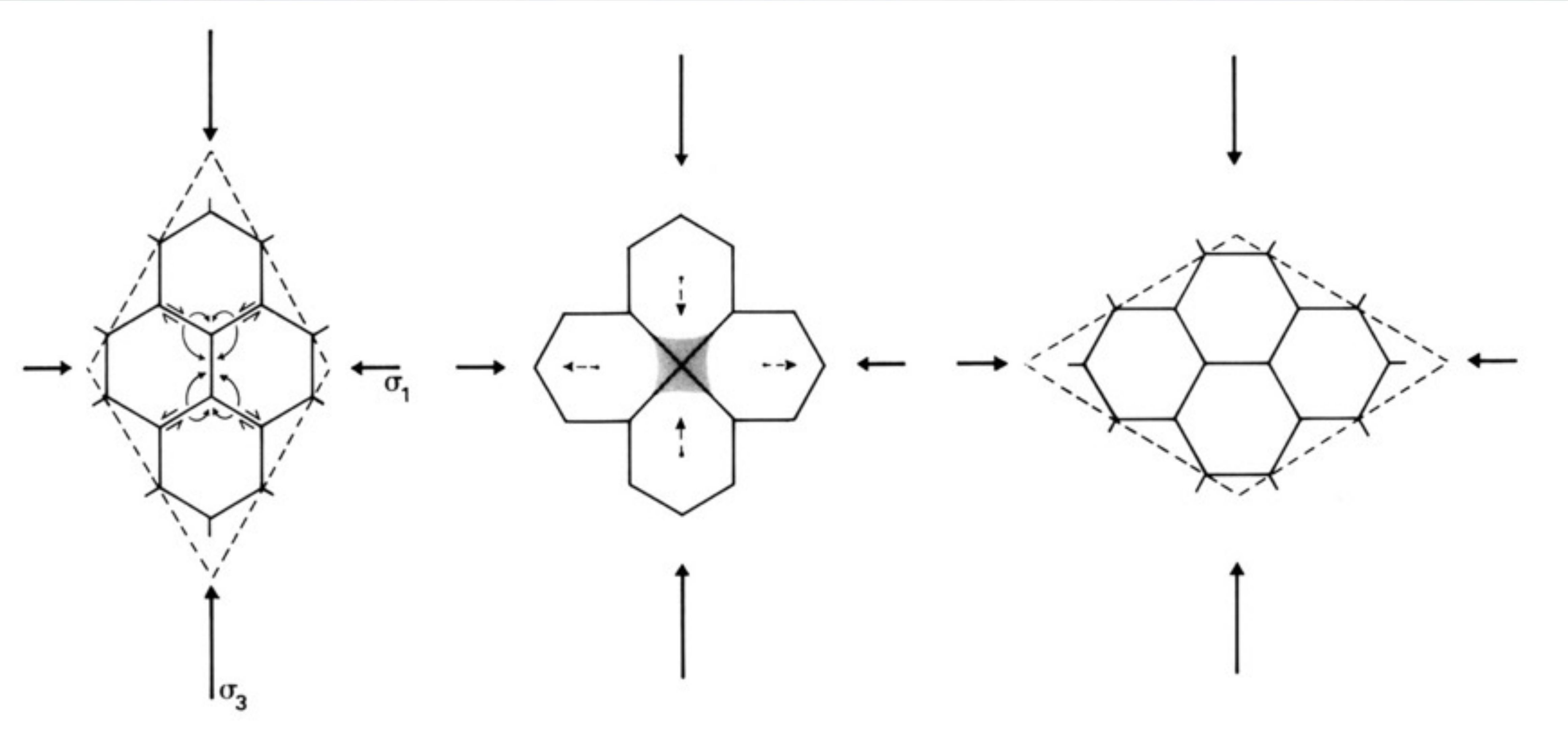




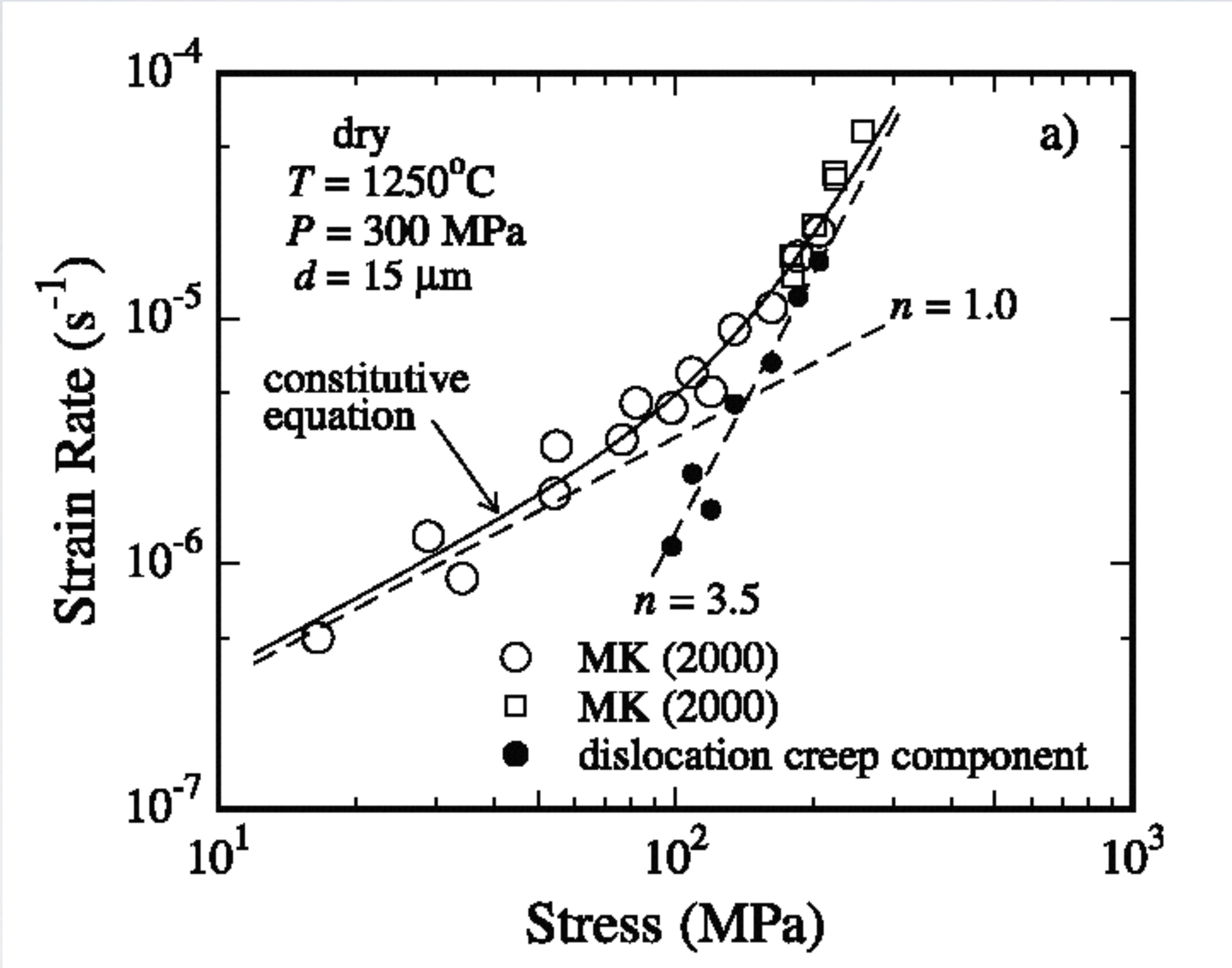
Observation: Grains remain equiaxed after deformation



Diffusionally accommodated grain boundary sliding (diffusion creep)



Macroscopic shape change without grain shape change



dry
T = 1250°C
P = 300 MPa
d = 15 µm
a)
constitutive equation
n = 1.0
n = 3.5
MK (2000)
MK (2000)
dislocation creep component
Strain Rate (s⁻¹)
10⁻⁴
10⁻⁵
10⁻⁶
10⁻⁷
10¹
10²
10³
Stress (MPa)

Hirth, 2003

Dislocation creep

(Weertman creep)

glide velocity: $v \sim b\, v_0 \exp(-E/kT)$

(b Burgers vector)

but dislocations get ‚stuck‘ (entangled): edge dislocations have to ‚climb‘ out of their glide plane, climb is rate limiting. Climb is a diffusive process.

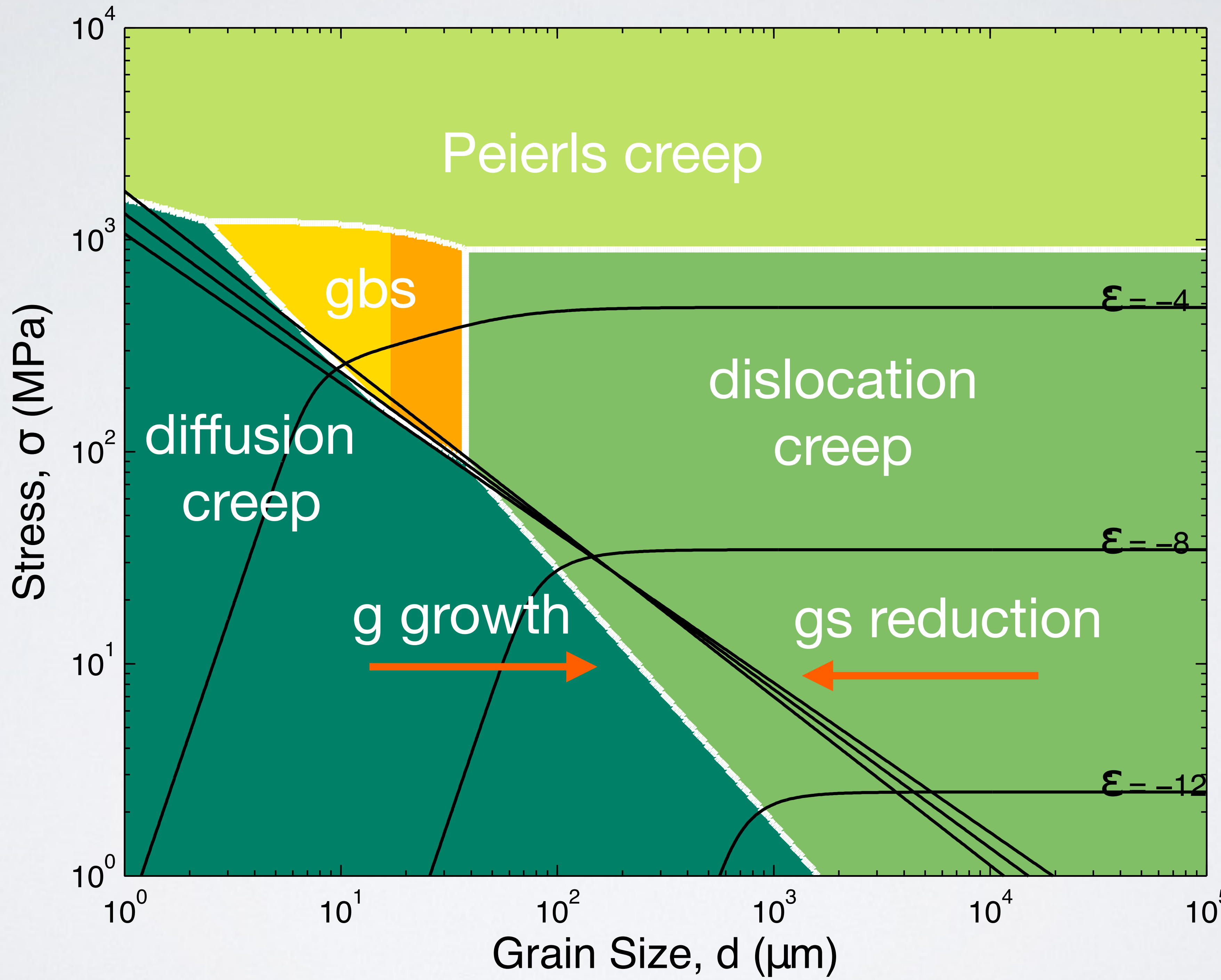
$$\dot{\varepsilon} = A\, \sigma^n \exp[-(E+PV^*)/RT]$$

dislocation density ~ stress: strain rate depends on stress^n

dislocations are intracrystalline, no grain size dependence

Deformation Mechanism Map

Olivine, 1250ºC, 0.4 GPa



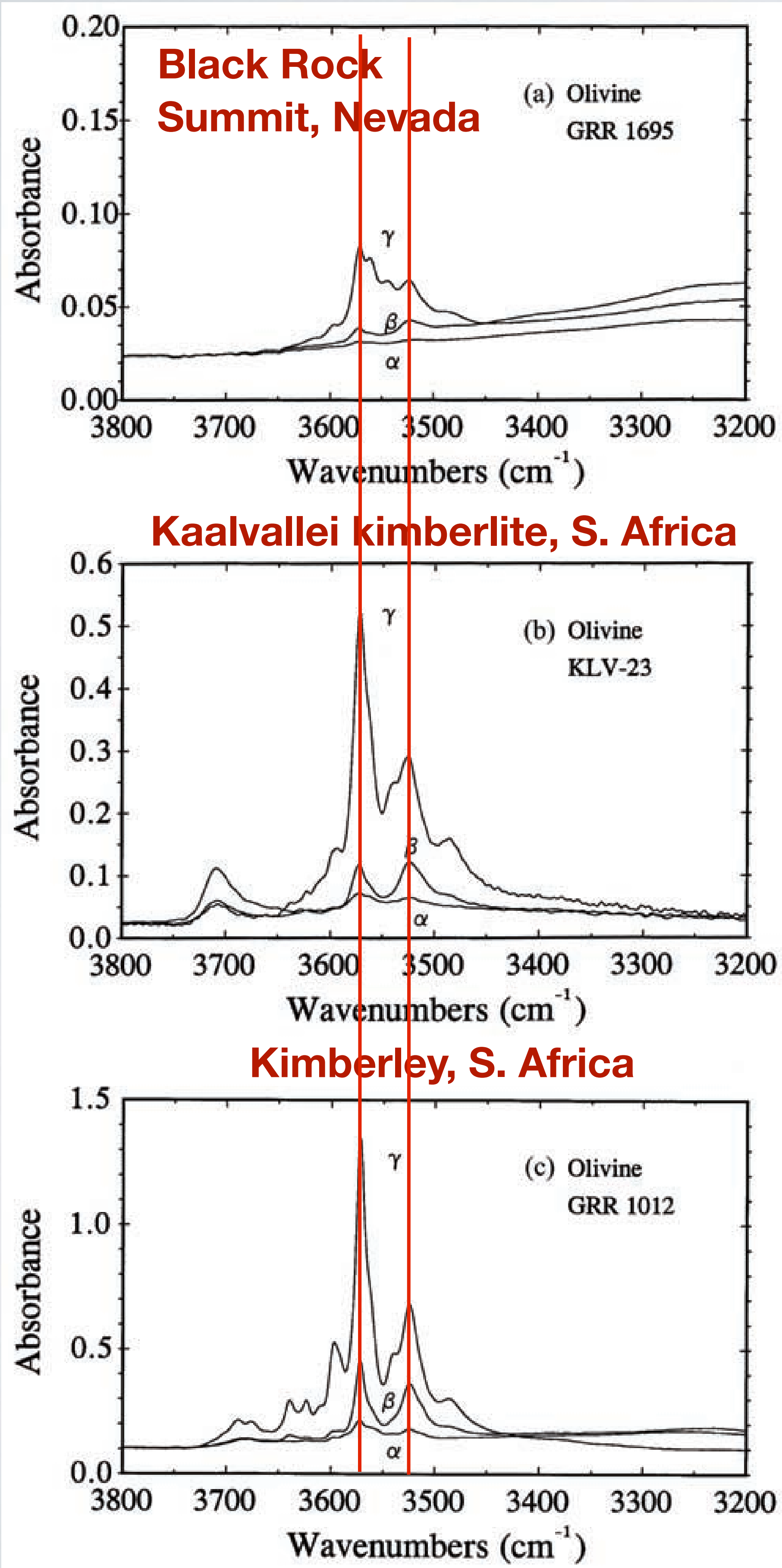
J. Warren

Influence of Water

	DC0212	DC0212	DC0212	San Carlos #1	San Carlos #1	San Carlos #1	San Carlos #2
	Laser ICP-MS	Ion probe	Solution ICP-MS	Laser ICP-MS	Laser ICP-MS	Ion probe	Ion probe
	Oxford	Edinburgh	GVC	Oxford	GVC	Edinburgh	Edinburgh
	($n=5$)	($n=15$)	($n=4$)	($n=2$)	($n=2$)	($n=2$)	($n=4$)
Li	1.46 (7)	1.09 (5)	1.4 (5)		1.9	1.17	1.08 (3)
Na	61 (4)	62 (4)	115 (59)	68	82	77	55 (15)
Al	39 (2)	39 (3)	77 (22)	196	174	172	104 (10)
P	46 (4)			42	32		
Ca	174 (9)	150 (2)	252 (42)	625	665	552	563 (7)
Sc	0.77 (8)	0.66 (9)	1.0 (3)	3.2	2.7	2.4	3.04 (6)
Ti	14.9 (6)	12 (1)	22 (2)	25	24	19	9.7 (5)
V	4.9 (3)	4.0 (3)	5.1 (3)	4.2	4.5	4.0	2.7 (3)
Cr	181 (10)	167 (8)	205 (26)	103	108	107	146 (2)
Mn	695 (13)	670 (12)	724 (23)	1057	1085	1098	1117 (20)
Co	137 (3)	157 (9)	143 (3)	142	143	186	179 (6)
Ni	3300 (200)	3450 (50)		2950	2840		
Cu	2.7 (2)	<5	6 (1)	0.9	1.7		
Zn	54.9 (8)	46 (5)		54	58		
Ga	0.049 (1)			0.062			
As	<0.1						
Rb	<0.008 [a]	0.06 (2)					
Sr	<0.01[a]	0.14 (5)		<0.01			
Y	<0.002	0.007 (2)	0.04 (1)	0.06	0.07	0.115	0.059 (7)
Zr	0.06 (2)	0.14 (5)	0.14 (7)	0.003	0.009	<0.1	<0.1
Nb	0.24 (3)	0.33 (6)	0.23 (3)	0.0005	<0.02	<0.05	<0.05
Mo	0.022 (5)			0.023			
In	0.006 (2)						
Sn	0.52 (3)						
Sb	0.022 (2)						
Ba	<0.006[a]	0.28 (6)			<0.005		
Ce	<0.003[a]	0.16 (4)			<0.002		
Pb	0.038 (5)[a]	1.2 (3)					
Th	0.006 (2)[a]	0.03 (1)					
U	0.0002 (1)[a]	0.14 (5)					

deHoog et al., 2010

Water in olivine:
a brief background



water (hydrogen) is a trace element

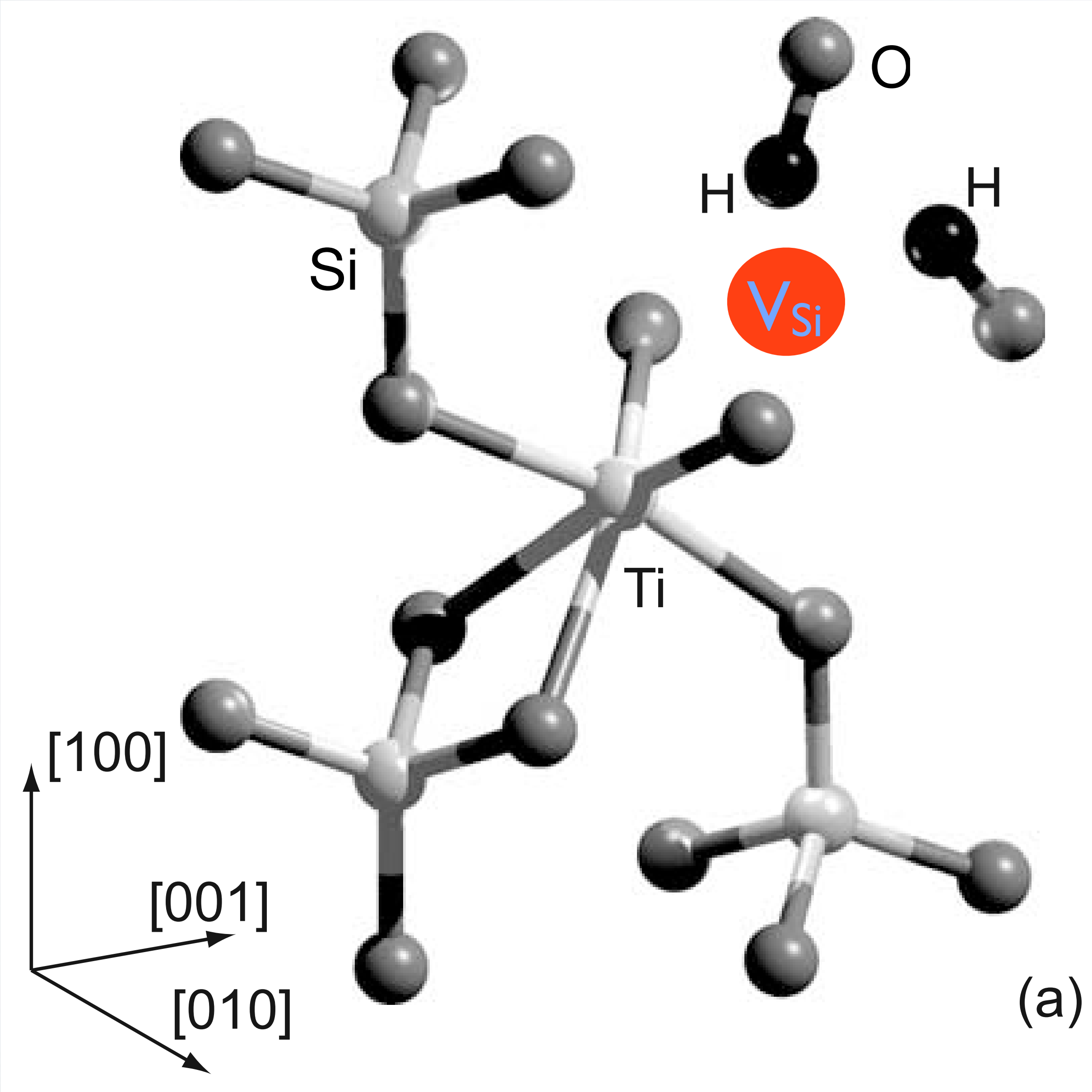
observable by Fourier Transform **Infrared Spectroscopy** (FTIR)
(absorption of IR light)
advantage: absorption depends on bonding environment

Most natural olivine has two dominant absorption bands in IR spectra, at 3525 and 3572 cm^{-1}

Bell et al., JGR, 2003

Where is the water?

Titanium clinohumite “Point Defect”: Coupled substitution of 6-fold coordinated Ti on M1 site with 2 H on Si vacancy.
Energetically the most stable.
Supported by synchrotron observations

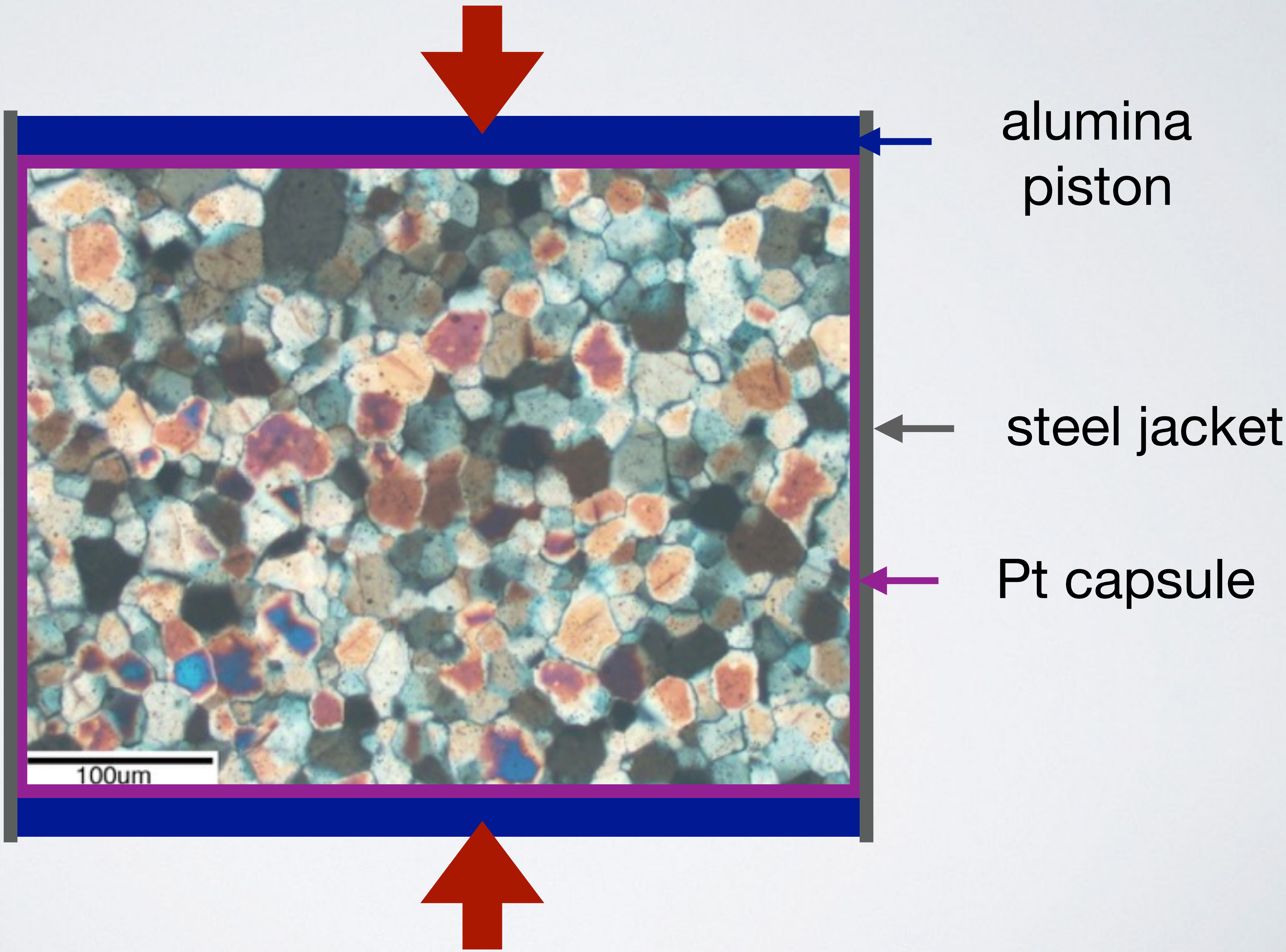


calculated OH - vibrational frequencies: 3572 and 3525 cm^{-1}

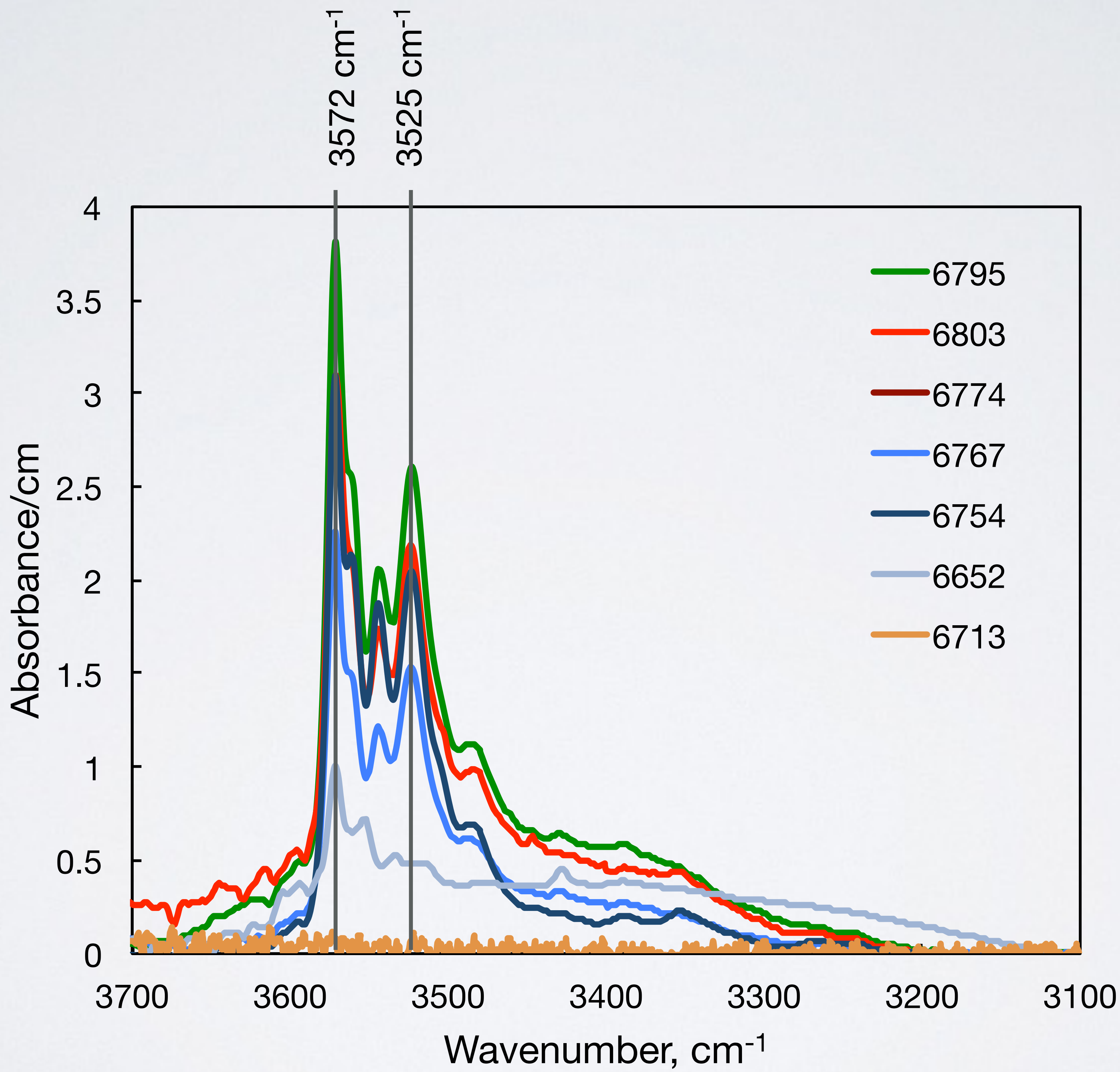
Walker et al., 2007

Experiments in Paterson (gas medium) apparatus, 300 MPa confining pressure, 1200 - 1350°C. Water not buffered, samples encapsulated in Pt.

Fo_{90} olivine doped with 0.04 wt% TiO_2

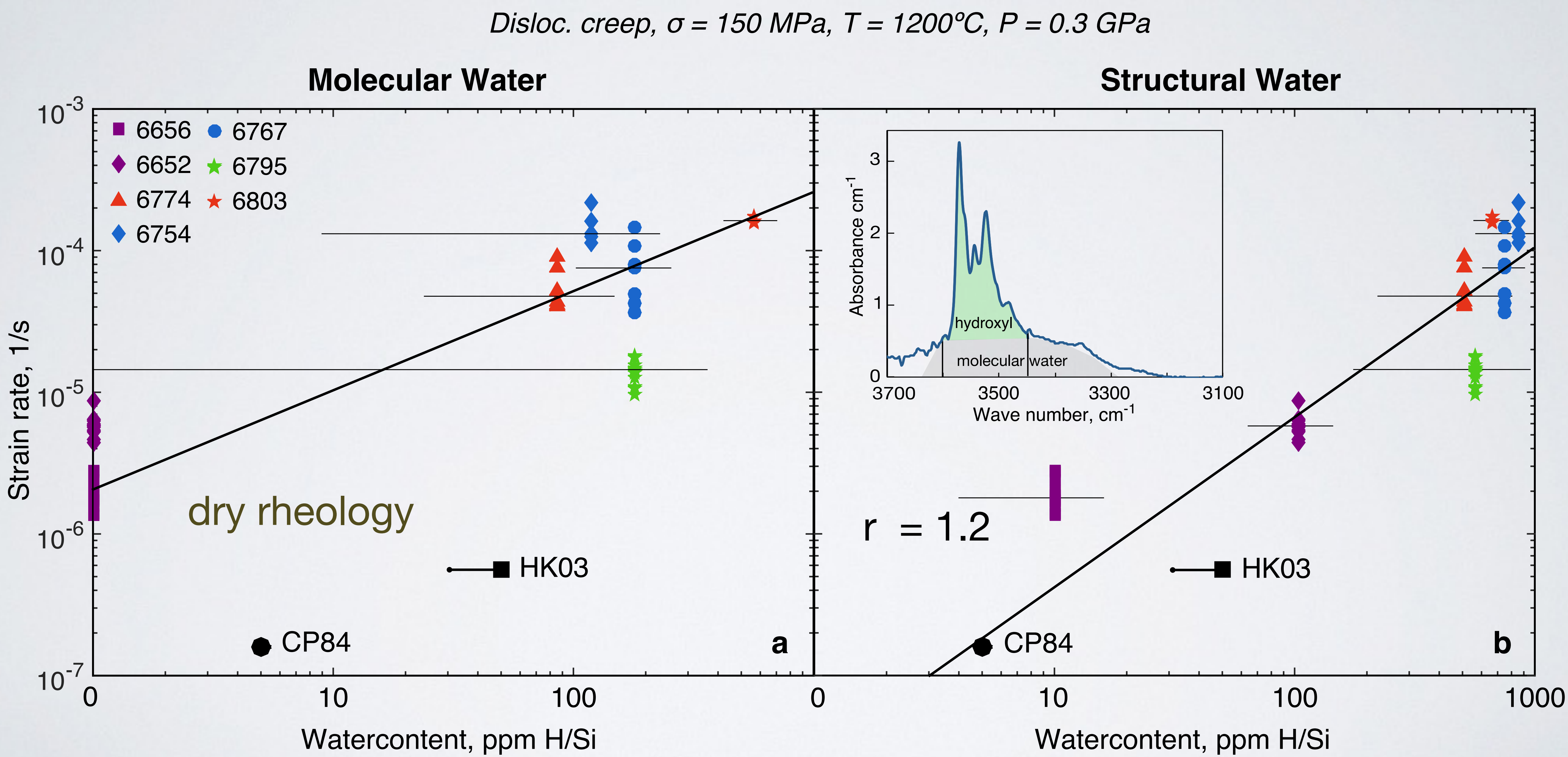


Absorption of infrared light reveals structural location of hydrogen in olivine crystal lattice



- rheology controlled by structurally bound water (hydroxyl)
- water incorporation linked to titanium:

extrinsic defects control water incorporation and rheology

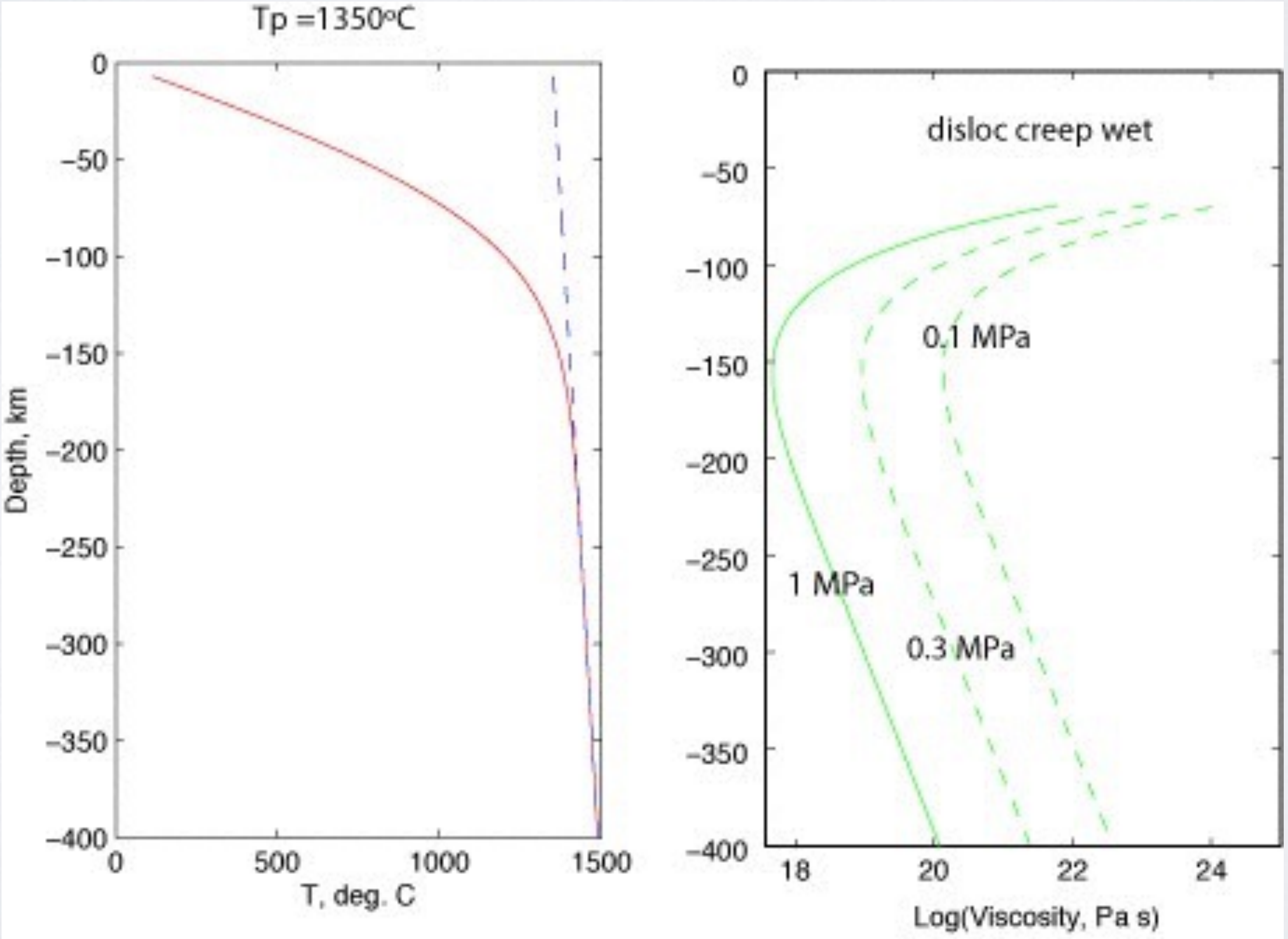


Faul et al. in review, EPSL

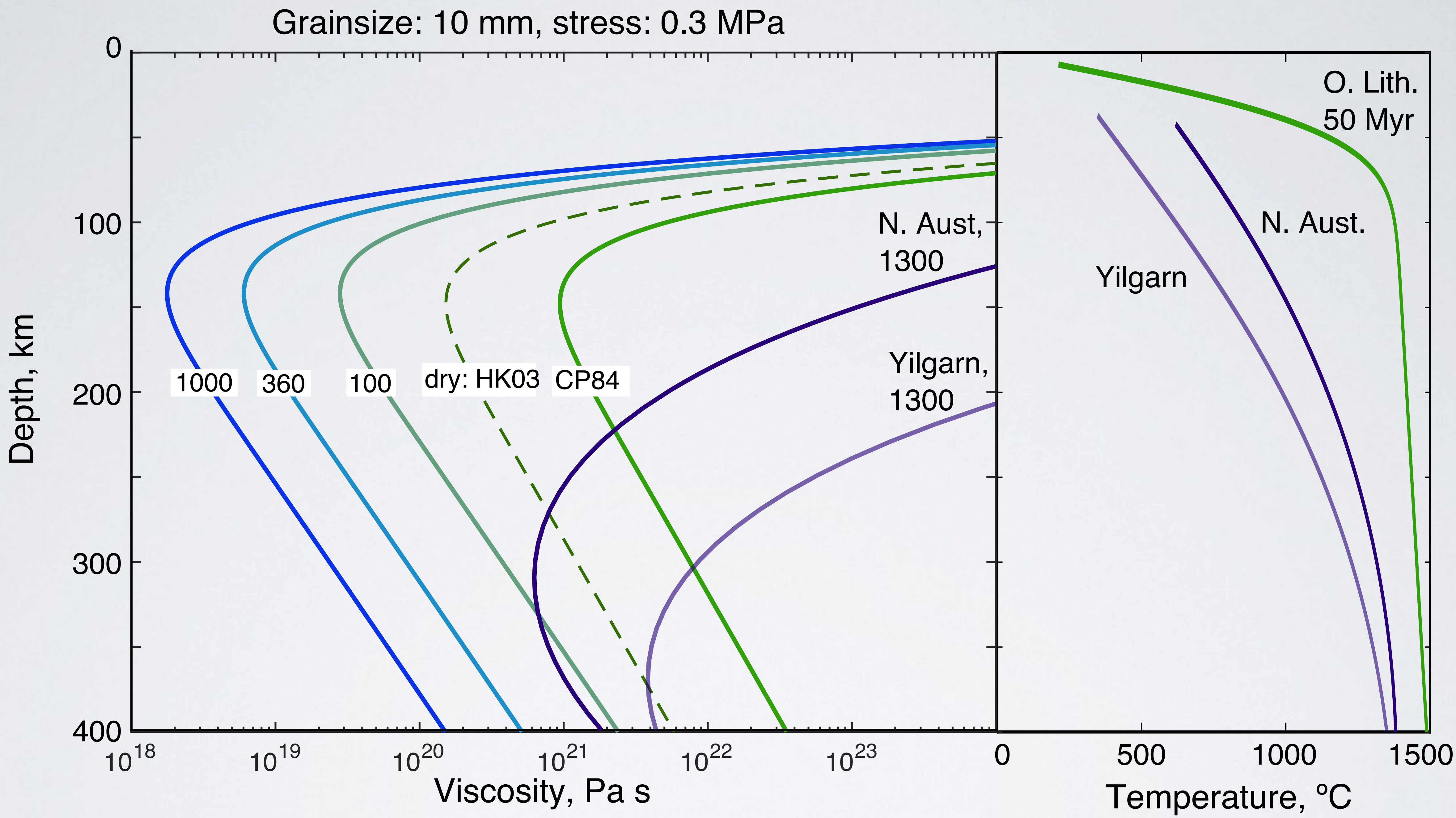
linear relationship between water content and strain rate:

$$\dot{\varepsilon} = A\sigma^{n}\, d^{-p}\, fH_2O^{r} \exp\left(\frac{-Q}{RT}\right)$$

viscosity $\eta = \sigma / 2\dot{\varepsilon}$ (Newtonian)



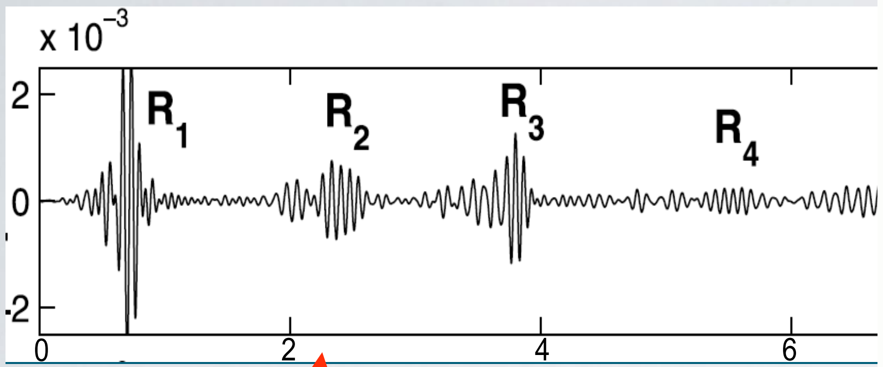
Upper mantle viscosity at water-undersaturated conditions

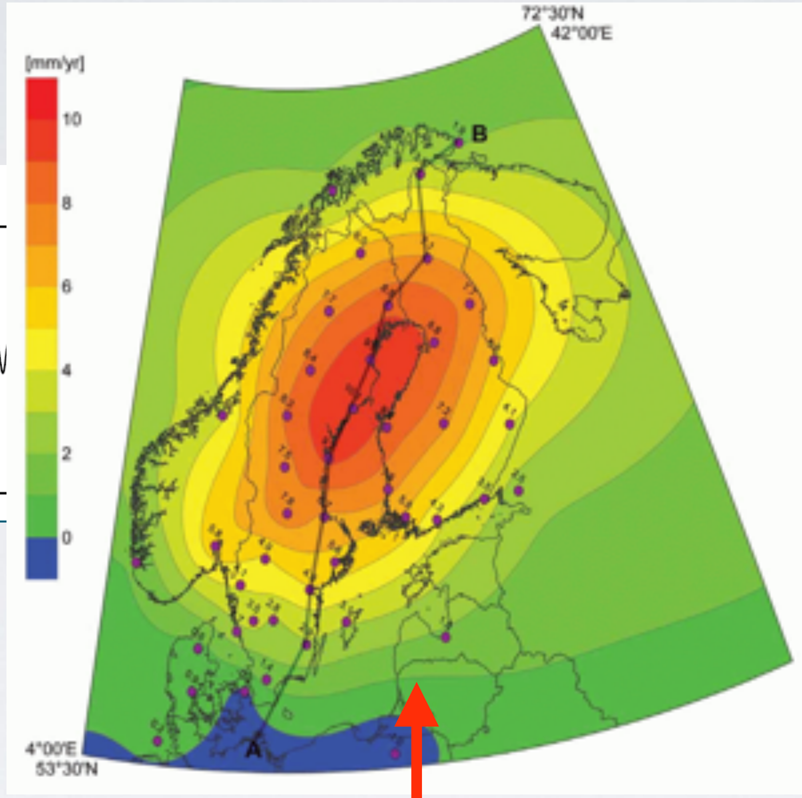


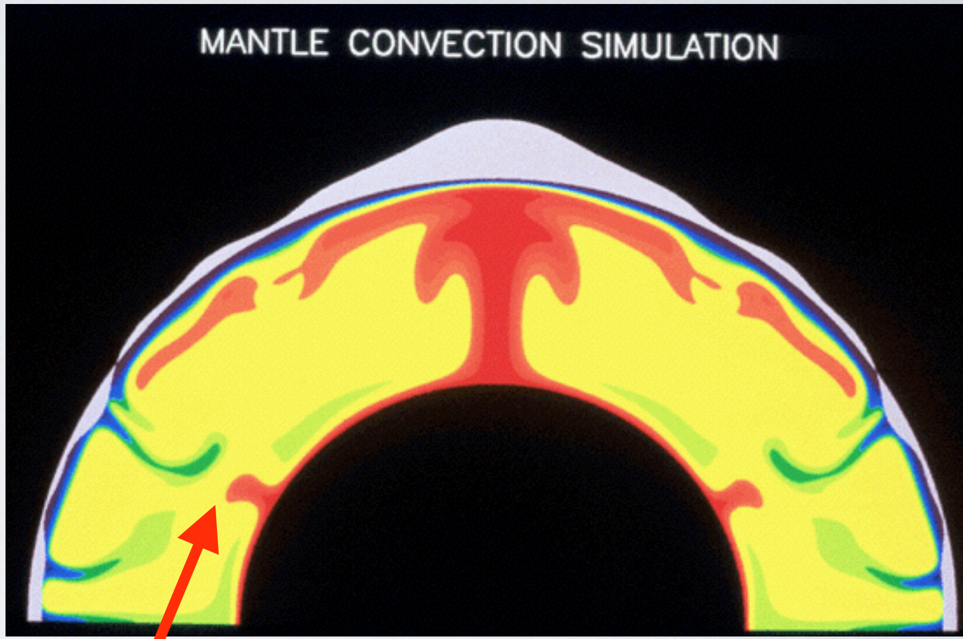
(water contents in ppm H/Si)

water content in olivine in ppm H/Si; 1000 ppm H/Si ≈ 60 ppm H_2O

Deformation at a range of time scales

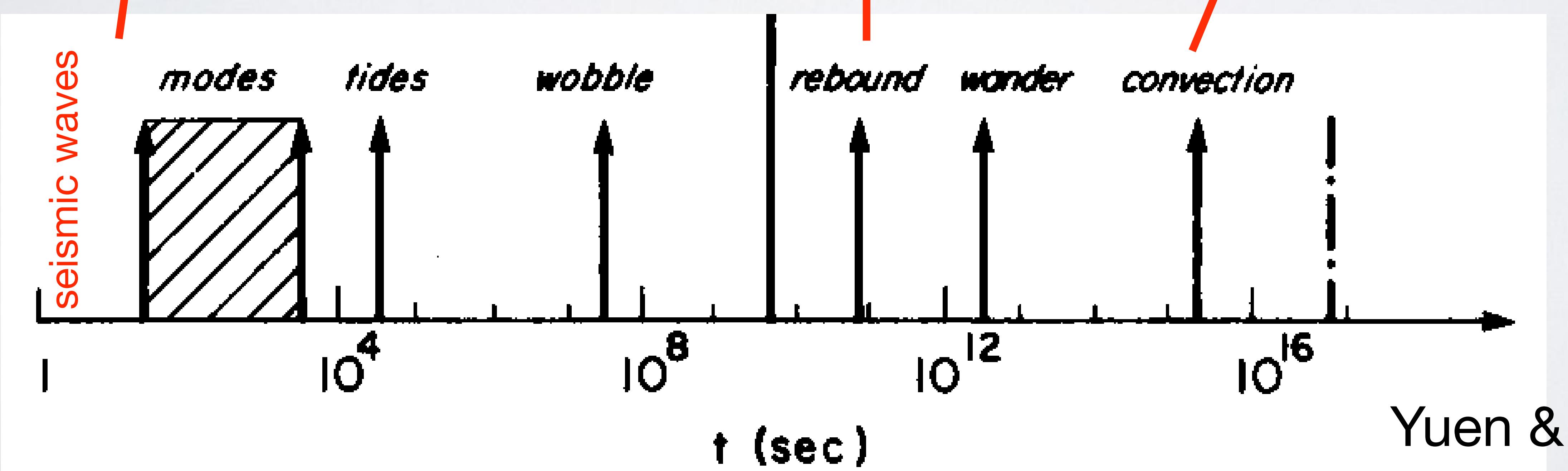






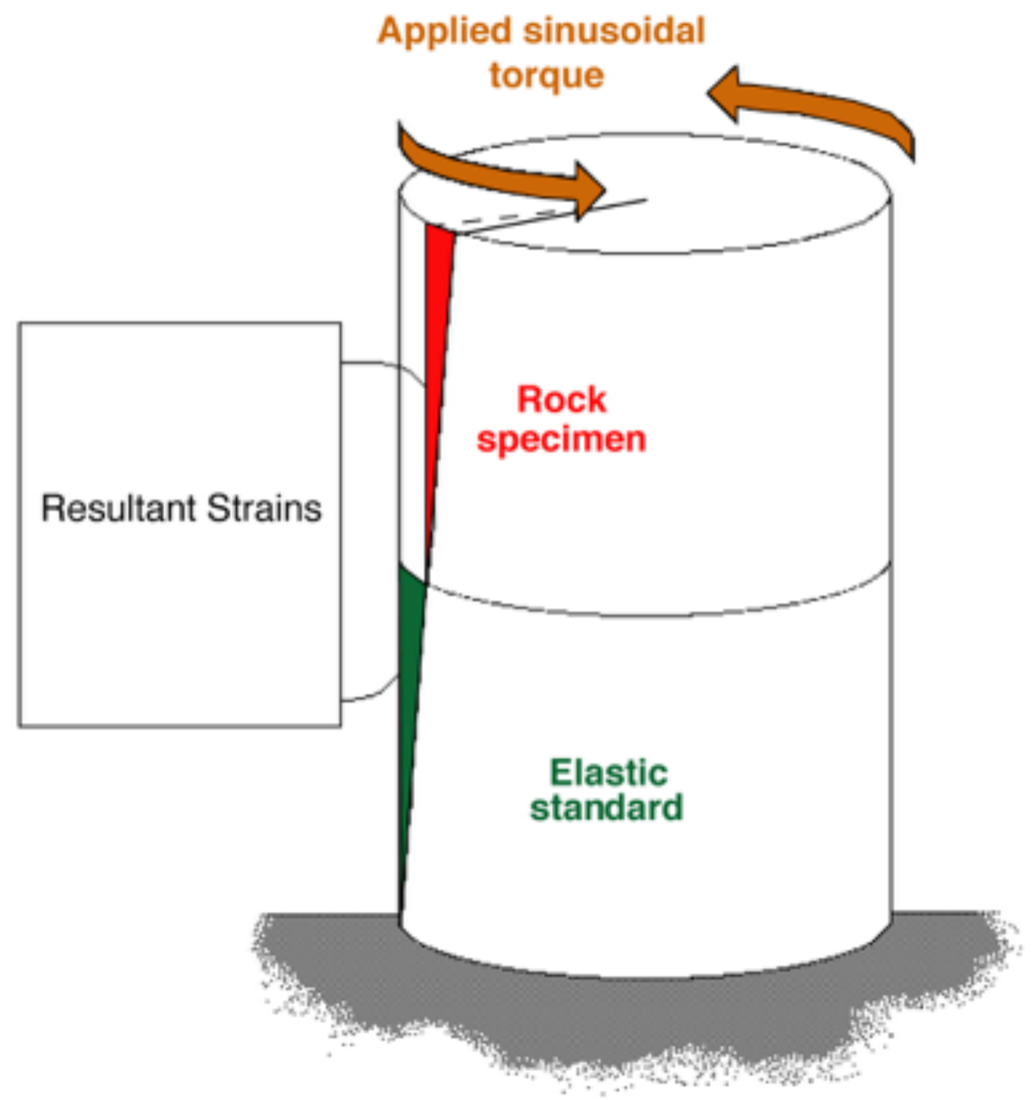
$\varepsilon < 10^{-4}$

$\varepsilon >> 1$



Yuen & Peltier, 1982

Experiments: Measurement of shear modulus (G) and attenuation (1/Q)



Research School of Earth Sciences, Australian National University

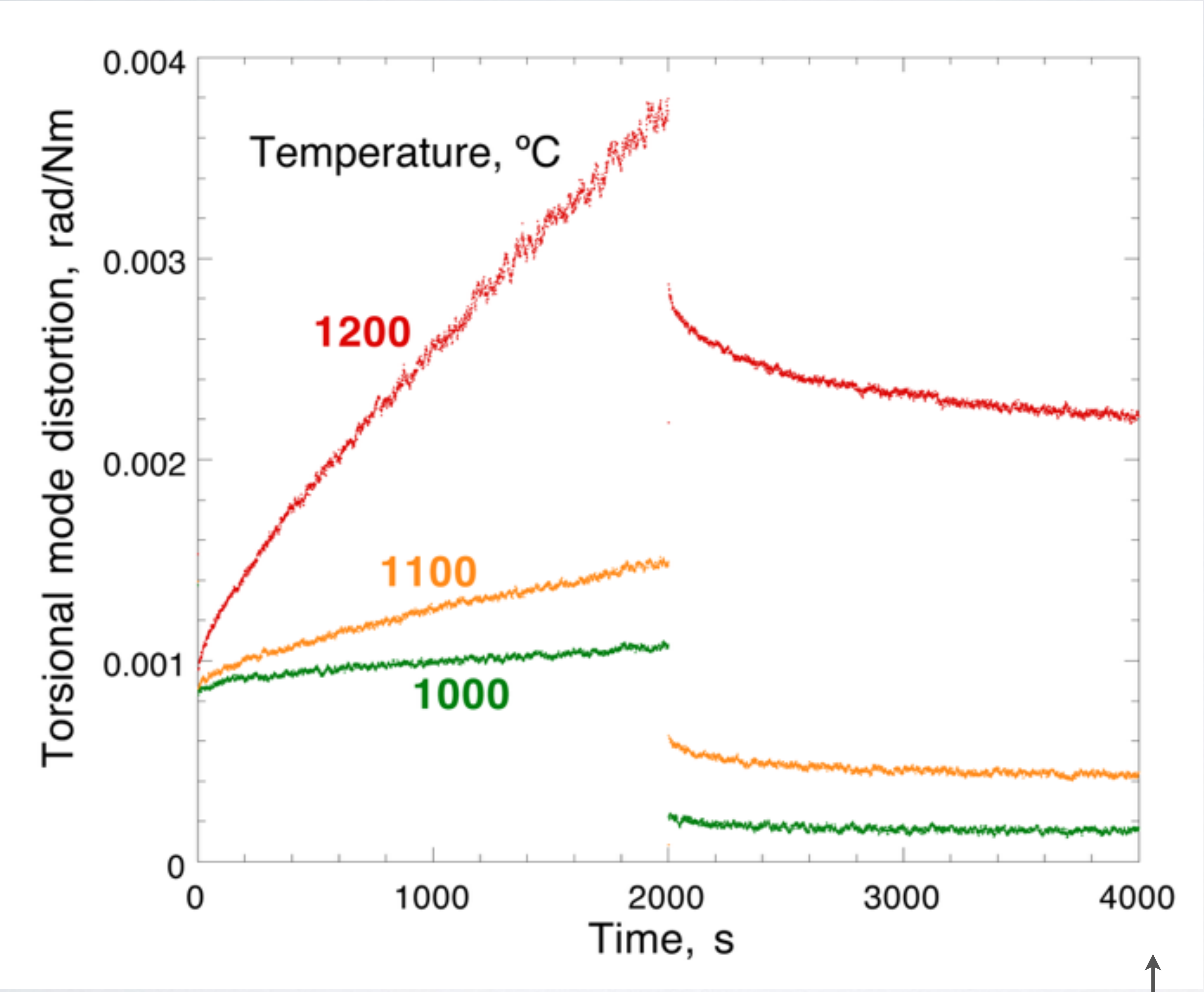
Experiments at

- temperatures to 1300°C
- periods 1 - 1000s
- 200 MPa confining pressure

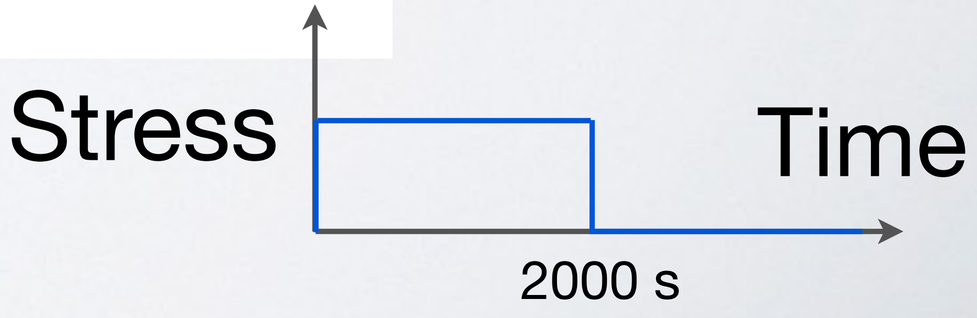
Measure shear modulus G and dissipation/attenuation

Attenuation (1/Q): energy loss per cycle

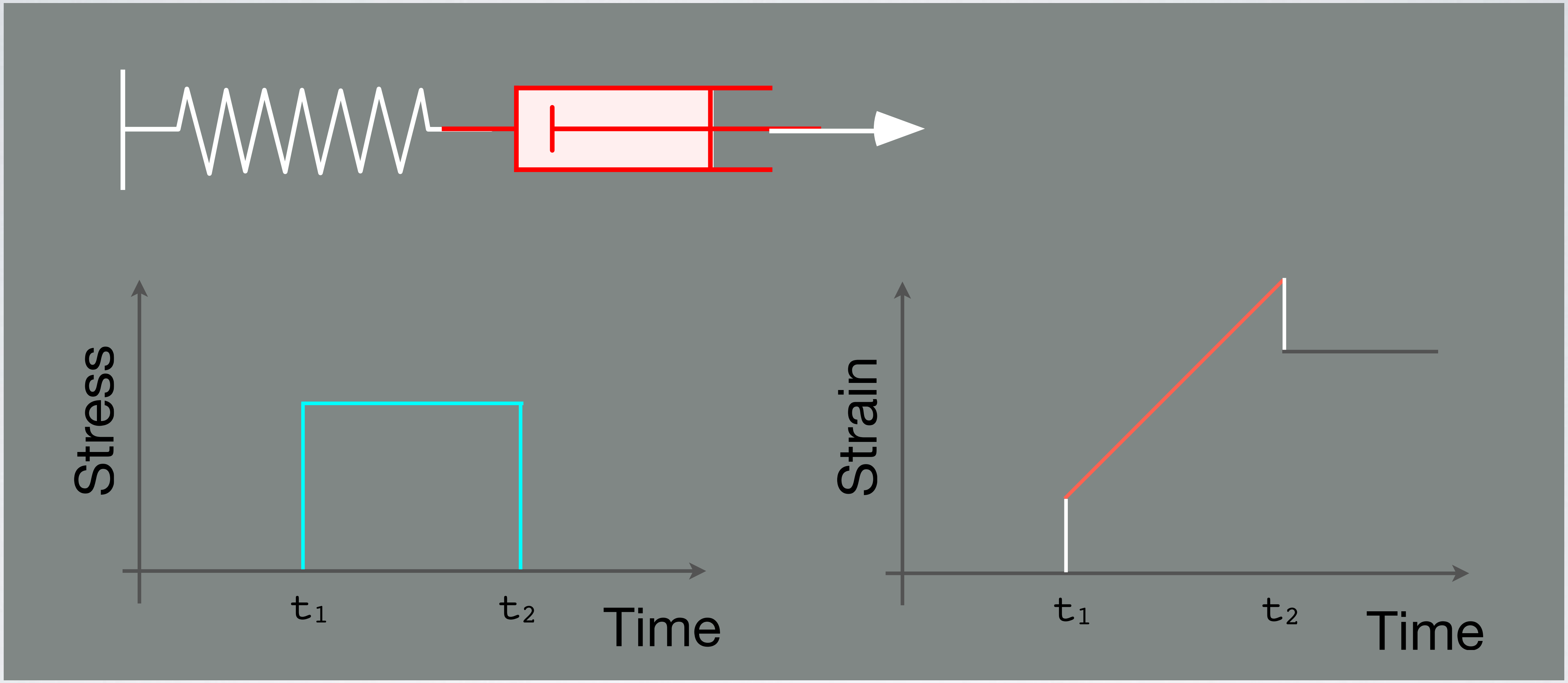
Microcreep experiments (time domain)



$\varepsilon < 10^{-4}$

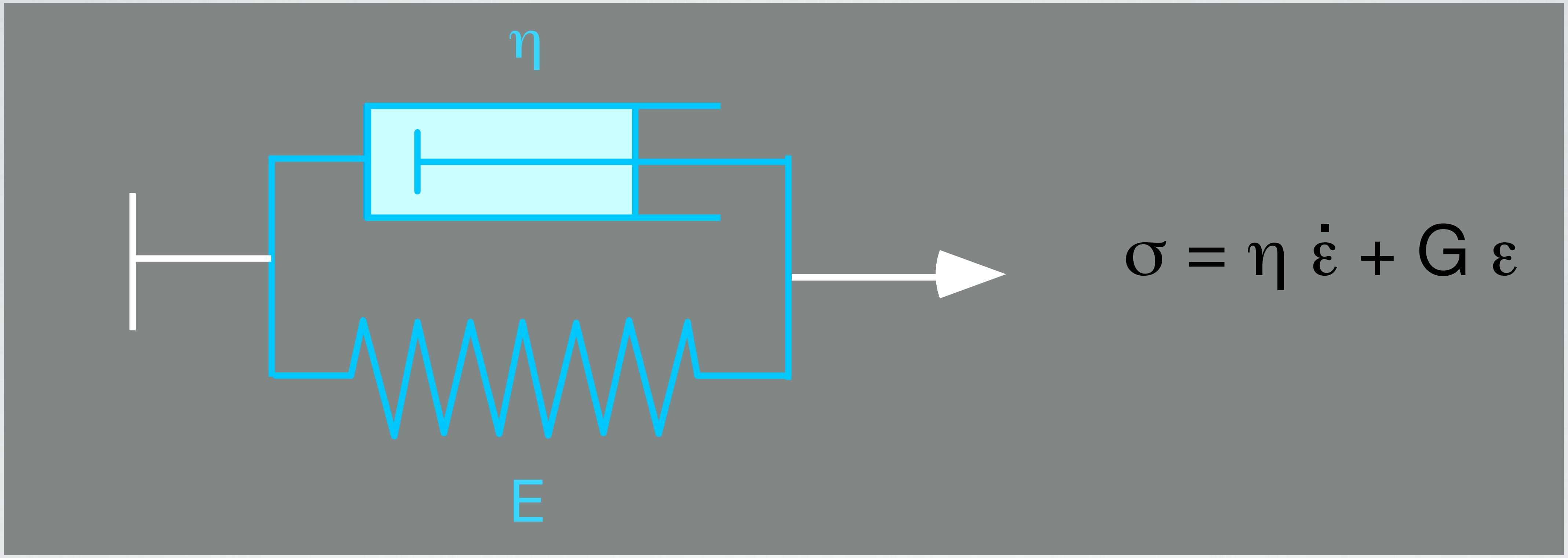


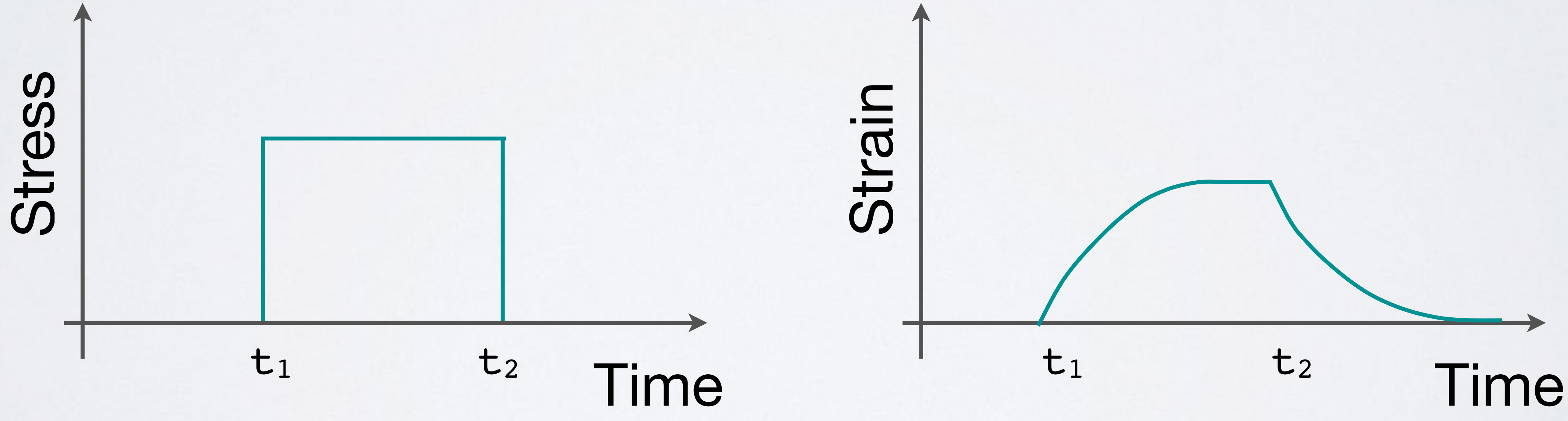
Maxwell body: viscoelastic



instantaneous, reversible + time-dependent permanent

Anelastic behavior (transient creep)

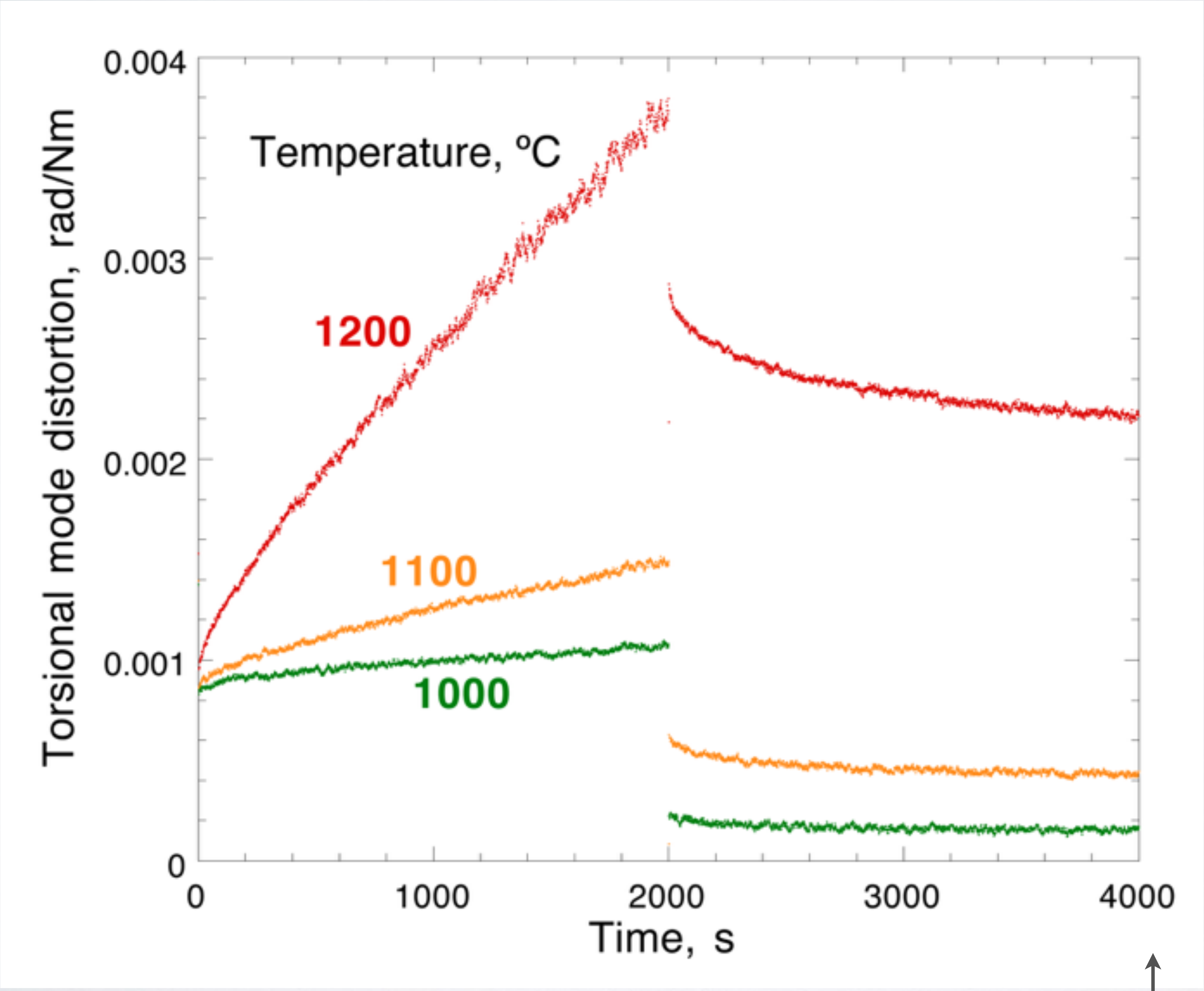


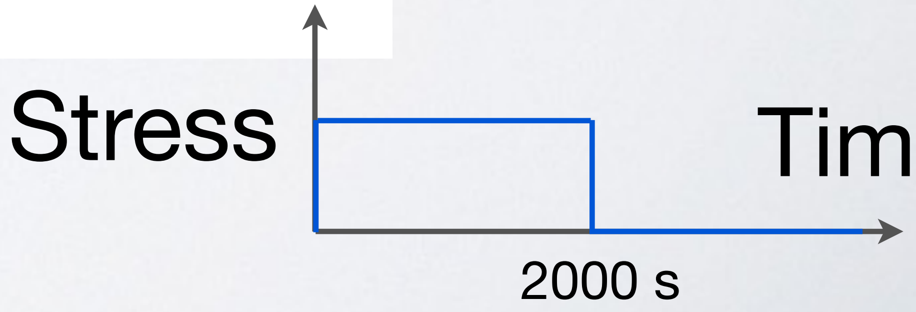


time-dependent, unique equilibrium, recoverable

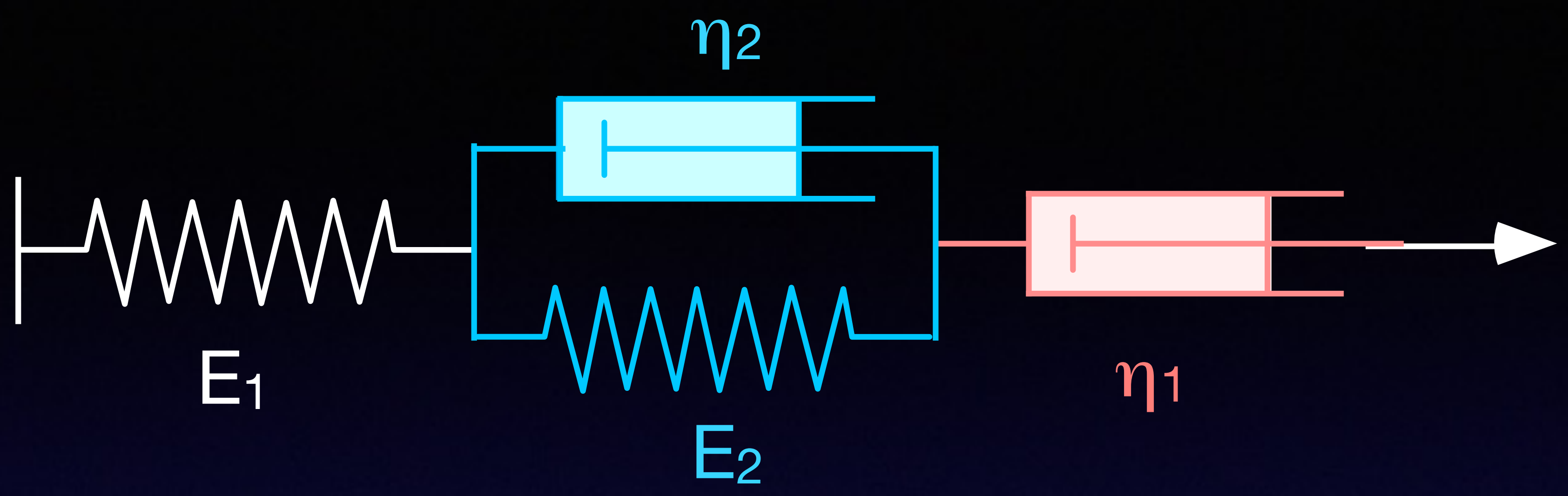
Microcreep experiments

$\varepsilon < 10^{-4}$

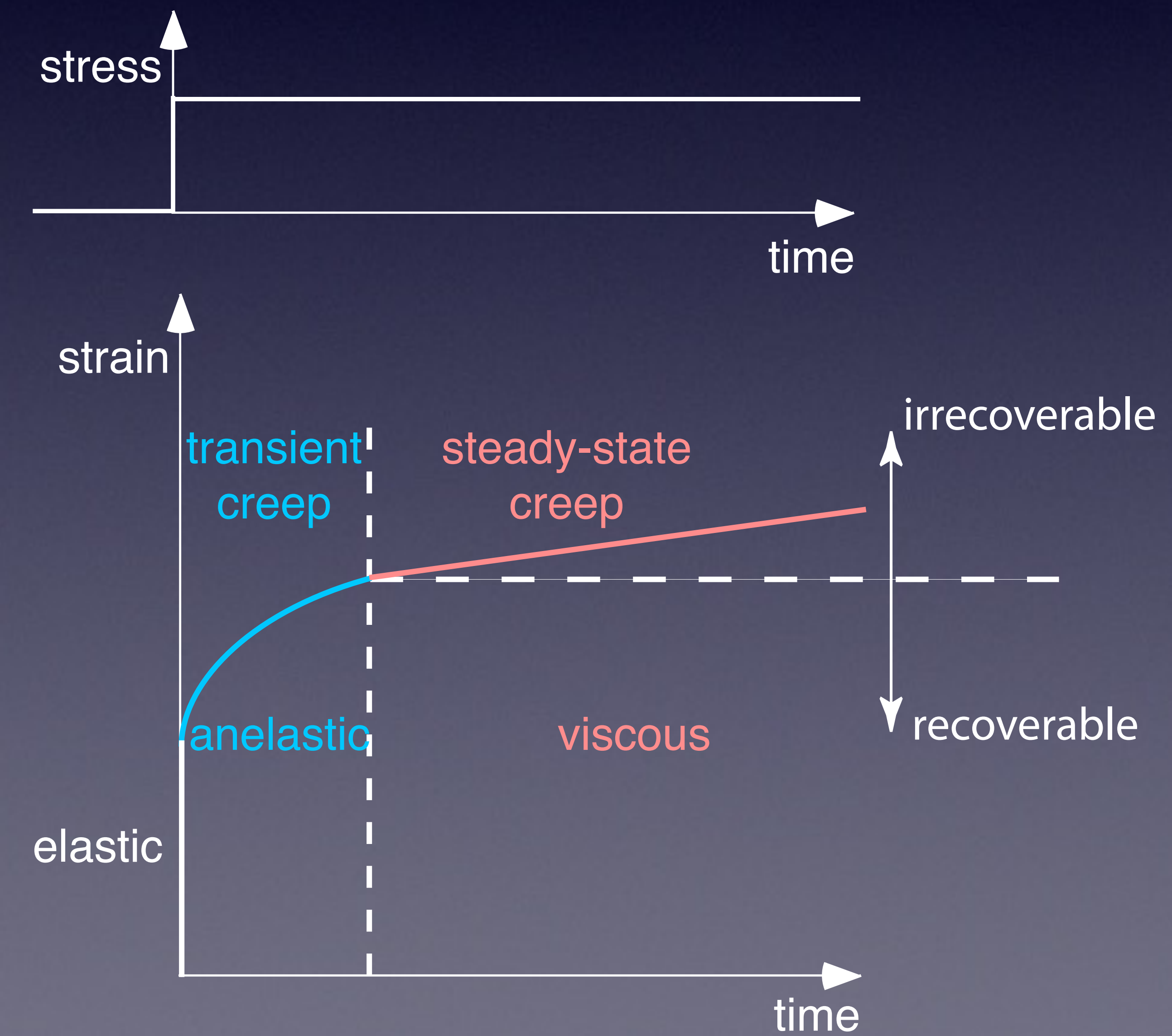




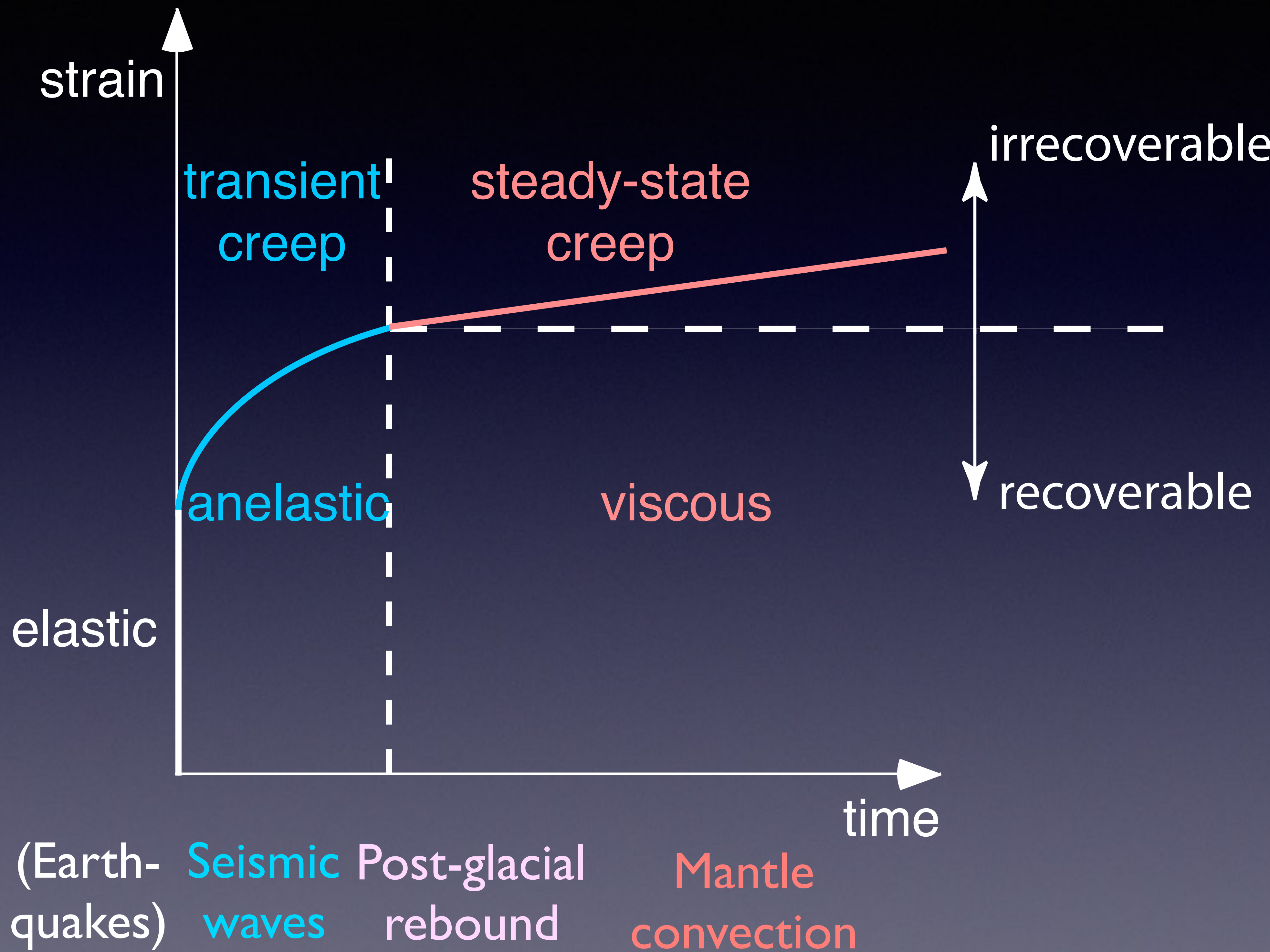
Viscoelastic behavior: Burgers Model



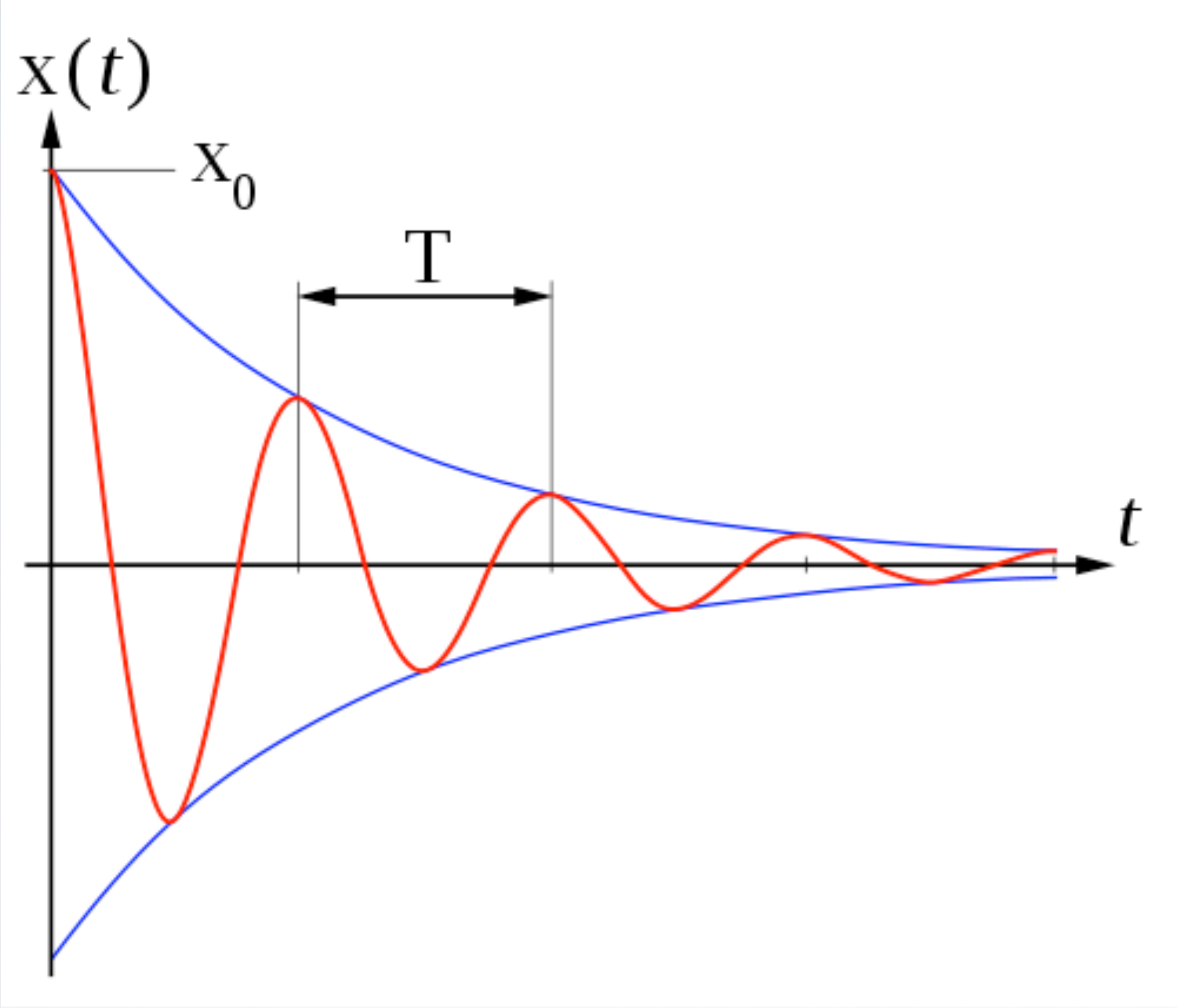
$$\varepsilon(t) = \varepsilon_e + \varepsilon_t(t) + \dot{\varepsilon}t$$



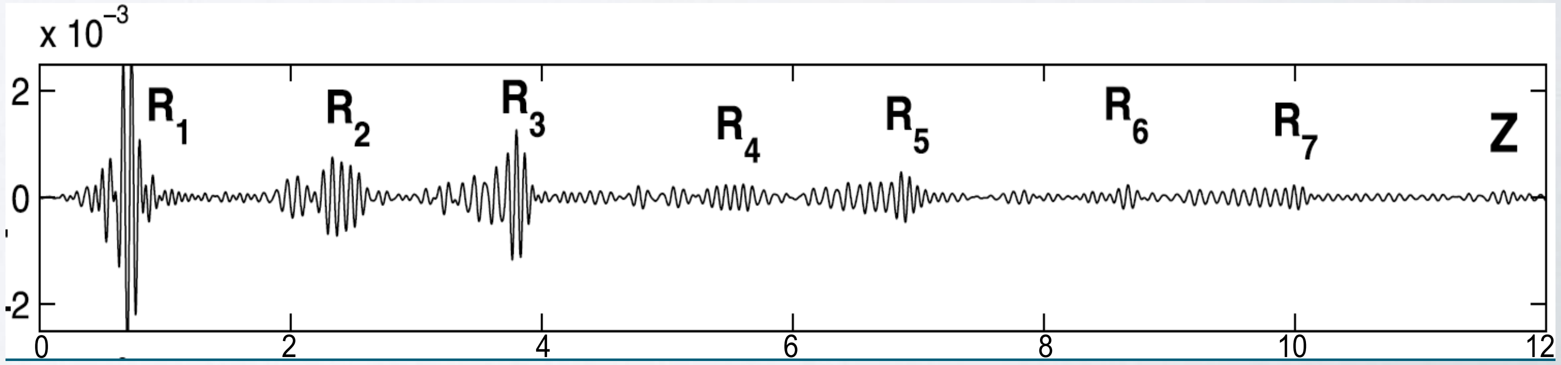
Timescales of deformation in the Earth



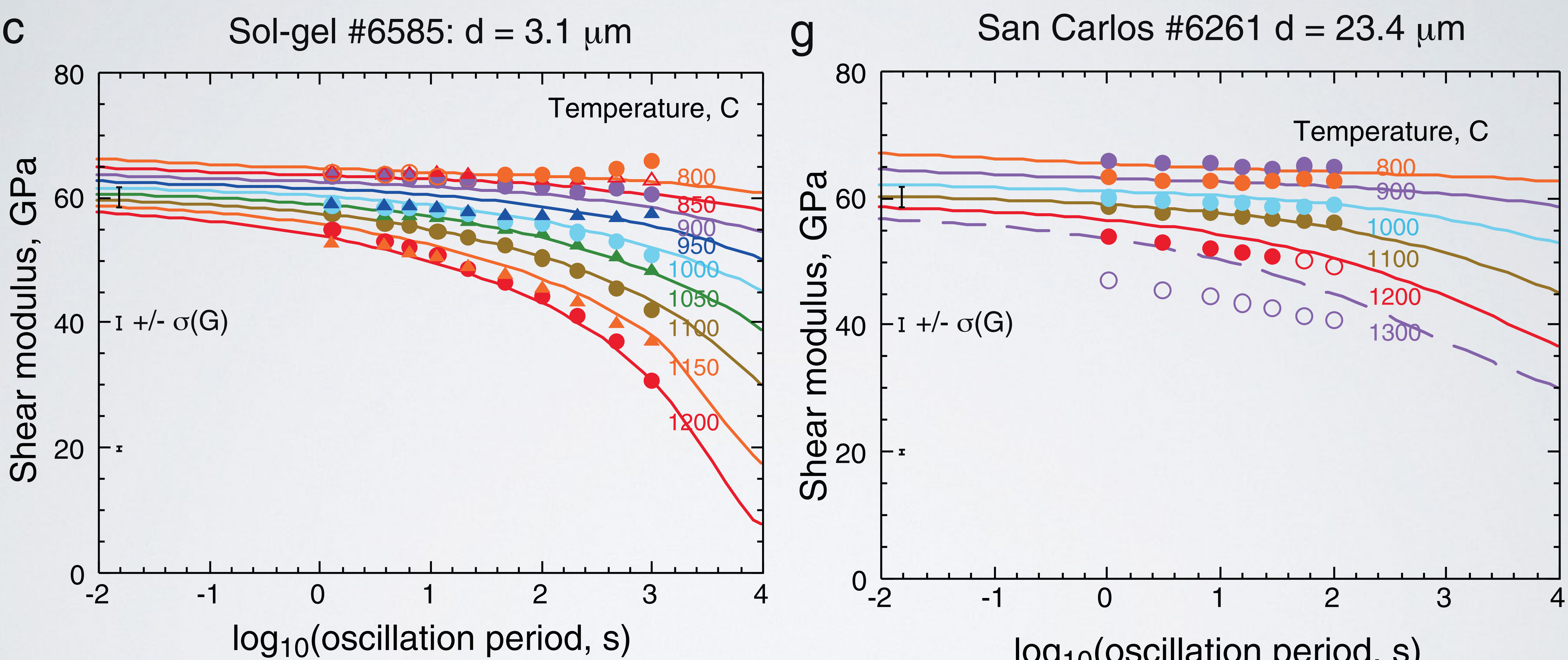
Attenuation/dissipation (frequency domain) amplitude decreases with each cycle



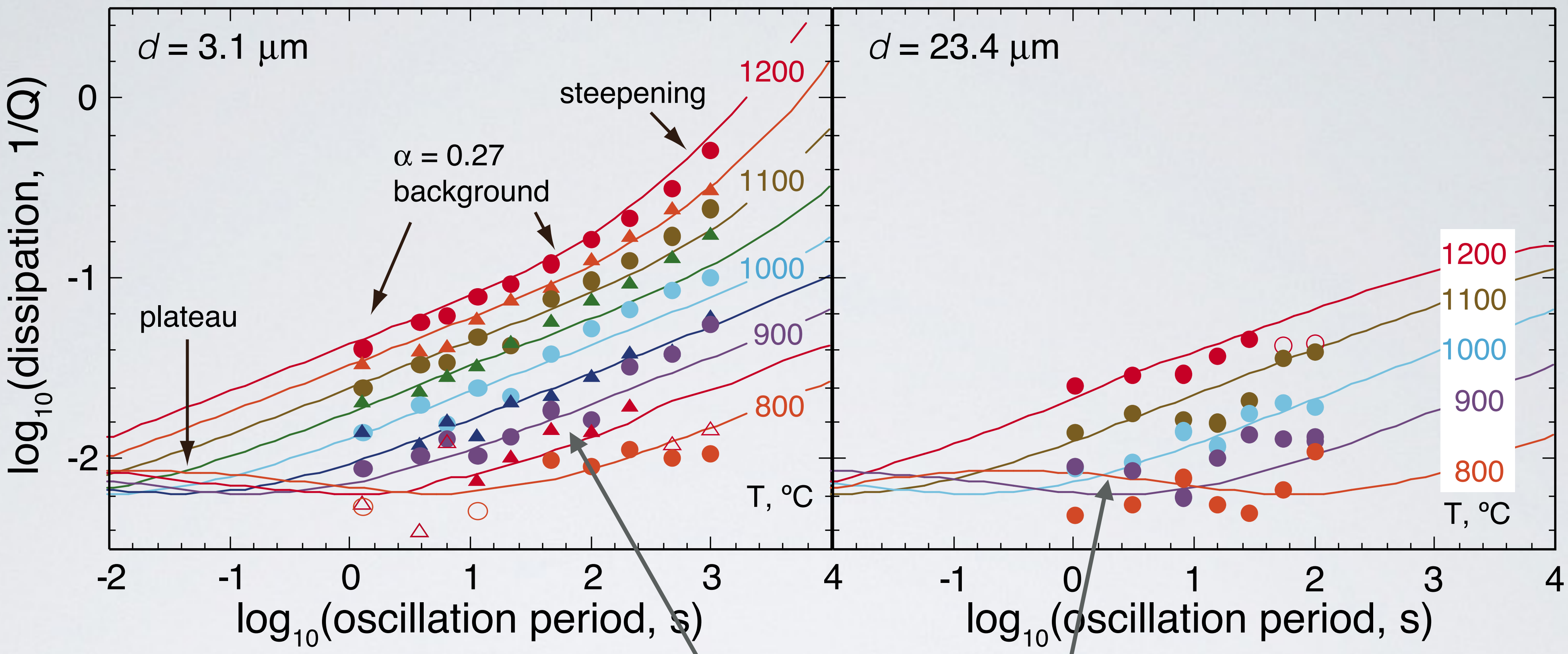
$Q^{-1} = \delta E/E * 1/2\pi$



Forced torsional oscillation (frequency domain): Temperature, grain size and frequency dependence of dry, melt-free polycrystalline olivine



Jackson and Faul, 2010



distribution of relaxation times

$$J_1(\omega) = J_U \left\{ 1 + \frac{\alpha \Delta_B}{\tau_H^\alpha - \tau_L^\alpha} \int_{\tau_L}^{\tau_H} \frac{\tau^{\alpha-1}}{1+\omega^2\tau^2} d\tau \right.$$

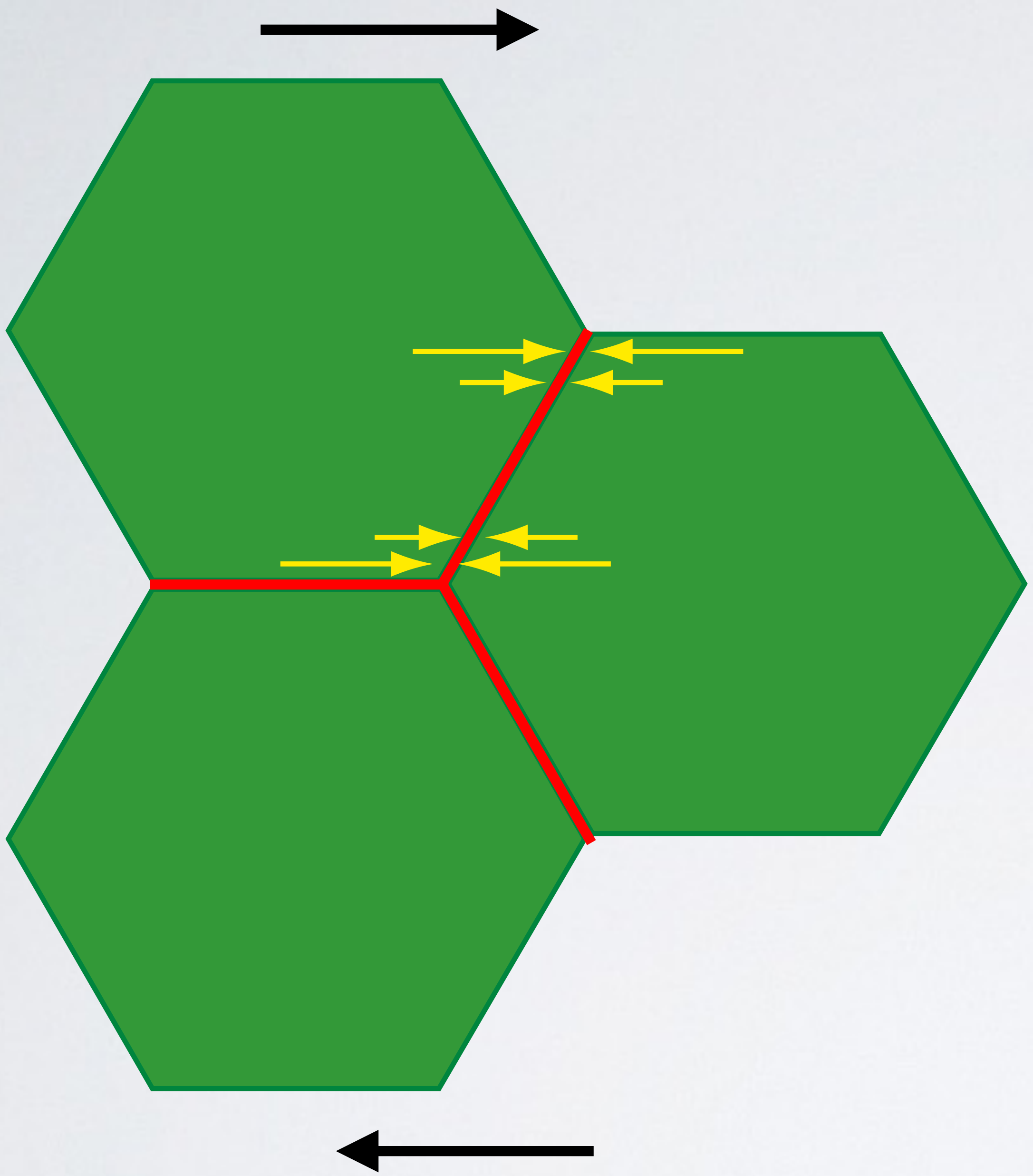
plateau

$$\left. + \frac{1}{\sigma\sqrt{(2\pi)}} \Delta_P \int_0^\infty \frac{1}{\tau} \frac{exp\left(\frac{-[ln(\tau/\tau_P)/\sigma]^2}{2}\right)}{1+\omega^2\tau^2} d\tau \right\}$$

$$J_2(\omega) = J_U \left\{ \frac{\omega \alpha \Delta_B}{\tau_H^\alpha - \tau_L^\alpha} \int_{\tau_L}^{\tau_H} \frac{\tau^{\alpha}}{1+\omega^2\tau^2} d\tau \right.$$

$$\left. + \frac{\omega}{\sigma\sqrt{(2\pi)}} \Delta_P \int_0^\infty \frac{exp\left(\frac{-[ln(\tau/\tau_P)/\sigma]^2}{2}\right)}{1+\omega^2\tau^2} d\tau + \frac{1}{\omega \tau_M} \right\}$$

1. Elastically accommodated sliding



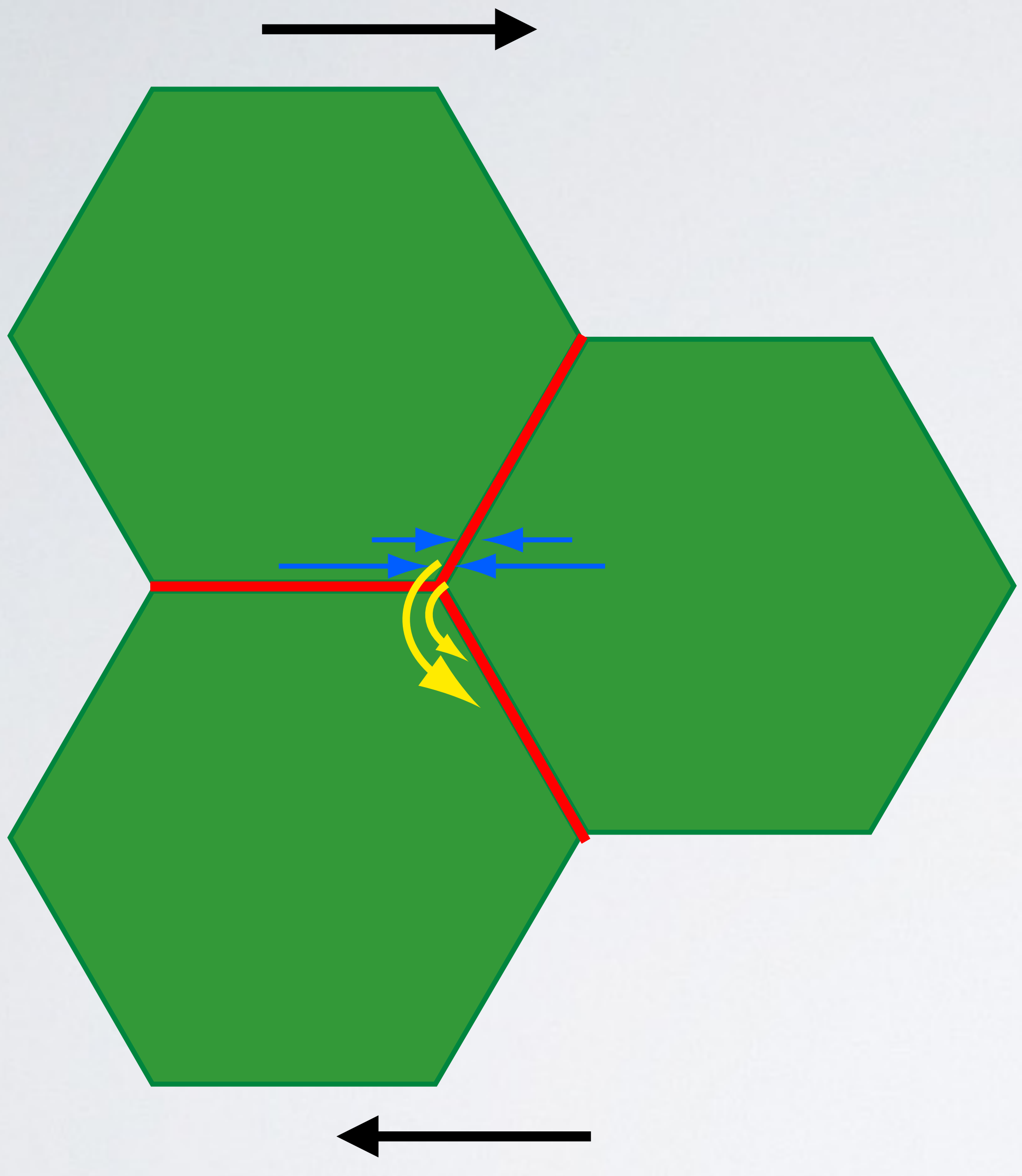
viscous sliding of grain boundaries leads to elastic stress concentrations at grain corners

time scale: $\tau_E = \ \eta_{gb}\, d/G\, \delta$

recoverable strain, anelastic process, dissipation peak

After Raj and Ashby, 1971; Raj, 1975

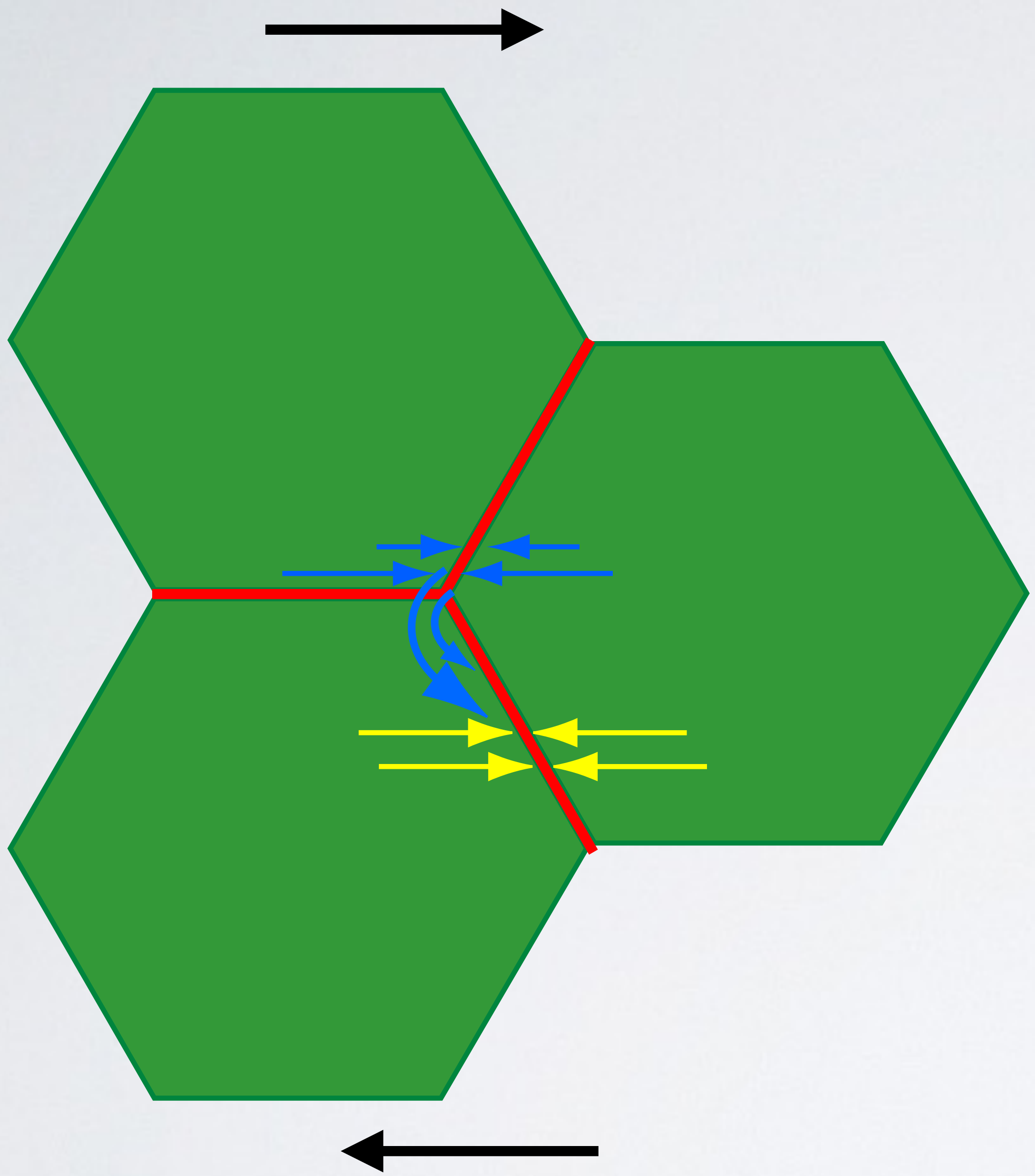
2. Diffusionally assisted sliding

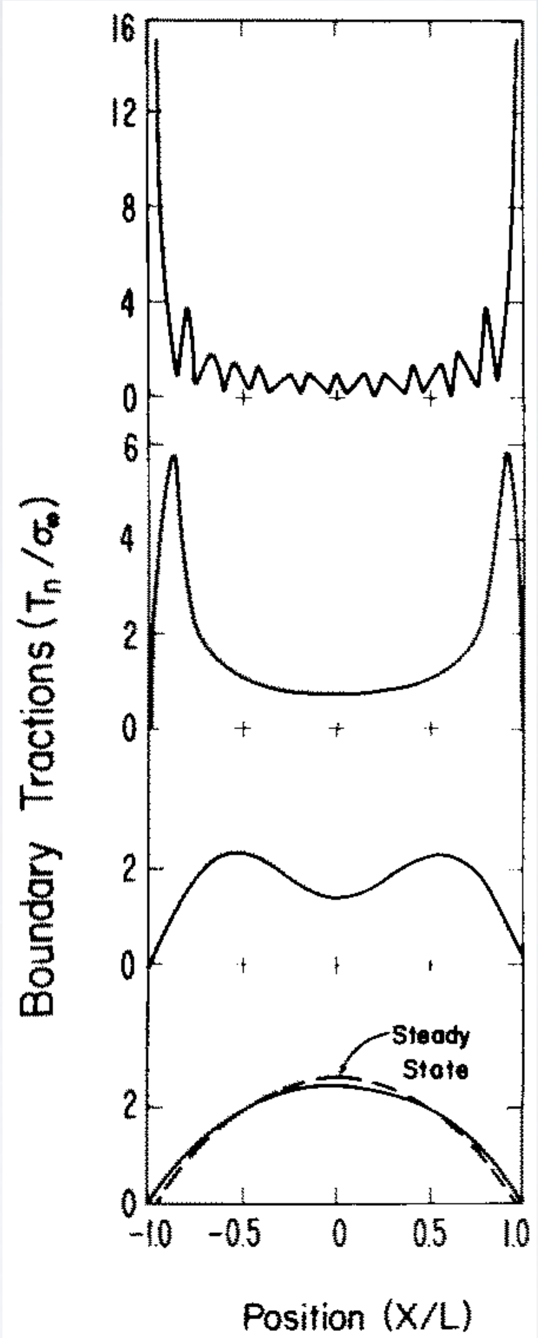


- stress concentrations cause diffusion away from corners
- transient phase is characterised by diffusion over increasing length scales

distribution of relaxation times, transient, recoverable

3. Diffusionally accommodated sliding (steady state)





1. end of elastically accommodated sliding

2. diffusionally assisted sliding

3. steady state creep

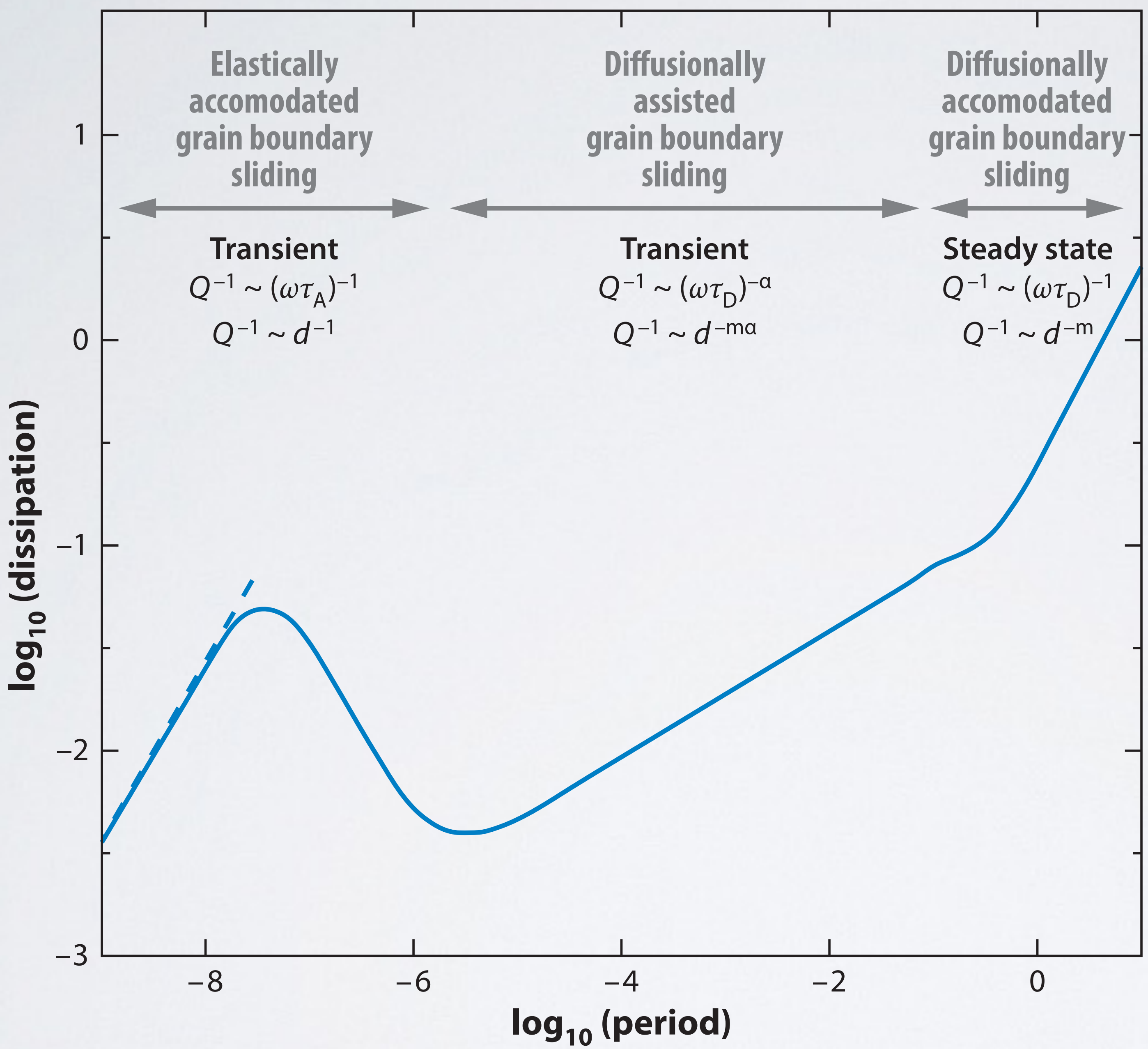
Raj 1975,
Gribb and Cooper, 1998

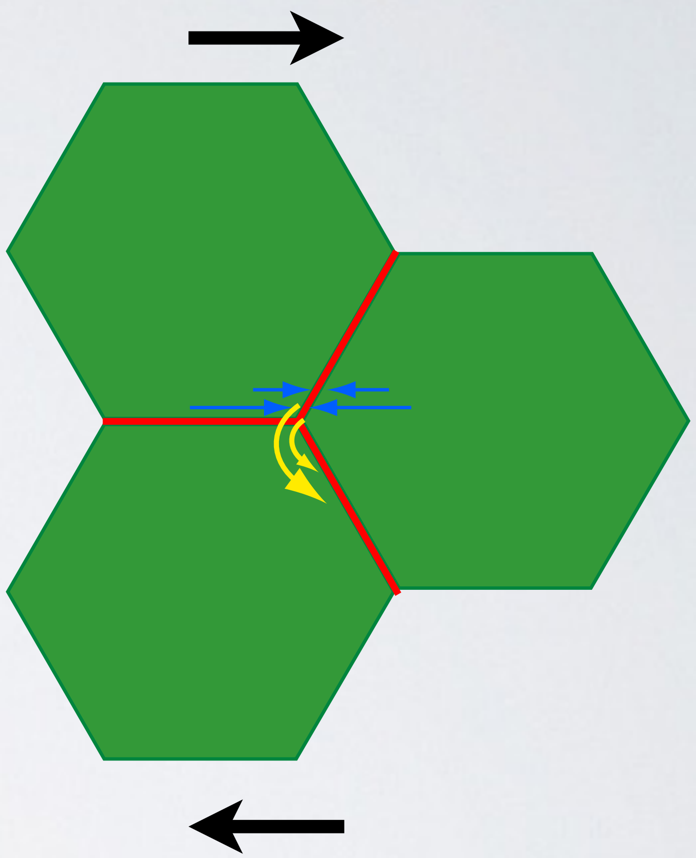
time scale: $\tau_D \sim \; T\, d^3/G\, \delta\, D_{gb}$

gb normal stresses are highest in center between grain corners (steady state diffusion creep)

Microphysical model: Continuum of relaxation times (absorption band) is required.

Diffusionally assisted grain boundary sliding

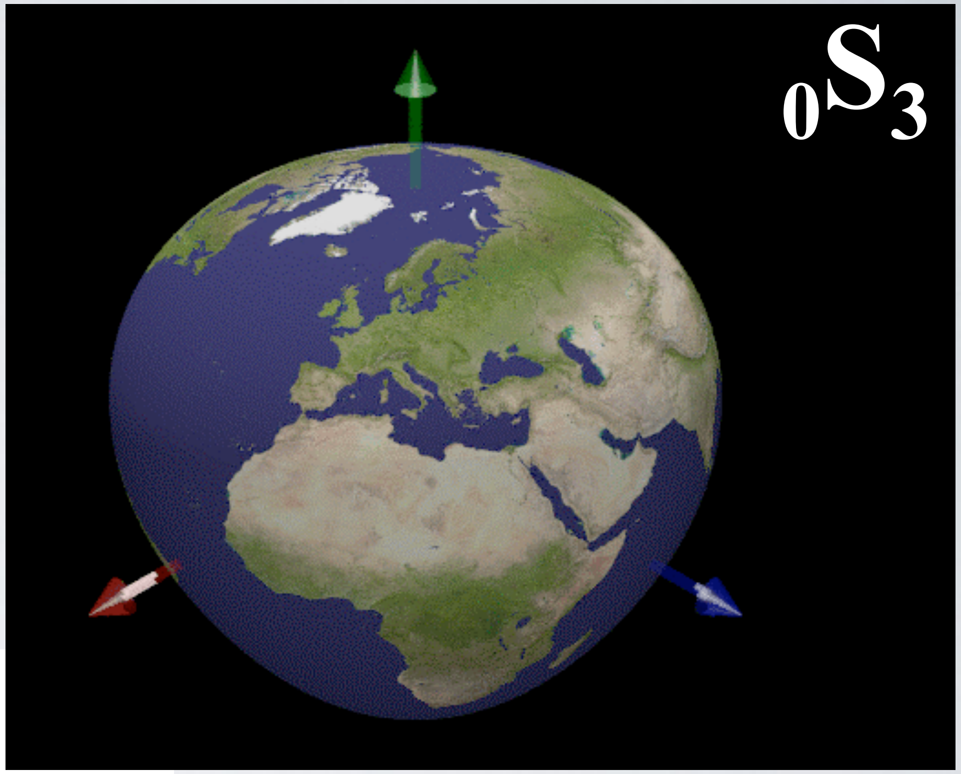


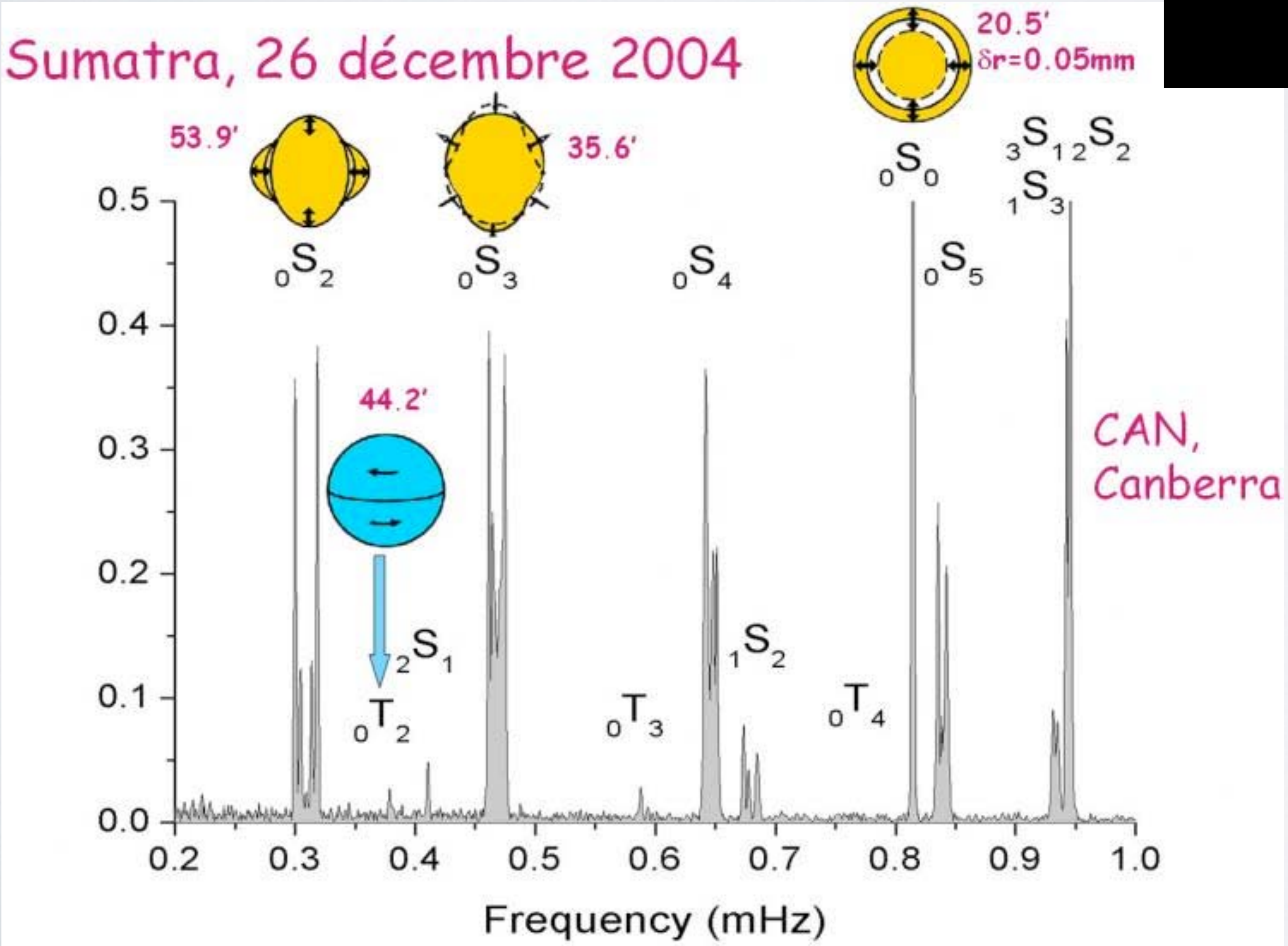


seamless transition from transient creep to steady-state deformation

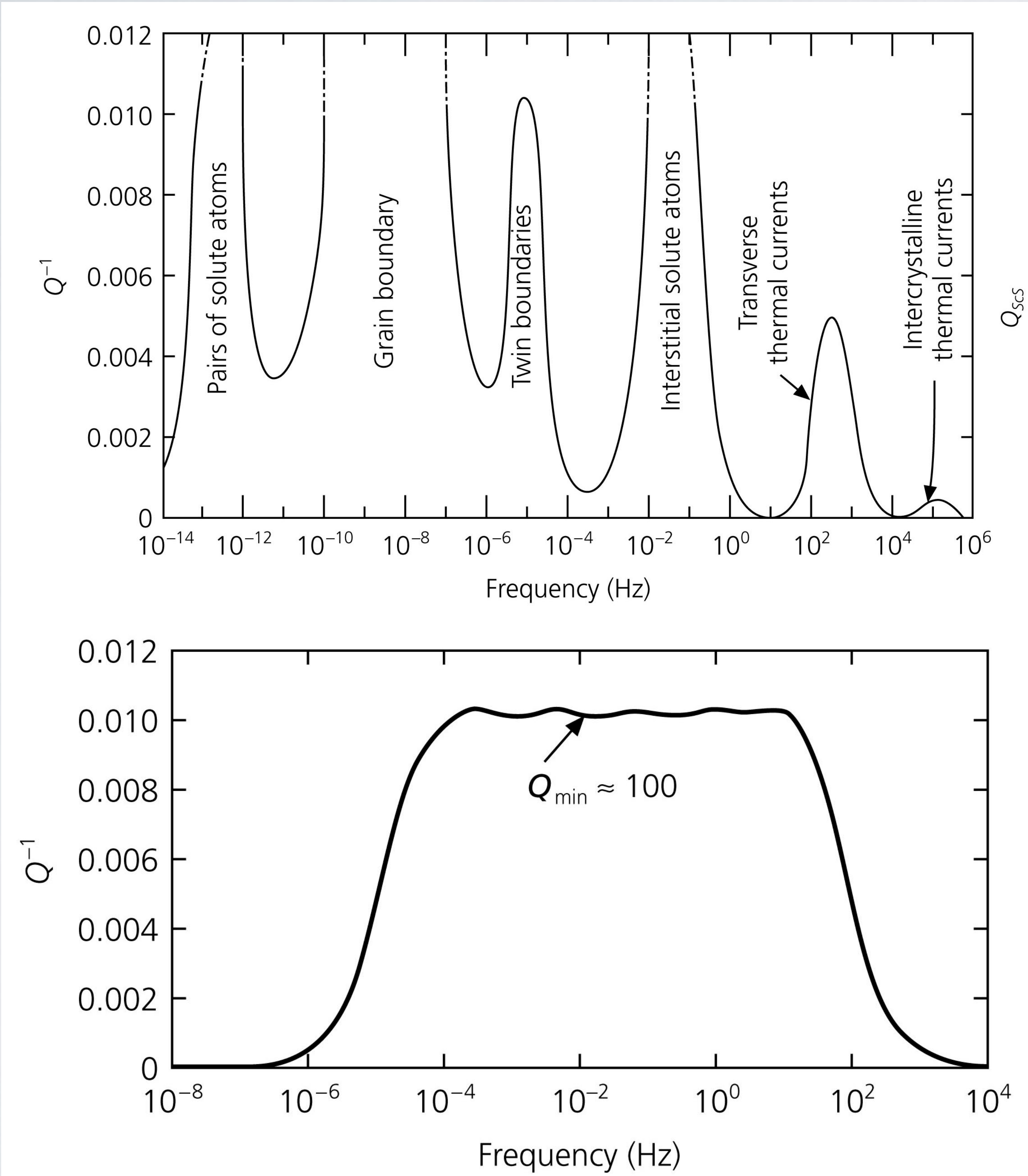
Morris and Jackson, 2009, Lee et al., 2011 Faul & Jackson, AREPS, 2015

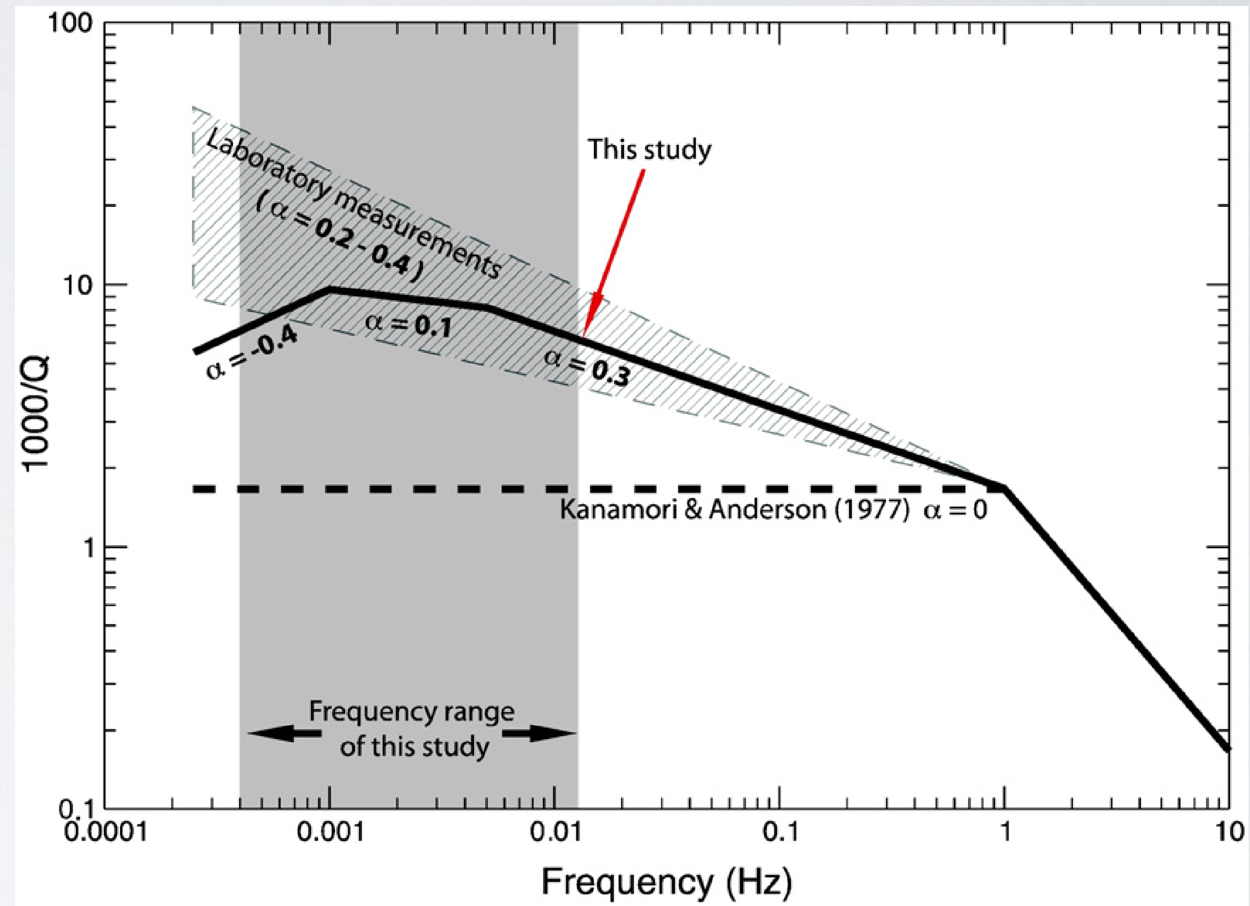
At longer periods: normal modes - whole Earth deformation



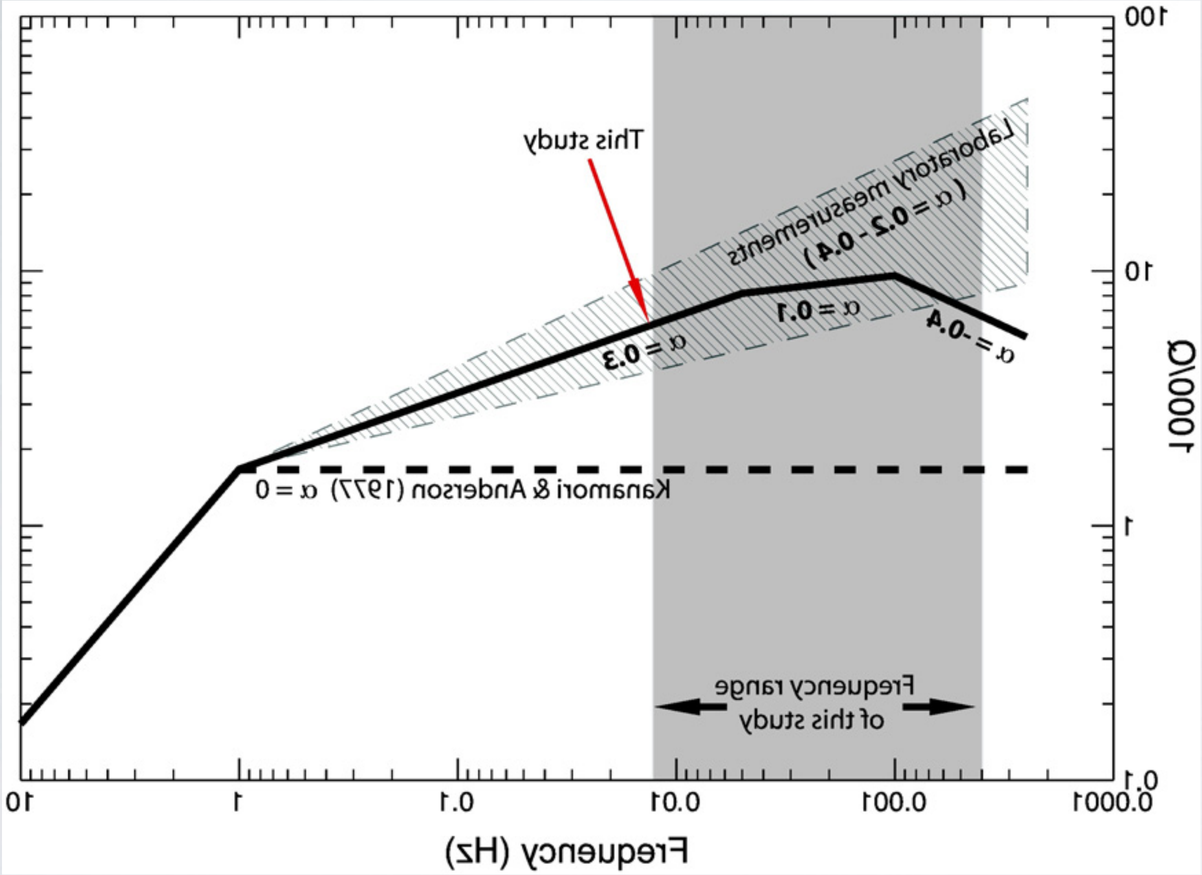


Park et al., 2005





Lekić et al., 2009

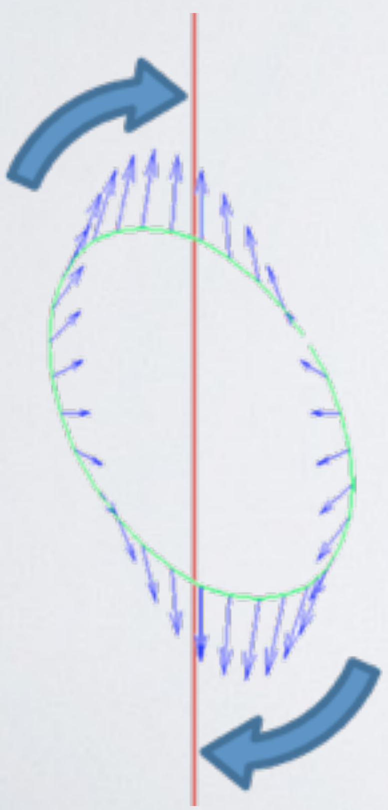


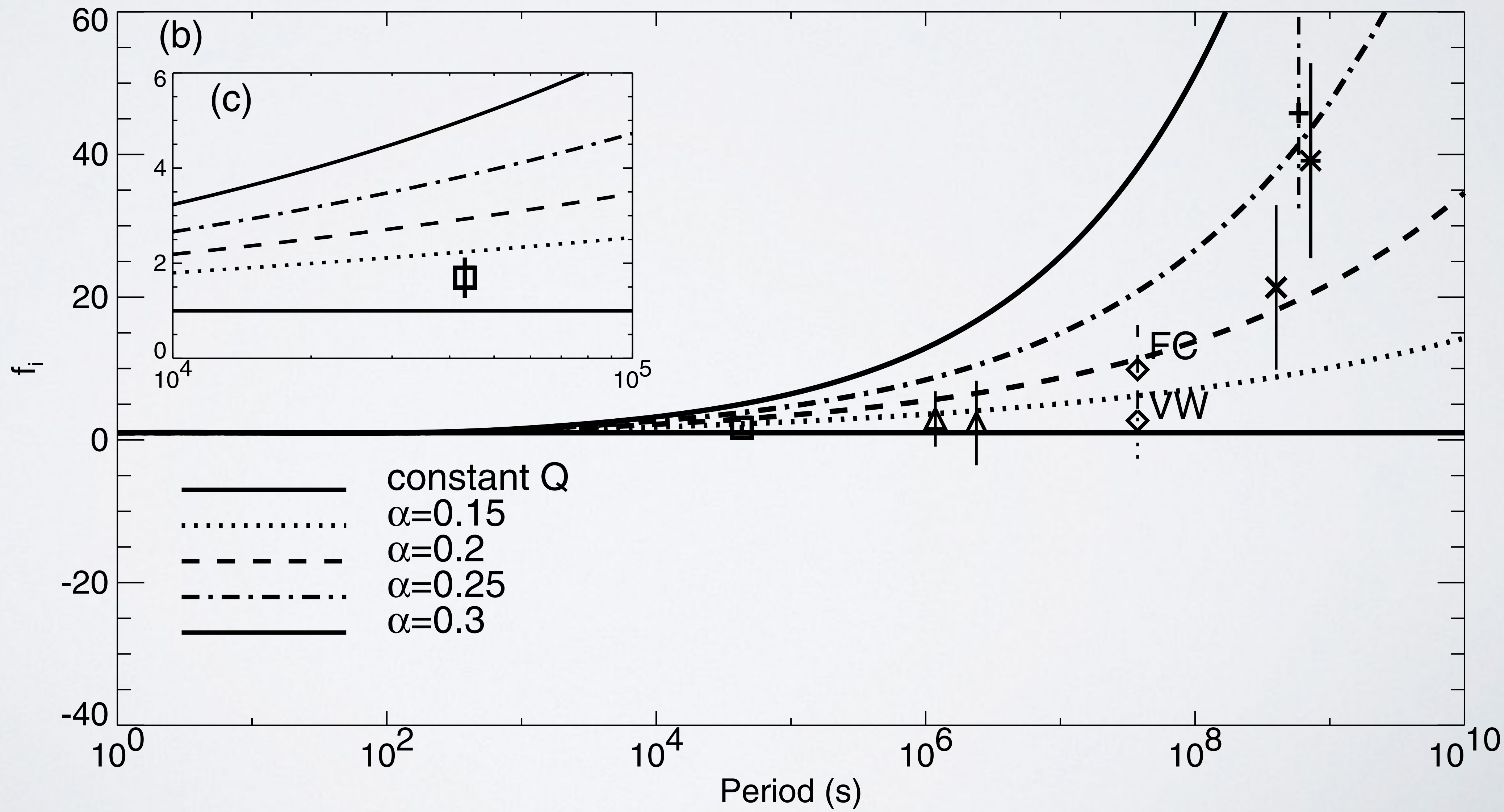
Modes

Lekić et al., 2009

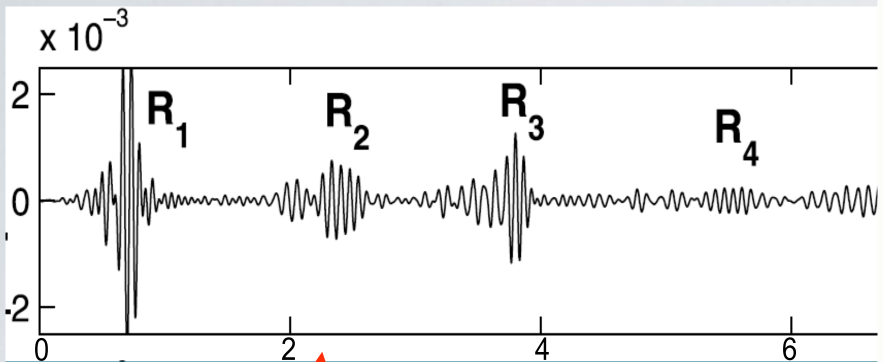
Tidal deformation:
12 h - 16 years

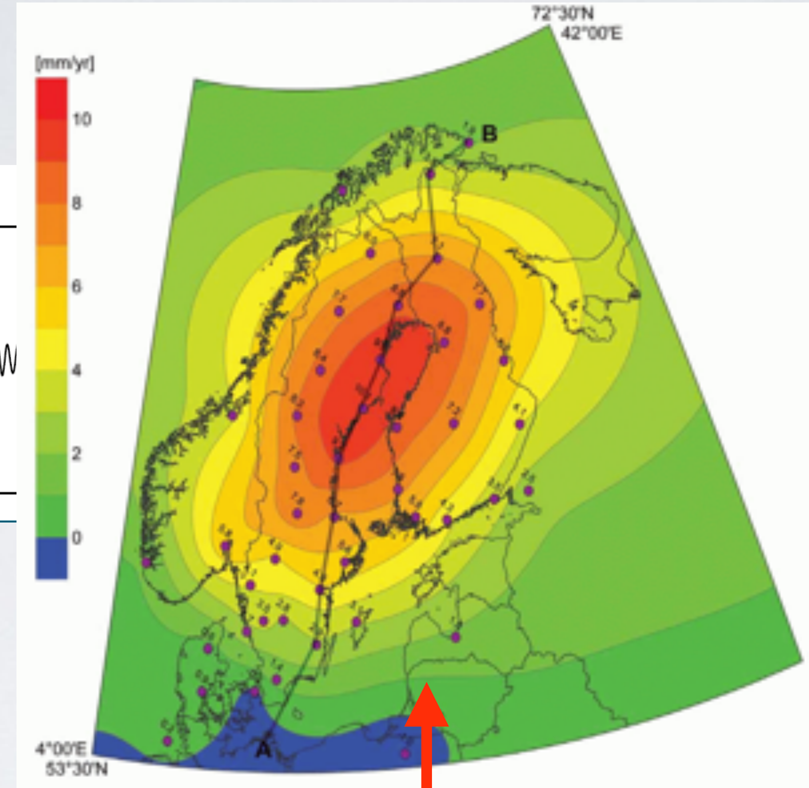
Benjamin et al., 2006

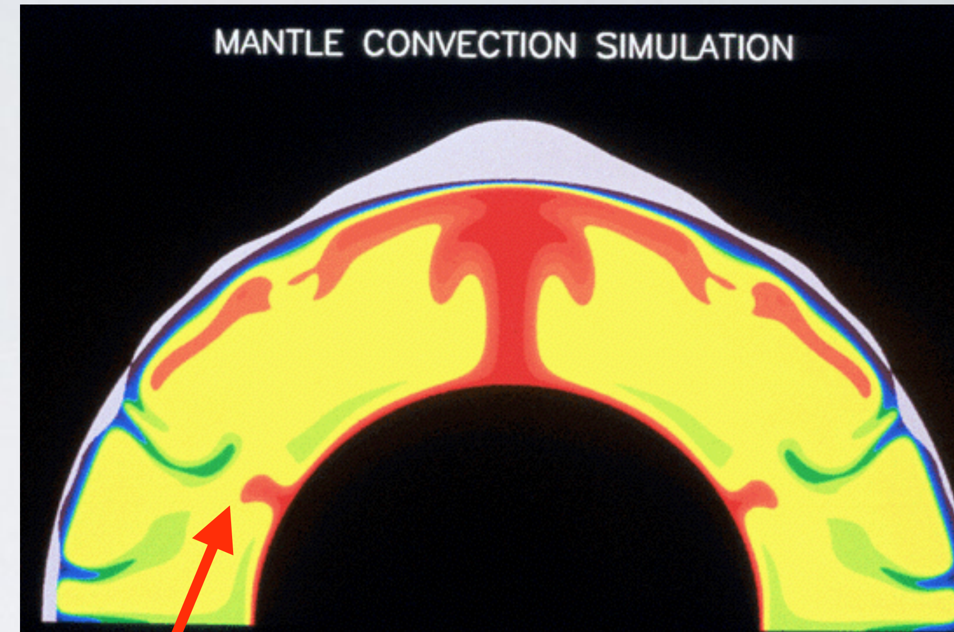




Deformation at a range of time scales

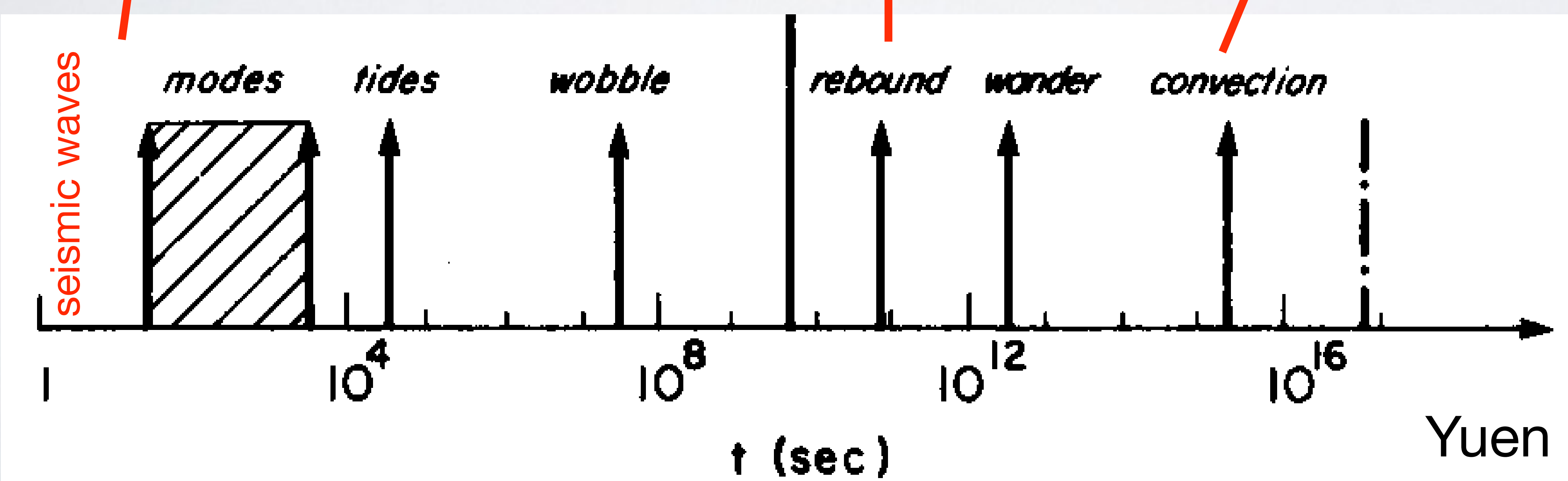






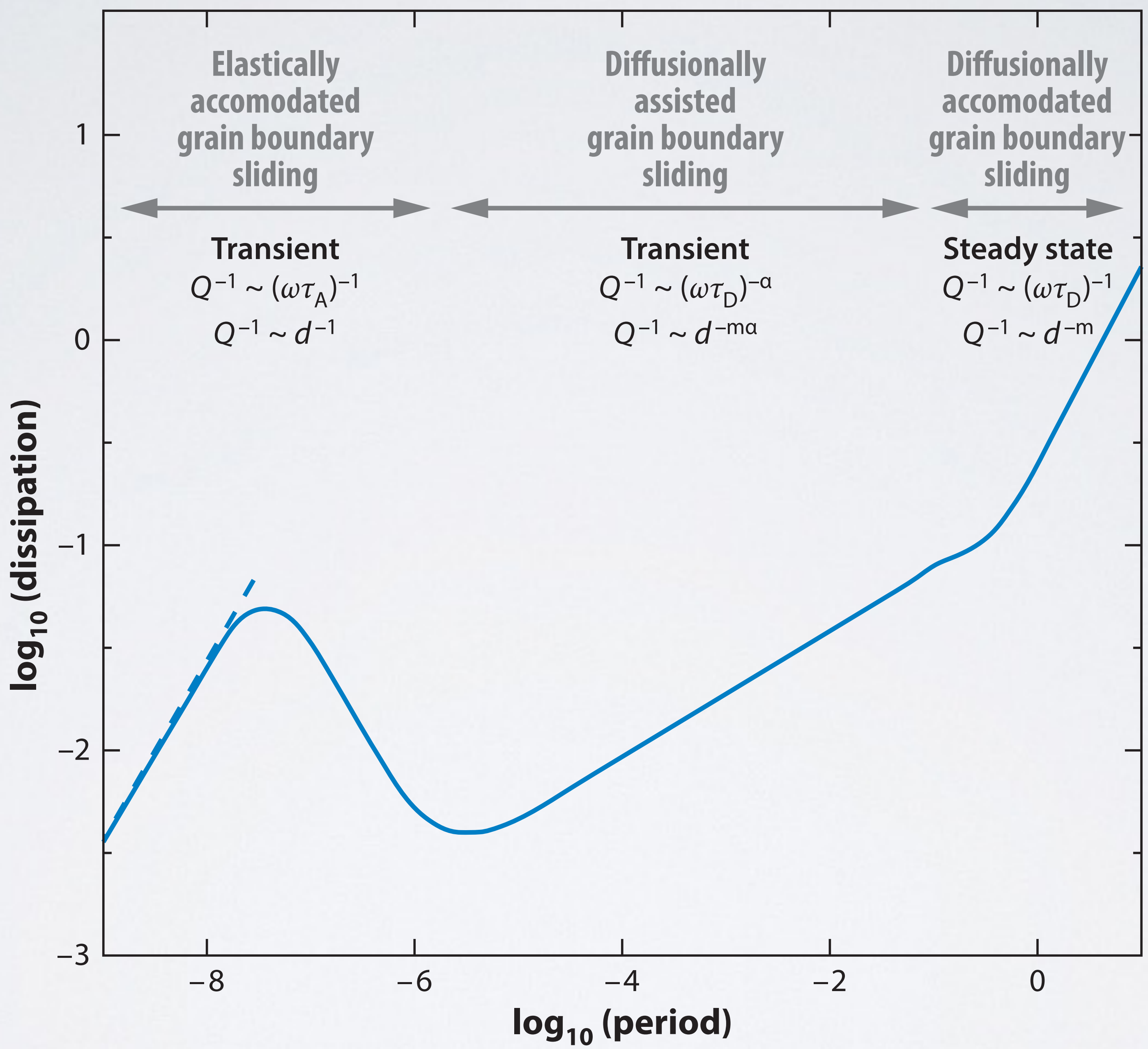
$\varepsilon < 10^{-4}$

$\varepsilon >> 1$

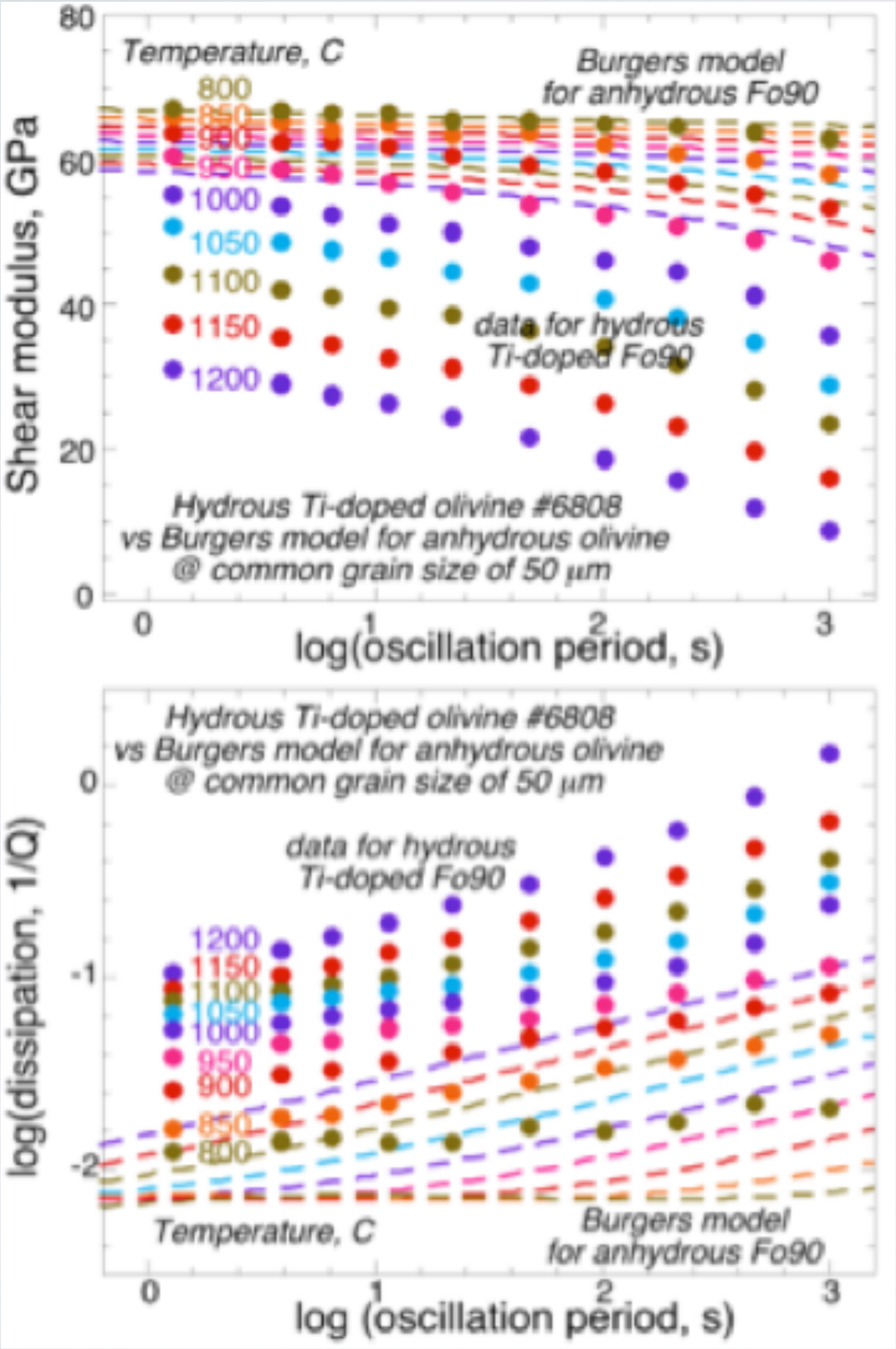


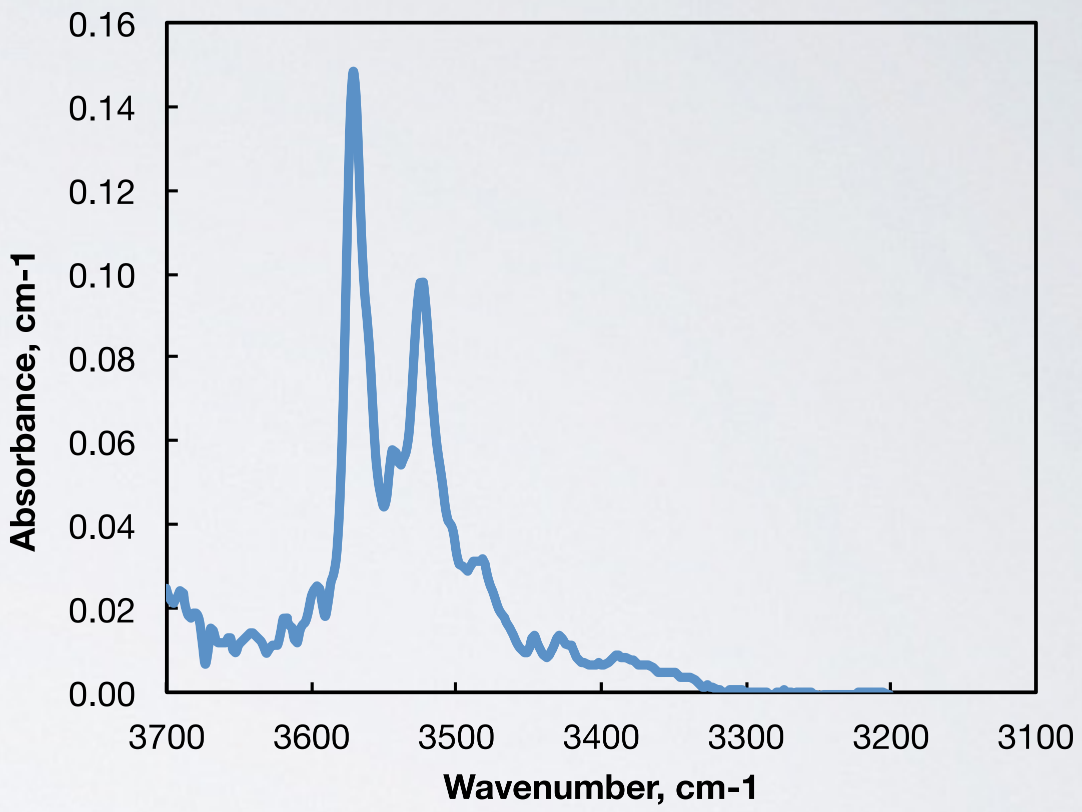
Yuen & Peltier, 1982

Continuum of relaxation times from seismic to convective time scales



Hydroxyl reduces seismic velocities and increases attenuation





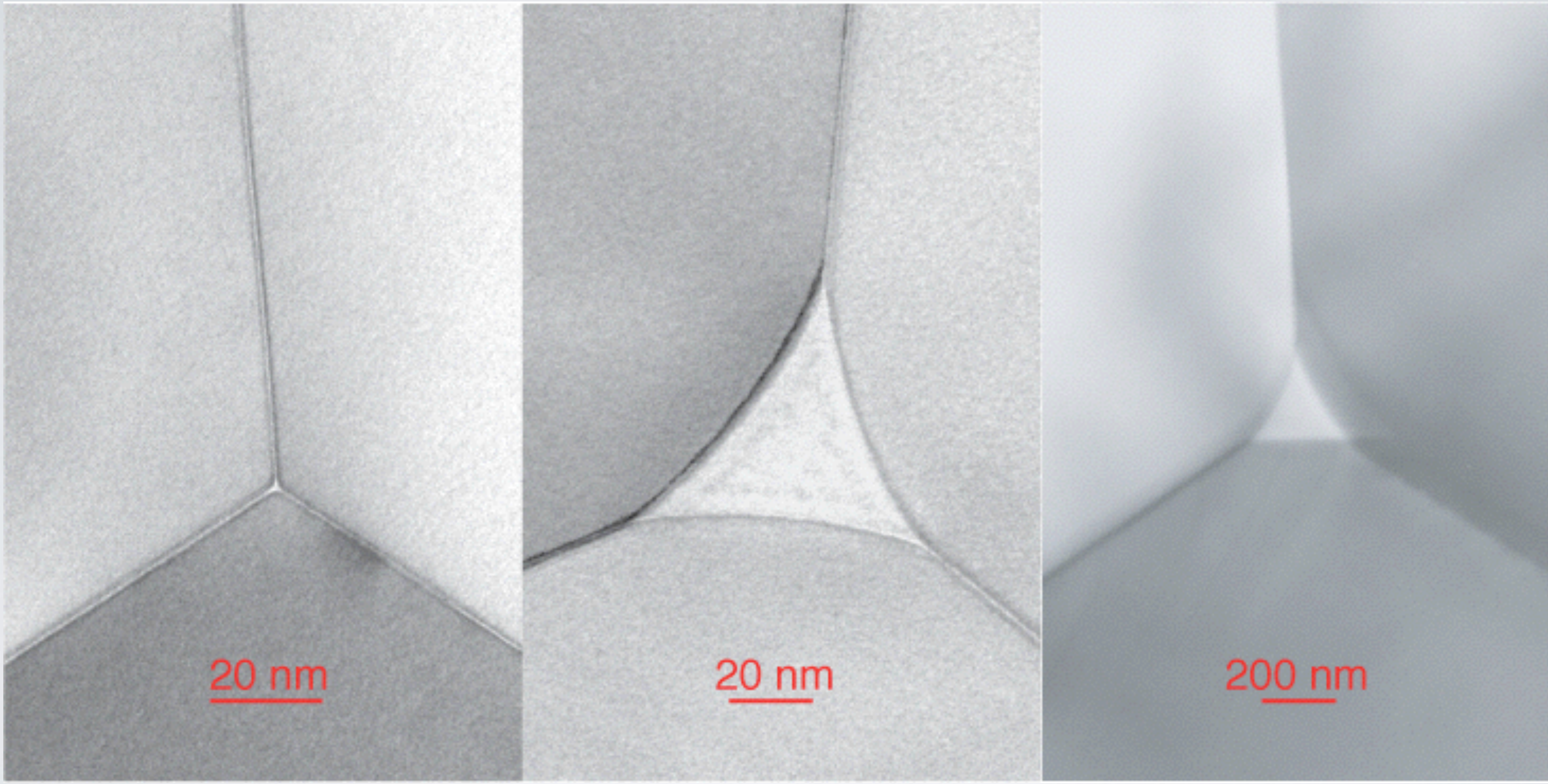
d ~ 50 µm, 70 wt. ppm
or ~ 1100 ppm H/Si

MORB source mantle: olivine contains
~ 350 - 800 ppm H/Si

Cline et al., in prep

Influence of melt on deformation

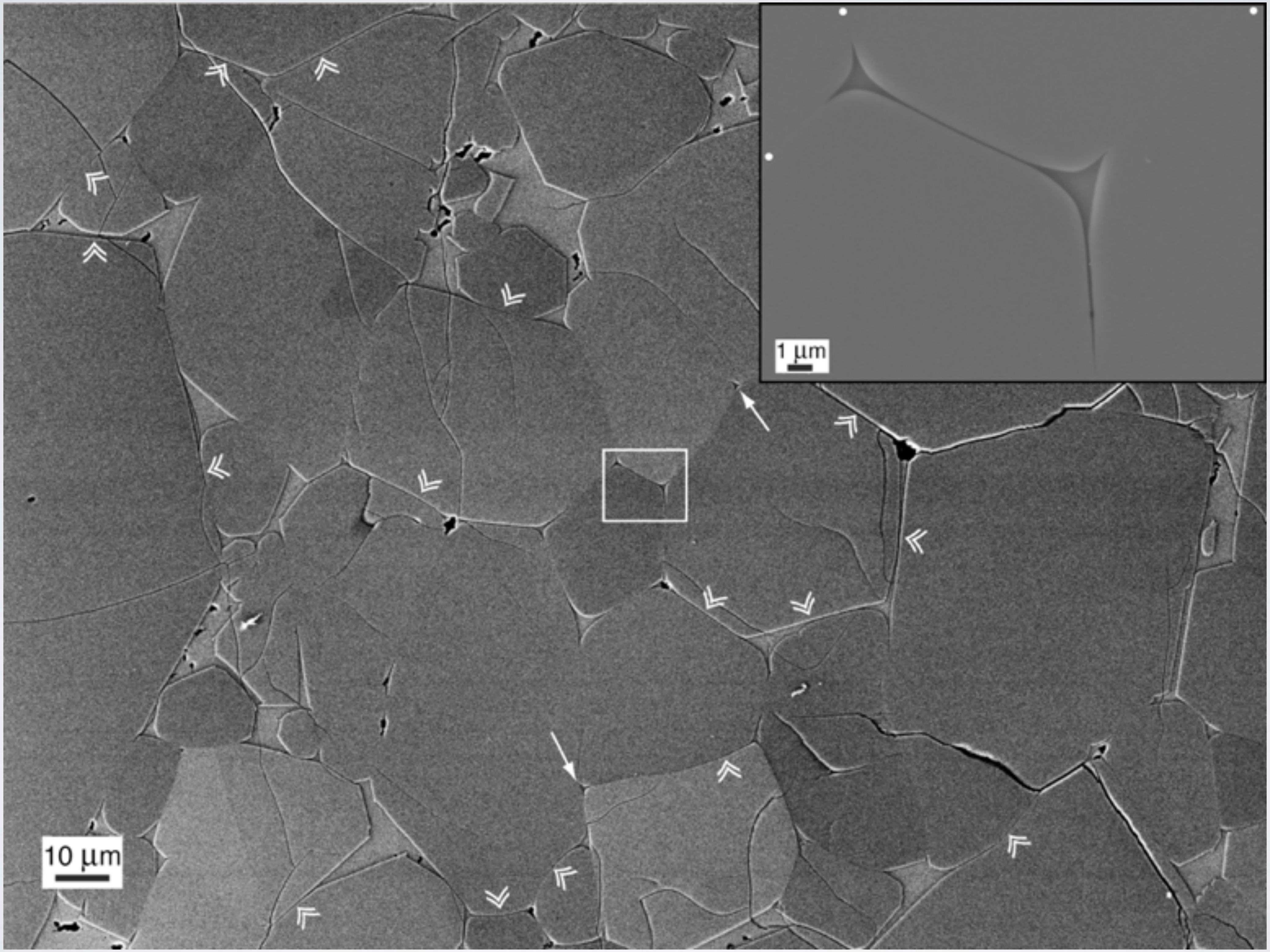
TEM images of three-grain edges



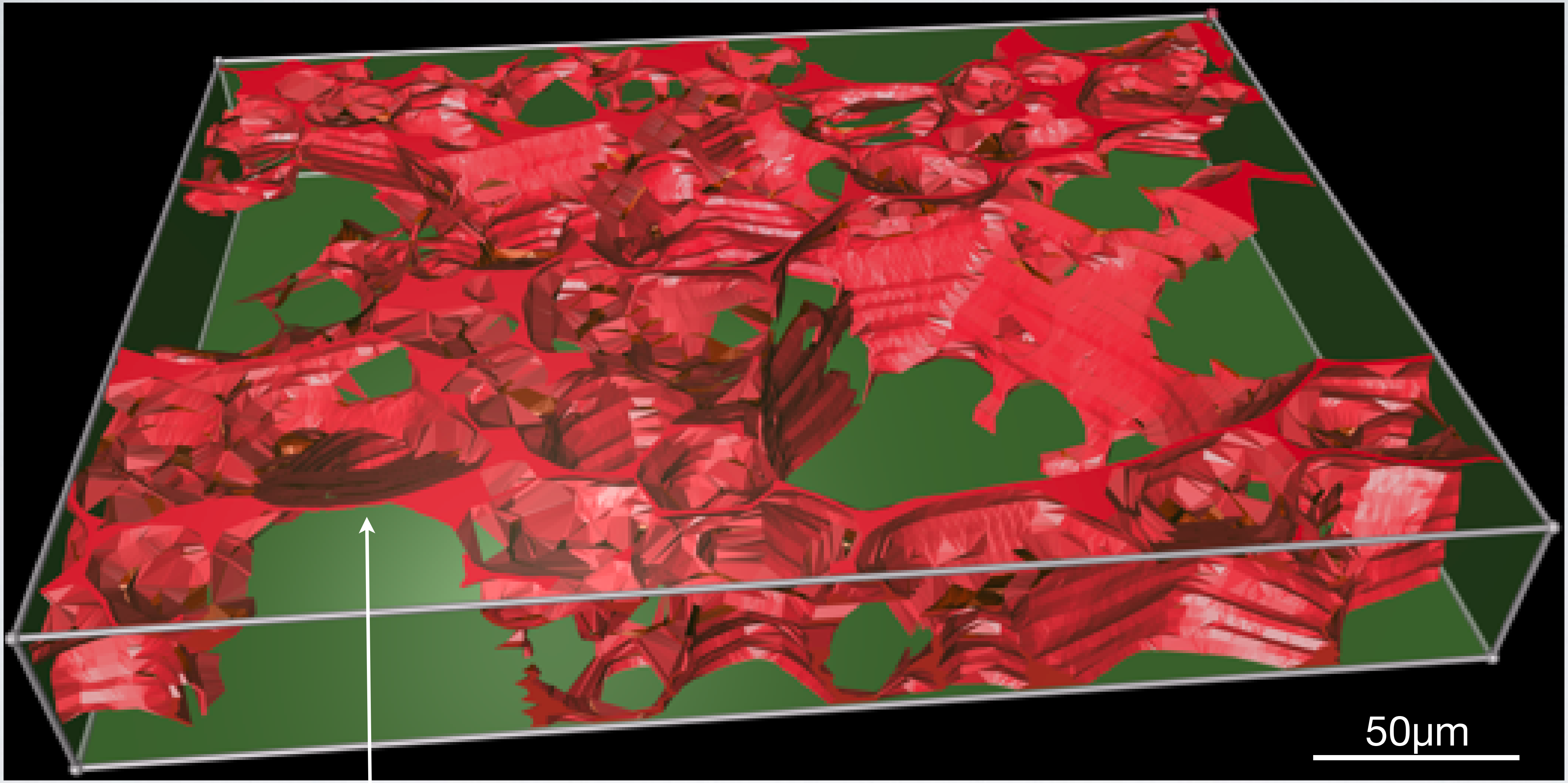
Solgel olivine

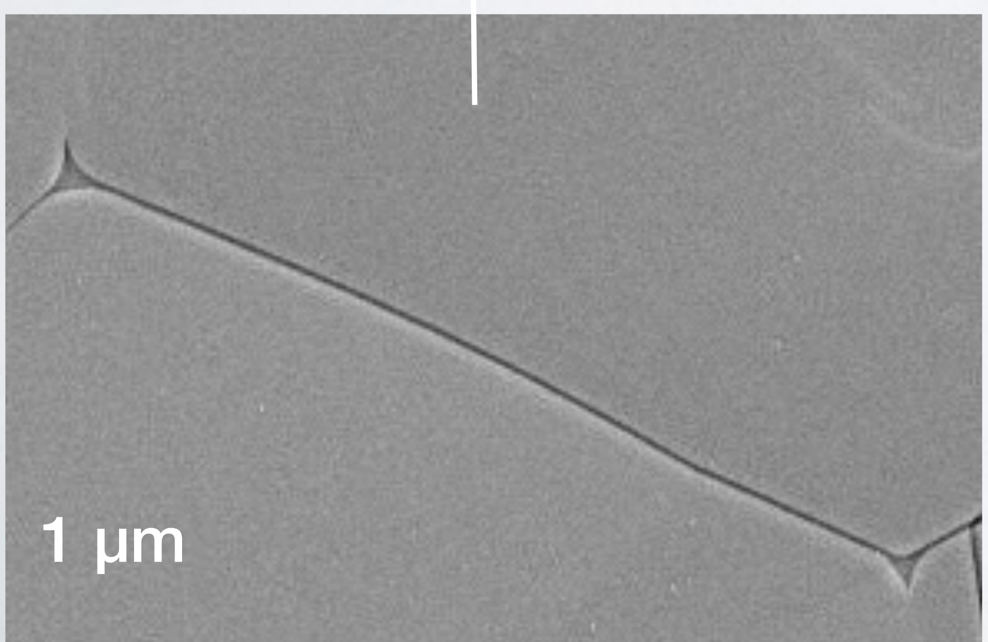
San Carlos olivine

Melt distribution in polycrystalline olivine



P = 1 GPa, 1350ºC, 432 hours



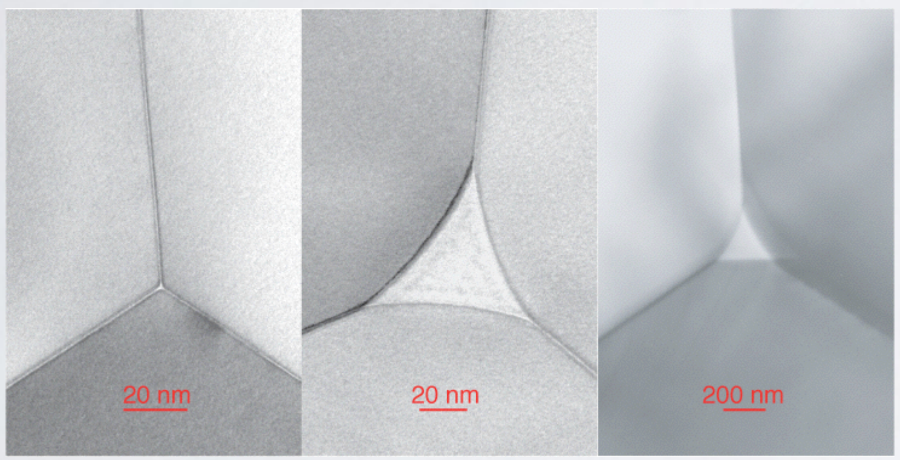


3-D view of the melt distribution
3.6% melt, 30 μm grain size

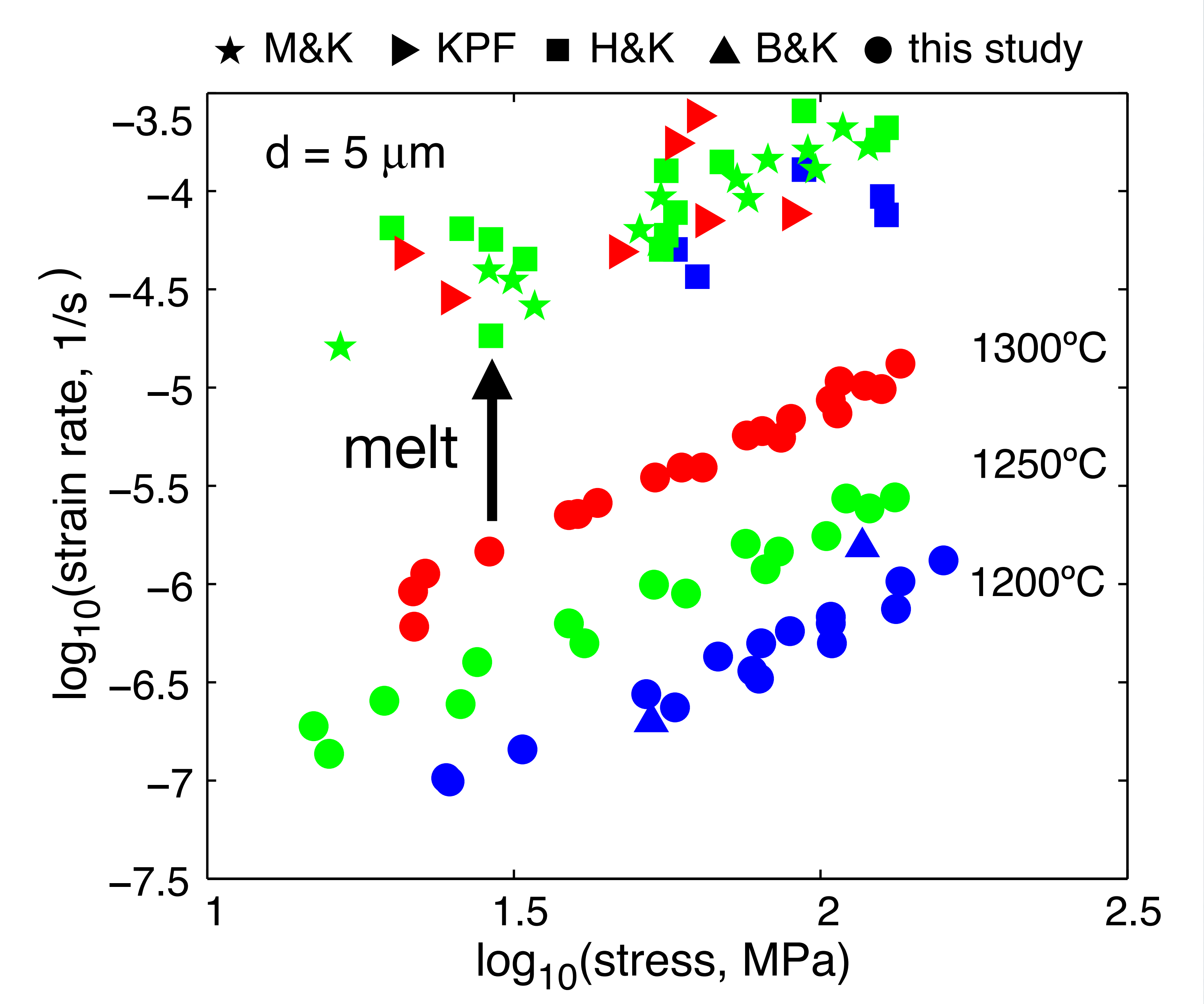
Garapić, Faul and Brisson, G[3], 2013

Influence of melt on rheology

Small amount (1%) of melt enhances strain rate by about an order of magnitude (in diffusion creep)

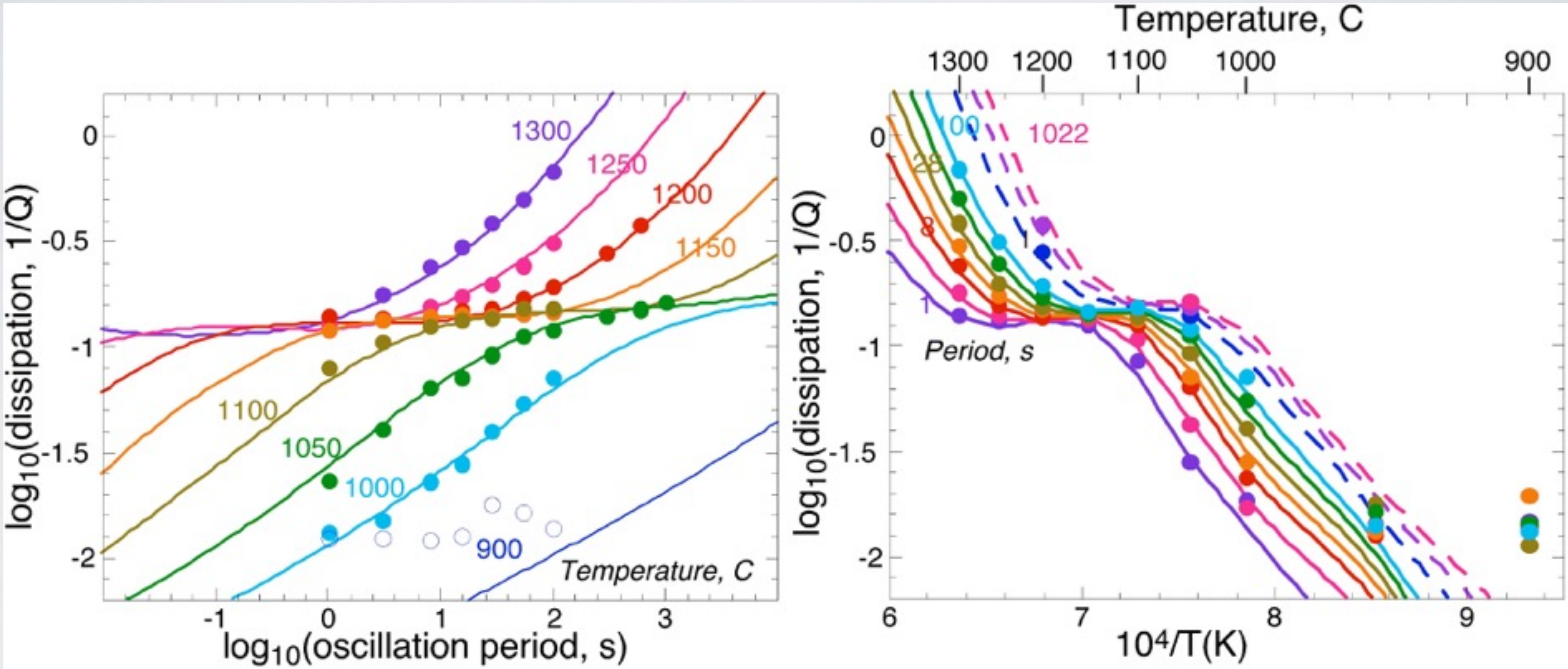


connected melt produces short circuit diffusion paths



Faul and Jackson, 2007

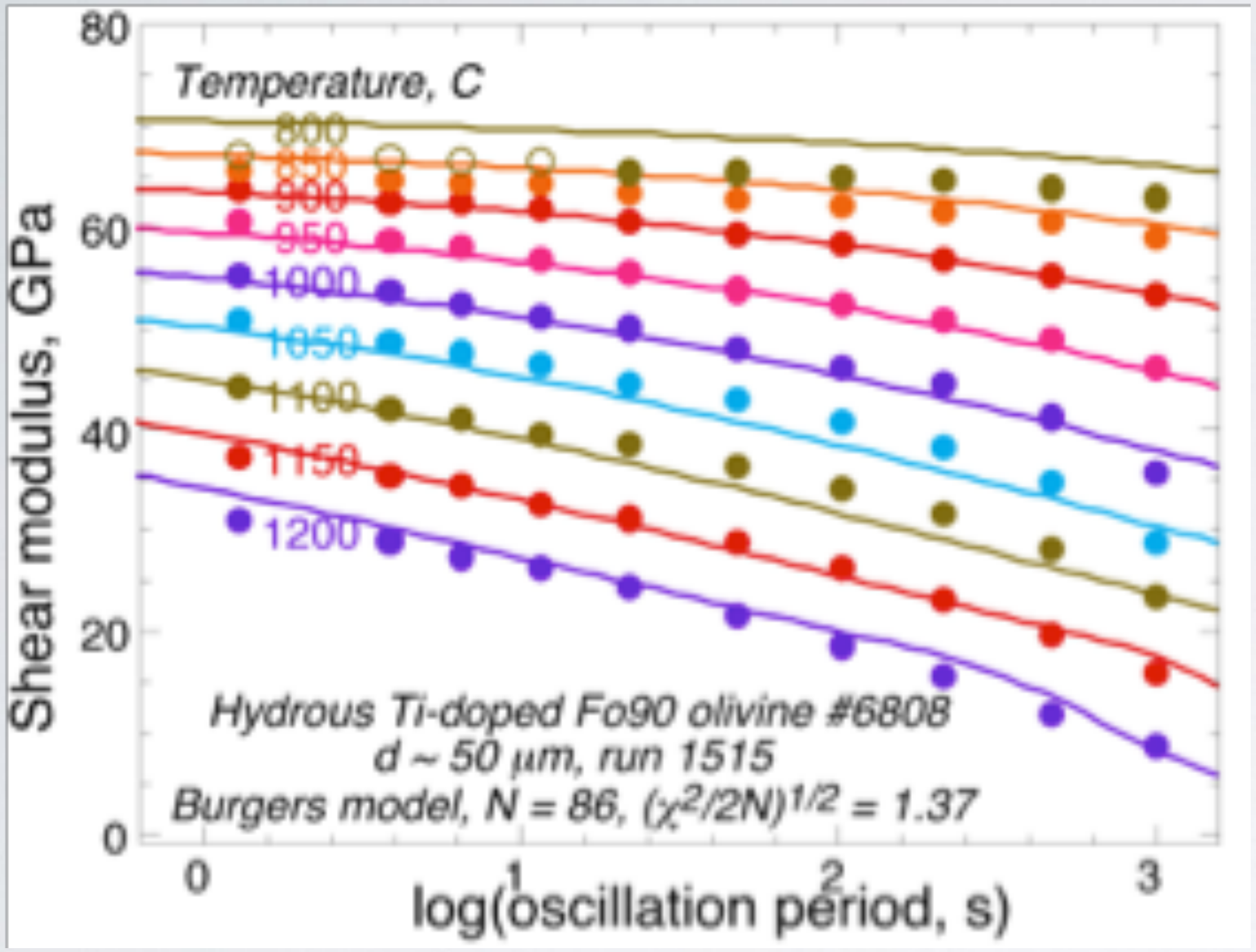
Melt present: dissipation peak due to melt 'squirt'

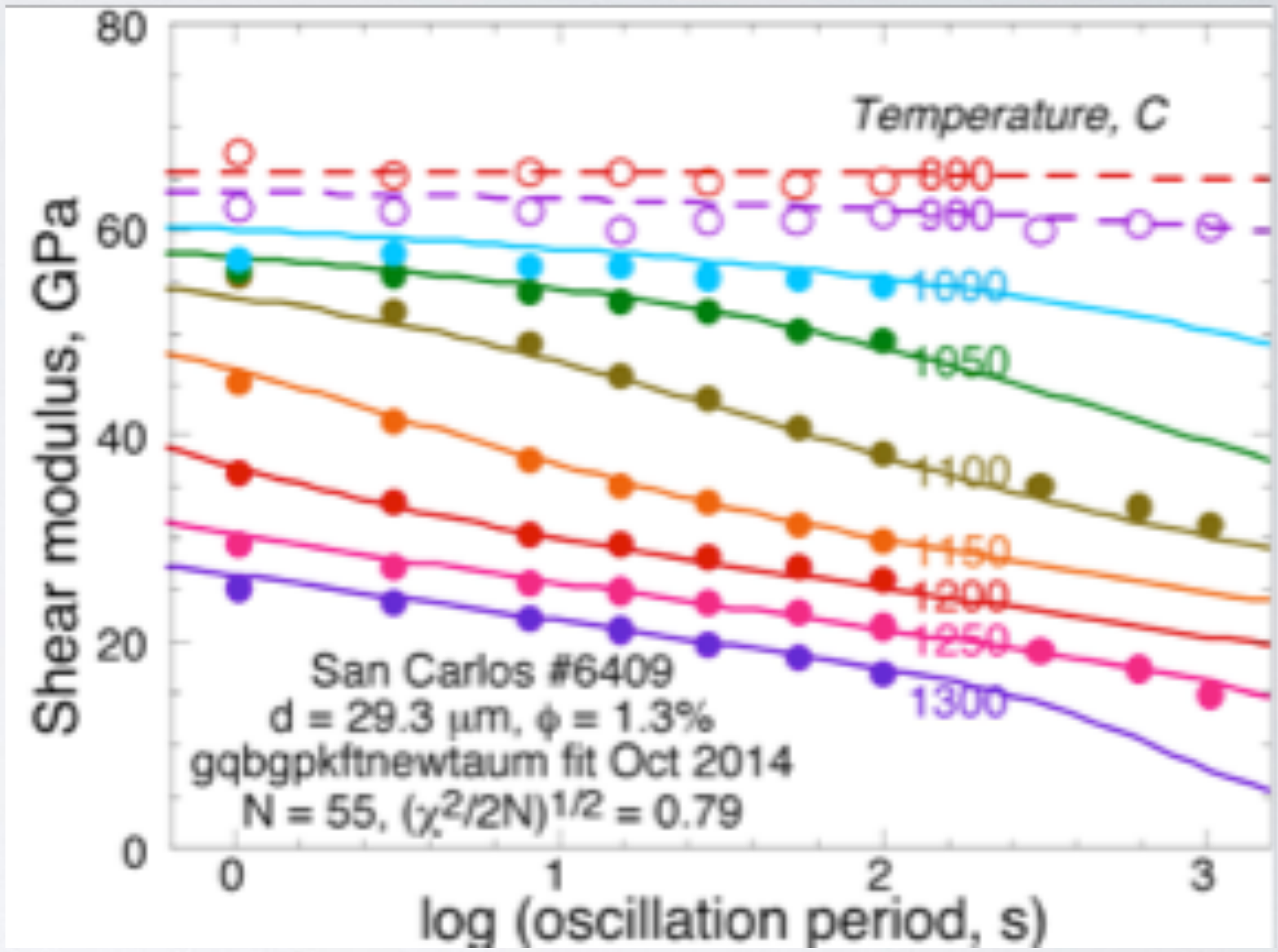


Jackson et al., 2004, Faul et al., 2004

~70 wt. ppm water

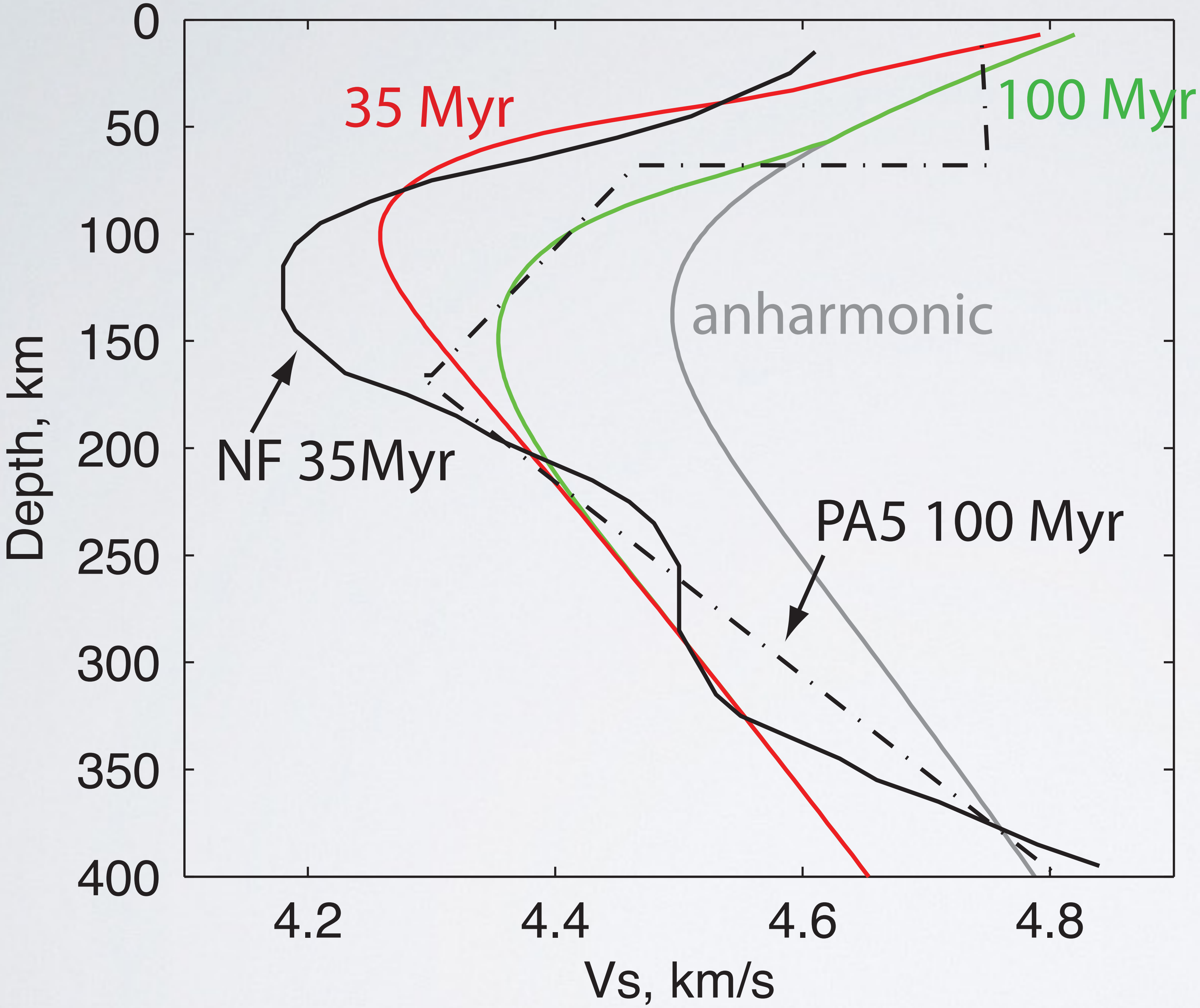
1.3 % melt

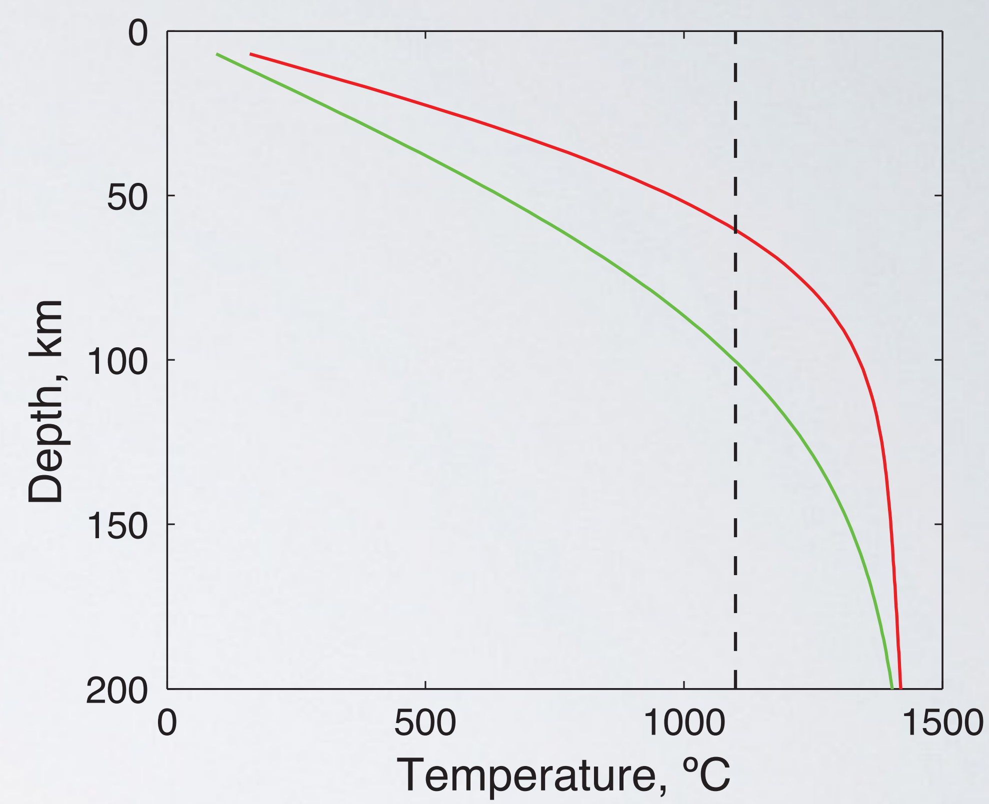




Seismic structure of the upper mantle: Water or melt?

Application to the Upper Mantle
Comparison with velocity models for the Pacific





d = 1 cm, Tp = 1350ºC

Nishimura & Forsyth, 1989
Gaherty et al., 1996